U(1) flavour symmetries as Peccei-Quinn symmetries

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ABSTRACT: We investigate to what extent a generic, generation-dependent U(1) symmetry acting on the quark Yukawa operators can reduce the number of free parameters by forcing some entries in the Yukawa matrices to vanish. The maximal reduction compatible with CP violation yields nine real parameters and one phase, which matches the number of physical observables, implying that such models have no free parameters. We derive a set of results: (i) the only possible structures have the form $M_4 \oplus M_5$, where the subscripts indicate the number of real parameters in the Yukawa matrices, (ii) there are only two inequivalent Yukawa structures, each one giving rise to six different models depending on quark flavour assignments, (iii) the U(1) symmetries that generate these textures all have a QCD anomaly, and hence are Peccei-Quinn symmetries, reinforcing the idea of a possible connection between the quark flavour puzzle and the axion solution to the strong CP problem, (iv) in some cases the contributions to the QCD anomaly of two generations cancels out, and this opens the possibility that the axion coupling to nucleons could be strongly suppressed. Flavour-violating axion couplings to quarks are completely fixed, up to the axion decay constant, providing a non-trivial complementarity between low-energy flavour-violating processes and standard axion searches.

KEYWORDS: Beyond Standard Model, Global Symmetries, Quark Masses and SM Parameters

ARXIV EPRINT: [1811.09637](https://arxiv.org/abs/1811.09637)

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1 Introduction

The origin of flavour remains one of the least understood aspects of the Standard Model (SM): the large majority of the SM free parameters are related to flavour and, at least in the quark sector, their values do not appear to be random. Quark masses are strongly hierarchical and, when the same ordering is chosen in the up and down sectors (for example from light to heavy), the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix is close to diagonal. This implies a surprising degree of alignment between the up and down Yukawa matrices: either the weak and mass eigenstates are nearly aligned in both sectors or some mechanism ensures that the misalignments are quantitatively similar.

Many efforts have been made to address the flavour puzzle, often by invoking flavour symmetries. These may be Abelian or non-Abelian, global or discrete. In particular, non-Abelian family symmetries have received much attention by model builders, as they allow for unification of the fermion generations. They are often discrete and are frequently used in conjunction with gauge unification, see e.g. [1, 2] for reviews. Another popular approach to accommodate quark mass hierarchies and small mixing angles is to postulate a global U(1) symmetry (or possibly a discrete $\mathbb{Z}_N$ subgroup). The canonical example is the Froggatt-Nielsen mechanism [3], whereby the symmetry forbids most fermion Yukawa couplings at the renormalizable level. The symmetry is spontaneously broken by the vacuum expectation value (vev) of a SM singlet flavon field, after which a set of effective operators arises that couples the SM fermions to the electroweak Higgs boson. The hierarchy of fermion

masses results from the dimensional hierarchy among higher-order operators, which are suppressed by powers of a dimensionless, and conveniently small, symmetry-breaking parameter, with the suppression powers determined by the Abelian charges assigned to the fermion fields.

A different approach, put forth already in the 1970s [4, 5], aims to reproduce the data with a reduced number of free parameters. While this generally allows one to compute some of the observables in the Yukawa sector, the more ambitious goal is to reveal some well-defined structure that could shed light on the mechanism at the origin of the flavour architecture. This strategy remained actively pursued during the last decades of the past century [6–9]. Since then, the viability of Yukawa matrices with a reduced set of non-vanishing entries and different numbers of texture zeros has been systematically studied (see e.g. [10–12]). However, these types of studies in general do not specify the detailed mechanism (presumably some symmetry) which enforces the vanishing of specific entries in the Yukawa matrices.

In this work we study to what extent a generation-dependent global $U(1)$ symmetry can serve as a generator of texture zeros in the Yukawa matrices, the maximal parameter reduction it can enforce consistently with experimental data, and which type of textures can arise. Throughout our analysis we assume two Higgs doublets carrying different $U(1)$ charges, the minimum number required to enforce symmetries of this type. Although generating quark Yukawa textures by imposing a $U(1)$ symmetry may seem an obvious pathway to explore, we are only aware of a couple of studies in this direction [13, 14]. These focus on the classification of all combinatorially allowed textures compatible with some (typically discrete) Abelian symmetry, with emphasis on combinatorics rather than minimality. Some of our findings are unexpected and non-trivial: the maximum parameter reduction is to nine non-vanishing entries which can only be arranged in $M_4 \oplus M_5$ structures, where the subscripts indicate the number of non-vanishing entries in the pair of Yukawa matrices. Among all the inequivalent structures of this type, only two are consistent with a $U(1)$ symmetry. Folding in the different possibilities for assigning flavour labels to the matrix rows and columns gives rise to twelve different ‘models’. Interestingly, in all cases the resulting $U(1)$ symmetry has a QCD anomaly, and can thus be interpreted as a Peccei-Quinn (PQ) symmetry. Indeed, the idea that a PQ symmetry might have a non-trivial relationship with flavour was proposed long ago by Wilczek [15] (see also [16–18]), and the possible connection between axion models and the SM flavour puzzle has recently triggered a revived interest [19–32]. Interestingly, in some cases only the light quark generation contributes to the $U(1)$ -QCD anomaly and, as was recently shown in [33], this feature opens up the possibility that axion couplings to nucleons are sizeably weaker than what is generally assumed.

The paper is structured as follows. In section 2 we derive a set of rules which provide the largest number of texture zeroes, compatible with a generation-dependent $U(1)$ symmetry and able to reproduce the observed quark masses and mixings. In section 3 we discuss the main phenomenological consequences of these constructions, focusing in particular on the predictions for axion couplings. We conclude in section 4. Ancillary results and numerical fits are deferred to a set of appendices.

2 U(1) symmetry and Yukawa textures

In order to accommodate an extra global U(1) symmetry in the renormalizable quark Yukawa Lagrangian, we assume the minimal content of two Higgs doublets $H_{1,2}$, taking their hypercharges to be $Y(H_{1,2}) = -1/2$, and which acquire vevs $\langle H_{1,2} \rangle = v_{1,2}$ such that $v_1^2 + v_2^2 \equiv v^2 \simeq (174 \text{ GeV})^2$. We assume that $H_{1,2}$ and the quark fields carry some new U(1) global charge $\mathcal{X}$. As discussed at the end of this section, the method for identifying viable textures does not rely on particular values of the Higgs charges, which could be left generic. On the other hand, a convenient choice of placeholder charges can simplify the analysis considerably. Hence we assign to the Higgs fields respectively the charges $\mathcal{X}_{1,2} \equiv \mathcal{X}(H_{1,2}) = \pm 1$, which is a particularly convenient choice. We take the quark charges to be generation-dependent, so that $H_{1,2}$ couple in a generation-dependent way to the quark bilinears $\bar{Q}_i u_j$ and $\bar{Q}_i d_j$ ($i, j = 1, 2, 3$) where Q_i denote the left-handed (LH) quark doublets and u_j, d_j the right-handed (RH) $SU(2)_L$ singlets. Here the labels i, j do not refer to any particular ordering, and u, d should also be understood as dummy flavour labels, so that permutations of the i, j indices and $u \leftrightarrow d$ relabeling can be performed freely to comply with a consistent physical interpretation. As will be explained at the end of this section, this choice is without loss of generality as far as the search for viable Yukawa structures is concerned.

To proceed, let us assign to the quark fields generic U(1) charges $\mathcal{X}(\bar{Q}) = \{x, y, 0\}$, $\mathcal{X}(u) = \{a, b, c\}$, and $\mathcal{X}(d) = \{m, n, p\}$, where the charge of one quark field (here Q_3) can be always set to zero by a redefinition of all charges, proportional to baryon number B . The U(1) charge structure of the Yukawa bilinears reads

$$\mathcal{X}_{\bar{Q}u} = \begin{pmatrix} a+x & b+x & c+x \\ a+y & b+y & c+y \\ a & b & c \end{pmatrix}, \quad \mathcal{X}_{\bar{Q}d} = \begin{pmatrix} m+x & n+x & p+x \\ m+y & n+y & p+y \\ m & n & p \end{pmatrix}. \quad (2.1)$$

Here and in the corresponding Yukawa matrices, the LH doublets Q_i label the rows, and the RH singlets u_i, d_i label the columns. It is straightforward to see that consistency of U(1) charge assignments yields constraints for the charge difference between pair of entries in $\mathcal{X}_{\bar{Q}u}, \mathcal{X}_{\bar{Q}d}$,

$$\begin{aligned} (\mathcal{X}_{\bar{Q}d})_{ij} - (\mathcal{X}_{\bar{Q}d})_{ik} &= \Delta_{jk}^d, \\ (\mathcal{X}_{\bar{Q}u})_{ij} - (\mathcal{X}_{\bar{Q}u})_{ik} &= \Delta_{jk}^u, \\ (\mathcal{X}_{\bar{Q}d})_{ji} - (\mathcal{X}_{\bar{Q}d})_{ki} &= (\mathcal{X}_{\bar{Q}u})_{ji} - (\mathcal{X}_{\bar{Q}u})_{ki} = \Delta_{jk}^Q, \end{aligned} \quad (2.2)$$

that is, the differences in eq. (2.2) are independent of the index i and, for example, $\Delta_{12}^Q = x - y$, $\Delta_{23}^Q = y$, etc. Clearly, the only non-zero entries in the Yukawa matrices $M_{d,u}$ will be the ones for which $\mathcal{X}_{\bar{Q}d}, \mathcal{X}_{\bar{Q}u} = \pm 1$ so that a U(1)-invariant coupling with one of the two Higgs is possible.

We want to establish what is the maximum reduction in the number of non-zero entries in $M_{d,u}$ that can be enforced by a U(1) symmetry, since the corresponding reduction in the number of free Yukawa parameters would yield models with enhanced predictivity. Viable constructions must have U(1) charge assignments consistent with eq. (2.2), as well as with

a set of phenomenological constraints: no massless quarks, no vanishing mixing angles in the CKM matrix V_{CKM} , and CP violation from a complex phase in V_{CKM} . These conditions can be formulated more precisely in terms of generic Yukawa matrices with complex entries M_d, M_u .¹ We require:

- A non-vanishing commutator $\mathcal{D} \equiv \det[M_d M_d^\dagger, M_u M_u^\dagger]$. $\mathcal{D} \neq 0$ is a necessary condition for a non-vanishing Jarlskog invariant $J \propto \text{Im}[\mathcal{D}]$ [34], which in turn ensures that all mixing angles and the CP-violating phase in V_{CKM} are non-vanishing. Since with nine quark fields there are eight relative phase redefinitions that can be used to remove complex phases in the Yukawa parameters, we can immediately conclude that a minimum of nine non-zero entries (of which eight can be made real) is a necessary condition for CP violation.²
- Non-zero determinants $\det[M_d] \neq 0$ and $\det[M_u] \neq 0$. This ensures no massless quarks, and has the immediate consequence that some structures are not viable. Denoting with a subscript the number of non-zero elements in a mass matrix, these are $M_0 \oplus M_9$, $M_1 \oplus M_8$ and $M_2 \oplus M_7$.
- Consistency of the U(1) charge assignment with eq. (2.1). In particular this implies that $M_3 \oplus M_6$ structures must be also discarded because, as shown in appendix A, they cannot be enforced by consistent U(1) assignments.

We conclude that the only viable structures with the minimum number of nine parameters have the form $M_4 \oplus M_5$. In our study we collect the large number of possible $M_4 \oplus M_5$ structures in equivalence classes containing pairs of matrices that, for a fixed set of numerical inputs, yield the same mass eigenvalues and CKM mixings. For example, independent permutations of the columns in M_4 and M_5 affect only RH mixing, which are not SM observables, while permuting the rows in both matrices in the same way amounts to relabeling the quark doublets $Q_{1,2,3}$ with no effect on V_{CKM} .

Let us now proceed to identify the possible inequivalent Yukawa textures. Let us start with M_4 , to which we assign the dummy variable ‘ d ’ so that the matrix of charges of the quark bilinears is $\mathcal{X}_{\bar{Q}d}$. To ensure $\det[M_4] \neq 0$, the first three entries can be arranged in $\frac{1}{3!}(9 \cdot 4 \cdot 1)$ ways. That is, the first entry can go anywhere (9 possibilities), the second in the 2×2 submatrix that does not contain the first entry, with the position of the last entry then fixed. The fourth entry can now go in any of the six remaining empty positions (the combinatorial factor does not change because this entry cannot contribute to the determinant). We can then permute the columns in six ways and the rows in six ways to

¹For convenience, $M_{d,u}$ denote two Yukawa matrices multiplied by the dimensional parameter v and with their non-zero entries appropriately rescaled by weight factors of $\sin\beta$ or $\cos\beta$, with $\tan\beta = v_2/v_1$. However, it is understood that $M_{d,u}$, and in particular their structures, are properly defined only at a scale well above the electroweak breaking scale.

²The total parameter freedom can be further reduced by assuming symmetric matrices [7, 8] in which case the number of free parameters is reduced while the number of non-vanishing entries is not. However, in the absence of a mechanism enforcing this condition (e.g. $\mathcal{C}$ -parity in left-right symmetric models), such an assumption is not justified.

get equivalent configurations, such that the final number of inequivalent textures for M_4 is $\frac{1}{3!}(9 \cdot 4 \cdot 1) \cdot 6 \cdot \frac{1}{6 \cdot 6} = 1$.

Let us choose as the representative structure for M_4 three nonzero entries on the diagonal plus one in $(M_4)_{12}$ to which we assign, without loss of generality, a complex phase, i.e. $(M_4)_{12} = |(M_4)_{12}|e^{i\alpha}$. M_5 will then be taken to be a real matrix. There are two possible charge assignments that can realize this texture, depending on if we choose $\text{diag}(\mathcal{X}_{\bar{Q}d}) = (s, s, -s)$ or $\text{diag}(\mathcal{X}_{\bar{Q}d}) = (s, s, s)$, where $s = \pm 1$. The remaining entries $\mathcal{X}_{\bar{Q}d}$ can be filled up by requiring charge consistency and that no additional entries besides the diagonal and $(M_4)_{12}$ are allowed in M_4 . This yields

$$\begin{aligned}\mathcal{X}_{\bar{Q}d} &= \begin{pmatrix} \mathbf{s} & -\mathbf{s} & -(k+2)s \\ 3s & \mathbf{s} & -ks \\ (k+2)s & ks & -\mathbf{s} \end{pmatrix}_{k \neq \pm 1, -3}, \\ \tilde{\mathcal{X}}_{\bar{Q}d} &= \begin{pmatrix} \mathbf{s} & -\mathbf{s} & -(k+2)s \\ 3s & \mathbf{s} & -ks \\ (k+4)s & (k+2)s & \mathbf{s} \end{pmatrix}_{k \neq \pm 1, -3, -5},\end{aligned}\tag{2.3}$$

where for clarity a boldface $\mathbf{s}$ has been used whenever the bilinears match a Higgs charge. Note that the opposite sign entry $(-s)$ cannot appear in one of the first two positions along the diagonal, as this implies that if $(M_4)_{12} \neq 0$ then also $(M_4)_{21}$ is non-vanishing.

Let us now study M_5 . Again, without loss of generality we fill the diagonal with three nonzero entries. The remaining two entries can be assigned according to three possible configurations: (i) block diagonal, in which one entry has no other non-vanishing entries in the row and column to which it belongs, (ii) a row or a column filled with three entries, (iii) ‘democratic’ textures that do not belong to (i) or (ii); see the following examples,

$$(i): M_5 = \begin{pmatrix} \times & & \\ & \times & \times \\ & \times & \times \end{pmatrix}, \quad (ii): M_5 = \begin{pmatrix} \times & \times & \\ & \times & \times \\ & & \times \end{pmatrix}, \quad (iii): M_5 = \begin{pmatrix} \times & \times & \\ & \times & \times \\ & & \times \end{pmatrix}.\tag{2.4}$$

Block diagonal textures (i) are not viable because, as we will now argue, they yield $\det[M_4 M_4^\dagger, M_5 M_5^\dagger] = 0$. The structure of M_4 preserves a flavour symmetry for Q_3 , see eq. (2.3). In order to break this symmetry and mix Q_3 with the other quark doublets then the 2×2 block must involve the Q_3 row, as in case (2.4 i). Let us now write $M_4 M_4^\dagger = \Phi S_4 \Phi^*$ with $\Phi = \text{diag}(e^{i\alpha}, 1, 1)$ and S_4 a real symmetric matrix. $M_5 M_5^\dagger = S_5$ is also real symmetric and satisfies $\Phi S_5 \Phi^* = S_5$. It follows that we can write the commutator as

$$[M_4 M_4^\dagger, M_5 M_5^\dagger] = \Phi [S_4, S_5] \Phi^*.\tag{2.5}$$

The commutator of two symmetric matrices is an antisymmetric matrix, and the determinant of an antisymmetric matrix of odd dimension vanishes. The other block-diagonal texture, with off-diagonal entries assigned to $(M_5)_{13}$ and $(M_5)_{31}$, can be brought to the same form as (2.4 i) by permuting the first two columns and rows in both M_4 and M_5 , and then the same argument applies.

Under the requirement of only five non-vanishing entries, filling a row or a column with three entries, case (ii), clashes with U(1) charge consistency. This can be understood as follows: as $s = \pm 1$ are the only possible choices for the charges of non-vanishing entries, at least two of the three allowed entries in the row/column must have the same charge, take $\mathcal{X}_{13} = \mathcal{X}_{33}$ in example (2.4 ii). The difference between their charges then vanishes. By taking a second row/column with a non-vanishing entry in the same column/row position ($\mathcal{X}_{11}$ for the case at hand) we see that to match the vanishing of the corresponding charge difference there must be another entry with the same charge (here $\mathcal{X}_{31}$) which implies a sixth non-vanishing entry.

The remaining possibilities are thus the democratic textures of type (iii) (and permutations), for which the only consistent possibility for charge assignments is

$$\mathcal{X}_{\bar{Q}u} = \begin{pmatrix} 1 & -1 & -3 \\ 3 & 1 & -1 \\ 5 & 3 & 1 \end{pmatrix}. \quad (2.6)$$

Here we have used explicit numerical values for the bilinear charges since the possibility of a relative minus sign with respect to the charges in $\mathcal{X}_{\bar{Q}d}$ is already accounted for in eq. (2.3) by $s = \pm 1$. Eq. (2.6) gives the row charge differences $\Delta_{12}^Q = \Delta_{23}^Q = -2$ which, for consistency, should be respected also by some permutation of the matrices in eq. (2.3). For $\mathcal{X}_{\bar{Q}d}$ we obtain $s = +1$ and $k = 3$ by straightforward inspection. However, there is also another possibility which is obtained by permuting the first and third row and gives $s = -1$ and $k = 3$. All other permutations yield either $s \neq \pm 1$ or $k = \pm 1, -3$ and hence can be discarded. For $\tilde{\mathcal{X}}_{\bar{Q}d}$ we obtain $s = +1$ and $k = 1$ which is forbidden, and similarly none of the additional five possibilities obtained by permuting the rows is viable. We conclude that there are only two possible charge assignments compatible with the requirement of maximal reduction in the number of Yukawa parameters (nine non-vanishing Yukawa couplings) and with U(1) charge consistency. These are eq. (2.6) (which gives M_5) together with

$$\mathcal{X}_{\bar{Q}d}^{(1)} = \begin{pmatrix} 1 & -1 & -5 \\ 3 & 1 & -3 \\ 5 & 3 & -1 \end{pmatrix}, \quad \mathcal{X}_{\bar{Q}d}^{(2)} = \begin{pmatrix} -5 & -3 & 1 \\ -3 & -1 & 3 \\ -1 & 1 & 5 \end{pmatrix}, \quad (2.7)$$

which give $M_4^{(1,2)}$. We denote the combined Yukawa structures by $\mathcal{T}_{1,2} = M_4^{(1,2)} \oplus M_5$, corresponding to the structures

$$\mathcal{T}_1 = \begin{pmatrix} \times & \times & 0 \\ 0 & \times & 0 \\ 0 & 0 & \times \end{pmatrix} \oplus \begin{pmatrix} \times & \times & 0 \\ 0 & \times & \times \\ 0 & 0 & \times \end{pmatrix}, \quad \mathcal{T}_2 = \begin{pmatrix} 0 & 0 & \times \\ 0 & \times & 0 \\ \times & \times & 0 \end{pmatrix} \oplus \begin{pmatrix} \times & \times & 0 \\ 0 & \times & \times \\ 0 & 0 & \times \end{pmatrix}, \quad (2.8)$$

where the “ $\times$ ” denote the non-vanishing Yukawa entries.

As was mentioned at the beginning of this section, the choice $\mathcal{X}_{1,2} = \pm 1$ for the Higgs charges represents just a simplification to carry out the analysis (since $\mathcal{X}(H_2^\dagger) = \mathcal{X}(H_1)$), but it does not imply any restriction for the correct identification of the viable textures, and the same result would have been obtained with generic Higgs charges. To see this, let us

choose the charge normalization $\mathcal{X}(H_1^\dagger H_2) = \mathcal{X}_2 - \mathcal{X}_1 = 1$, so that $\mathcal{X}_1 = -s_\beta^2 \equiv -\sin^2 \beta$ and $\mathcal{X}_2 = c_\beta^2 \equiv \cos^2 \beta$. Eqs. (2.6) and (2.7) can be rewritten in terms of these generic charges by making the following substitutions: in eq. (2.6), $\mathbf{1} \rightarrow -\mathcal{X}_1$ and $-\mathbf{1} \rightarrow -\mathcal{X}_2$, while in eq. (2.7), to match the conjugate Higgs doublets, $-\mathbf{1} \rightarrow \mathcal{X}_1$ and $\mathbf{1} \rightarrow \mathcal{X}_2$.³ Comparing these entries with the corresponding entries in eq. (2.1) and solving the linear system of nine equations determines the values of the individual charges $\{x, y\}$, $\{a, b, c\}$, and $\{m, n, p\}$, which in turn allows us to reconstruct the complete charge matrices in terms of s_β^2, c_β^2 .

3 Phenomenological constraints and predictions

As we have argued, the maximal reduction in the number of Yukawa parameters yields nine real parameters and one complex phase, thus matching one-to-one the number of independent observables: six quark masses, three mixing angles, and one CKM phase. This ensures that any $M_4 \oplus M_5$ Yukawa texture can successfully fit the experimental data and, most importantly, that any other quantity which depends on the Yukawa matrices is predicted. More precisely, while in the SM only one combination of the LH and RH quark mixing matrices $\mathcal{V}_{L,R}^{u,d}$ is observable, namely $V_{\text{CKM}} = \mathcal{V}_L^u \mathcal{V}_L^{d\dagger}$, new physics (NP) processes might be sensitive to other combinations of the diagonalizing matrices. In the case at hand, the spontaneous breaking of the U(1) symmetry at the scale f_a will lead to a QCD axion, whose couplings to quarks (in particular the flavour-violating ones) depend on the individual mixing matrices and hence can be univocally predicted, modulo an overall factor $1/f_a$ suppressing the coupling strength.

3.1 Reconstruction of the Yukawa matrices

The exact match between the numbers of fundamental Yukawa parameters and flavour observables ensures that a complete reconstruction of the fundamental Yukawa matrices in terms of measured quantities is always possible. This is an important step in computing the individual $\mathcal{V}_{L,R}^{u,d}$ mixing matrices that control NP processes. However, in practice, carrying out such a task is not completely straightforward, and we will now illustrate the main steps that allow for $M_{u,d}$ reconstruction.

Clearly, identifying M_4 with the down or with the up quark Yukawa matrix will yield different physics. We also note that, for example, $M_4^{(1)}$ has a single entry in the third row and third column, which can be arbitrarily identified with one of the six quark flavours d, s, b , or u, c, t . Since this particular flavour will not mix with the other two quark flavours of the same electric charge, different choices will yield qualitatively different physics. We will refer to this flavour as the “sequestered” quark. Meanwhile, a different labeling for the remaining entries, for which all the quarks of the same electric charge do mix, is equivalent to a trivial reshuffling of the corresponding quark labels. In summary, we have two different structures, $\mathcal{T}_{1,2}$ in eq. (2.8), two ways to assign the up and down quarks to $M_{4,5}$, and three ways to identify the sequestered quark, for a total of 12 different models. For each one of

³This fixes the flavour label u to refer unambiguously to the up sector; if $\mathcal{X}_{\bar{Q}u}$ is associated instead with the down quark bilinears, we have the substitutions: in eq. (2.6), $\mathbf{1} \rightarrow \mathcal{X}_2$ and $-\mathbf{1} \rightarrow \mathcal{X}_1$, while in eq. (2.7), $-\mathbf{1} \rightarrow -\mathcal{X}_2$ and $\mathbf{1} \rightarrow -\mathcal{X}_1$.

the two textures $\mathcal{T}_{1,2}$ we label the six possible models by a superscript (q), where $q = u, c, t, d, s, b$ labels the sequestered quark in M_4 , e.g. $\mathcal{T}_1^{(t)}$ labels the model where the (3, 3) element of M_4 corresponds to the top quark.

As above, we use the labels $M_d = M_4$ and $M_u = M_5$ with the understanding that u and d will remain dummy labels until the analytical expressions are matched to the flavour observables. Let us study the case of $\mathcal{T}_1 = M_4^{(1)} \oplus M_5$ (the same analysis can be straightforwardly carried out for $\mathcal{T}_2$). The entries of the corresponding Yukawa matrices are labeled as

$$M_d = \begin{pmatrix} m_{11}^d & m_{12}^d e^{i\alpha} & 0 \\ 0 & m_{22}^d & 0 \\ 0 & 0 & m_{33}^d \end{pmatrix}, \quad M_u = \begin{pmatrix} m_{11}^u & m_{12}^u & 0 \\ 0 & m_{22}^u & m_{23}^u \\ 0 & 0 & m_{33}^u \end{pmatrix}, \quad (3.1)$$

where $m_{ij}^{u,d}$ are real parameters. Let us define the Hermitian matrices $\mathcal{D} = M_d M_d^\dagger$ and $\mathcal{U} = M_u M_u^\dagger$. They are diagonalized by the two unitary matrices $\mathcal{V}_L^{u,d}$, such that

$$\mathcal{D} = \mathcal{V}_L^{d\dagger} \hat{M}_d^2 \mathcal{V}_L^d, \quad \mathcal{U} = \mathcal{V}_L^{u\dagger} \hat{M}_u^2 \mathcal{V}_L^u, \quad (3.2)$$

where $\hat{M}_d = \text{diag}(m_{d1}, m_{d2}, m_{d3})$ and $\hat{M}_u = \text{diag}(m_{u1}, m_{u2}, m_{u3})$ are diagonal Yukawa matrices proportional to the physical masses. $\mathcal{V}_L^d$ takes the form

$$\mathcal{V}_L^d = \begin{pmatrix} c_\tau & -e^{i\alpha} s_\tau & 0 \\ e^{-i\alpha} s_\tau & c_\tau & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (3.3)$$

where $c_\tau = \cos \tau$, $s_\tau = \sin \tau$, and the phase has been fixed by means of the first relation in eq. (3.2). The CKM matrix is defined as $V_{\text{CKM}} \equiv \mathcal{V}_L^u \mathcal{V}_L^{d\dagger}$, and thus we can write $\mathcal{U} = \mathcal{V}_L^{d\dagger} V_{\text{CKM}}^\dagger \hat{M}_u^2 V_{\text{CKM}} \mathcal{V}_L^d$. The texture zeroes in M_u imply $\mathcal{U}_{13} = \mathcal{U}_{31} = 0$. Explicitly,

$$\mathcal{U}_{13} = c_\tau \left(V_{\text{CKM}}^\dagger \hat{M}_u^2 V_{\text{CKM}} \right)_{13} + e^{i\alpha} s_\tau \left(V_{\text{CKM}}^\dagger \hat{M}_u^2 V_{\text{CKM}} \right)_{23} = 0, \quad (3.4)$$

which yields

$$e^{i\alpha} \tan \tau = - \frac{\left(V_{\text{CKM}}^\dagger \hat{M}_u^2 V_{\text{CKM}} \right)_{13}}{\left(V_{\text{CKM}}^\dagger \hat{M}_u^2 V_{\text{CKM}} \right)_{23}}. \quad (3.5)$$

This completely determines $\mathcal{V}_L^d$ in terms of measured quantities. With $\mathcal{V}_L^d$ fixed it is straightforward to determine the couplings m_{ij}^d by means of the first relation in eq. (3.2). We obtain

$$m_{33}^d = m_{d3}, \quad m_{22}^d = \sqrt{m_{d2}^2 c_\tau^2 + m_{d1}^2 s_\tau^2}, \quad m_{12}^d = \frac{(m_{d2}^2 - m_{d1}^2) c_\tau s_\tau}{m_{22}^d}, \quad m_{11}^d = \sqrt{m_{d1}^2 c_\tau^2 + m_{d2}^2 s_\tau^2 - (m_{12}^d)^2}. \quad (3.6)$$

$\mathcal{U}$ is also completely determined in terms of known parameters, as $\mathcal{U} = \mathcal{V}_L^{d\dagger} V_{\text{CKM}}^\dagger \hat{M}_u^2 V_{\text{CKM}} \mathcal{V}_L^d$. By comparing this expression with $\mathcal{U} = M_u M_u^\dagger$ (with M_u as in eq. (3.1)), the fundamental couplings m_{ij}^u can be easily determined, at least numerically, according to

$$m_{33}^u = \sqrt{\mathcal{U}_{33}}, \quad m_{23}^u = \frac{\mathcal{U}_{23}}{m_{33}^u}, \quad m_{22}^u = \sqrt{\mathcal{U}_{22} - (m_{23}^u)^2}, \quad m_{12}^u = \frac{|\mathcal{U}_{12}|}{m_{22}^u}, \quad m_{11}^u = \sqrt{\mathcal{U}_{11} - (m_{12}^u)^2}. \quad (3.7)$$

Observable	Value	Observable	Value
m_u/MeV	$0.61^{+0.19}_{-0.18}$	θ_{12}	0.22735 ± 0.00072
m_c/GeV	$0.281^{+0.02}_{-0.04}$	θ_{13}	0.00364 ± 0.00013
m_t/GeV	82.6 ± 1.4	θ_{23}	0.04208 ± 0.00064
m_d/MeV	1.27 ± 0.22	δ	1.208 ± 0.054
m_s/MeV	26^{+8}_{-5}		
m_b/GeV	$1.16^{+0.07}_{-0.02}$		

Table 1. Input values of the quark masses and CKM parameters taken from [40, 41] (see text).

Note that in general $\mathcal{U}_{12}$, written in terms of $\mathcal{V}_L^d$ and V_{CKM} , will not turn out to be real; a real value, in agreement with the matrix in eq. (3.1), can be obtained by shifting the phase in M_d as $\alpha \rightarrow \alpha - \arg \mathcal{U}_{12}$. Finally, the RH mixing matrices $\mathcal{V}_R^{d,u}$ can be straightforwardly obtained (numerically) by diagonalization of the Hermitian matrices $M_d^\dagger M_d$ and $M_u^\dagger M_u$.

3.2 Observable parameters and stability

Since the Higgs doublets carry U(1) charges it is clear that the symmetry will be spontaneously broken. In fact, as we will see below, the breaking must occur at a scale much larger than the electroweak scale (via a SM-singlet scalar field ϕ) to sufficiently suppress the couplings of the U(1) Goldstone boson to the SM fields. After U(1) breaking the texture zeroes are no longer protected, and will be lifted to non-zero values by renormalization group (RG) running effects.⁴ This means that, for consistency, the non-zero entries in the Yukawa matrices should be determined in terms of the high-scale values of the SM observables. RG running of the SM parameters from low energy to various high-energy scales has been performed by various groups [38–41]. While the exact scale of U(1) breaking is to some extent arbitrary, we anticipate that it has to be compatible with axion phenomenology, thus fixing the value of the symmetry-breaking order parameter $v_\phi \sim f_a \gtrsim 10^8 \text{ GeV}$. We then use the values for the quark masses at $\mu = 4 \times 10^{12} \text{ GeV}$ given in [40]. In this reference, the corresponding high scale values of the CKM mixing angles and CP phase are not given. However, it is known that these quantities do not run much. We have hence adopted the results given in [41] which correspond to a scale $\mu = M_Z$. The values of our input parameters are given in table 1.

As noted above, for each of the two textures identified in section 2 there are six physically distinct arrangements of the quark flavours, for a total of twelve sets of input parameters, all of which reproduce the SM data but in general yield different NP effects. The complete sets of numerical solutions are given in appendix B. It is natural to ask to what extent these solutions are numerically fine-tuned, or in other words how stable they are under small perturbations, as for example when the reference values of the observables

⁴In principle the texture zeroes can also be lifted by higher-dimensional operators that preserve the U(1) symmetry, or even that violate it if they are gravity-induced [35–37]. Assuming a cutoff scale for these operators of $\mathcal{O}(M_{\text{Planck}})$ renders their effect subdominant with respect to the effects of RG running.

are varied within their respective experimental uncertainties. Equivalently, it would be desirable if Yukawa matrices that do not differ too much from the exact solutions would still yield acceptable values for the observables.

The simplest mechanism that can yield large fluctuations is when a small number arises from a tuned cancellation between the values of two large parameters. Clearly this requires at least one pair of parameters with sufficiently close values. Since there are no pairs of experimental observables with close values, nor among the numerical entries in the matrices in appendix B, we conclude that there is no such simple source of instability in the correspondence between observables $\mathcal{O}$ and fundamental parameters p of the Yukawa matrices. This is true for both the direct ($\mathcal{O} = \mathcal{O}(p)$) and inverse ($p = p(\mathcal{O})$) correspondence. However, given that the equations for the inverse problem are highly nonlinear, more complicated sources of fine-tuned cancellations are possible. A robust way to assess if fine-tuning is present in the correspondences is to define, analogous to the Barbieri-Giudice measure of fine-tuning [42], the quantities

$$Q_{ij} = \left| \frac{p_j}{\mathcal{O}_i} \frac{\Delta \mathcal{O}_i}{\Delta p_j} \right|, \quad (3.8)$$

expressing the relative change in the value of an observable $\mathcal{O}_i$ for a given relative change in the value of a fundamental parameter p_j . If, for any i, j , $Q_{ij} \gg 1$ (when varying p_j) or $Q_{ij} \ll 1$ (when varying $\mathcal{O}_i$), we expect a tuned solution. We have verified that for all the numerical solutions listed in appendix B, $1/2 \lesssim Q_{ij} \lesssim 2$ for all $i, j = 1, 2, \dots, 10$, confirming that the numerical solutions of the direct and inverse problems are remarkably stable, and absent of fine-tunings.

It is well known that models with two Higgs doublets with generic couplings to the fermions can give rise to scalar-mediated flavour-changing neutral currents (FCNCs), for which stringent experimental bounds exist. These effects are ubiquitous in multi-Higgs doublets models and not specific to our scenario, hence their phenomenology, and various mechanisms to keep them under control, are well documented in the literature. For simplicity we will assume here that the Higgs sector can realize the decoupling limit [43, 44], wherein the lightest Higgs mass eigenstate has SM-like couplings, while the other scalars are much more heavy and sufficiently decoupled.⁵

3.3 The U(1) flavour symmetries are Peccei-Quinn symmetries

With a field content consisting only of the SM and two Higgs doublets, the U(1) flavour symmetry gets spontaneously broken by the vevs $v_{1,2}$. To identify the physical U(1) Goldstone mode we must ensure that it is not mixed with the Z boson, or more precisely with the Goldstone mode of $U(1)_Y$ of hypercharge. This fixes the ratio of the two Higgs charges to $\mathcal{X}_1/\mathcal{X}_2 = -\tan^2 \beta$ [49], where $\tan \beta = v_2/v_1$ is a free parameter. With the charge normalization $\mathcal{X}_1 = -s_\beta^2$ and $\mathcal{X}_2 = c_\beta^2$ already introduced at the end of section 2, the

⁵Another popular approach to suppress scalar-mediated FCNCs is Yukawa alignment, where for a given fermion sector the Yukawa coupling matrices associated with each Higgs are simultaneously diagonalizable. Specific realizations using flavour symmetries have been considered in [45–48]. This strategy, however, cannot be straightforwardly implemented in our case.

U(1) charges of the quark bilinears in eqs. (2.6)–(2.7) correspond to the following charge assignments:

Texture	$\mathcal{X}(Q)$	$\mathcal{X}(u)$	$\mathcal{X}(d)$
$\mathcal{T}_1^{(d,s,b)}$	$\{2, 1, 0\}$	$\{3 - c_\beta^2, 2 - c_\beta^2, 1 - c_\beta^2\}$	$\{2 + c_\beta^2, 1 + c_\beta^2, -1 + c_\beta^2\}$
$\mathcal{T}_2^{(d,s,b)}$			$\{-1 + c_\beta^2, c_\beta^2, 2 + c_\beta^2\}$
$\mathcal{T}_1^{(u,c,t)}$		$\{3 - c_\beta^2, 2 - c_\beta^2, -c_\beta^2\}$	$\{2 + c_\beta^2, 1 + c_\beta^2, c_\beta^2\}$
$\mathcal{T}_2^{(u,c,t)}$		$\{-c_\beta^2, 1 - c_\beta^2, 3 - c_\beta^2\}$	

(3.9)

The QCD anomaly number is given by

$$2N = \sum_i [\mathcal{X}(u) + \mathcal{X}(d) - 2\mathcal{X}(Q)]_i, \quad (3.10)$$

so that we obtain $2N(\mathcal{T}_1) = 2(\mathcal{X}_2 - \mathcal{X}_1) = 2$, and $2N(\mathcal{T}_2) = (\mathcal{X}_2 - \mathcal{X}_1) = 1$. In both cases there is an anomaly; the two U(1) flavour symmetries therefore have the correct properties for being identified with PQ symmetries, and in turn the Goldstone mode of the new global symmetry can be identified with an axion. As in the usual Dine-Fischler-Srednicki-Zhitnitsky (DFSZ) [49, 50] model, this axion can be compatible with low energy phenomenology by rendering it *invisible*, by introducing a new scalar field ϕ , a singlet under the SM gauge group but carrying a U(1) charge $\mathcal{X}_\phi$, and acquiring a large vev $v_\phi \gtrsim 10^8$ GeV. In this way v_ϕ becomes the dominant order parameter for the spontaneous breaking of U(1), the QCD axion $a(x)$ dominantly emerges from the phase of the SM singlet $\phi \supset \frac{v_\phi}{\sqrt{2}} e^{ia/v_\phi}$, and all the axion couplings get suppressed by $1/v_\phi$.

The periodicity of the anomaly under a U(1) transformation of the quark fields $q \rightarrow e^{i\mathcal{X}_q\theta} q$ relative to the periodicity of the axion potential under the shift $a \rightarrow a + v_\phi \mathcal{X}_\phi \theta$ determines the number of inequivalent minima of the axion potential, i.e. the number of domain walls $N_{\text{DW}} = 2N/\mathcal{X}_\phi$ that form in the early Universe around the QCD phase transition. The same quantity also enters in the definition of the axion decay constant $f_a = v_\phi/N_{\text{DW}}$. In turn, $\mathcal{X}_\phi$ can take the two values, $\mathcal{X}_\phi = 1$ or $\mathcal{X}_\phi = 1/2$, depending on whether the renormalizable non-Hermitian operator that couples the scalar singlet to the Higgs doublets is of dimension three ($H_2^\dagger H_1 \phi$) or four ($H_2^\dagger H_1 \phi^2$). Thus we have the following possibilities: $N_{\text{DW}}(\mathcal{T}_1) = 2$ or 4, and $N_{\text{DW}}(\mathcal{T}_2) = 1$ or 2.

3.4 Axion couplings

The axion couplings to quarks are given by

$$\mathcal{L}_{aq} = \frac{\partial_\mu a}{2f_a} \left[\bar{\mathbf{u}} \gamma^\mu V^u \mathbf{u} + \bar{\mathbf{u}} \gamma^\mu \gamma_5 (A^u - F^u) \mathbf{u} + \bar{\mathbf{d}} \gamma^\mu V^d \mathbf{d} + \bar{\mathbf{d}} \gamma^\mu \gamma_5 (A^d - F^d) \mathbf{d} \right], \quad (3.11)$$

where $\mathbf{u} = (u, c, t)$ and $\mathbf{d} = (d, s, b)$ are vectors of the quark mass eigenstates. $F^{u,d} = \text{diag}(f_{u,d}, 0, 0)$, with $f_u + f_d = 1$, are model-independent contributions to the light quark couplings, originating from the $aG\tilde{G}$ term. Defining $z = m_u/m_d$ and choosing $f_u = 1/(1+z)$

avoids tree-level axion-pion mixing (see e.g. [51]). The coupling matrices V^q and A^q , for $q = u, d$, are given by

$$V^q = \frac{\mathcal{X}_\phi}{2N} \left(\mathcal{V}_R^q \mathcal{X}_{qR} \mathcal{V}_R^{q\dagger} + \mathcal{V}_L^q \mathcal{X}_{qL} \mathcal{V}_L^{q\dagger} \right), \quad A^q = \frac{\mathcal{X}_\phi}{2N} \left(\mathcal{V}_R^q \mathcal{X}_{qR} \mathcal{V}_R^{q\dagger} - \mathcal{V}_L^q \mathcal{X}_{qL} \mathcal{V}_L^{q\dagger} \right). \quad (3.12)$$

Note that as V^q and A^q refer to couplings between mass eigenstate quarks, the rows and columns are now explicitly ordered, and we choose the usual ordering according to mass hierarchy, e.g. V_{12}^u always refers to the axion coupling to an up and a charm quark. The axion couplings to quarks depend both on LH ($\mathcal{V}_L^q$) and RH ($\mathcal{V}_R^q$) mixing matrices. Moreover, as the charges $\mathcal{X}_q$ are not universal, V^q and A^q are non-diagonal, leading to flavour violation (FV).

The off-diagonal couplings can be probed in decays of heavy mesons; a recent review of current experiments and their constraints on the PQ scale is found in [19] (see in particular table 2 of that work). In the current generation of experiments, searches for kaon decays of the type $K^+ \rightarrow \pi^+ a$ provide the best sensitivity to a flavour-violating QCD axion. The NA62 experiment at the CERN SPS, whose primary goal is measuring $\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$, could potentially probe scales up to $f_a \gtrsim 10^{12} \text{ GeV}$. Conversely, if we wish to avoid the strongest bound from FV, we must ensure that, for either the down or the strange quark, the FV interactions with the axion are particularly strongly suppressed. From the discussion in section 3.1 it is clear that this can be obtained by assigning the down quarks to $M_4^{(1,2)}$, and by choosing d or s as the sequestered quark, which will then mix with the other same-type quarks only due to subleading RG effects, which eventually lift the zero texture. If, for example, the sequestered quark is the strange, both the $\bar{d}sa$ and $\bar{s}ba$ couplings would be strongly suppressed, which in turn provides a strong suppression of $K \rightarrow \pi a$ and $B \rightarrow Ka$ decays. $B \rightarrow \pi a$ decays are still allowed at the leading order, yielding a limit $f_a \gtrsim 10^8 |V_{31}^d| \text{ GeV}$, which is safely compatible the traditional axion window.

Constraints on the axion mass and axion-photon coupling $g_{a\gamma}$ are plotted in figure 1. Experimental bounds from FV are denoted by vertical lines, each referring to one texture, with couplings fixed by flavour data (see table 2 of appendix B). Of the twelve lines plotted, eight constrain $m_a \lesssim 10^{-4} \text{ eV}$, corresponding to the experimental bound on $K^+ \rightarrow \pi^+ a$. The remaining four lines correspond to the cases when either the s or the d quarks are sequestered, and $K \rightarrow \pi a$ transitions are only induced by the RG effects on the Yukawa matrices. For $\mathcal{T}_{1,2}^{(s)}$ the bounds from $B \rightarrow Ka$ or $B \rightarrow \pi a$ decays are typically stronger than the ones from RG-induced K decays, while for $\mathcal{T}_{1,2}^{(d)}$ we find instead that the limits from RG-induced K decays and tree-level B decays can be comparable. So as to not clutter the plot, for these cases we show only the bounds from B decays. As can be seen from figure 1, in all cases where s or d are sequestered, the limits remain rather weak, implying only $m_a \lesssim 1 \text{ eV}$.

The axion-photon coupling is given by $g_{a\gamma} = \frac{\alpha_{\text{em}}}{2\pi f_a} [E/N - 1.92(4)]$ [51] and depends on the ratio E/N between the electromagnetic and QCD anomalies, which can acquire values within a rather large range, see [52, 53]. However, given that the leptons carry electromagnetic charge, the value of this ratio cannot be determined without first establishing how the $U(1)$ symmetry acts in the lepton sector. A detailed exploration of the lepton

Figure 1. Bounds on the axion-photon coupling $g_{a\gamma}$ and axion mass m_a . The vertical colored lines denote the upper bound on m_a from FV, for the two different textures and quark assignments. Also shown are astrophysical/cosmological bounds, and experimental sensitivities from helioscopes (in blue) and haloscopes (in green); see the text for more details. The prediction for $g_{a\gamma}$ for the models discussed in the text, in which the PQ symmetry acts also on the lepton sector, lie between the two oblique yellow lines, where the highlighted segment indicates the favored mass region for axion DM.

sector is beyond the scope of this paper (see e.g. [53] for a recent reassessment), but for illustration we consider the two simplest scenarios, wherein all leptons couple either to $\tilde{H}_1$ or to $\tilde{H}_2$. For each scenario we have up to four possible values of E/N : since the up- and down-type quarks have different electric charges, their contribution to E depends on how they are assigned to $M_{4,5}$, as well as on the choice of Yukawa structure $\mathcal{T}_{1,2}$. If leptons couple to $\tilde{H}_1$, $E/N = 2/3, -1/3$ or $-10/3$, while if they couple to $\tilde{H}_2$, $E/N = 8/3, 11/3$ or $20/3$. As these values yield similar predictions for $g_{a\gamma}$, in figure 1 we plot $|g_{a\gamma}|$ only for the extremal cases, corresponding to $E/N = 8/3$ and $20/3$. The highlighted segments on these lines correspond to $m_a \in [25, 150] \mu\text{eV}$, which is favored by the calculation of the axion DM relic density in the post-inflationary PQ-breaking scenarios.⁶ Observationally,

⁶A recent computation of the decay of topological defects and their contribution to the axion relic DM density in the post-inflation scenarios with $N_{\text{DW}} = 1$ predicts a range $m_a \in [60, 150] \mu\text{eV}$ [54, 55]. Another study claims a more definite and lower prediction $m_a = 26.5 \pm 3.4 \mu\text{eV}$ [56], while ref. [57] argues that the uncertainty on the relic axion mass is much larger due to the required extrapolation of the axion spectrum from strings till the time of the QCD crossover.

$g_{a\gamma}$ is constrained by measurements of stellar cooling rates, and chiefly from the evolution of horizontal branch (HB) stars in globular clusters [58]. Upper bounds on $|g_{a\gamma}|$ (depicted in figure 1 with full lines) are also set by CAST [59] and ADMX [60] for different m_a . The sensitivities of future axion experiments are shown with dashed lines. Projections from IAXO [61] are given in blue, while in green we have, from left to right, projections from KHASH [62], ACTION [63], ADMX, CULTASK [64], and MADMAX [65]. For a recent review of axion experiments, see [66]. A separate upper bound on m_a is given by limits on hot DM abundance from structure formation [67] which is depicted in the figure with a vertical grey line.

3.5 Astrophobia

It was recently observed [33] that in DFSZ-like axion models with non-universal quark PQ charges, it is possible to suppress the axion couplings to both protons (C_p) and neutrons (C_n), provided certain conditions are met for the axion coupling to the lightest quarks. This is important since, contrary to common belief, it implies that the oft-quoted bound on the axion mass, $m_a \lesssim 20$ meV, from the neutrino burst duration of the Supernova SN1987A [68, 69] can be significantly relaxed, thus calling for exploration of the $\mathcal{O}(0.1)$ eV axion mass region.

Conforming to standard notation, we define $C_u \equiv A_{11}^u$ and $C_d \equiv A_{11}^d$, respectively for the axion coupling to up and down quarks. Following [33], let us consider the combinations $C_p \pm C_n$, given by

$$\begin{aligned} C_p + C_n &= 0.50(5) (C_u + C_d - 1) - 2\delta_s, \\ C_p - C_n &= 1.273(2) \left(C_u - C_d - \frac{1}{3} \right), \end{aligned} \quad (3.13)$$

where we used the relations $f_u + f_d = 1$ (exact) and $f_u - f_d \simeq 1/3$ (approximate). $|\delta_s| \lesssim 0.04$ is a correction dominated by the strange sea quark [51]. The conditions for nucleophobia read $C_p \pm C_n \approx 0$. While the sum does not depend on the vev angle β , the difference does, i.e. there exist specific values of $\tan \beta$ (see below) for which $C_p - C_n \approx 0$. In the class of models discussed here all charges and couplings are known, and we find that, provided we sequester either the up or down quark, the charge assignments yielding the $\mathcal{T}_2$ structure allow us to enforce such a cancellation, i.e. nucleophobic models can be constructed by choosing $\mathcal{T}_2^{(q)}$ with $q = u, d$. To show that this is indeed the case, let us consider the limit where weak and mass eigenstates coincide. In this limit, up to independent reordering of their columns and concurrent reordering of the rows, the Yukawa matrices $M_{4,5}$ are diagonal. We can then extract the individual contributions from each generation to the anomaly from the positions of the non-zero entries in the mass matrices. Let us consider the case of $\mathcal{T}_2^{(d)}$. We are still free to choose the ordering of the weak eigenstates with respect to the mass basis, and we choose the usual one, introducing an index $i = 1, 2, 3$ that labels the mass eigenstates $\hat{u}_{Li}$, $\hat{u}_{Ri}$, $\hat{d}_{Li}$, and $\hat{d}_{Ri}$ from light to heavy. As M_u is diagonal, and given the charges in eq. (3.9), we have $\mathcal{X}(\hat{u}_{L1}) \equiv \mathcal{X}(\hat{d}_{L1}) = \mathcal{X}(Q)_1 = 2$ and $\mathcal{X}(\hat{u}_{R1}) = \mathcal{X}(u)_1 = 3 - c_\beta^2$, and so on for $i = 2, 3$. For $\mathcal{T}_2$, M_d is instead anti-diagonal, so we have $\mathcal{X}(\hat{d}_{R1}) = \mathcal{X}(d)_3 = 2 + c_\beta^2$, $\mathcal{X}(\hat{d}_{R2}) = \mathcal{X}(d)_2 = c_\beta^2$, and $\mathcal{X}(\hat{d}_{R3}) = \mathcal{X}(d)_1 = -1 + c_\beta^2$. The

contribution of the single generations to the anomaly then are $2N_1 = 1$ and $2N_2 = 2N_3 = 0$. (For $\mathcal{T}_1$ we would instead obtain $2N_1 = 2N_2 = 1$ and $2N_3 = 0$.) The first nucleophobia condition $C_u + C_d = 1$ is realized when $N_l/N = 1$, where N_l is the anomaly contribution of the lightest generation. This implies that the contributions of the two heavier generations must add up to zero. This can be satisfied for $\mathcal{T}_2$ (but not for $\mathcal{T}_1$). It is sufficient to arrange for $N_l = N_1$ which can be done by sequestering one of the two light quarks. Taking into account the effects of quark mixing, we obtain $C_u + C_d \simeq 1.1$ which might even result in a more accurate cancellation also against the δ_s correction. Barring fine-tunings between mixings and strange quark corrections, we see that just from the choice of a specific model, and without any tuning of the parameters, we can obtain a suppression of the combination $C_p + C_n$ at the level of 10%. The second condition for nucleophobia is satisfied for $C_u - C_d \approx 1/3$. This can be obtained by an appropriate choice for the ratio of Higgs doublet vevs. We need $\tan \beta \approx 1/\sqrt{2}$ for $\mathcal{T}_2^{(u)}$, and $\tan \beta \approx \sqrt{2}$ for $\mathcal{T}_2^{(d)}$, tuned to within about 4%, in order to match the accuracy of the first condition.

For standard DFSZ models, in which the leptons and in particular the electrons are also coupled to the axion, bounds from white dwarf cooling rates and from red giant evolution apply, and yield limits on the axion mass which are not much weaker than those from SN1987A [68]. However, in some models also the axion-electron coupling can be suppressed, thus making the axion “astrophobic” [33], in the sense that all the strongest bounds from astrophysical considerations can be relaxed. There are at least two ways to achieve electrophobia. In the first case, the Higgs sector is not enlarged, and the leptons couple to the same Higgs doublets as do quarks. The electrons carry a PQ charge and thus couple to the axion. However, the lepton sector is characterized by large flavour mixings and, as was shown in [33], by an appropriate choice of lepton U(1) charges and Yukawa matrix structures one can enforce a cancellation between the contribution of the unmixed couplings and large corrections from mixing, while maintaining agreement with the lepton sector experimental data. The second approach is to couple all leptons exclusively to a third Higgs doublet H_3 with charge $\mathcal{X}_3$, which in turn couples to $H_{1,2}$ via judiciously chosen non-Hermitian invariants in the scalar potential. It can be shown that after imposing orthogonality of the axion with respect to the hypercharge Goldstone mode, it is possible to arrange the ratios of the vevs such that the redefined $\mathcal{X}_3$ charge becomes arbitrarily small, effectively decoupling the whole lepton sector from the physical axion.

4 Conclusion

In this paper we have explored a class of models for the quark Yukawa sector which are characterized by having the maximum possible reduction in the number of fundamental Yukawa operators, while still ensuring compatibility with all experimental data. This demand for minimality can be motivated by Occam’s Razor: to explain the same set of data, a generic SM Yukawa structure involves twice as many Yukawa operators as we have assumed here, and therefore our Yukawa structures are simpler by far. To comply with another well-known paradigm, namely Gell-Mann’s totalitarian principle stating “everything not forbidden is compulsory”, we have appealed to a simple, generation-dependent

U(1) symmetry, that forbids in total nine Yukawa operators, and generates corresponding texture zeros in the Yukawa matrices, at the cost of introducing a second Higgs doublet. We have shown that there are only two Yukawa structures, inequivalent under row and column permutations, that can ensure all quark masses, CKM mixings and CP phase are non-zero, while also corresponding to a consistent set of U(1) charge assignments for the quarks and Higgs fields. Of the nine non-zero Yukawa couplings, all but one can be made real by quark redefinitions, and we have provided a method to determine univocally the numerical values of these 9+1 input parameters in terms of experimental observables. We have found that the generation-dependent U(1) symmetries which enforce these minimal Yukawa structures all have a QCD anomaly, and thus can be straightforwardly interpreted as PQ symmetries. The resulting axion is characterized by couplings that feel the flavour content of the quarks to which it couples, mediating flavour-changing meson decays that may be visible in future experiments.

Interestingly, two specific Yukawa textures allow for the construction of models wherein the axion couplings to nucleons can be suppressed by one order of magnitude, with a moderate amount of tuning in the parameters. This gives rise to a nucleophobic axion, for which the strong constraints from the neutrino burst duration of the SN1987A are relaxed. We have also described two different ways to suppress the axion couplings to electrons. The first one does not require enlarging the Higgs sector, but it relies on a tuned cancellation in the axion-electron coupling. The second one requires a third Higgs doublet, but with no need for additional tunings in the parameters beyond what is already required to enforce nucleophobia. Such axions truly deserve the title of “astrophobic” [33], since for all of them the strongest astrophysical bounds are sizeably relaxed, which renders a region of relatively large axion masses, $0.1 \lesssim m_a/\text{eV} \lesssim 1$, generally believed to be ruled out, indeed viable. Finally, it would be interesting to extend the present study to the lepton sector, where the scale of U(1) breaking could be naturally connected with the seesaw scale.

Acknowledgments

We acknowledge valuable discussions with Paolo Panci and Robert Ziegler in the early stages of this work. FB and EN are supported in part by the INFN “Iniziativa Specifica” Theoretical Astroparticle Physics (TAsP-LNF). The work of LDL was supported by the ERC grant NEO-NAT. This work is supported by MINECO grant FPA2016-76005-C2-1-P, by Maria de Maetzu program grant MDM-2014-0367 of ICCUB and 2017 SGR 929. The work of EN was performed in part at the Aspen Center for Physics, which is supported by National Science Foundation grant PHY-1607611. The participation of EN at the Aspen Center for Physics was supported by the Simons Foundation.

A 3+6 textures

In this appendix, we show that the $M_3 \oplus M_6$ Yukawa structures are not compatible with a consistent U(1) charge assignment of the quarks. Let us choose $M_d = M_3$, with three non-zero Yukawa textures, and $M_u = M_6$. By appropriate choice of U(1) charges $\mathcal{X}(d)$,

we arrange the only diagonal elements of $\mathcal{X}_{\bar{Q}d}$ to be $\pm s$, where $s = \pm 1$ correspond to the two Higgs charges. All off-diagonal elements must be $\neq \pm s$, leading to constraints on the quark U(1) charges. There are two possibilities for the diagonal entries of $\mathcal{X}_{\bar{Q}d}$, namely all diagonal charges equal, i.e. (s, s, s) or one charge different, i.e. $(s, s, -s)$ or some permutation thereof. Denoting the row charge difference between two rows i and j as Δ_{ij}^Q , we have

$$\mathcal{X}_{\bar{Q}d} = \begin{pmatrix} s & s + \Delta_{12}^Q & \pm s + \Delta_{13}^Q \\ s - \Delta_{12}^Q & s & \pm s + \Delta_{23}^Q \\ s - \Delta_{13}^Q & s - \Delta_{23}^Q & \pm s \end{pmatrix}. \quad (\text{A.1})$$

Regardless of the sign in the third column, we thus require $\Delta_{12}^Q \neq 0, \pm 2s$, $\Delta_{23}^Q \neq 0, 2s$, and $\Delta_{13}^Q \neq 0, 2s$. Additionally if $\text{diag}(\mathcal{X}_{\bar{Q}d}) = (s, s, s)$, we also have $\Delta_{23}^Q \neq -2s$ and $\Delta_{13}^Q \neq -2s$.

We will now see that these constraints exclude all physically viable textures in $M_u = M_6$. We must fill six entries; it is immediately clear that at least two columns of M_6 will have at least two filled entries each. In fact, given that each row in M_3 (and thereby also in M_6) is different, each column has exactly two non-zero entries. In a given column, in order to have non-zero Yukawa couplings, the bilinear U(1) charges must be $\pm s$ and the difference between them, i.e. $\mathcal{X}_{ik} - \mathcal{X}_{jk}$, must be 0 or $\pm 2s$. Most of these choices are disallowed by the constraints on M_3 . The only possible options are $\Delta_{13}^Q = -2s$ and $\Delta_{23}^Q = -2s$. Choosing $\Delta_{23}^Q = -2s$, we can arrange two identical columns in a charge consistent way, like

$$\mathcal{X}_{\bar{Q}u} = \begin{pmatrix} ks & ks & \cdot \\ -s & -s & \cdot \\ s & s & \cdot \end{pmatrix}_{k \neq \pm 1}. \quad (\text{A.2})$$

In order to ensure a mass matrix with a non-zero determinant, i.e. no rows or columns of zeroes in M_6 , the final column must take the form $(\pm s, s, k's)$ or $(\pm s, k's, -s)$, where $k' \neq \pm 1$. However, any of these configurations violates one of the constraints on Δ_{12}^Q or Δ_{13}^Q . We reach the same conclusion if we choose $\Delta_{13}^Q = -2s$.

Physically speaking, the reason 3+6 textures are excluded lies in the fact that enforcing a strictly diagonal M_3 texture places severe constraints on the allowed charges of the LH quarks. The constraints in turn are not compatible with a much richer mass matrix structure in M_6 .

B Numerical fits

Recall that there are two structures, $\mathcal{T}_1$ and $\mathcal{T}_2$, given in eq. (2.8). For each, we add a superscript (q) , where $q = u, c, t, d, s, b$ labels the quark in M_4 that is “sequestered”. The model parameters can be obtained by the method outlined in section 3.1. The vector (V^q) and axial (A^q) coupling matrices of the axion to quarks ($q = u, d$ labels the sectors) are defined in eq. (3.12). Notably, the sequestered quark has no FV couplings. Table 2 presents the model parameters corresponding to the correct quark masses at $\mu = 4 \times 10^{12}$ GeV, which is close to the presumed U(1)-breaking scale. It also gives the off-diagonal elements of V^q and A^q , which describe flavour-violating interactions.

Texture	M_u/GeV	M_d/GeV	(i, j)	V_{ij}^u	A_{ij}^u	V_{ij}^d	A_{ij}^d
$\mathcal{T}_1^{(u)}$	$\begin{bmatrix} 0.28 & 4.9e^{i\rho_u} & 0 \\ 0 & 83 & 0 \\ 0 & 0 & 0.00061 \end{bmatrix}$	$\begin{bmatrix} 0.0055 & 0.043 & 0 \\ 0 & 0.95 & 0.67 \\ 0 & 0 & 0.0073 \end{bmatrix}$	(1,2)	0	0	0.39	-0.051
			(1,3)	0	0	0.047	0.05
			(2,3)	-0.03	0.03	-0.25	-0.22
$\mathcal{T}_1^{(c)}$	$\begin{bmatrix} 0.00061 & 0.43e^{i\rho_c} & 0 \\ 0 & 83 & 0 \\ 0 & 0 & 0.28 \end{bmatrix}$	$\begin{bmatrix} 0.0013 & 0.0066 & 0 \\ 0 & 0.54 & 1.0 \\ 0 & 0 & 0.055 \end{bmatrix}$	(1,2)	0	0	0.23	-0.21
			(1,3)	-0.003	0.003	0.006	-0.001
			(2,3)	0	0	-0.23	-0.18
$\mathcal{T}_1^{(t)}$	$\begin{bmatrix} 0.00061 & 0.024e^{i\rho_t} & 0 \\ 0 & 0.28 & 0 \\ 0 & 0 & 83 \end{bmatrix}$	$\begin{bmatrix} 0.0013 & 0.0055 & 0 \\ 0 & 0.025 & 0.049 \\ 0 & 0 & 1.2 \end{bmatrix}$	(1,2)	-0.043	0.043	-0.11	0.098
			(1,3)	0	0	0.004	-0.004
			(2,3)	0	0	0.021	-0.020
$\mathcal{T}_1^{(d)}$	$\begin{bmatrix} 0.0027 & 0.28 & 0 \\ 0 & 7.1 & 82 \\ 0 & 0 & 0.74 \end{bmatrix}$	$\begin{bmatrix} 0.026 & 0.048e^{i\rho_d} & 0 \\ 0 & 1.2 & 0 \\ 0 & 0 & 0.0013 \end{bmatrix}$	(1,2)	0.22	-0.22	0	0
			(1,3)	0.005	-0.004	0	0
			(2,3)	-0.044	-0.042	-0.021	0.020
$\mathcal{T}_1^{(s)}$	$\begin{bmatrix} 0.00063 & 0.064 & 0 \\ 0 & 6.6 & 82 \\ 0 & 0 & 3.4 \end{bmatrix}$	$\begin{bmatrix} 0.0013 & 0.010e^{i\rho_s} & 0 \\ 0 & 1.2 & 0 \\ 0 & 0 & 0.026 \end{bmatrix}$	(1,2)	0.22	-0.219	0	0
			(1,3)	0.005	-0.005	-0.004	0.004
			(2,3)	-0.06	-0.020	0	0
$\mathcal{T}_1^{(b)}$	$\begin{bmatrix} 0.00061 & 0.024 & 0 \\ 0 & 0.28 & 3.5 \\ 0 & 0 & 83 \end{bmatrix}$	$\begin{bmatrix} 0.0013 & 0.0055e^{i\rho_b} & 0 \\ 0 & 0.025 & 0 \\ 0 & 0 & 1.2 \end{bmatrix}$	(1,2)	-0.043	0.043	-0.11	0.097
			(1,3)	-0.002	0.002	0	0
			(2,3)	-0.021	0.021	0	0
$\mathcal{T}_2^{(u)}$	$\begin{bmatrix} 0 & 0 & 0.00061 \\ 0 & 83 & 0 \\ 0.28 & 4.9e^{i\rho_u} & 0 \end{bmatrix}$	$\begin{bmatrix} 0.0013 & 0.0072 & 0 \\ 0 & 0.68 & 0.94 \\ 0 & 0 & 0.043 \end{bmatrix}$	(1,2)	0	0	0.45	-0.424
			(1,3)	0	0	0.009	0.002
			(2,3)	-0.06	0.06	-0.51	-0.44
$\mathcal{T}_2^{(c)}$	$\begin{bmatrix} 0 & 0 & 0.28 \\ 0 & 83 & 0 \\ 0.00061 & 0.43e^{i\rho_c} & 0 \end{bmatrix}$	$\begin{bmatrix} 0.0055 & 0.055 & 0 \\ 0 & 1.0 & 0.52 \\ 0 & 0 & 0.0067 \end{bmatrix}$	(1,2)	0	0	0.8	-0.076
			(1,3)	-0.005	0.005	0.076	0.09
			(2,3)	0	0	-0.44	-0.35
$\mathcal{T}_2^{(t)}$	$\begin{bmatrix} 0 & 0 & 83 \\ 0 & 0.28 & 0 \\ 0.00061 & 0.024e^{i\rho_t} & 0 \end{bmatrix}$	$\begin{bmatrix} 0.139 & 1.15 & 0 \\ 0 & 0.049 & 0.025 \\ 0 & 0 & 0.0056 \end{bmatrix}$	(1,2)	-0.086	0.086	0.64	0.22
			(1,3)	0	0	-0.12	-0.11
			(2,3)	0	0	-0.068	0.014
$\mathcal{T}_2^{(d)}$	$\begin{bmatrix} 0.00063 & 0.74 & 0 \\ 0 & 82 & 7.1 \\ 0 & 0 & 0.28 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0.0013 \\ 0 & 1.2 & 0 \\ 0.026 & 0.048e^{i\rho_d} & 0 \end{bmatrix}$	(1,2)	0.44	-0.44	0	0
			(1,3)	-0.009	0.009	0	0
			(2,3)	-0.088	-0.084	-0.042	0.041
$\mathcal{T}_2^{(s)}$	$\begin{bmatrix} 0.0027 & 3.4 & 0 \\ 0 & 82 & 6.6 \\ 0 & 0 & 0.064 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 0.026 \\ 0 & 1.2 & 0 \\ 0.0013 & 0.010e^{i\rho_s} & 0 \end{bmatrix}$	(1,2)	0.46	-0.42	0	0
			(1,3)	-0.008	0.01	-0.009	0.009
			(2,3)	-0.12	-0.039	0	0
$\mathcal{T}_2^{(b)}$	$\begin{bmatrix} 0.17 & 83 & 0 \\ 0 & 3.5 & 0.28 \\ 0 & 0 & 0.024 \end{bmatrix}$	$\begin{bmatrix} 0 & 0 & 1.2 \\ 0 & 0.025 & 0 \\ 0.0013 & 0.0055e^{i\rho_b} & 0 \end{bmatrix}$	(1,2)	0.14	-0.036	-0.22	0.19
			(1,3)	-0.006	0.002	0	0
			(2,3)	-0.042	0.042	0	0

Table 2. Input parameters m_{ij}^q fitted to flavour data at $\mu = 4 \times 10^{12} \text{ GeV}$, and associated off-diagonal (flavour-violating) axion-quark couplings $V_{ij}^q = V_{ji}^q$ and $A_{ij}^q = A_{ji}^q$. The phases ρ_i are as follows: $\rho_u = 0.7179$, $\rho_c = -0.7162$, $\rho_t = 1.5681$, $\rho_d = -0.3994$, $\rho_s = 0.3988$, and $\rho_b = -1.5693$.

Texture	i	V_{ii}^u	A_{ii}^u	V_{ii}^d	A_{ii}^d
$\mathcal{T}_1^{(u)}$	1	$0.0 - c_\beta^2/2$	$0.0 - c_\beta^2/2$	$1.0 + c_\beta^2/2$	$0.91 + c_\beta^2/2$
	2	$2.5 - c_\beta^2/2$	$0.5 - c_\beta^2/2$	$1.2 + c_\beta^2/2$	$-0.75 + c_\beta^2/2$
	3	$1.5 - c_\beta^2/2$	$0.5 - c_\beta^2/2$	$0.83 + c_\beta^2/2$	$-0.17 + c_\beta^2/2$
$\mathcal{T}_1^{(c)}$	1	$2.5 - c_\beta^2/2$	$0.5 - c_\beta^2/2$	$1.9 + c_\beta^2/2$	$0.051 + c_\beta^2/2$
	2	$0.0 - c_\beta^2/2$	$0.0 - c_\beta^2/2$	$0.45 + c_\beta^2/2$	$0.34 + c_\beta^2/2$
	3	$1.5 - c_\beta^2/2$	$0.5 - c_\beta^2/2$	$0.61 + c_\beta^2/2$	$-0.39 + c_\beta^2/2$
$\mathcal{T}_1^{(t)}$	1	$2.5 - c_\beta^2/2$	$0.5 - c_\beta^2/2$	$2.0 + c_\beta^2/2$	$0.022 + c_\beta^2/2$
	2	$1.5 - c_\beta^2/2$	$0.5 - c_\beta^2/2$	$1.0 + c_\beta^2/2$	$-0.021 + c_\beta^2/2$
	3	$0.0 - c_\beta^2/2$	$0.0 - c_\beta^2/2$	$0.0009 + c_\beta^2/2$	$-0.0009 + c_\beta^2/2$
$\mathcal{T}_1^{(d)}$	1	$1.6 - c_\beta^2/2$	$1.4 - c_\beta^2/2$	$-0.5 + c_\beta^2/2$	$-0.5 + c_\beta^2/2$
	2	$1.9 - c_\beta^2/2$	$0.047 - c_\beta^2/2$	$2.0 + c_\beta^2/2$	$8.6 \cdot 10^{-4} + c_\beta^2/2$
	3	$1.0 - c_\beta^2/2$	$0.0037 - c_\beta^2/2$	$1.0 + c_\beta^2/2$	$-8.6 \cdot 10^{-4} + c_\beta^2/2$
$\mathcal{T}_1^{(s)}$	1	$2.4 - c_\beta^2/2$	$0.55 - c_\beta^2/2$	$2.0 + c_\beta^2/2$	$4.0 \cdot 10^{-5} + c_\beta^2/2$
	2	$1.0 - c_\beta^2/2$	$0.95 - c_\beta^2/2$	$-0.5 + c_\beta^2/2$	$-0.5 + c_\beta^2/2$
	3	$1.0 - c_\beta^2/2$	$0.004 - c_\beta^2/2$	$1.0 + c_\beta^2/2$	$-4.0 \cdot 10^{-5} + c_\beta^2/2$
$\mathcal{T}_1^{(b)}$	1	$2.5 - c_\beta^2/2$	$0.5 - c_\beta^2/2$	$2.0 + c_\beta^2/2$	$0.022 + c_\beta^2/2$
	2	$1.5 - c_\beta^2/2$	$0.5 - c_\beta^2/2$	$1.0 + c_\beta^2/2$	$-0.022 + c_\beta^2/2$
	3	$0.5 - c_\beta^2/2$	$0.5 - c_\beta^2/2$	$-0.5 + c_\beta^2/2$	$-0.5 + c_\beta^2/2$
$\mathcal{T}_2^{(u)}$	1	$5.0 - c_\beta^2$	$1.0 - c_\beta^2$	$3.9 + c_\beta^2$	$0.1 + c_\beta^2$
	2	$0.0036 - c_\beta^2$	$-0.0036 - c_\beta^2$	$0.76 + c_\beta^2$	$0.55 + c_\beta^2$
	3	$2.0 - c_\beta^2$	$0.0036 - c_\beta^2$	$1.3 + c_\beta^2$	$-0.66 + c_\beta^2$
$\mathcal{T}_2^{(c)}$	1	$0.00003 - c_\beta^2$	$-0.00003 - c_\beta^2$	$2.0 + c_\beta^2$	$1.8 + c_\beta^2$
	2	$5.0 - c_\beta^2$	$1 - c_\beta^2$	$2.2 + c_\beta^2$	$-1.6 + c_\beta^2$
	3	$2.0 - c_\beta^2$	$0.00003 - c_\beta^2$	$1.8 + c_\beta^2$	$-0.21 + c_\beta^2$
$\mathcal{T}_2^{(t)}$	1	$0.0074 - c_\beta^2$	$-0.0074 - c_\beta^2$	$1.9 + c_\beta^2$	$1.8 + c_\beta^2$
	2	$2.0 - c_\beta^2$	$0.0074 - c_\beta^2$	$1.1 + c_\beta^2$	$-0.86 + c_\beta^2$
	3	$5.0 - c_\beta^2$	$1.0 - c_\beta^2$	$3.0 + c_\beta^2$	$-0.98 + c_\beta^2$
$\mathcal{T}_2^{(d)}$	1	$4.9 - c_\beta^2$	$1.1 - c_\beta^2$	$4.0 + c_\beta^2$	c_β^2
	2	$1.1 - c_\beta^2$	$0.91 - c_\beta^2$	$-1.0 + c_\beta^2$	$-1.0 + c_\beta^2$
	3	$3.0 - c_\beta^2$	$0.99 - c_\beta^2$	$1.0 + c_\beta^2$	$-1.0 + c_\beta^2$
$\mathcal{T}_2^{(s)}$	1	$3.1 - c_\beta^2$	$2.9 - c_\beta^2$	$-1.0 + c_\beta^2$	$-1.0 + c_\beta^2$
	2	$2.9 - c_\beta^2$	$-0.89 - c_\beta^2$	$4.0 + c_\beta^2$	c_β^2
	3	$3.0 - c_\beta^2$	$0.99 - c_\beta^2$	$1.0 + c_\beta^2$	$-1.0 + c_\beta^2$
$\mathcal{T}_2^{(b)}$	1	$3.0 - c_\beta^2$	$3.0 - c_\beta^2$	$-0.96 + c_\beta^2$	$-1.0 + c_\beta^2$
	2	$2.0 - c_\beta^2$	$0.0069 - c_\beta^2$	$0.96 + c_\beta^2$	$-0.96 + c_\beta^2$
	3	$4.0 - c_\beta^2$	$0.0018 - c_\beta^2$	$4.0 + c_\beta^2$	c_β^2

Table 3. Diagonal vector (V_{ii}^q) and axial-vector (A_{ii}^q) couplings of the axion to quarks.

Table 3 gives the corresponding diagonal elements of the coupling matrices, which depend explicitly on $\beta = \arctan(v_2/v_1)$. Note that the couplings are all real: this follows from the highly constrained phase structure of the Yukawa matrices, with only one non-zero phase in the off-diagonal element of M_4 , which ultimately cancels in V^q and A^q .

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