Probing the EVH limit of supersymmetric AdS black holes

Kevin Goldstein,^a Vishnu Jejjala,^a Yang Lei,^{a,b} Sam van Leuven^a and Wei Li^c

^aMandelstam Institute for Theoretical Physics, School of Physics, NITheP and CoE-MaSS, University of the Witwatersrand, 1 Jan Smuts Avenue, Johannesburg, South Africa

^bThe Niels Bohr Institute, Copenhagen University, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark

^cInstitute of Theoretical Physics, Chinese Academy of Sciences, Zhongguancun East Rd 55, 100190 Beijing, P.R. China

E-mail: kevin.goldstein@wits.ac.za, vishnu@neo.phys.wits.ac.za, yang.lei@wits.ac.za, svleuven@xs4all.nl, weili@itp.ac.cn

ABSTRACT: Extremal black holes in general dimensions are well known to contain an AdS_2 factor in their near-horizon geometries. If the extremal limit is taken in conjunction with a specific vanishing horizon limit, the so-called Extremal Vanishing Horizon (EVH) limit, the AdS_2 factor lifts to a locally AdS_3 factor with a pinching angular direction. In this paper, we study the EVH limit of asymptotically AdS black holes which preserve some supersymmetry. The primary example we consider is the $1/16^{\text{th}}$ BPS asymptotically AdS_5 black hole, whose EVH limit has an AdS_3 factor in its near-horizon geometry. We also consider the near-EVH limit of this black hole, in which the near-horizon geometry instead contains an extremal BTZ factor. We employ recent results on the large- N limit of the superconformal index of the dual CFT_4 to understand the emergence of a CFT_2 in the IR of the CFT_4 , which is the field theory dual to the emergence of the locally AdS_3 factor in the near-horizon geometry. In particular, we show that the inverse Laplace transform of the superconformal index, yielding the black hole entropy, becomes equivalent to the derivation of a Cardy formula for the dual CFT_2 . Finally, we examine the EVH limit of supersymmetric black holes in other dimensions.

KEYWORDS: AdS-CFT Correspondence, Black Holes in String Theory, Supersymmetric Gauge Theory

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1 Introduction

A remarkable realization of holography is the well-studied gauge/gravity correspondence [1–3]. String theory on $\text{AdS}_{d+1} \times X_{9-d}$, with X_{9-d} compact, is dual to a d -dimensional conformal field theory living on the boundary of the anti-de Sitter space. The example of primary interest for us is the duality between type IIB string theory on $\text{AdS}_5 \times S^5$ and $\mathcal{N} = 4$ super-Yang-Mills theory. Avatars in other dimensions include: the duality between type IIB string theory on $\text{AdS}_3 \times S^3 \times X_4$ and the D1-D5 CFT_2 living on X_4 (for a review see [4]), the duality between type IIA string theory on $\text{AdS}_4 \times \mathbb{CP}^3$ and the ABJM theory [5], and the duality between the $(0, 2)$ superconformal field theory in six dimensions and M-theory on $\text{AdS}_7 \times S^4$ (see e.g. [6–8]).

AdS/CFT provides a fundamental laboratory for the study of the physics of black holes. More precisely, since the CFT provides a non-perturbative definition of string theory on AdS backgrounds, one can study questions about (asymptotically AdS) black holes in the well-defined setting of the quantum field theory. The prototypical example is the BTZ black hole in AdS_3 [9, 10], whose entropy can be understood in terms of the Cardy formula in the associated CFT_2 [11–13]. Generic extremal black holes have an AdS_2 factor in

their near-horizon geometry and have a vanishing temperature but finite horizon area [14]. Often, this is enhanced or lifted to an AdS_3 near-horizon, and the determination of the entropy utilizes known results about the BTZ geometry.

One lesson of AdS/CFT is that the entropy of a black hole can be calculated either from the point of view of the CFT corresponding to the AdS asymptopia (the UV CFT) or from a lower dimensional CFT corresponding to the near-horizon AdS factor (the IR CFT). As entropy counts microscopic degrees of freedom, we would like to identify black hole microstates in both the UV CFT and the IR CFT and determine how one ensemble of states evolves smoothly to the other under the holographic renormalization group. This is in general a challenging problem. We specialize to settings in which algebraic features of the IR CFT are realized in terms of the UV CFT.

Recently, the entropy for certain supersymmetric asymptotically AdS_4 and AdS_5 black holes with finite horizon area has been accounted for from the dual CFT point of view [15–31]. In this paper, we employ the results on $1/16^{\text{th}}$ BPS AdS_5 black holes [32–36] to study the so-called extremal vanishing horizon (EVH) black holes [37–43]. The EVH black holes of interest to us are defined by first taking a limit of a generic (non-BPS) AdS_5 black hole in which the horizon area A scales with a positive integer power of the temperature as $T \rightarrow 0$:

$$A \sim T^k, \quad k > 0. \quad (1.1)$$

In fact, we restrict to the case $k = 1$ in which, upon replacing A with the entropy S , the thermodynamic relation (1.1) is precisely what one expects for a CFT_2 . On the gravity side, it is then expected that a near-horizon (pinching) AdS_3 geometry emerges which decouples from the rest of the bulk directions. This setup is what is known as the EVH/ CFT_2 correspondence, where the CFT_2 represents the dual to the decoupled AdS_3 geometry. We will also consider near-EVH black holes which have a (pinching) BTZ factor in their near-horizon geometry. Such black holes arise when we in addition to the EVH limit take $G_3 \rightarrow 0$, such that both the entropy and temperature are finite in the limit [41, 42, 44]. These black holes correspond to excited states in the putative CFT_2 .

Our work aims to achieve a detailed understanding of the emergence of a CFT_2 in the IR from the CFT_4 in the UV from purely field theoretic considerations.¹ In particular, we calculate the entropy for (near-)EVH black holes using the methods of [19]. These results apply to the $1/16^{\text{th}}$ BPS AdS_5 black holes only, so we will restrict our attention to a (smooth) BPS limit of the EVH black holes. One of our main findings is that the computation of the entropy in the four-dimensional CFT reduces to the derivation of the Cardy formula of a two-dimensional CFT with precisely the central charge and conformal dimension as expected from the EVH/ CFT_2 proposal. We take this as further evidence for the EVH/ CFT correspondence derived from a purely field theoretical perspective. We comment on how this derivation may help in understanding the field theory perspective in more detail.

Given the success of understanding the EVH limit of AdS_5 BPS black holes, it is interesting to know whether the method can also be used to study BPS black hole in other dimensions. The entropy-temperature scaling definition (1.1) of the EVH black holes

¹See also [45, 46] for some results in this direction. Also, similar work has been done in [47, 48].

requires knowledge about non-extremal AdS geometries at strong coupling. Limitations of known AdS black hole solutions in general dimensions restricts our capacity to explore this further. We find that among the set of known AdS_{d+1} ($d > 2$) black hole solutions, the AdS_5 BPS black hole [34, 36, 42] is the best example to explore the EVH-BPS limit.

This paper is organized as follows. In section 2, we review the most general black hole solutions in AdS_5 and subsequently restrict to supersymmetric solutions and study their (near-)EVH limits. In section 3, we review the microscopic derivation of the supersymmetric AdS_5 black hole entropy. We then use this calculation to derive the entropy of the (near-)EVH black holes and show how this computation is related to the derivation of the Cardy formula in two-dimensional CFT. In section 4, we introduce a method to determine whether AdS_{k+1} could appear in the near-horizon limit of EVH-BPS black holes in AdS_{d+1} for $k < d$ for general d and $k < d$. Finally, in section 5 we discuss our results and conclude with directions for future research.

2 Supersymmetric black hole solution in AdS_5

In this section, we will review the general AdS_5 black hole solutions. In addition, we will take both the EVH and BPS limits of these solutions and derive their near-horizon geometries. Finally, we review the constraint on the chemical potentials, which will be important to connect the geometry to its field theory dual. The study of the EVH-BPS limit of black holes in general dimensions will be postponed to section 4.

2.1 Black hole solutions in 5d supergravity

The aim of this section is to set notation. As we write out the solutions explicitly, certain formulae are somewhat intricate. We will first consider a supersymmetric AdS_5 black hole as the solution of five-dimensional gauged supergravity. The generic five-dimensional $\text{U}(1)^n$ gauged supergravity action [36] is:²

$$S_{5d} = \frac{1}{16\pi G_5} \times \int \left(R_5 \star 1 - Q_{IJ} F^I \wedge \star F^J - Q_{IJ} dX^I \wedge \star dX^J - \frac{1}{6} C_{IJK} F^I \wedge F^J \wedge A^K + 2\mathcal{V} \star 1 \right), \quad (2.1)$$

where the n real X^I obey the constraint

$$\frac{1}{6} C_{IJK} X^I X^J X^K = 1 \quad \text{with} \quad I, J, K = 1, \dots, n, \quad (2.2)$$

and C_{IJK} is symmetric under permutation of (IJK) . In addition

$$F^I \equiv dA^I, \quad X_I \equiv \frac{1}{6} C_{IJK} X^J X^K, \quad Q_{IJ} \equiv \frac{9}{2} X_I X_J - \frac{1}{2} C_{IJK} X^K. \quad (2.3)$$

We will focus on $\text{U}(1)^3$ gauged supergravity, which contains $\text{U}(1)^3$ gauge fields and two independent real scalars. Now $C_{IJK} = 1$ when (IJK) is a permutation of (123) and zero

²We set the coupling $g = 1$.

otherwise. In terms of X^I with $I = 1, 2, 3$, the constraint (2.2) and eq. (2.3) become

$$X^1 X^2 X^3 = 1, \quad X_I \equiv \frac{1}{3X^I}, \quad Q_{IJ} \equiv \frac{\delta_{IJ}}{2(X^I)^2}. \quad (2.4)$$

Furthermore, the potential $\mathcal{V}$ is given by:

$$2\mathcal{V} \star 1 = 4\sqrt{-G} \sum_{i=1}^3 \frac{1}{X^I} dV. \quad (2.5)$$

The scalars X^I can be written in terms of two independent scalars φ_i :

$$X^1 = e^{-\frac{1}{\sqrt{6}}\varphi_1 - \frac{1}{\sqrt{2}}\varphi_2}, \quad X^2 = e^{-\frac{1}{\sqrt{6}}\varphi_1 + \frac{1}{\sqrt{2}}\varphi_2}, \quad X^3 = e^{\frac{2}{\sqrt{6}}\varphi_1}. \quad (2.6)$$

In terms of the φ_i , the bosonic part of Lagrangian of this $U(1)^3$ theory becomes [34]:

$$\frac{1}{\sqrt{|g|}} \mathcal{L} = R - \frac{1}{2}((\partial\varphi_1)^2 + (\partial\varphi_2)^2) - \sum_{I=1}^3 \frac{1}{(2X^I)^2} (F^I)^2 + 4 \sum_{I=1}^3 \frac{1}{X^I} + \frac{1}{6} \epsilon^{\mu\nu\rho\sigma\lambda} F_{\mu\nu}^1 F_{\rho\sigma}^2 A_\lambda^3. \quad (2.7)$$

The action associated to (2.7) has a non-BPS black hole solution, described by four free parameters (a, b, m, q) [34]:

$$\begin{aligned} ds_5^2 &= H^{-\frac{4}{3}} \left[-\frac{X}{\rho^2} (dt - a \sin^2 \theta \frac{d\phi}{\Xi_a} - b \cos^2 \theta \frac{d\psi}{\Xi_b})^2 + \frac{C}{\rho^2} \left(\frac{ab}{f_3} dt - \frac{b}{f_2} \sin^2 \theta \frac{d\phi}{\Xi_a} - \frac{a}{f_1} \cos^2 \theta \frac{d\psi}{\Xi_b} \right)^2 \right. \\ &\quad \left. + \frac{Z \sin^2 \theta}{\rho^2} \left(\frac{a}{f_3} dt - \frac{1}{f_2} \frac{d\phi}{\Xi_a} \right)^2 + \frac{W \cos^2 \theta}{\rho^2} \left(\frac{b}{f_3} dt - \frac{1}{f_1} \frac{d\psi}{\Xi_b} \right)^2 \right] + H^{\frac{2}{3}} \left[\frac{\rho^2}{X} dr^2 + \frac{\rho^2}{\Delta_\theta} d\theta^2 \right], \\ H &= \frac{\tilde{\rho}^2}{\rho^2}, \quad \rho^2 = r^2 + a^2 \cos^2 \theta + b^2 \sin^2 \theta, \quad \tilde{\rho}^2 = \rho^2 + q, \\ f_1 &= a^2 + r^2, \quad f_2 = b^2 + r^2, \quad f_3 = (a^2 + r^2)(b^2 + r^2) + qr^2, \\ \Delta_\theta &= 1 - a^2 \cos^2 \theta - b^2 \sin^2 \theta, \quad X = \frac{(a^2 + r^2)(b^2 + r^2)}{r^2} - 2m + (a^2 + r^2 + q)(b^2 + r^2 + q), \\ C &= f_1 f_2 (X + 2m - \frac{q^2}{\rho^2}), \quad \Xi_a = 1 - a^2, \quad \Xi_b = 1 - b^2, \\ Z &= -b^2 C + \frac{f_2 f_3}{r^2} [f_3 - r^2(a^2 - b^2)(a^2 + r^2 + q) \cos^2 \theta], \\ W &= -a^2 C + \frac{f_1 f_3}{r^2} [f_3 + r^2(a^2 - b^2)(b^2 + r^2 + q) \sin^2 \theta]. \end{aligned} \quad (2.8)$$

The scalars are related to the warp factors in the metric:

$$X_1 = X_2 = H^{-\frac{1}{3}}, \quad X_3 = H^{\frac{2}{3}}, \quad (2.9)$$

and the gauge fields are given by:

$$\begin{aligned} A^1 &= A^2 = \frac{\sqrt{q^2 + 2mq}}{\tilde{\rho}^2} \left(dt - a \sin^2 \theta \frac{d\phi}{\Xi_a} - b \cos^2 \theta \frac{d\psi}{\Xi_b} \right), \\ A^3 &= \frac{q}{\rho^2} \left(b \sin^2 \theta \frac{d\phi}{\Xi_a} + a \cos^2 \theta \frac{d\psi}{\Xi_b} \right). \end{aligned} \quad (2.10)$$

The black hole solution is written in the asymptotic rotating frame. One can transform the solution to the asymptotic static frame by the coordinate transformation [34]:

$$\phi^S = \phi + at, \quad \psi^S = \psi + bt. \quad (2.11)$$

In the asymptotic static frame, the chemical potentials and angular velocities are [34]:

$$\begin{aligned} \Omega_a &= \frac{a(r_+^4 + r_+^2 b^2 + r_+^2 q + b^2 + r_+^2)}{(r_+^2 + a^2)(r_+^2 + b^2) + qr_+^2}, & \Omega_b &= \frac{b(r_+^4 + r_+^2 a^2 + r_+^2 q + a^2 + r_+^2)}{(r_+^2 + a^2)(r_+^2 + b^2) + qr_+^2}, \\ \Phi_1 = \Phi_2 &= \frac{\sqrt{q^2 + 2mqr_+^2}}{(r_+^2 + a^2)(r_+^2 + b^2) + qr_+^2}, & \Phi_3 &= \frac{qab}{(r_+^2 + a^2)(r_+^2 + b^2) + qr_+^2}, \end{aligned} \quad (2.12)$$

where the Newton constant G_5 is expressed in terms of the rank of the $SU(N)$ gauge group of the dual field theory by $\frac{\pi}{2G_5} = N^2$. Moreover, r_+ is the outer horizon radius for which X as defined in (2.8) vanishes: $X(r_+) = 0$. In addition, the angular momenta, U(1) charges, entropy, temperature, and mass of the black hole (2.8) are given by [34, 42]:

$$\begin{aligned} J_a &= \frac{N^2 a(2m + q\Xi_b)}{2\Xi_b \Xi_a^2}, & J_b &= \frac{N^2 b(2m + q\Xi_a)}{2\Xi_a \Xi_b^2}, \\ Q_1 = Q_2 &= \frac{N^2 \sqrt{q^2 + 2mq}}{2\Xi_a \Xi_b}, & Q_3 &= -\frac{N^2 abq}{2\Xi_a \Xi_b}, \\ S &= \frac{N^2 \pi [(r_+^2 + a^2)(r_+^2 + b^2) + qr_+^2]}{\Xi_a \Xi_b r_+}, \\ T_H &= \frac{2r_+^6 + r_+^4(1 + a^2 + b^2 + 2q) - a^2 b^2}{2\pi r_+ [(r_+^2 + a^2)(r_+^2 + b^2) + qr_+^2]}, \\ E &= \frac{N^2}{4\Xi_a^2 \Xi_b^2} [2m(2\Xi_a + 2\Xi_b - \Xi_a \Xi_b) + q(2\Xi_a^2 + 2\Xi_b^2 + 2\Xi_a \Xi_b - \Xi_a^2 \Xi_b - \Xi_b^2 \Xi_a)], \end{aligned} \quad (2.13)$$

where J_a and J_b measure the angular momenta around the ϕ and ψ directions respectively,³ whereas Q_I measure the electric charges of the black hole with respect to the U(1) gauge fields A^I .

The black hole solution preserves a supersymmetry when its charges satisfy the following BPS condition [34]:

$$E - J_a - J_b - Q_1 - Q_2 - Q_3 = 0. \quad (2.14)$$

In terms of the parameters (a, b, m, q) , this condition is equivalent to:

$$q = \frac{2m}{(a+b)(2+a+b)}. \quad (2.15)$$

However, even when this constraint is satisfied, it turns out that the spacetime could contain closed timelike curves. To avoid those, and in addition naked singularities, one also has to constrain the parameter m in terms of a and b [49]. The result is:

$$m = \frac{(a+b)^2(1+a)(1+b)(2+a+b)}{2(1+a+b)}. \quad (2.16)$$

³We adopt this notation because $J_a \equiv J_\phi$ vanishes when $a = 0$, and $J_b \equiv J_\psi$ vanishes when $b = 0$.

For these values, it turns out that the horizon radius is given by:

$$r_+^2 = r_0^2 \equiv \frac{ab}{1+a+b}. \quad (2.17)$$

Plugging in these values, the function X simplifies to:

$$X(r) = \frac{(r^2 - r_0^2)^2 (r^2 + (1+a+b)^2)}{r^2}, \quad (2.18)$$

showing that the solution has become extremal. Namely, a causally well-behaved BPS black hole is also extremal. At the BPS-extremal point, the charge q simplifies to:

$$q = \frac{(a+b)(1+a)(1+b)}{1+a+b}. \quad (2.19)$$

Therefore, this BPS black hole only depends on the parameters a and b .

The supersymmetric solution we study here looks different from the most general supersymmetric black holes found in [36], which we review in appendix A. The supersymmetric solution in [36] chooses a t coordinate adapted to the null Killing field ∂_t on the horizon. Thus, the solution (A.1) is corotating with horizon.⁴ To embed the supersymmetric limit of solution (2.8) into the most general class (A.1), we need to transform the metric into the corotating frame via:

$$\tilde{\phi} = \phi + (a-1)t, \quad \tilde{\psi} = \psi + (b-1)t. \quad (2.20)$$

Finally, the black hole solution (2.8) can be embedded into ten-dimensional type IIB supergravity [50] and also into eleven dimensions supergravity [51]. The ansatz for the ten-dimensional supergravity solution is

$$ds_{10}^2 = \sqrt{\tilde{\Delta}} ds_5^2 + \frac{1}{\sqrt{\tilde{\Delta}}} \sum_{i=1}^3 \frac{1}{X_i} (d\mu_i^2 + \mu_i^2 (d\psi_i + A^i)^2). \quad (2.21)$$

The extra five-dimensional manifold can be considered as a deformation of S^5 , which is parametrized by:

$$d\Omega_{S^5}^2 = \sum_i (d\mu_i^2 + \mu_i^2 d\psi_i^2). \quad (2.22)$$

Here, we think of S^5 as a three-torus T^3 fibration over the two-sphere S^2 , where the μ_i parametrize the S^2 :

$$\mu_1 = \sin \alpha \cos \beta, \quad \mu_2 = \sin \alpha \sin \beta, \quad \mu_3 = \cos \alpha. \quad (2.23)$$

On the other hand, the ψ_i parametrize T^3 . Finally, $\tilde{\Delta}$ only depends on the scalar fields X_i through $\tilde{\Delta} = \sum_i \mu_i^2 X_i$.

⁴We thank James Lucietti for clarifying this point for us.

2.2 (Near-)EVH limit of the BPS black hole

The BPS black hole we constructed above generically has a finite horizon area. In particular, its near-horizon geometry contains an AdS_2 throat [16]. In order for an AdS_3 to arise in the near-horizon geometry, the horizon size of the black hole has to vanish. The AdS_3 geometry reflects the vanishing horizon area in that its angular direction is pinching. In general, this will result into an infinitely gapped system, since the energy gap above the ground state scales like $1/R$, with R the (conformal) boundary circle radius. However, as studied in [41, 44], one may keep non-trivial dynamics by combining the vanishing entropy limit with a $G_3 \rightarrow 0$ (i.e., $c \rightarrow \infty$) limit. This will be the type of double scaling limit we consider for the near-EVH black holes.

Let us first revisit the strict EVH black hole obtained in [42]. The EVH condition can be imposed independently of the supersymmetry condition. For the solution (2.8) with a ten-dimensional embedding (2.21), the EVH condition is

$$X(r_+ = b = 0) = 0. \quad (2.24)$$

This gives:

$$m = \frac{q^2 + a^2(1+q)}{2}. \quad (2.25)$$

The near-horizon limit is approached by expanding (2.8) in terms of $r = \epsilon\rho$. After some work and suitable coordinate redefinitions, [42] obtains the following geometry:

$$\begin{aligned} ds^2 = & h_1 h_2 \left[-\frac{x^2}{\ell_3^2} d\tau^2 + \frac{\ell_3^2}{x^2} dx^2 + x^2 d\tilde{\chi}^2 \right] \\ & + \frac{(a^2 + q)h_1 h_2}{\Delta_\theta} d\theta^2 + \frac{\cos^2 \alpha \cos^2 \theta}{K^2 h_1 h_2} d\xi^2 + \frac{a^2 + q}{\Xi_a^2} \frac{h_2}{h_1^3} \Delta_\theta \sin^2 \theta d\tilde{\phi}^2 \\ & + \frac{h_2}{h_1} d\alpha^2 + \frac{h_1}{h_2} \sin^2 \alpha d\beta^2 + \frac{h_1}{h_2} \left[\mu_1^2 (d\tilde{\psi}_1 - A d\tilde{\phi})^2 + \mu_1^2 (d\tilde{\psi}_2 - A d\tilde{\phi})^2 \right]. \end{aligned} \quad (2.26)$$

Apart from the definitions of the various functions that we will collect below, let us start by summarizing the most important aspects of this geometry. First of all, the metric describes a warped product of a locally AdS_3 geometry (first line (2.26)) and some seven-dimensional manifold (second and third line in (2.26)). The time and radial coordinates τ and x are directly related to the time and radial coordinates of the original AdS_5 black hole geometry. In addition, the new angular coordinates $\tilde{\chi}$ and ξ are related to the original angles ψ and ϕ of the AdS_5 geometry via:

$$\tilde{\chi} = \epsilon\psi, \quad \xi = \frac{1}{\sqrt{2}}(\psi_3 + \psi). \quad (2.27)$$

This implies in particular that the periodicity of $\tilde{\chi}$ is $2\pi\epsilon$, showing the local AdS_3 geometry is the pinching orbifold of [44]. Moreover, the coordinate transformation shows that the AdS_3 geometry is embedded completely inside the AdS_5 geometry, as opposed to its angular

part originating from an S^5 direction as in e.g., [37]. Finally, note that the generator of rotations around the angular direction in AdS_3 will be proportional to $\partial_{\tilde{\chi}} = \epsilon^{-1}(\partial_{\psi} + \partial_{\psi_3})$.⁵

Let us now briefly collect the definitions of the various functions appearing in the metric. Some were already defined in (2.8) and (2.23). For the rest, $h_{1,2}$ are given by [42]:

$$h_1^2 = \frac{a^2 \cos^2 \theta + q}{a^2 + q}, \quad h_2^2 = \frac{a^2 \cos^2 \theta + q \mu_3^2}{a^2 + q}. \quad (2.28)$$

Furthermore, we have:

$$\Delta_\theta = 1 - a^2 \cos^2 \theta, \quad A = \frac{a \sqrt{q \mathbf{Y}_s} \sin^2 \theta}{\ell_3 \Xi_a \sqrt{\mathbf{V}} h_1^2}. \quad (2.29)$$

Finally, we have:

$$K = \sqrt{\frac{a^2 + q}{a^2 + q^2}}, \quad \ell_3^2 = \frac{a^2 + q}{\mathbf{V}}. \quad (2.30)$$

At the BPS point, when we are imposing conditions (2.15) and (2.16), $\mathbf{V}$, q and $\mathbf{Y}_s$ take the values:

$$\mathbf{V} = (1 + a)^2, \quad q = a, \quad \mathbf{Y}_s = 1 + a. \quad (2.31)$$

Finally, let us write down the charges of this specific black hole. First of all, note that $b = 0$ implies that $J_b = Q_3 = 0$. The other charges are finite and given by:

$$J_a = \frac{N^2 a^2}{2(1 - a)^2}, \quad Q_1 = Q_2 = \frac{a}{2(1 - a)} N^2, \quad (2.32)$$

and satisfy:

$$\frac{N^2}{2} J_a = Q_1^2. \quad (2.33)$$

We will come back to this relation in section 3.

To obtain the near-horizon metric for the near-EVH black hole, instead of $b = 0$ one takes $b = \lambda \epsilon^2$. The analysis that leads to the near-horizon geometry is otherwise similar to the strict EVH case, so we will not repeat it here and refer the interested reader to [42]. Suffice to say that the near-horizon geometry describes a similar warped product as in (2.26) but now instead of an AdS_3 geometry there is a pinching extremal BTZ in the BPS case:

$$ds_3^2 = -\frac{(x^2 - x_0^2)^2}{\ell_3^2 x^2} d\tau^2 + \frac{\ell_3^2 x^2 dx^2}{(x^2 - x_0^2)^2} + x^2 \left(d\tilde{\chi} - \frac{x_0^2}{\ell_3^2 x^2} d\tau \right)^2, \quad (2.34)$$

where:

$$\ell_3^2 = \frac{a}{1 + a}, \quad x_0^2 = \frac{\lambda}{2}. \quad (2.35)$$

Moreover, the periodicity of $\tilde{\chi}$ is again $2\pi\epsilon$, reflecting the fact that the BTZ is pinching.

⁵This will explain a relation we discuss below in the context of the near-EVH black hole, which relates the L_0 eigenvalue of the CFT_2 to the near-EVH black hole charges $L_0 \sim J_b + Q_3$ (since in the BPS case $\bar{L}_0 = 0$).

By the relation between the parent central charge of the $\mathcal{N} = 4$ theory to the ten-dimensional Newton constant and compactifying the warped geometry to the BTZ in three dimensions, we obtain [42]:

$$\frac{1}{G_3} = 2\sqrt{2}N^2 \frac{a}{1-a} \sqrt{\frac{a}{1+a}}, \quad (2.36)$$

with N the rank of the gauge group. This allows us to compute the entropy of the BTZ black hole:

$$S_{\text{BTZ}} = \frac{2\pi\epsilon x_0}{4G_3} = \frac{\pi a}{1-a} \sqrt{\frac{\lambda a}{1+a}} N^2 \epsilon. \quad (2.37)$$

As mentioned at the beginning of this section, we see from this formula that to allow for a finite entropy we need to take the double scaling limit $\epsilon \rightarrow 0$ and $N \rightarrow \infty$ keeping $N^2\epsilon$ fixed. This then defines the near-EVH limit. We can also read off the mass of the BTZ black hole. It is given by:

$$M_{\text{BTZ}}\ell_3 = L_0 - \frac{c}{24} = \frac{a\lambda}{2\sqrt{2}(1-a)} N^2 \epsilon = \frac{1}{\sqrt{2}\epsilon} (J_b + Q_3). \quad (2.38)$$

In the first equality, we remind the reader how the (extremal) BTZ mass is related to the conformal dimension in the dual CFT_2 , whereas the last equality can be checked by plugging in $b = \lambda\epsilon^2$ into the black hole charges (2.13). Note that we anticipated this relation below (2.27). We will give a precise CFT_4 interpretation of both (2.37) and (2.38) in section 3.

Let us end this section by noting that our near-EVH black hole geometry (2.34) has two free parameters (a, λ) . The most general supersymmetric AdS_5 black hole [36] has four free parameters, which still has a vanishing entropy if $Q_3 = J_b = 0$ [19]. This implies the EVH black hole also exists in these solutions. We will show in appendix A that the near-horizon geometry of these most general supersymmetric AdS_5 black holes still contain AdS_3 factors in the EVH limit.

2.3 Chemical potentials

In this section, we will define the various potentials for the supersymmetric black holes by taking the BPS limit of the potentials for the generic non-BPS black hole (2.12). The BPS limit can be taken through a zero temperature limit accompanied by specific limits for the various potentials as discussed in [52] (see also [18] for a more recent discussion in our context). The fact that one has to take limits of the chemical potentials as well can be illustrated by a simple example [52]. Consider the partition function

$$Z = \sum_{E,J} e^{-\beta E - \beta \Omega J} = \sum e^{-\beta(E-J) + \beta(1-\Omega)J}, \quad (2.39)$$

where β is the inverse temperature and Ω is the chemical potential associated to the charge J . In the zero temperature limit $\beta \rightarrow \infty$, we see that the partition function localizes on the states satisfying a BPS equation $E = J$. However, for the partition function to be well defined, one should also make sure that $\Delta \equiv \beta(1-\Omega)$ is finite in the limit $\beta \rightarrow \infty$. It is Δ that then defines the BPS limit of the chemical potential.

To find the BPS values of the chemical potentials, we follow [18]. We can start from the non-BPS black hole (2.8). The parameter m can be expressed in terms of the horizon size r_+ as:

$$m = \frac{(r_+^2 + a^2)(r_+^2 + b^2)}{2r_+^2} + \frac{(r_+^2 + a^2 + q)(r_+^2 + b^2 + q)}{2}. \quad (2.40)$$

Imposing the supersymmetry condition (2.15) without the causality condition (2.16), we can solve for q as:⁶

$$q = \frac{(a - ir_+)(b - ir_+)(1 - ir_+)}{-ir_+} = a + b + ab - r_+^2 - \frac{i(1 + a + b)}{r_+}(r_+^2 - r_0^2). \quad (2.41)$$

The parameter q is only real when $r_+ = r_0$ with r_0 determined by (2.17), which reproduces the BPS-extremal value (2.19). For all the other values, the BPS black hole has a naked singularity and is causally pathological [34]. However, to take the BPS limit it turns out to be convenient to momentarily ignore the constraint $r_+ = r_0$ and take q as in (2.41). We can then write the black hole chemical potentials Φ_I and angular velocities Ω_i in (2.12) as follows:

$$\begin{aligned} \Phi_1 = \Phi_2 &= \frac{(1 + a + b)r_+(r_+ + i)}{i(1 + a + b)r_+ + ab}, & \Phi_3 &= \frac{ab(i + r_+)}{r_+(ab + i(1 + a + b)r_+)}, \\ \Omega_a &= \frac{a(1 - ir_+)[(1 + a + b)r_+ - ib]}{(a - ir_+)[(1 + a + b)r_+ - iab]}, & \Omega_b &= \frac{b(1 - ir_+)[(1 + a + b)r_+ - ia]}{(b - ir_+)[(1 + a + b)r_+ - iab]}. \end{aligned} \quad (2.42)$$

One verifies that these potentials obey the equation:

$$\beta(\Omega_a + \Omega_b - \Phi_1 - \Phi_2 - \Phi_3 + 1) = 2\pi i, \quad (2.43)$$

where β corresponds to the inverse Hawking temperature of the non-BPS black hole given in (2.13). We now want to consider the $\beta \rightarrow \infty$ limit, obtained by taking $r_+ \rightarrow r_0$ with r_0 defined in (2.17). The chemical potentials in this limit simplify to:

$$\Phi_1 = \Phi_2 = \Phi_3 = \Omega_a = \Omega_b = 1. \quad (2.44)$$

As in the simple example discussed above, this motivates us to define finite BPS potentials as:

$$\omega_i = \lim_{r_+ \rightarrow r_0} \beta(1 - \Omega_i), \quad \Delta_I = \lim_{r_+ \rightarrow r_0} \beta(1 - \Phi_I), \quad (2.45)$$

with $i = a, b$ and $I = 1, 2, 3$. Explicitly, ω_i and Δ_I are given by:

$$\begin{aligned} \Delta_1 = \Delta_2 &= -\frac{\pi r_0(a + b)[(1 + a + b)r_0 + iab]}{iab(1 + a + b - ir_0)}, \\ \Delta_3 &= \frac{\pi(a + b)[(1 + a + b)r_0 + iab]}{ab(1 + a + b - ir_0)}, \\ \omega_a &= \frac{\pi(1 - a)(ib + r_0)}{ir_0(1 + a + b - ir_0)}, & \omega_b &= \frac{\pi(1 - b)(ia + r_0)}{ir_0(1 + a + b - ir_0)}. \end{aligned} \quad (2.46)$$

⁶We have chosen one branch of the solution. The other can be obtained by $i \rightarrow -i$.

The BPS potentials then satisfy the constraint:

$$\Delta_1 + \Delta_2 + \Delta_3 - \omega_a - \omega_b = \beta - \beta(\Phi_1 + \Phi_2 + \Phi_3 - \Omega_a - \Omega_b) = 2\pi i. \quad (2.47)$$

Therefore, even though the original chemical potentials (2.42) become real in the limit $r_+ = r_0$, the properly defined BPS potentials satisfy a complex constraint! This observation underlies all recent derivations of the AdS₅ BPS black hole entropy from the supersymmetric partition function of the $\mathcal{N} = 4$ SYM theory [17–19].

In section 3, we will derive the values of the chemical potentials in the (near-) EVH limit from the dual CFT. Therefore, we will now take the EVH limit of these potentials in order to compare to the field theory analysis later. It turns out that in this comparison there is a slight subtlety which is related to the fact that in the strict EVH limit $b \rightarrow 0$, the EVH limit and BPS limit of the chemical potentials do not commute. To see this, let us first compute the EVH limit of the BPS potentials in (2.45):

$$\lim_{b \rightarrow 0} \Delta_1 = \lim_{b \rightarrow 0} \Delta_2 = \frac{\pi i a}{1 + a}, \quad \lim_{b \rightarrow 0} \omega_a = -\frac{\pi i(1 - a)}{1 + a}. \quad (2.48)$$

This shows that in the EVH limit $\Delta_{1,2}$ and ω_a satisfy the following condition:

$$\Delta_1 + \Delta_2 - \omega_a = \pi i. \quad (2.49)$$

In addition:

$$\lim_{b \rightarrow 0} \omega_b = \frac{\pi}{\sqrt{b}} \sqrt{\frac{a}{1 + a}} + \mathcal{O}(b^0). \quad (2.50)$$

On the other hand, if we first take the EVH limit, such that in particular $J_b = Q_3 = 0$, the BPS condition (2.14) reads:

$$E = Q_1 + Q_2 + J_a \quad (2.51)$$

This condition may be viewed as the consequence of imposing two independent BPS conditions:

$$\begin{aligned} E &= Q_1 + Q_2 + Q_3 + J_a + J_b, \\ E &= Q_1 + Q_2 - Q_3 + J_a - J_b. \end{aligned} \quad (2.52)$$

For the black hole solutions we have studied, the first condition results in the constraint (2.47) for generic r_+ , as we have seen above. On the other hand, a similar analysis shows that black hole solutions obeying the second condition have chemical potentials that obey another constraint:

$$\Delta_1 + \Delta_2 - \Delta_3 - \omega_a + \omega_b = 2\pi i. \quad (2.53)$$

These two constraints combine to give:

$$\Delta_1 + \Delta_2 - \omega_a = 2\pi i, \quad (2.54)$$

which is in clear contradiction to (2.49). The explicit potentials we derive for $b \rightarrow 0$ in this case are given by:

$$\lim_{b \rightarrow 0} \Delta_1 = \lim_{b \rightarrow 0} \Delta_2 = \frac{2\pi i a}{1 + a}, \quad \lim_{b \rightarrow 0} \omega_a = -\frac{2\pi i(1 - a)}{1 + a}. \quad (2.55)$$

As we will see in the next section, from the field theory point of view the latter potentials emerge in the strict EVH limit. However, in the near-EVH limit we do find the values (2.48) and (2.50).

We will call the black holes satisfying the constraint (2.49) the BPS black hole in the EVH limit. On the other hand, the black holes satisfying the constraint (2.54) is called the EVH-BPS black hole. The constraint (2.54) is the result of enhancement of supersymmetry in the strict EVH limit, as we will see in the following section.

3 $\mathcal{N} = 4$ SYM and the superconformal index

To study the microscopic origin of the EVH/CFT correspondence, we now turn to the CFT dual description of the AdS_5 black holes discussed in the previous section. This dual description is provided by the four-dimensional $\mathcal{N} = 4$ $\text{SU}(N)$ super-Yang-Mills theory. One of the most basic entries in the dictionary is the relation between the rank of the gauge group and the AdS radius and Newton's constant in five dimensions:

$$N^2 = \frac{\pi}{2G_5 g^3}, \quad (3.1)$$

where we put $g = L_{\text{AdS}}^{-1} = 1$ in the following. To arrive at a microscopic derivation of the entropy of the (near-)EVH black holes, we will follow the recent work on a microscopic accounting of the entropy of general AdS_5 black holes [19].⁷

The partition function studied in [19] is defined by:

$$Z(\beta, \Delta_I, \omega_i) = \text{Tr}_{\mathcal{H}} \left[e^{-\sum_{I=1}^3 \Delta_I Q_I} e^{-\omega_a J_a} e^{-\omega_b J_b} e^{-\beta E_{\text{susy}}} \right]. \quad (3.2)$$

In this expression, $\mathcal{H}$ is the Hilbert space of the theory quantized on S^3 (the conformal boundary of AdS_5), which via the operator-state correspondence is constructed by applying gauge invariant local operators on the vacuum. Furthermore, the Q_I charges correspond to the Cartan generators of the $\text{SO}(6)$ R-symmetry and the J_i charges correspond to the Cartan generators of the $\text{SO}(4)$ rotational symmetry of S^3 . The operator E_{susy} is taken to be:

$$E_{\text{susy}} \equiv \{\mathcal{Q}, \mathcal{S}\} = E - \left(\sum_{I=1}^3 Q_I \right) - J_a - J_b, \quad (3.3)$$

where $\mathcal{Q}$ is a supercharge with charge $+\frac{1}{2}$ under the Q_I and charge $-\frac{1}{2}$ under the J_i , and E is the generator of time translations of the theory on S^3 . Finally, the chemical potentials Δ_I and ω_i can in general be complex, as we will see below, but are periodic under translations by $4\pi i$.

As it stands, the partition function in (3.2) receives contributions from all states in the theory. This is because $\mathcal{Q}$ does not anti-commute with the operator in the trace. Indeed, we have:

$$e^{-\Delta_I Q_I - \omega_i J_i} \mathcal{Q} = e^{-\frac{(\sum_{I=1}^3 \Delta_I) - \omega_a - \omega_b}{2}} \mathcal{Q} e^{-\Delta_I Q_I - \omega_i J_i}. \quad (3.4)$$

⁷See also [17, 18] for similar results which were obtained through different methods.

To resolve this issue, it is proposed in [19] (see also [18]) to restrict the theory to the hypersurface:

$$\left(\sum_{I=1}^3 \Delta_I\right) - \omega_a - \omega_b = 2\pi i \pmod{4\pi i}. \quad (3.5)$$

Note that this is also the constraint obeyed by black holes as reviewed in section 2.3. On this hypersurface, we see that the supercharge $\mathcal{Q}$ anticommutes with the full operator in the trace and therefore the partition function localizes on the space of $1/16^{\text{th}}$ BPS states:

$$\mathcal{H}_{\mathcal{Q}} = \{|\psi\rangle \in \mathcal{H} \mid \mathcal{Q}|\psi\rangle = 0\}. \quad (3.6)$$

This is achieved without the need for an explicit insertion of $(-1)^F$. Since these BPS states are annihilated by $\{\mathcal{Q}, \mathcal{S}\}$ as well, their quantum numbers saturate the unitarity bound:

$$E \geq Q_1 + Q_2 + Q_3 + J_a + J_b. \quad (3.7)$$

In [19], the index (3.2) is explicitly computed in the weakly coupled $\mathcal{N} = 4$ theory, even though the final result holds for arbitrary coupling due to the fact that only BPS states contribute. As is standard, one deals with the gauge invariance constraint on states by including holonomies for the gauge field along the temporal circle in the trace (3.2). The projection on the gauge invariant states can then be performed by integration over the gauge group with the Haar measure:

$$\int [dU] = \frac{1}{N!(2\pi)^{N-1}} \int \prod_{i=1}^{N-1} d\alpha_i \prod_{i < j} 4 \sin^2 \frac{1}{2}(\alpha_i - \alpha_j). \quad (3.8)$$

The resulting expression for the index is given by [19, 53]:

$$Z = \frac{1}{N!} \int \prod_{i=1}^N \frac{d\alpha_i}{2\pi} \prod_{i < j} \left(2 \sin \frac{\alpha_{ij}}{2}\right)^2 \exp \left[\sum_{n=1}^{\infty} \frac{1}{n} \left(1 - \frac{\prod_{I=1}^3 2 \sinh \frac{n\Delta_I}{2}}{2 \sinh \frac{n\omega_a}{2} 2 \sinh \frac{n\omega_b}{2}}\right) \sum_{i,j=1}^N e^{in\alpha_{ij}} \right]. \quad (3.9)$$

Here, the α_i correspond to the weights of $\text{SU}(N)$ and $\alpha_{ij} \equiv \alpha_i - \alpha_j$ to the roots, which reflect the gauge charges of the various adjoint valued fields.

At large- N , this index can be computed through a saddle point approximation. The early result of [54] showed that the leading term is only of order $\mathcal{O}(1)$ at large- N . The failure to reproduce an $\mathcal{O}(N^2)$ scaling, as expected from the existence of large AdS_5 BPS black holes, is believed to be due to the large cancellation between bosonic and fermionic degrees of freedom.

The crucial ingredient of the new analysis in [19] is to allow for complex chemical potentials (as also emphasized in [17, 18]). This is in fact required since the black hole chemical potentials satisfy a complex constraint (3.5). The complex phase could potentially obstruct the large cancellation between bosonic and fermionic degrees of freedom. This turns out to be the case. The leading order contribution to the partition function (3.9)

at large- N is computed through a saddle point approximation in the “generalized Cardy” limit $\omega_i \ll 1$ and is given by the simple $\mathcal{O}(N^2)$ expression:

$$\log Z = \frac{N^2}{2} \frac{\Delta_1 \Delta_2 \Delta_3}{\omega_a \omega_b}. \quad (3.10)$$

To arrive at this expression, it is assumed that the matrix integral in (3.9) is dominated by the “maximally deconfining” saddle $\alpha_1 = \alpha_2 = \dots = \alpha_N$. Note that the expression is formally identical to the supersymmetric Casimir energy [16, 55, 56], but with chemical potential subject to the constraint (3.5). To compute the degeneracy of states for fixed charges, one should now compute the inverse Laplace transform:

$$d = \int d\Delta_1 d\Delta_2 d\omega_a d\omega_b Z(\Delta_I, \omega_i) e^{\omega_a(J_a+Q_3)} e^{\omega_b(J_b+Q_3)} e^{\Delta_1(Q_1-Q_3)} e^{\Delta_2(Q_2-Q_3)} (-1)^F, \quad (3.11)$$

where $(-1)^F = e^{2\pi i Q_3}$. At large- N and large charges (i.e., the charges scale with N^2), the inverse Laplace transform becomes a Legendre transform of $\log Z$. As shown in [16] (see also [18, 19]), this reproduces the expected BPS AdS₅ black hole entropy:

$$S = 2\pi \sqrt{Q_1 Q_2 + Q_2 Q_3 + Q_3 Q_1 - \frac{N^2}{2}(J_a + J_b)}. \quad (3.12)$$

3.1 Entropy of (near-)EVH black holes

We now turn to our black holes of interest, as described in section 2.2. This will not be a trivial specialization of the analysis reviewed above, since for the near-EVH black hole the charges J_b and Q_3 are not of order N^2 and therefore it is not directly clear whether one can perform a saddle point approximation with respect to the associated chemical potentials.

The EVH limit. Since (near-)EVH black holes have $Q_1 = Q_2 \equiv Q$, the expression (3.11) becomes:

$$d = \int d\Delta_1 d\Delta_2 d\omega_a d\omega_b \exp \left(\frac{N^2}{2} \frac{\Delta_1 \Delta_2 \Delta_3}{\omega_a \omega_b} + \omega_a(J_a + Q_3) + \omega_b(J_b + Q_3) + (\Delta_1 + \Delta_2)(Q - Q_3) + 2\pi i Q_3 \right). \quad (3.13)$$

The saddle point equations for Δ_1 and Δ_2 are identical in this case, implying $\Delta_1 = \Delta_2 \equiv \Delta$ on the saddle. In the following, we will use this fact to write (3.13) as:

$$d = \int d\Delta d\omega_a d\omega_b \exp \left(\frac{N^2}{2} \frac{\Delta^2 \Delta_3}{\omega_a \omega_b} + \omega_a(J_a + Q_3) + \omega_b(J_b + Q_3) + 2\Delta(Q - Q_3) + 2\pi i Q_3 \right), \quad (3.14)$$

which is allowed as long as we restrict to the saddle point approximation of the integrand.

The computation of (3.14) is simplified further since the integrand in the strict EVH limit does not depend on ω_b . This is due to the fact that the EVH black holes preserve an additional supercharge. To understand this in more detail, let us first write down the index (3.2) after having solved the constraint (3.5):

$$Z(\Delta_I, \omega_i) = \text{Tr}_{\mathcal{H}_Q} \left[e^{-\Delta_1(Q_1-Q_3)} e^{-\Delta_2(Q_2-Q_3)} e^{-\omega_a(J_a+Q_3)} e^{-\omega_b(J_b+Q_3)} e^{-2\pi i Q_3} e^{-\beta E_{\text{susy}}} \right]. \quad (3.15)$$

Since EVH black holes have $J_b = Q_3 = 0$, we can take $\omega_b \rightarrow \infty$ in the index without losing their contributions. Indeed, in this limit the index localizes onto configurations with $J_b + Q_3 = 0$, of which the EVH black holes form a subset. The $\omega_b \rightarrow \infty$ limit is known as the Macdonald limit of the superconformal index, which can be defined for generic four-dimensional $\mathcal{N} = 2$ SCFTs. One can explicitly check that the supercharge $\mathcal{Q}'$ with charges:

$$Q_1 = Q_2 = -Q_3 = +\frac{1}{2}, \quad J_a = -J_b = -\frac{1}{2} \quad (3.16)$$

anticommutes with the remaining operator in the trace. Therefore, the trace localizes on the Hilbert space of $1/8^{\text{th}}$ BPS states annihilated by both $\mathcal{Q}$ and $\mathcal{Q}'$. One may verify the additional charge constraint in the Macdonald sector by noting that:

$$\{\mathcal{Q}', S'\} = E - Q_1 - Q_2 + Q_3 - J_a + J_b. \quad (3.17)$$

The vanishing of this expression, combined with the $1/16^{\text{th}}$ BPS condition (3.7), indeed leads to $J_b + Q_3 = 0$.

The Macdonald limit of the index was also considered in [19]. Solving the constraint (3.5) in the limit $\omega_b \rightarrow \infty$ for Δ_3 , one finds that $\Delta_3 \rightarrow \infty$ also. This leaves Δ and ω_a unconstrained.⁸ In the limit that $\Delta, \omega_a \ll 1$, [19] shows that the partition function becomes equal to the naive $\omega_b, \Delta_3 \rightarrow \infty$ limit of (3.10):

$$\log Z = \frac{N^2}{2} \frac{\Delta^2}{\omega_a}. \quad (3.18)$$

Since the charges J_a and Q are of order $\mathcal{O}(N^2)$, we may in complete analogy with the analysis leading to (3.12) compute the Legendre transform of (3.18) with respect to Δ and ω_a , i.e., we extremize

$$S = \left(\frac{N^2}{2} \frac{\Delta^2}{\omega_a} + \omega_a J_a + 2\Delta Q \right) \Big|_{\hat{\Delta}, \hat{\omega}_a}, \quad (3.19)$$

with respect to Δ and ω_a , where the hatted potentials denote those values at which the extremum is reached. One finds that this leads to $S = 0$, as expected for an EVH black hole. However, this extremization procedure only fixes the ratio of $\hat{\Delta}$ and $\hat{\omega}_a$ to be:

$$\frac{\hat{\Delta}}{\hat{\omega}_a} = -\frac{2Q}{N^2} = -\frac{a}{1-a}, \quad (3.20)$$

where we have used the EVH value of $Q = Q_1 = Q_2$ given in eq. (2.13). Even though this is consistent with the chemical potentials of the EVH black hole given in (2.48), we will show that we can do better by correctly taking into account the constraint on Δ and ω_a .

To continue, we take a different approach to compute the Macdonald limit. Instead of taking the chemical potential $\omega_b \rightarrow \infty$, we will require instead that apart from $\mathcal{Q}$ also $\mathcal{Q}'$ anticommutes with the operator in the trace. This will guarantee that the trace restricts to the Macdonald sector $\mathcal{H}_{\mathcal{Q}, \mathcal{Q}'}$ of the theory. In doing this, we derive an additional constraint

⁸This claim will be re-examined below. However, even at this stage one should note that only the *real* parts of the chemical potentials $\Delta_{1,2}, \omega_a$ are unconstrained, since any finite real part is insignificant with respect to $\omega_b, \Delta_3 \rightarrow \infty$.

on the chemical potentials, similar to how (3.5) was derived. We start from the index in which we have solved (3.5) for Δ_3 , as given in (3.15). Demanding that $\mathcal{Q}'$ anticommutes with the operator in the trace and using (3.16), we derive the following constraint:

$$\Delta_1 + \Delta_2 - \omega_a = 2\pi i n \mod 4\pi i, \quad n \in \mathbb{Z}. \quad (3.21)$$

Combining this with the original constraint, we note that in this case we also have:

$$\Delta_3 - \omega_b = 2\pi i m \mod 4\pi i, \quad m \in \mathbb{Z} \quad \text{such that} \quad m + n \in 2\mathbb{Z} + 1. \quad (3.22)$$

Solving the new constraint for ω_a while taking n odd and m even, we find:

$$Z(\Delta_1, \Delta_2) = \text{Tr}_{\mathcal{H}_{\mathcal{Q}, \mathcal{Q}'}} \left[e^{-\Delta_1(Q_1 + J_a)} e^{-\Delta_2(Q_2 + J_a)} e^{2\pi i J_a} e^{-\beta E_{\text{susy}}} \right]. \quad (3.23)$$

Note that we used the fact that $J_b + Q_3 = 0$ in $\mathcal{H}_{\mathcal{Q}, \mathcal{Q}'}$ and consequently the index does not depend on ω_b .

We will now work out the same extremization procedure as above, but taking into account the constraint (3.21). We implement the constraint by a Lagrange multiplier and thus extremize:

$$S = \left(\frac{N^2}{2} \frac{\Delta^2}{\omega_a} + \omega_a J_a + 2\Delta Q + \Lambda(2\Delta - \omega_a - 2\pi i) \right) \Big|_{\hat{\Delta}, \hat{\omega}_a, \hat{\Lambda}}. \quad (3.24)$$

This leads again to $S = 0$, as expected. However, due to the additional constraint, we are able to solve for the chemical potentials exactly. They turn out to be given by:⁹

$$\hat{\Delta} = \frac{2\pi i a}{1+a}, \quad \hat{\omega}_a = -\frac{2\pi i(1-a)}{1+a}, \quad \hat{\Lambda} = 0. \quad (3.25)$$

Note that these values reproduce the gravity calculation (2.55) exactly. For consistency, we should check that these chemical potentials fall within the regime of validity of the analysis in [19] leading to (3.18). To compare with their analysis, we first shift $\Delta \rightarrow \Delta - i\pi$ such that the constraint (3.21) reads (for odd n):

$$2\Delta - \omega = 0 \mod 4\pi i. \quad (3.26)$$

The values of the shifted chemical potentials that extremize S then read:

$$\hat{\Delta} = \frac{\pi i(a-1)}{1+a}, \quad \hat{\omega}_a = -\frac{2\pi i(1-a)}{1+a}, \quad \hat{\Lambda} = 0. \quad (3.27)$$

Since the regime of validity is dictated by $|\Delta|, |\omega_a| \ll 1$, we find that our analysis is only consistent for $a \rightarrow 1$, or in other words very fast spinning black holes.¹⁰

Finally, we note that our chemical potentials are purely imaginary, whereas a positive real part is required for convergence of the index. We take the point of view that these chemical potentials reach 0^+ , as required for convergence, in the EVH limit, as we will further show when considering the near-EVH limit.

⁹There is another branch of solution, which gives an imaginary entropy, and hence is discarded.

¹⁰The fast spinning limit of EVH black holes also appeared in [45, 46], and was crucial in understanding the appearance of a dual chiral two-dimensional CFT. We will further comment on this below.

The near-EVH limit. We will now study the near-EVH limit of the index. In this case, J_a and $Q_1 = Q_2$ are of order N^2 . However, the charges Q_3 and J_b are of order $N^2\epsilon^2$, as reviewed in section 2. Thus, in the large- N limit, we cannot *a priori* justify a saddle point approximation for the evaluation of the ω_b integral in (3.11). So let us first perform saddle point approximations for the ω_a and Δ integrals in:

$$d = \int d\Delta d\omega_a d\omega_b \exp \left(\frac{N^2}{2} \frac{\Delta^2 \Delta_3}{\omega_a \omega_b} + \omega_a (J_a + Q_3) + \omega_b (J_b + Q_3) + 2\Delta(Q - Q_3) + 2\pi i Q_3 \right). \quad (3.28)$$

Understanding the constraint (3.5) as fixing Δ_3 as a function of the other chemical potentials, the saddle point equations for Δ and ω_a become respectively:

$$\begin{aligned} \frac{N^2}{2} \frac{\Delta \Delta_3 - \Delta^2}{\omega_a \omega_b} + Q - Q_3 &= 0, \\ \frac{N^2}{2} \left(-\frac{\Delta^2 \Delta_3}{\omega_a^2 \omega_b} + \frac{\Delta^2}{\omega_a \omega_b} \right) + J_a + Q_3 &= 0. \end{aligned} \quad (3.29)$$

We will denote the solutions to these equations by $\hat{\Delta}$ and $\hat{\omega}_a$. Using the saddle point equations to eliminate the charges $Q - Q_3$ and $J_a + Q_3$, we find:

$$d = \int d\omega_b \exp \left[\frac{N^2}{2} \frac{\hat{\Delta}^2}{\omega_b} \left(\frac{2\hat{\Delta}}{\hat{\omega}_a} - 1 \right) + 2\pi i Q_3 \right] e^{\omega_b (J_b + Q_3)}. \quad (3.30)$$

To understand how to proceed, we will need to determine the precise expressions corresponding to $\hat{\Delta}$ and $\hat{\omega}_a$, in particular their dependence on ω_b . Let us therefore analyze (3.29) in more detail. First of all, the analysis will be simplified by reparametrizing the chemical potentials in terms of unconstrained variables z_I , $I = 1, \dots, 4$ [16, 19]:

$$\Delta_I = \frac{2\pi i z_I}{1 + z_1 + z_2 + z_3 + z_4}, \quad \omega_a = \frac{-2\pi i}{1 + z_1 + z_2 + z_3 + z_4}, \quad \omega_b = -\frac{2\pi i z_4}{1 + z_1 + z_2 + z_3 + z_4}. \quad (3.31)$$

In our case, we use $z_1 = z_2 = z$, for which the saddle point equations become:

$$\begin{aligned} \frac{N^2}{2} \frac{z(z_3 - z)}{z_4} &= Q_3 - Q, \\ \frac{N^2}{2} \frac{z^2(z_3 + 1)}{z_4} &= -J_a - Q_3. \end{aligned} \quad (3.32)$$

Let us first solve for z_3 in terms of z and z_4 . We find:

$$z_3 = -\frac{(J_a + Q)z_4}{\frac{N^2}{2}z(z+1)}. \quad (3.33)$$

This leads to a third order equation to be satisfied by z :

$$\frac{N^2}{2} \frac{z^2(z+1)}{z_4} + (Q_3 - Q)(z+1) + J_a + Q = 0. \quad (3.34)$$

This equation can be solved exactly, but the solutions are not particularly illuminating. Instead, we will analyze the equation in the near-EVH limit, meaning we take $b = \lambda\epsilon^2$, and

take the limit in which $\epsilon \rightarrow 0$ and $N \rightarrow \infty$ keeping fixed $N^2\epsilon$. From the gravity solution, we know that $\omega_b \sim \epsilon^{-1}$ for $\epsilon \rightarrow 0$, as can be seen from (2.50). In this limit, it turns out that z_4 is proportional to ω_b ,¹¹ so we may solve (3.34) near $z_4 \rightarrow \infty$ and keep only finite terms in the limit. Note that for consistency, we should check that the resulting values of the chemical potentials indeed provide a *dominant* saddle for the integral (3.28).

In the limit $z_4 \rightarrow \infty$, two of the three solutions to (3.34) blow up, whereas the finite solution reads:

$$z = \frac{a}{1-a} + \frac{a}{(1-a)^2} \frac{1}{z_4} + \mathcal{O}(z_4^{-2}). \quad (3.35)$$

Feeding this back into (3.33), we also find the solution for z_3 :

$$z_3 = -z_4 + \frac{1+a}{1-a} + \mathcal{O}(z_4^{-1}). \quad (3.36)$$

Using (3.31) and taking $z_4 = \frac{i\sqrt{a(1+a)}}{\sqrt{\lambda\epsilon(1-a)}}$, one can verify that these values give the expected chemical potentials for $(\Delta, \omega_a, \omega_b)$ in the near-EVH limit, as featured in (2.48) and (2.50):

$$\Delta = \frac{i\pi a}{1+a} + \mathcal{O}(z_4^{-1}), \quad \omega_a = -\frac{\pi i(1-a)}{1+a} + \mathcal{O}(z_4^{-1}), \quad \omega_b = \frac{\pi}{\sqrt{\lambda\epsilon}} \sqrt{\frac{a}{1+a}} + \mathcal{O}(z_4^0). \quad (3.37)$$

We now plug the solutions into the integrand and take $b = \lambda\epsilon^2$ to obtain:

$$d = \int d\omega_b \exp \left[\frac{N^2}{2} \frac{\pi^2 a^2}{1-a^2} \frac{1}{\omega_b} + \frac{N^2 \epsilon^2}{2} \frac{\lambda a}{1-a} \omega_b + \mathcal{O}(\epsilon) \right]. \quad (3.38)$$

Let us first note that it may seem as if the second term is $\mathcal{O}(\epsilon)$, and we should drop it. However, since we expect the saddle to have $\omega_b \sim \epsilon^{-1}$, this term is finite in the near-EVH limit and in fact of the same order as the first term. To justify a saddle point analysis of this integral, large- N is not enough. This leaves us with two choices: either we fix $N^2\epsilon$ to be a large number, which is allowed in the near-EVH limit, or we take the fast-spinning limit $a \rightarrow 1$. Whichever we choose, we can then solve for the saddle point to find:

$$\hat{\omega}_b = \frac{\pi}{\sqrt{\lambda\epsilon}} \sqrt{\frac{a}{1+a}}, \quad (3.39)$$

which confirms that the saddle lies at the near-EVH value of ω_b and in particular shows that the integral for either $N^2\epsilon$ large or $a \rightarrow 1$ is dominated by a saddle at large ω_b . Evaluating the integrand at the saddle then finally gives us the entropy:

$$S = \frac{\pi a}{1-a} \sqrt{\frac{\lambda a}{1+a}} N^2 \epsilon. \quad (3.40)$$

This reproduces exactly the near-EVH entropy computed from gravity (2.37). Finally, one can check that (3.40) reproduces, at this order, the near-EVH limit of the 1/16th BPS

¹¹To see this, we note that to the order in epsilon we are considering, $z_3 = -z_4 + \mathcal{O}(z_4^0)$ (as in (3.36)). This implies that in the denominator of ω_b in terms of the z_I (3.31), the diverging pieces in z_3 and z_4 cancel. Therefore, $\omega_b \rightarrow \infty$ implies $z_4 \rightarrow \infty$.

black hole entropy in (3.12), with ϵ -expansion of charges

$$\begin{aligned} Q_1 = Q_2 &= \frac{a}{2(1-a)} N^2 + \frac{(1+a)}{2(1-a)} \lambda N^2 \epsilon^2 + \mathcal{O}(\epsilon^4), & Q_3 &= -\frac{a^2}{2(1-a^2)} \lambda N^2 \epsilon^2 + \mathcal{O}(\epsilon^4), \\ J_a &= -\frac{a^2}{2(1-a)^2} N^2 + \frac{a(1+3a+a^2)}{2(1-a)^2(1+a)} \lambda N^2 \epsilon^2 & J_b &= \frac{a(1+2a)}{2(1-a^2)} \lambda N^2 \epsilon^2 + \mathcal{O}(\epsilon^4). \end{aligned} \quad (3.41)$$

$$+ \mathcal{O}(\epsilon^4),$$

Connection to Cardy formula. As discussed in [42], the near-EVH entropy can be understood as the entropy of a BTZ black hole that arises in the near-horizon limit of the near-EVH black hole. This automatically means the entropy can be written as a Cardy formula [57] for the putative two-dimensional CFT dual. The central charge and the conformal dimension for the (BPS) case at hand read [42]:

$$c = 3\sqrt{2} \frac{a^2}{1-a^2} N^2 \epsilon, \quad L_0 - \frac{c}{24} = \frac{1}{2\sqrt{2}} \frac{a}{1-a} \lambda N^2 \epsilon. \quad (3.42)$$

Before making the connection between the derivation of the Cardy formula and our computation of (3.38), let us briefly review the derivation of the Cardy formula in two-dimensional CFTs. Using the fact that the partition function obeys the property $Z(\beta) = Z(\frac{4\pi^2}{\beta})$ and its low temperature expansion reads $Z \approx e^{\frac{\beta c}{24}}$, one has for the high temperature expansion:

$$Z(\beta) \approx \exp\left(\frac{\pi^2 c}{6\beta}\right), \quad \beta \ll 1. \quad (3.43)$$

The degeneracy at high temperature can then be estimated through a saddle point approximation of the inverse Laplace transform:

$$d \approx \int d\beta \exp\left[\frac{\pi^2 c}{6\beta} + \beta\left(L_0 - \frac{c}{24}\right)\right]. \quad (3.44)$$

The saddle lies at:

$$\hat{\beta} = \pi \sqrt{\frac{c}{6(L_0 - \frac{c}{24})}} = \frac{\sqrt{2}\pi}{\sqrt{\lambda}} \sqrt{\frac{a}{1+a}}, \quad (3.45)$$

where in the last step we have inserted (3.42). The saddle point evaluation of (3.44) then yields the entropy at high temperature $S = \log d$, and in our case reproduces (3.40).

The naive regime of validity is as usual $L_0 \gg c$, which in this case would correspond to $\lambda \gg 1$, such that the saddle indeed lies at high temperature. However, it has been established that for “holographic CFTs” with $c \gg 1$, the Cardy formula applies already for $L_0 \sim c$ (see e.g., [13], also for a more precise formulation of what defines a holographic CFT). This regime can be reached by taking either $a \rightarrow 1$ or $N^2 \epsilon$ large, as can be seen from (3.42).¹² In particular, this implies that the Cardy formula applies for states with

¹²These limits were already encountered when we had to justify the saddle point approximation of the ω_b integral around (3.39). We will see below that this justification precisely brings us into the extended regime of validity of the Cardy formula for holographic CFTs.

temperature $\hat{\beta} = \mathcal{O}(1)$ and that the high-temperature expansion (3.43) extends to $\beta = \mathcal{O}(1)$ up to $\mathcal{O}(c^0)$ corrections [13].¹³

We are now ready to compare our derivation of the near-EVH entropy from the index with the usual Cardy formula. For this, let us compare the integrands of (3.38) and (3.44) and their respective saddles (3.39) and (3.45). We note that if we make the following redefinition:

$$\omega_b = \frac{\tilde{\omega}_b}{\sqrt{2}\epsilon}, \quad (3.46)$$

the integrand in (3.38) is precisely of the form of (3.44) for the near-EVH values of the central charge and conformal dimension quoted in (3.42). The conclusion we draw from this is that a careful saddle point analysis of the four-dimensional superconformal index for near-EVH black hole charges becomes equivalent to the derivation of a two-dimensional Cardy formula for a two-dimensional holographic CFT.¹⁴

Let us say more about the meaning of the redefinition (3.46), especially the factor ϵ . To understand it, we first assume that the two terms in the integrand in (3.38) can be understood as the central charge and conformal dimension of some two-dimensional CFT, i.e.:

$$\tilde{c} = 3 \frac{a^2}{1-a^2} N^2, \quad \tilde{L}_0 - \frac{\tilde{c}}{24} = \frac{1}{2} \frac{a}{1-a} \lambda N^2 \epsilon^2. \quad (3.47)$$

In the near-EVH limit, $\tilde{L}_0 - \frac{\tilde{c}}{24} = 0$ meaning that we would be looking at a massless BTZ black hole. However, from the near-horizon geometry, as studied in section 2.2, we know that in addition there is still a conical defect of order $\frac{1}{\epsilon}$, see (2.27) and (2.38). Following [44], this results in a new Virasoro symmetry of the boundary theory with a rescaled central charge and fractionated spectrum:¹⁵

$$c = \epsilon \tilde{c}, \quad L_0 - \frac{c}{24} = \frac{1}{\epsilon} \left(\tilde{L}_0 - \frac{\tilde{c}}{24} \right). \quad (3.48)$$

Note that the Cardy formula is invariant under such a transformation; this should be understood as the statement that states appearing in the fractionated spectrum provide the dominant contribution to the entropy. In particular, this gives us the required factors of ϵ to compare with (the derivation of) the Cardy formula for the near-EVH CFT.

Hence, apart from the factor $\sqrt{2}$,¹⁶ the redefinition (3.46) may be understood as resulting from a “long string phenomenon” (3.48) in the IR two-dimensional CFT. It would

¹³Already in the famous Strominger-Vafa computation [58], the Cardy formula correctly reproduced the entropy when temperatures are strictly speaking $\mathcal{O}(1)$. Indeed, the Cardy formula is broadly applicable when there is a supergravity description of the system. See also [59–61].

¹⁴See [62] for similar observations in the context of the Schur index.

¹⁵If the CFT allows a symmetric product description, the conical defect translates into a twist operator in the CFT which projects onto a twisted sector of order $\frac{1}{\epsilon}$.

¹⁶From the gravitational point of view, this factor arises due to the specific relation between the AdS₅ and AdS₃ time coordinates (see e.g., (4.17) and (5.15) in [42]). It is not clear to us presently how this factor could arise from the CFT₄ superconformal index. Perhaps like the famous factor of $\frac{3}{4}$ in the thermal free energy of $\mathcal{N} = 4$ super-Yang-Mills [63, 64], it arises due to strong coupling effects to which our weak coupling analysis is blind.

be very interesting to find a four-dimensional interpretation of the transformation (3.48). We will further comment on this in the next section and the discussion.

We will conclude with some further comments on the appearance of a fractionated spectrum encountered above. Such fractionation is well-known to occur for symmetric product CFTs via the so-called long string phenomenon. Assuming that the EVH CFT is continuously connected to some symmetric product description, we would like to understand the appearance of the factor N^2 in the central charge (3.47) of the symmetric product CFT. Since the conical defect is of order N^2 as well, it is implied that the symmetric product would have to be of order N^2 at least. Some of the more well-known situations in which symmetric product CFTs arise from gauge theories is in the low energy limit of N D1 branes (matrix string theory) [65] or in the D1-D5 system [58]. Naively, since we have started from just N D3 branes, our setup seems more in the spirit of matrix string theory. However, the resulting symmetric product in that case would only be of order N , which is related to the fact that the Weyl group of the $SU(N)$ gauge symmetry is S_N . On the other hand, the D1-D5 system has central charge $c \sim Q_1 Q_5$. Thus taking $Q_1 Q_5 \sim N^2$, it would be possible to generate a central charge of $\mathcal{O}(N^2)$. However, at the moment it is not clear to us if and how an analogy with the D1-D5 system in our present setup would work.¹⁷

Another, heuristic explanation for the origin of the factor N^2 in the central charge is that it arises from the fact that we consider the SYM theory at its maximally deconfining saddle. This means that all degrees of the freedom of the $SU(N)$ matrix become free, which should somehow translate into the fact that there are N^2 independent copies of the two-dimensional CFT. The symmetric product should then reflect the indistinguishability of these degrees of freedom.

3.2 4d operators for 2d EVH CFT

In this section, we would like to make some comments on the question of which operators in the four-dimensional theory contribute to the entropy of the near-EVH black hole. We begin by reviewing the result of [45], who propose an answer to the same question in the strict EVH limit, and then comment on some important differences in the near-EVH case.

Let us first briefly summarize the basic operator content in the $\mathcal{N} = 4$ SYM theory. It contains six scalars (X, Y, Z and conjugates), four (complex) Weyl fermions and two independent components of the gauge field. The classification of the superconformal multiplets is denoted as $[k, q, p]_{(j_1, j_2)}$, as first explored in a beautiful paper [66]. We list all single letter operators subject to the BPS condition $E = J_a + J_b + Q_1 + Q_2 + Q_3$ in table 1, following the notation of [67]. These operators may be combined to compose a generic $1/16^{\text{th}}$ BPS operator. The $1/16^{\text{th}}$ BPS sector is also called the $SU(1, 2|3)$ subsector in [67].

Certain operators preserve more supersymmetry. In the strict EVH limit, as discussed in section 3.1, the black hole is $1/8^{\text{th}}$ BPS. These states are part of the reduced subsector known as the $SU(1, 1|2)$ subsector, composed by the single letter operators $(\chi_1, \bar{\chi}_7, \partial_{++}, Z, X)$, which satisfy the additional charge constraint $J_b + Q_3 = 0$. Since EVH black holes have $Q_3 = J_b = 0$, they in fact comprise only a subsector of the $SU(1, 1|2)$

¹⁷Perhaps ideas along the lines of [37] could be relevant here as well.

Name in [67]	SO(4)[J_a, J_b]	Name in [54]	Q	Q_3	E_0	E
Z	$[0, 0]$	Z	$\frac{1}{2}$	0	1	1
X	$[0, 0]$	X	$\frac{1}{2}$	0	1	1
W	$[0, 0]$	Y	0	1	1	1
$\bar{F}_+$	$[1, 1]$	F_{++}	0	0	2	2
χ_1	$[\frac{1}{2}, -\frac{1}{2}]$	$\psi_{0,+,+++}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{3}{2}$
χ_2	$[-\frac{1}{2}, \frac{1}{2}]$	$\psi_{0,-,+++}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{3}{2}$
$\bar{\chi}_3$	$[\frac{1}{2}, \frac{1}{2}]$	$\psi_{+,0,-++}$	0	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{3}{2}$
$\bar{\chi}_5$	$[\frac{1}{2}, \frac{1}{2}]$	$\psi_{+,0,+--}$	0	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{3}{2}$
$\bar{\chi}_7$	$[\frac{1}{2}, \frac{1}{2}]$	$\psi_{+,0,++-}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{3}{2}$	$\frac{3}{2}$
d_1	$[1, 0]$	∂_{++}	0	0	1	1
d_2	$[0, 1]$	∂_{+-}	0	0	1	1

Table 1. The single letter operators making up $1/16^{\text{th}}$ BPS operators in [54] correspond to the single letter operators comprising the SU(1,2|3) subsector as defined in [67, 68]. They satisfy the $E = E_0$ BPS condition, where E_0 is the BPS value $E_0 = J_a + J_b + 2Q + Q_3$, $J_a = J_1 + J_2$, $J_b = J_1 - J_2$ and $2Q = Q_1 + Q_2$. Note that we are working with the SYM theory on $S^1 \times S^3$, dual to AdS₅ in global coordinates. In radial quantization, the states on S^3 are in one-to-one correspondence with the operators on $\mathbb{R}^4$ made up from the letters in the table.

sector. In [45], it is proposed that the EVH black hole is dual to the fermionic subsector of SU(1,1|2), the so-called SU(1,1) subsector, which contains fermions $\chi_1, \bar{\chi}_7$ and the derivative ∂_{++} . Partial evidence for this relies on the fact that one can construct states using $\chi_1, \bar{\chi}_7, \partial_{++}$ which satisfy the (BPS) EVH condition:

$$\frac{N^2}{2} J_a = Q_1^2. \quad (3.49)$$

This state can be written schematically as [45]:

$$\text{Sym} \left[\prod_{a,b=1}^{N^2} \prod_{j=0}^{\frac{K}{2}-1} \psi_j^a \prod_{m=\frac{K}{2}}^{K-1} \bar{\psi}_m^b \right], \quad (3.50)$$

where $\psi_k \equiv \partial_{++}^k \chi_1$ and $\bar{\psi}_k \equiv \partial_{++}^k \bar{\chi}_7$. Furthermore, the symmetrization is with respect to ψ and $\bar{\psi}$. Using the data in table 1, one may verify that the charges of this operator are

given by:

$$Q_1 = Q_2 = \frac{N^2 K}{2}, \quad (3.51)$$

$$Q_3 = 0, \quad (3.52)$$

$$\frac{J_a + J_b}{2} = j_1 = N^2 \left(\sum_{j=0}^{\frac{K}{2}-1} \frac{j+1}{2} + \sum_{m=\frac{K}{2}}^{K-1} \frac{m}{2} \right) = \frac{N^2 K^2}{4}, \quad (3.53)$$

$$\frac{J_a - J_b}{2} = j_2 = N^2 \left(\sum_{j=0}^{\frac{K}{2}-1} \frac{j}{2} + \sum_{m=\frac{K}{2}}^{K-1} \frac{m+1}{2} \right) = \frac{N^2 K^2}{4}. \quad (3.54)$$

These charges satisfy the condition (3.49) and $J_b = Q_3 = 0$, as required for the BPS EVH black hole. Comparing with (2.32), we identify:

$$K = \frac{a}{1-a}. \quad (3.55)$$

In [45], the authors propose an explicit description of the EVH CFT₂ in the large K or fast rotating ($a \rightarrow 1$) limit. In the free limit of the $\mathcal{N} = 4$ SYM theory, the two-dimensional CFT is described by free chiral adjoint fermions, or more precisely the (chiral) WZW models:

$$\mathrm{SU}(N)_N \oplus \mathrm{SU}(N)_N, \quad (3.56)$$

up to a Gauss law constraint projecting onto gauge invariant states. Including loop corrections for finite Yang-Mills coupling, the primary effect is that the CFT becomes a gauged WZW model. This provides a specific realization of the fact that the thermodynamics of the near-horizon BTZ black hole can be derived as the IR limit of the thermodynamics of the AdS₅ black hole [42].

For (BPS) near-EVH black holes instead, we have in general $J_b + Q_3 \neq 0$, so that we should now take into account in principle the full $\mathrm{SU}(1,2|3)$ subsector. More precisely, a near-EVH black hole has the charges:

$$J_b = \frac{1}{2} \left(\frac{a}{1-a} + \frac{a^2}{1-a^2} \right) \lambda N^2 \epsilon^2, \quad Q_3 = -\frac{1}{2} \frac{a^2}{1-a^2} \lambda N^2 \epsilon^2. \quad (3.57)$$

Since we keep fixed $N^2 \epsilon$, apparently $J_b = Q_3 = 0$ in the limit. However, as we have seen in the previous section, the spectrum of the two-dimensional CFT fractionates, as expressed by the long string transformation (3.48). Since $L_0 \sim J_b + Q_3$, we denote with $\tilde{J}_b + \tilde{Q}_3$ the charges measured with respect to the fractionated spectrum, i.e.:

$$\tilde{J}_b + \tilde{Q}_3 = \frac{1}{2} \frac{a}{1-a} \lambda N^2 \epsilon. \quad (3.58)$$

This is finite in the near-EVH limit. However, presently it is not clear to us how such a fractionation of the spectrum could be described from the perspective of the four-dimensional CFT. Note that since these charges do not scale with N^2 , as opposed to Q and J_a , it is suggested that we should effectively attach single-trace operators with non-vanishing values for J_b, Q_3 to the EVH operator (3.50) to find the description of states responsible for the near-EVH entropy. We leave further exploration of these speculations to future work.

4 Probing EVH-BPS limit in general AdS_{d+1}

In this section, we explore the EVH-BPS limit of black holes in AdS_{d+1} ($d > 2$). It is already known from [39] that certain AdS_4 black holes have an EVH limit, whose field theory dual is proposed to be generalized BMN limit of CFT_3 . As in the case of the AdS_5 black hole, an AdS_3 geometry emerges in the near-horizon limit. Other known AdS black hole solutions include AdS_6 black holes [49, 69–71] and AdS_7 black holes [49, 72]. An interesting question is whether the success of probing the EVH-BPS limit of AdS_5 black hole from the field theory point of view can be extended to general dimensional AdS black holes. Two situations will appear as we focus on the EVH-BPS limit:

- The black hole reduces to a naked singularity.
- An EVH limit of the black hole appears with entropy-temperature scaling $S \sim T^k$ ($1 < k < d - 1$).

The scaling relation indicates the emergence of a possible $\text{AdS}_{k+2}/\text{CFT}_{k+1}$ correspondence in the far IR of the UV $\text{AdS}_{d+1}/\text{CFT}_d$. We find that only for the supersymmetric AdS_5 black holes, one can probe the EVH-BPS limit by using $\text{AdS}_3/\text{CFT}_2$. We should note that these problems may originate from the fact that for other dimensions than 5, the most general AdS black hole solutions are not (yet) known.

4.1 Revisiting AdS_5

We have already shown in section 2.1 that the AdS_5 black holes have a well-defined EVH-BPS limit. However, we will now propose a simpler method of probing this limit without referring to the (complicated) metric. The EVH black hole with a pinching AdS_3 in its near-horizon is characterized by the scaling relation $S \sim T$. However, the situation is a bit more subtle for BPS black holes, which have vanishing temperature. The correct way to understand the EVH-BPS black hole in general dimensions is to note that S/T^k is finite for some k as $T \rightarrow 0$. Therefore, it is essential to understand how temperature and entropy scale as one take near-horizon and BPS limits. This requires knowledge of both the non-extremal black hole solutions and their thermodynamics.

Let us focus on the entropy and temperature in (2.13). Approaching the near-horizon region defines ϵ by $r = \epsilon\rho$. For the black hole to have a vanishing entropy, we may take the ansatz

$$a \sim \epsilon^\alpha, \quad b \sim \epsilon^\beta, \quad r_+ \sim \epsilon. \quad (4.1)$$

Without loss of generality, we may assume $\alpha \leq \beta$. We are interested in the EVH limit with supersymmetry. Then α and β are constrained by consistency with the BPS limit (2.15) and (2.17). The horizon size $r_0^2 \sim ab$ requires $\alpha + \beta = 2$. Our assumption $a \geq b$ then implies $\alpha \leq 1 \leq \beta$. In addition, the BPS value of q (2.19) shows that $q \sim \epsilon^\alpha$. Combining these together, we can see in the limit $\epsilon \rightarrow 0$ that:

$$S \sim \frac{a^2 r_+^2 + q r_+^2}{r_+} \sim q r_+ \sim \epsilon^{\alpha+1}, \quad (4.2)$$

$$T \sim \frac{r_+^4(1+a^2) - a^2 b^2}{q r_+^3} \sim \epsilon^{1-\alpha}. \quad (4.3)$$

The emergence of a near-horizon CFT_2 would require $S \sim T$, which fixes $\alpha = 0$. This is consistent with the solution we studied in section 2.

4.2 AdS_4

Certain types of static AdS_4 black hole with $\text{U}(1)^4$ gauge fields can have a well-defined EVH limit [39]. We revisit this problem in the case of a supersymmetric black hole. We focus on rotating AdS_4 black holes [49]. The example we will study is the rotating AdS_4 black holes have equal pairs of R-charges $Q_1 = Q_3$, $Q_2 = Q_4$. Let us collect some relevant formulas from [70]. The energy, charges and angular momentum of the black hole are determined by four free parameters (δ_i, a, m) , $i = 1, 2$, as follows:

$$\begin{aligned} E &= \frac{m}{2\Xi^2 G} (\cosh 2\delta_1 + \cosh 2\delta_2), & J &= \frac{ma}{2\Xi^2 G} (\cosh 2\delta_1 + \cosh 2\delta_2), \\ Q_1 = Q_3 &= \frac{m}{4G\Xi} \sinh 2\delta_1, & Q_2 = Q_4 &= \frac{m}{4G\Xi} \sinh 2\delta_2. \end{aligned} \quad (4.4)$$

The BPS condition requires:

$$e^{2\delta_1+2\delta_2} = 1 + \frac{2}{a}. \quad (4.5)$$

Similar to the AdS_5 case, further constraints are required to prevent the naked singularity. These read:

$$m^2 = \frac{\cosh^2(\delta_1 + \delta_2)}{e^{\delta_1+\delta_2} \sinh^3(\delta_1 + \delta_2) \sinh(2\delta_1) \sinh(2\delta_2)}. \quad (4.6)$$

The entropy in the BPS limit satisfies:

$$\begin{aligned} 2(Q_1 + Q_2)S &= \frac{\pi}{G}J, \\ S^2 + \frac{\pi}{G}S - 16\pi^2 Q_1 Q_2 &= 0. \end{aligned} \quad (4.7)$$

The EVH condition forces $J = Q_1 Q_2 = 0$. Without loss of generality, we can assume $Q_2 = \delta_2 = 0$. $J = 0$ implies $a = 0$. We then find there is no solution to the singularity free condition (4.6). This means that all the BPS solutions in the EVH limit become solutions with naked singularities, as opposed to the non-BPS solutions of [39].

This does not mean there is no hope of studying EVH limit of BPS AdS_4 black holes. In the above EVH limit, we notice that $\delta_2 = 0$ forces two of four $\text{U}(1)$ charges to vanish. However, we should expect at least three non-vanishing charges to support a near-horizon AdS_3 geometry [39]. Consequently, we expect that studying the EVH limit requires the knowledge of general rotating AdS_4 black hole solutions with unequal $\text{U}(1)^4$ charges. Solutions with more general charges were recently found in [73]. It's interesting to revisit EVH-BPS limit in this recently found solution.

4.3 AdS_6

Another interesting example is the AdS_6 black hole solution [69]. A CFT_5 account of the entropy has been studied in [70]. The field theory dual of AdS_6 black hole should be an

$\mathcal{N} = 1$ five-dimensional SCFT. The black hole solution has energy, two angular momenta, and one R-charge:

$$\begin{aligned}
 E &= \frac{2\pi m}{3G\Xi_a\Xi_b} \left[\frac{1}{\Xi_a} + \frac{1}{\Xi_b} + \frac{q}{2m} \left(1 + \frac{\Xi_b}{\Xi_a} + \frac{\Xi_a}{\Xi_b} \right) \right], & Q &= \frac{\pi\sqrt{q^2 + 2mq}}{G\Xi_a\Xi_b}, \\
 J_a &= \frac{2\pi ma}{3G\Xi_a^2\Xi_b} \left(1 + \frac{\Xi_b q}{2m} \right), & J_b &= \frac{2\pi mb}{3G\Xi_a\Xi_b^2} \left(1 + \frac{\Xi_a q}{2m} \right), \\
 S &= \frac{2\pi^2[(r_+^2 + a^2)(r_+^2 + b^2) + qr_+]}{3G\Xi_a\Xi_b}, \\
 T &= \frac{2r_+^2(1 + r_+^2)(2r_+^2 + a^2 + b^2) - (1 - r_+^2)(r_+^2 + a^2)(r_+^2 + b^2) + 4qr_+^3 - q^2}{4\pi r_+[(r_+^2 + a^2)(r_+^2 + b^2) + qr_+]}.
 \end{aligned} \tag{4.8}$$

The BPS condition is very similar to what we have discussed for AdS₅ black hole:

$$q = \frac{2m}{(2 + a + b)(a + b)}. \tag{4.9}$$

The parameter m is related to horizon size r_+ via:

$$2m = \frac{(r_+^2 + a^2)(r_+^2 + b^2)}{r_+} + \frac{1}{r_+} [r_+(r_+^2 + a^2) + q][r_+(r_+^2 + b^2) + q]. \tag{4.10}$$

We can solve for q in terms of r_+ as follows:

$$q = (a + b + ab)r_+ - r_+^3 - i(1 + a + b)(r_+^2 - r_0^2), \tag{4.11}$$

where r_0 is the same as in (2.17). Thus the singularity free condition is given by $r_+ = r_0$, similar to the AdS₅ case. The result is:

$$q = \frac{(1 + a)(1 + b)(a + b)r_+}{1 + a + b}. \tag{4.12}$$

Let us again make the ansatz (4.1) for a, b in the EVH limit. It remains true that $\alpha + \beta = 2$, since we still have $r_0^2 \sim ab$. We then find $(r_+^2 + a^2)(r_+^2 + b^2) \sim a^2 r_+^2 \sim \epsilon^{2\alpha+2}$ and $qr_+ \sim \epsilon^{2+\alpha}$. As a result, $S \sim \epsilon^{2+\alpha}$. The scaling of temperature is more complicated. Each separate term in the temperature scales as:

$$\begin{aligned}
 2r_+^2(1 + r_+^2)(2r_+^2 + a^2 + b^2) &\sim \epsilon^{2\alpha+2}, \\
 (1 - r_+^2)(r_+^2 + a^2)(r_+^2 + b^2) &\sim \epsilon^{2\alpha+2}, \\
 4qr_+^3 &\sim \epsilon^{4+\alpha}, \\
 q^2 &\sim \epsilon^{2\alpha+2}, \\
 r_+[(r_+^2 + a^2)(r_+^2 + b^2) + qr_+] &\sim \epsilon^{3+\alpha}.
 \end{aligned} \tag{4.13}$$

Naively, the leading term in the $\epsilon \rightarrow 0$ limit implies that $T \sim \epsilon^{2\alpha+2-(3+\alpha)} = \epsilon^{\alpha-1}$, but this cannot be correct since an extremal black hole should have a vanishing temperature (recall that $\alpha < 1$).

This can be resolved by noting that in the BPS limit, there is a cancellation between the three terms of order $\epsilon^{2\alpha+2}$. To see this we should first notice that $q \sim ar_+$. One may

verify that all the $r_+^2 a^2$ terms in the numerator of T will cancel. The subleading term in the numerator of T comes from r_+^4 terms, resulting in $T \sim \epsilon^{4-(3+\alpha)} = \epsilon^{1-\alpha}$. Setting $\alpha = 0$, as we did in AdS₅ case, will result in a potential EVH black hole $S \sim T^2$. Since $\alpha \leq 1$, $S \sim T$ cannot arise for any allowed α .

In summary, the $T \sim \epsilon$ limit ($\alpha = 0$) can be achieved by

$$q = (a + \lambda b)r_+, \quad r_+ \sim \sqrt{b} \sim \epsilon, \quad (4.14)$$

for some general values of λ . The EVH-BPS limit of the AdS₆ black hole should be understood as the extremal vanishing horizon limit while keeping S/T^2 fixed, which gives rise to an AdS₄ factor in the near-horizon limit of the EVH-BPS AdS₆ black hole.

4.4 AdS₇

AdS₇ black holes have three angular momenta $J_{1,2,3}$ and two different R-charges Q_1, Q_2 . There are two types of AdS₇ black hole solutions known so far. The first type is the solution with equal angular momenta [49]. The BPS and EVH condition result in naked singularities, similar to the AdS₄ case. Thus we do not attempt to study the EVH-BPS limit here. The other type is the solution with equal charges but different angular momenta [72]. This case is rather similar to AdS₆ discussed above. Let us collect some relevant formulas from [72]. The energy, entropy and temperature are given by:

$$\begin{aligned} E &= \frac{\pi^2}{8\Xi_1\Xi_2\Xi_3} \left[\sum_i \frac{2m}{\Xi_i} - m + \frac{5q}{2} + \frac{q}{2} \sum_i \left(\sum_{j \neq i} \frac{2\Xi_j}{\Xi_i} - \Xi_i - \frac{2(1+2a_1a_2a_3)}{\Xi_i} \right) \right], \\ S &= \frac{\pi^3}{4\Xi_1\Xi_2\Xi_3 r_+} [(r_+^2 + a_1^2)(r_+^2 + a_2^2)(r_+^2 + a_3^2) + q(r_+^2 - a_1a_2a_3)], \\ T &= \frac{(1+r_+^2)r_+^2 \sum_i \prod_{j \neq i} (r_+^2 + a_j^2) - \prod_i (r_+^2 + a_i^2) + 2q(r_+^4 + a_1a_2a_3) - q^2}{2\pi r_+ [(r_+^2 + a_1^2)(r_+^2 + a_2^2)(r_+^2 + a_3^2) + q(r_+^2 - a_1a_2a_3)]}. \end{aligned} \quad (4.15)$$

Just as in the AdS_{5,6} cases, the supersymmetric solution is only singularity free if the horizon size reads:

$$r_+^2 = r_0^2 = \frac{a_1a_2 + a_2a_3 + a_3a_1 - a_1a_2a_3}{1 - a_1 - a_2 - a_3}. \quad (4.16)$$

The corresponding value for q is given by:

$$q = - \frac{(1-a_1)(1-a_2)(1-a_3)(a_1+a_2)(a_2+a_3)(a_3+a_1)}{(1-a_1-a_2-a_3)^2}. \quad (4.17)$$

We now make the ansatz: $a_i \sim \epsilon^{n_i}$ and $n_1 \leq n_2 \leq n_3$. The dominant term of r_0^2 in the $\epsilon \rightarrow 0$ limit is $r_0^2 \sim a_1a_2 \sim \epsilon^{n_1+n_2}$. So we can derive:

$$n_1 + n_2 = 2, \quad n_1 \leq 1 \leq n_2 \leq n_3. \quad (4.18)$$

This implies q should scale as $q \sim a_1^2 a_2 \sim \epsilon^{2+n_1}$. One finds that the entropy in the limit is dominated by:

$$S \sim \frac{1}{r_+} (a_1^2 r_+^4 + q r_+^2) \sim q r_+ \sim \epsilon^{3+n_1}. \quad (4.19)$$

Similar to the AdS_6 case, each term in the numerator of the temperature can be shown to scale as:

$$\begin{aligned}
 (1 + r_+^2)r_+^2 \sum_i \prod_{j \neq i} (r_+^2 + a_j^2) &\sim a_1^2 r_+^4 \sim \epsilon^{2n_1+4}, \\
 \prod_i (r_+^2 + a_i^2) &\sim q^2 \sim \epsilon^{2n_1+4}, \\
 2q(r_+^4 + a_1 a_2 a_3) &\sim \epsilon^{n_1+6} \quad \text{or} \quad \epsilon^{4+n_1+n_3}.
 \end{aligned}
 \tag{4.20}$$

The dependence on ϵ of the last term depends on whether $n_3 \geq 2$ or $n_3 < 2$. Again, there is a cancellation between the order ϵ^{2n_1+4} terms. The subleading terms in the numerator come from r_+^6 , which results in $T \sim \epsilon^{1-n_1}$. Setting $n_1 = 0$ would result in $S \sim T^3$. Therefore, a straightforward generalization of EVH-BPS limit in AdS_7 black hole cannot give rise to AdS_3 black hole in the near-horizon. However, the result shows that there may well be an AdS_5 geometry in the near-horizon limit of the EVH-BPS AdS_7 black hole.

We should note that our result does not imply that AdS_7 black hole cannot have an EVH limit with AdS_3 appearing in the near-horizon limit. Indeed, to fully establish the (non-)existence of an AdS_3 factor for the EVH black hole, one needs to know the general AdS_7 solutions with unequal charges.

5 Discussion

In this paper, we have employed the recent results for a microscopic enumeration of the entropy of $1/16^{\text{th}}$ BPS black holes in AdS_5 from the $\mathcal{N} = 4$ $\text{SU}(N)$ super Yang-Mills theory [19] to study the (near-)EVH/CFT₂ correspondence [42]. Our main result is that the entropy calculation for large AdS_5 black holes can be extended to (near-)EVH black holes in the fast rotating limit $a \rightarrow 1$ or for $N^2\epsilon$ large. Moreover, the computation of the Legendre transform of the superconformal index, yielding the black hole entropy, becomes equivalent to the derivation of the Cardy formula in two-dimensional CFT with a central charge and conformal dimension that precisely match the values expected from general considerations of the EVH/CFT₂ proposal. In addition, we have introduced a method to determine the existence of possible BPS EVH black holes in $\text{AdS}_{4,6,7}$ and their respective near-horizon geometries. We found indications that in the cases of $\text{AdS}_{6,7}$, the near-horizon geometries contain $\text{AdS}_{4,5}$ factors.

There are many directions for further exploration. First of all, we would like to better understand the underlying mechanism for the appearance of infinite dimensional symmetry algebras, associated with the two-dimensional CFT, from a four-dimensional CFT in the near-EVH limit. In particular, the EVH/CFT correspondence implies that a subset of the free operators in four-dimensional $\mathcal{N} = 4$ SYM can organize themselves into representations of the Virasoro algebra. Our derivation of the two-dimensional Cardy formula from the superconformal index sheds some light on this question. In particular, since the Virasoro zero mode is proportional to $J_b + Q_3$, the operators that account for the near-EVH entropy should lie in the full $\text{SU}(1, 2|3)$ subsector, as opposed to operators of $\text{SU}(1, 1|2)$ subsector which comprise the strict EVH black holes. Perhaps there exists a connection to [47], which derives a chiral algebra from four-dimensional operators through a cohomological

construction, and produces the full Virasoro algebra in the BPS near-EVH case we have been studying. However, this cannot be the complete story because the full $SU(1,2|3)$ subsector is much larger than the Schur $SU(1,1|2)$ sector, which is apparently the only subsector for which the construction of [47] holds. Moreover, the non-unitarity of their chiral algebras is difficult to reconcile with the expected unitarity of the EVH CFT.

In addition, an important element in our derivation of the entropy is the fractionation of the spectrum of the two-dimensional CFT, as implied by the presence of a conical defect. In two dimensions, such fractionation can be usefully understood in terms of a long string phenomenon [74, 75]. In the four-dimensional CFT, however, a mechanism behind the fractionation is less clear. It would be very interesting to find an interpretation of the fractionation in four dimensions, which we expect to be possible to achieve at least in the BPS case we have been exploring.

In our investigation of the AdS_2 or AdS_3 corresponding to EVH black holes, coordinates for the lower dimensional AdS space are realized within the asymptotic AdS_5 . In a generic setting, in flowing from $AdS_5 \times S^5$ asymptopia to the $AdS_d \times X$ geometry in the near-horizon region, the coordinates from the AdS and sphere mix as we move away from the boundary toward the interior. (This happens, for example, in [37].) In other words, generators of the Virasoro algebra in the IR arise as non-trivial combinations of generators of the $SO(2,4)$ conformal symmetry and generators of the $SU(4)$ R-symmetry of the $\mathcal{N} = 4$ super-Yang-Mills theory. Extending the analysis to such cases is work in progress.

In connecting the UV CFT associated to the asymptotic AdS geometry to the IR CFT associated to the near-horizon geometry, we expect certain c -functions are extremized along the RG flow. This can be made explicit in various contexts [76] using the attractor mechanism in $\mathcal{N} = 2$ supergravity [77–84]. Perhaps these c -functions can help characterize the relation between the Virasoro generators in the IR and the gauge theory operators in the UV. Indeed, as holographic RG integrates out gravitational degrees of freedom, the c -function is a monotonically decreasing function. However, the entropy of the black hole remains the same from either the UV or the IR perspective; the two are different descriptions of the same system. Our understanding of entropy is ultimately so far couched in terms of gravitational thermodynamics. A precise map between microstates in the UV and the IR is necessary to advance the discussion to the statistical physics of gravitational systems.

Finally, the scaling relations $S \sim T^2$ and $S \sim T^3$ in $AdS_{6,7}$ black hole solutions indicate the appearance of $d > 2$ dimensional CFTs in their respective EVH limits. To understand the physical consequences, we recall what happens if an AdS_3 appears in the EVH limit, where $S \sim T$. The emergent IR geometry (known as the pinching AdS_3) is a locally AdS_3 geometry with a pinching angular direction. In the field theory, such pinching translates into an infinitely gapped system unless the central charge is scaled to infinity [44] such that non-trivial dynamics may remain. For symmetric product CFTs, these dynamics can be traced to the very low energy modes which arise in this limit due to the long string phenomenon, or more generally momentum fractionation [44]. It is less clear how to think of pinching $AdS_{4,5}$ geometries which arise in (near-)EVH limits of $AdS_{6,7}$ black holes respectively, and whether their dual IR CFT_{3,4} spectra in an appropriate $c \rightarrow \infty$ limit exhibit a similar fractionation. We hope to answer these questions in future work.

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A EVH limit of general supersymmetric black holes

When the supersymmetry condition is satisfied, there exists a more general class of black hole solutions of the action (2.1) with four independent parameters. The BPS solution (2.8) should be viewed as a two-parameter solution with free parameters (a, b) . The more general solution is given by:

$$ds_5^2 = -f^2(dt + \omega_{\tilde{\phi}}d\tilde{\phi} + \omega_{\tilde{\psi}}d\tilde{\psi})^2 + f^{-1}h_{mn}dx^m dx^n, \quad (\text{A.1})$$

where h_{mn} is the metric on the base space. Supersymmetry requires the base space to be Kähler. The claim in [36] is that the h_{mn} for solutions in [34] at BPS point is given by:

$$\begin{aligned} h_{mn}dx^m dx^n = (r^2 - r_0^2) & \left[\frac{dr^2}{X} + \frac{d\theta^2}{\Delta_\theta} + \frac{\cos^2 \theta}{\Xi_b^2} [\Xi_b + \cos^2 \theta (\rho^2 + 2(1+b)(a+b))] d\tilde{\psi}^2 \right. \\ & + \frac{\sin^2 \theta}{\Xi_a^2} [\Xi_a + \sin^2 \theta (\rho^2 + 2(1+a)(a+b))] d\tilde{\phi}^2 \\ & \left. + \frac{2\sin^2 \theta \cos^2 \theta}{\Xi_a \Xi_b} [\rho^2 + 2(a+b) + (a+b)^2] d\tilde{\psi} d\tilde{\phi} \right]. \end{aligned} \quad (\text{A.2})$$

Note that $X(r)$ in the denominator of the dr^2 component is exactly as in (2.18). Therefore, we can compare the metric (2.8) to ansatz above. One subtlety is that the metric (A.1) is in the frame corotating with the horizon. It is related to the metric (2.8) by the coordinate transformation (2.20). Another subtlety is that the r_0^2 dependence of $X(r)$ is redundant, and it can be conveniently set to zero [36]. The trick is to realize that one may redefine the parameters a, b . Let us denote (a, b, r_0) as the set of parameters which after redefinition turn into $(\bar{a}, \bar{b}, 0)$. It has been shown in [36] that such redefinitions are subject to keeping A^2, B^2 fixed, with $A < B$ defined as:

$$A^2 = \frac{\Xi_a}{\alpha^2}, \quad B^2 = \frac{\Xi_b}{\alpha^2}, \quad \alpha^2 = r_0^2 + (1+a+b)^2. \quad (\text{A.3})$$

For $U(1)^3$ black hole, in new parameters $(\bar{a}, \bar{b}, 0)$, we have

$$\begin{aligned}
 H_I &= \frac{\rho^2 + 3e_I}{r^2} = 1 + \frac{\sqrt{\Xi_{\bar{a}}\Xi_{\bar{b}}}(1 + \mu_I) - \Xi_{\bar{a}}\cos^2\theta - \Xi_{\bar{b}}\sin^2\theta}{r^2}, \quad f = (H_1H_2H_3)^{-\frac{1}{3}}, \\
 \omega_\phi &= -\frac{\sin^2\theta}{r^2\Xi_{\bar{a}}} \left[\rho^4 + (2r_m^2 + \bar{a}^2)\rho^2 + \frac{1}{2}(\beta_2 - \bar{a}^2\bar{b}^2 - (\bar{a}^2 - \bar{b}^2)) \right], \\
 \omega_\psi &= -\frac{\cos^2\theta}{r^2\Xi_{\bar{b}}} \left[\rho^4 + (2r_m^2 + \bar{b}^2)\rho^2 + \frac{1}{2}(\beta_2 - \bar{a}^2\bar{b}^2 + (\bar{a}^2 - \bar{b}^2)) \right], \\
 r_m^2 &= \bar{a} + \bar{b} + \bar{a}\bar{b}, \quad X^I = \frac{(H_1H_2H_3)^{\frac{1}{3}}}{H_I}, \quad X(r) = r^2(r^2 + (1 + \bar{a} + \bar{b})^2).
 \end{aligned} \tag{A.4}$$

Through a simple calculation, we can show that the BPS limit of solution (2.8) can be set in the form of (A.1) by an appropriate choice of the ϕ and ψ coordinates. The conserved charges of the $\frac{1}{16}$ -BPS black hole are:¹⁸

$$\begin{aligned}
 Q_1 &= \frac{N^2}{2} \left[\mu_1 + \frac{1}{2}(\mu_1\mu_2 + \mu_1\mu_3 - \mu_2\mu_3) \right], \\
 Q_2 &= \frac{N^2}{2} \left[\mu_2 + \frac{1}{2}(\mu_3\mu_2 + \mu_1\mu_2 - \mu_1\mu_3) \right], \\
 Q_3 &= \frac{N^2}{2} \left[\mu_3 + \frac{1}{2}(\mu_3\mu_2 + \mu_1\mu_3 - \mu_1\mu_2) \right], \\
 \mathcal{J} &\equiv (1 + \mu_1)(1 + \mu_2)(1 + \mu_3), \\
 J_\phi &= \frac{N^2}{2} \left[\frac{1}{2}(\mu_1\mu_2 + \mu_1\mu_3 + \mu_2\mu_3) + \mu_1\mu_2\mu_3 + \mathcal{J} \left(\sqrt{\frac{\Xi_{\bar{b}}}{\Xi_{\bar{a}}}} - 1 \right) \right], \\
 J_\psi &= \frac{N^2}{2} \left[\frac{1}{2}(\mu_1\mu_2 + \mu_1\mu_3 + \mu_2\mu_3) + \mu_1\mu_2\mu_3 + \mathcal{J} \left(\sqrt{\frac{\Xi_{\bar{a}}}{\Xi_{\bar{b}}}} - 1 \right) \right],
 \end{aligned} \tag{A.5}$$

under the constraint

$$\mu_1 + \mu_2 + \mu_3 = \frac{1}{\sqrt{\Xi_{\bar{a}}\Xi_{\bar{b}}}} \left[2r_m^2 + 3(1 - \sqrt{\Xi_{\bar{a}}\Xi_{\bar{b}}}) \right]. \tag{A.6}$$

Therefore, this black hole has five parameters $(\mu_I, \bar{a}, \bar{b})$ with one constraint (A.6). The black hole entropy is found to be [16, 19, 36]:

$$S = 2\pi \sqrt{Q_1Q_2 + Q_2Q_3 + Q_3Q_1 - \frac{N^2}{2}(J_\phi + J_\psi)}. \tag{A.7}$$

The general charge formula can be related to the BPS limit of the equal charge black hole solution (2.13) by identifying $\mu_1 = \mu_2 = \mu$, $\mu_3 = \sigma$ and:¹⁹

$$\mu = -1 + \frac{(1+a)(1+b)}{\sqrt{\Xi_a\Xi_b}}, \quad \sigma = -1 + \frac{(1+a)(1+b)}{\sqrt{\Xi_a\Xi_b}(1+a+b)}. \tag{A.8}$$

¹⁸The J_ϕ , J_ψ will reduce to J_a, J_b after (A.8) conditions are taken. Generically, $a = 0$ does not indicate $J_\phi=0$ and similar for J_b .

¹⁹Note we are using (a, b, r_0) instead of $(\bar{a}, \bar{b}, 0)$ for (2.13)!

These parameters satisfy:

$$2\mu + \sigma = \frac{1}{\sqrt{\Xi_a \Xi_b}} \left[2r_m^2 + r_0^2 + 3(1 - \sqrt{\Xi_a \Xi_b}) \right]. \quad (\text{A.9})$$

This is consistent with (A.6) after reparametrisation.

The EVH limit of the equal charge black hole (2.8) is approached by setting $Q_3 = J_\psi = 0$, which as shown by [19] implies the black hole has zero entropy. Then for a more general class of supersymmetric black holes satisfying (A.5), we can expect to have an AdS_3 as the near-horizon geometry after imposing condition $Q_3 = J_\psi = 0$.

This turns out to be true. By solving $Q_3 = J_\psi = 0$, we get:

$$\begin{aligned} u_1 + u_2 &= -2 + \sqrt{\frac{1 - \bar{b}^2}{1 - \bar{a}^2}} + \frac{3 + 2\bar{b} + 2\bar{a} + 2\bar{a}\bar{b}}{\sqrt{(1 - \bar{a}^2)(1 - \bar{b}^2)}}, \\ u_1 u_2 &= \frac{(2 + 2\bar{b} + \bar{b}^2 + 2\bar{a} + 2\bar{a}\bar{b})(\bar{a}^2 - 1 + \sqrt{(1 - \bar{a}^2)(1 - \bar{b}^2)})}{(1 - \bar{a}^2)^{\frac{3}{2}}(1 - \bar{b}^2)^{\frac{1}{2}}}, \\ u_3 &= -1 + \sqrt{\frac{1 - \bar{b}^2}{1 - \bar{a}^2}}. \end{aligned} \quad (\text{A.10})$$

To work out the near-horizon geometry, we need to rotate the metric (A.1) and (A.2) to the coordinate system which is static at the horizon. Let us define $\hat{\psi} = \tilde{\psi} - \lambda t$, and look for the value of λ which makes the $d\hat{\psi}dt$ term vanish. The answer turns out to be $\lambda = 1$. We verify that under the condition $Q_3 = J_\psi = 0$, the $d\hat{\psi}d\tilde{\phi}$ term automatically vanishes. Then by this we successfully decouple AdS_3 part from AdS_5 . Then at the end, we are ready to set $r = \epsilon\rho$ and take $\epsilon \rightarrow 0$. The final metric of AdS_5 in the near-horizon limit is

$$\begin{aligned} ds^2 &= h_\theta \left(-\frac{\rho^2}{(1 - \bar{b}^2)(\bar{a}^2 - \bar{b}^2)} \epsilon^2 dt^2 + \frac{d\rho^2}{\rho^2(1 + \bar{a} + \bar{b})^2} + \frac{\rho^2}{(1 - \bar{b}^2)(\bar{a}^2 - \bar{b}^2)} \epsilon^2 d\hat{\psi}^2 \right), \\ &+ h_\theta \frac{d\theta^2}{\Delta_\theta} + h_\theta f(\theta) d\tilde{\phi}^2, \end{aligned} \quad (\text{A.11})$$

where

$$\begin{aligned} h_\theta &= \frac{1}{2} \left[(\bar{a}^2 - \bar{b}^2)^2 \cos^2 \theta (8 + 8\bar{a} + 8\bar{b} + 8\bar{a}\bar{b} + 3\bar{a}^2 + \bar{b}^2 \right. \\ &\quad \left. + 4(\bar{a}^2 + 2\bar{a} + 2\bar{b} + 2\bar{a}\bar{b} + 2\bar{b}^2) \cos 2\theta + (\bar{a}^2 - \bar{b}^2) \cos 4\theta \right]^{\frac{1}{3}} \\ f(\theta) &= \frac{8(1 + \bar{a} + \bar{b})^2 \Delta_\theta \sin^2 \theta}{(1 - \bar{a}^2)^2} (8 + 8\bar{a} + 8\bar{b} + 8\bar{a}\bar{b} + 3\bar{a}^2 + \bar{b}^2 \\ &\quad + 4(\bar{a}^2 + 2\bar{a} + 2\bar{b} + 2\bar{a}\bar{b} + 2\bar{b}^2) \cos 2\theta + (\bar{a}^2 - \bar{b}^2) \cos 4\theta)^{-1}. \end{aligned} \quad (\text{A.12})$$

This means that once $Q_3 = J_\psi = 0$ we may expect an AdS_3 geometry in the near-horizon limit even for the most general class of supersymmetric black holes [36].

We can expect that the BTZ like geometry can appear as near-EVH limit for the general supersymmetric black hole if we excite the charges Q_3 and J_ψ . In fact, we can

deform the charge constraint (A.10) to be:

$$\begin{aligned}
u_1 + u_2 &= -2 + \sqrt{\frac{1 - \bar{b}^2}{1 - \bar{a}^2}} + \frac{3 + 2\bar{b} + 2\bar{a} + 2\bar{a}\bar{b}}{\sqrt{(1 - \bar{a}^2)(1 - \bar{b}^2)}} + \lambda\epsilon^2, \\
u_1 u_2 &= \frac{(2 + 2\bar{b} + \bar{b}^2 + 2\bar{a} + 2\bar{a}\bar{b})(\bar{a}^2 - 1 + \sqrt{(1 - \bar{a}^2)(1 - \bar{b}^2)})}{(1 - \bar{a}^2)^{\frac{3}{2}}(1 - \bar{b}^2)^{\frac{1}{2}}}, \\
u_3 &= -1 + \sqrt{\frac{1 - \bar{b}^2}{1 - \bar{a}^2}} - \lambda\epsilon^2.
\end{aligned}
\tag{A.13}$$

The new solution also keeps the constraint (A.6) unaffected, but makes the charges $Q_3, J_\psi \sim N^2\epsilon^2$. We have checked using Mathematica that the near-horizon geometry is BTZ like.

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