Supersymmetric AdS_6 black holes from matter coupled $F(4)$ gauged supergravity

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ABSTRACT: In matter coupled $F(4)$ gauged supergravity in six dimensions, we study supersymmetric AdS_6 black holes with various horizon geometries. We find new $AdS_2 \times \Sigma_{g_1} \times \Sigma_{g_2}$ horizons with $g_1 > 1$ and $g_2 > 1$, and present the black hole solution numerically. The full black hole is an interpolating geometry between the asymptotically AdS_6 boundary and the $AdS_2 \times \Sigma_{g_1} \times \Sigma_{g_2}$ horizon. We also find black holes with horizons of Kähler four-cycles in Calabi-Yau fourfolds and Cayley four-cycles in $Spin(7)$ manifolds.

KEYWORDS: AdS-CFT Correspondence, Supergravity Models

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Contents

1	Introduction and conclusions	1
2	Matter coupled $F(4)$ gauged supergravity	2
2.1	$F(4)$ gauged supergravity coupled to three vector multiplets	5
3	Black holes with $AdS_2 \times \Sigma_{g_1} \times \Sigma_{g_2}$ horizon	5
3.1	The $U(1) \times U(1)$ -invariant truncation	5
3.2	The supersymmetry equations	6
3.3	The AdS_2 solutions	8
3.4	Numerical black hole solutions	9
4	Black holes with other horizons	11
4.1	Kähler four-cycles in Calabi-Yau fourfolds	11
4.2	Cayley four-cycles in $Spin(7)$ manifolds	13
4.2.1	The $SU(2)$ -invariant truncation	13
4.2.2	The supersymmetry equations	13
4.2.3	The AdS_2 solutions	15
4.2.4	Numerical black hole solutions	16
A	The equations of motion	17

1 Introduction and conclusions

In $F(4)$ gauged supergravity in six dimensions [1], there is a unique supersymmetric fixed point which is dual to 5d superconformal $USp(2N)$ gauge theory [2, 3]. As it was shown in [4], $F(4)$ gauged supergravity is a consistent truncation of massive type IIA supergravity [5]. The fixed point uplifts to $AdS_6 \times_w S^4$ near-horizon geometry of the D4-D8 brane system [6, 7]. In the spirit of [8], supergravity solutions of wrapped D4-branes on various supersymmetric cycles were studied in $F(4)$ gauged supergravity. D4-branes wrapped on two- and three-cycles were studied in [9]. They found AdS_4 and AdS_3 fixed point solutions. See [10] for more recent results.

Recently, by considering D4-branes wrapped on supersymmetric four-cycles, we found supersymmetric AdS_6 black holes of $F(4)$ gauged supergravity in [11].¹ To be specific, we found the full black hole solutions which is an interpolating geometry between the asymptotically AdS_6 boundary and the $AdS_2 \times H^2 \times H^2$ horizon. Via the AdS/CFT

¹Previously, D4-branes wrapped on supersymmetric four-cycles were studied in [12], but the equations and solutions were incorrect.

correspondence, [13], analogous to the AdS_4 black hole cases in [14–16], the Bekenstein-Hawking entropy of the black holes nicely matched with the topologically twisted index of 5d $USp(2N)$ gauge theory on $\Sigma_{g_1} \times \Sigma_{g_2} \times S^1$ in the large N limit [17, 18]. We also considered black hole horizons of Kähler four-cycles in Calabi-Yau fourfolds and Cayley four-cycles in $Spin(7)$ manifolds.

Pure $F(4)$ gauged supergravity is a consistent truncation of massive type IIA supergravity [4] and type IIB supergravity [19–21] on a four-hemisphere. Although it is not known whether it is also a consistent truncation of ten-dimensional supergravity, one can couple vector multiplets to pure $F(4)$ gauged supergravity [22]. In this theory, new fixed points and holographic RG flows were studied in [23–25]. See [26–28] also for other studies in this theory.

In this paper, in matter coupled $F(4)$ gauged supergravity, we continue our study on supersymmetric AdS_6 black holes. We consider $F(4)$ gauged supergravity coupled to three vector multiplets, and its $U(1) \times U(1)$ -invariant truncation first considered in [25]. We consider black hole solutions with a horizon which is a product of two Riemann surfaces, $AdS_2 \times \Sigma_{g_1} \times \Sigma_{g_2}$. We derive supersymmetry equations and obtain new AdS_2 solutions. The AdS_2 horizon exists only for the $H^2 \times H^2$ background, and not for the $H^2 \times S^2$ or $S^2 \times S^2$ backgrounds. We present the full black hole solutions numerically.

We also consider black holes with horizons of Kähler four-cycles in Calabi-Yau fourfolds and Cayley four-cycles in $Spin(7)$ manifolds. For Cayley four-cycles in $Spin(7)$ manifolds, we consider the $SU(2)_{\text{diag}}$ -invariant truncation of $F(4)$ gauged supergravity coupled to three vector multiplets. We find new AdS_2 horizons. It will be interesting to have a field theory interpretation of this AdS_2 solution.

In section 2, we review matter coupled $F(4)$ gauged supergravity in six dimensions. In section 3, we consider $F(4)$ gauged supergravity coupled to three vector multiplets, and its $U(1) \times U(1)$ -invariant truncation. We consider supersymmetric black hole solutions with a horizon which is a product of two Riemann surfaces. In section 4, we consider supersymmetric black hole solutions with horizons of Kähler four-cycles in Calabi-Yau fourfolds and Cayley four-cycles in $Spin(7)$ manifolds. In appendix A, we present the equations of motion for the $U(1) \times U(1)$ -invariant truncation.

Note added. In the final stage of this work, we became aware of [29] which has some overlap with the results presented. The solutions presented in sections 3 and 4.1 are identical to the solutions found in sections 5 and 6 of [29], respectively.

2 Matter coupled $F(4)$ gauged supergravity

We review matter coupled $F(4)$ gauged supergravity in six dimensions [22]. The gravity multiplet consists of

$$(e_\mu^a, \psi_\mu^A, A_\mu^\alpha, B_{\mu\nu}, \chi^A, \sigma) , \quad (2.1)$$

where they denote the graviton, gravitini, four vector fields, a two-form gauge potential, dilatini, and a real scalar field, respectively. The vector fields, A_μ^α , $\alpha = 0, 1, 2, 3$, can be

used to gauge the $SU(2)_R \times U(1)$ gauge symmetry. The vector multiplet consists of

$$(A_\mu, \lambda_A, \varphi^\alpha)^I, \quad (2.2)$$

where they denote a vector field, gaugini, and four real scalar fields, respectively, and $I = 1, \dots, n$ labels the vector multiplets. The fermionic fields are eight-dimensional pseudo-Majorana spinors and transform in the fundamental representation of the $SU(2)_R \sim USp(2)_R$ R-symmetry denoted by indices, $A, B = 1, 2$. We denote the coupling constants of gauge fields from the gravity and vector multiplets by g_1 and g_2 , respectively, and the mass parameter of the two-form gauge potential by m .

When there is no vector multiplet, the theory reduces to pure $F(4)$ gauged supergravity [1]. In pure $F(4)$ gauged supergravity, there are five inequivalent theories: $\mathcal{N} = 4^+$ ($g_1 > 0, m > 0$), $\mathcal{N} = 4^-$ ($g_1 < 0, m > 0$), $\mathcal{N} = 4^g$ ($g_1 > 0, m = 0$), $\mathcal{N} = 4^m$ ($g_1 = 0, m > 0$), $\mathcal{N} = 4^0$ ($g_1 = 0, m = 0$). The $\mathcal{N} = 4^+$ theory admits a supersymmetric AdS_6 fixed point when $g_1 = 3m$. At the supersymmetric AdS_6 fixed point, all the fields are vanishing except the AdS_6 metric.

The scalar fields from the gravity and vector multiplets parametrize each factor of the coset manifold,

$$\mathcal{M} = SO(1, 1) \times \frac{SO(4, n)}{SO(4) \times SO(n)}, \quad (2.3)$$

respectively. The coset representative of the second factor is given by

$$P^I{}_\alpha = (P^I{}_0, P^I{}_r) = (\Omega^I{}_0, \Omega^I{}_r), \quad (2.4)$$

where we define

$$\Omega^\Lambda{}_\Sigma = (L^{-1})^\Lambda{}_\Pi \nabla L^\Pi{}_\Sigma, \quad \nabla L^\Lambda{}_\Sigma = dL^\Lambda{}_\Sigma - f_\Gamma{}^\Lambda{}_\Pi A^\Gamma L^\Pi{}_\Sigma, \quad (2.5)$$

and $\Lambda = (\alpha, I)$ or $\Lambda = (0, r, I)$ with $r = 1, 2, 3$. The indices, $\Lambda, \Sigma, \dots$, are raised and lowered by

$$\eta_{\Lambda\Sigma} = \text{diag}(1, 1, 1, 1, -1, \dots, -1). \quad (2.6)$$

The bosonic Lagrangian is given by

$$\begin{aligned} e^{-1}\mathcal{L} = & -\frac{1}{4}R + \partial_\mu\sigma\partial^\mu\sigma - \frac{1}{4}P_{I\alpha\mu}P^{I\alpha\mu} - V - \frac{1}{8}e^{-2\sigma}\mathcal{N}_{\Lambda\Sigma}F^\Lambda_{\mu\nu}F^\Sigma{}^{\mu\nu} + \frac{3}{64}e^{4\sigma}H_{\mu\nu\lambda}H^{\mu\nu\lambda} \\ & - \frac{1}{64}\epsilon^{\mu\nu\rho\sigma\tau\kappa}B_{\mu\nu}\left(\eta_{\Lambda\Sigma}F^\Lambda_{\rho\sigma}F^\Sigma{}_{\tau\kappa} + mB_{\rho\sigma}F^0_{\tau\kappa} + \frac{1}{3}m^2B_{\rho\sigma}B_{\tau\kappa}\right), \end{aligned} \quad (2.7)$$

and

$$\begin{aligned} V = & -e^{2\sigma}\left(\frac{1}{36}A^2 + \frac{1}{4}B^tB_t + \frac{1}{4}C^I{}_tC_{It} + D^I{}_tD_{It}\right) \\ & + m^2e^{-6\sigma}\mathcal{N}_{00} - me^{-2\sigma}\left(\frac{2}{3}AL_{00} - 2B^tL_{0t}\right), \end{aligned} \quad (2.8)$$

where we define

$$\mathcal{N}_{\Lambda\Sigma} = L^0{}_\Lambda(L^{-1})_{0\Sigma} + L_\Lambda{}^r(L^{-1})_{r\Sigma} - L_\Lambda{}^I(L^{-1})_{I\Sigma}, \quad (2.9)$$

and

$$A = \epsilon^{rst} K_{rst}, \quad B^t = \epsilon^{trs} K_{rs0}, \quad C_I{}^t = \epsilon^{trs} K_{rIs}, \quad D_{It} = K_{0It}. \quad (2.10)$$

The field strengths are collectively defined by²

$$F_{\mu\nu}^\Lambda = \partial_{[\mu} A_{\nu]}^\Lambda - m \delta_0^\Lambda B_{\mu\nu}, \quad H_{\mu\nu\lambda} = \partial_{[\mu} B_{\nu\lambda]}. \quad (2.11)$$

The supersymmetry variations of the fermionic fields are given by³

$$\begin{aligned} \delta\psi_{\mu A} = & D_\mu \epsilon_A + \frac{i}{24} [Ae^\sigma + 6me^{-3\sigma}(L^{-1})_{00}] \gamma_\mu \epsilon_A - \frac{i}{8} [B_t e^\sigma - 2me^{-3\sigma}(L^{-1})_{t0}] \gamma^7 \gamma_\mu \sigma_{AB}^t \epsilon^B \\ & + \frac{1}{16} e^{-\sigma} [(L^{-1})_{0\Lambda} (F_{\nu\lambda}^\Lambda - m B_{\nu\lambda} \delta_0^\Lambda) \gamma_7 \epsilon_{AB} - (L^{-1})_{r\Lambda} F_{\nu\lambda}^\Lambda \sigma_{AB}^r] \left(\gamma_\mu{}^{\nu\lambda} - 6\delta_\mu^\nu \gamma^\lambda \right) \epsilon^B \\ & + \frac{i}{32} e^{2\sigma} H_{\nu\lambda\rho} \gamma_7 \gamma^{\nu\lambda\rho} \gamma_\mu \epsilon_A, \end{aligned} \quad (2.12)$$

$$\begin{aligned} \delta\chi_A = & \frac{i}{2} \gamma^\mu \partial_\mu \sigma \epsilon_A + \frac{1}{24} [Ae^\sigma - 18me^{-3\sigma}(L^{-1})_{00}] \epsilon_A + \frac{1}{8} [B_t e^\sigma + 6me^{-3\sigma}(L^{-1})_{t0}] \gamma^7 \sigma_{AB}^t \epsilon^B \\ & + \frac{i}{16} e^{-\sigma} [(L^{-1})_{0\Lambda} (F_{\mu\nu}^\Lambda - m B_{\mu\nu} \delta_0^\Lambda) \gamma_7 \epsilon_{AB} + (L^{-1})_{r\Lambda} F_{\mu\nu}^\Lambda \sigma_{AB}^r] \gamma^{\mu\nu} \epsilon^B \\ & + \frac{1}{32} e^{2\sigma} H_{\mu\nu\lambda} \gamma_7 \gamma^{\mu\nu\lambda} \epsilon_A, \end{aligned} \quad (2.13)$$

$$\begin{aligned} \delta\lambda_A^I = & i P_{\mu r}^I \gamma^\mu \sigma_{AB}^r \epsilon^B - i P_{\mu 0}^I \gamma^7 \gamma^\mu \epsilon_A - e^\sigma [C^I{}_t - 2i\gamma^7 D^I{}_t] \sigma_{AB}^t \epsilon^B - 2me^{-3\sigma} (L^{-1})^I{}_0 \gamma^7 \epsilon_A \\ & + \frac{i}{2} e^{-\sigma} (L^{-1})^I{}_\Lambda F_{\mu\nu}^\Lambda \gamma^{\mu\nu} \epsilon_A, \end{aligned} \quad (2.14)$$

where we define

$$\begin{aligned} K_{rst} &= g_1 \epsilon_{lmn} L^l{}_r (L^{-1})_s{}^m L^n{}_t + g_2 C_{IJK} L^I{}_r (L^{-1})_s{}^J L^K{}_t, \\ K_{rs0} &= g_1 \epsilon_{lmn} L^l{}_r (L^{-1})_s{}^m L^n{}_0 + g_2 C_{IJK} L^I{}_r (L^{-1})_s{}^J L^K{}_0, \\ K_{rIt} &= g_1 \epsilon_{lmn} L^l{}_r (L^{-1})_I{}^m L^n{}_t + g_2 C_{LJK} L^L{}_r (L^{-1})_I{}^J L^K{}_t, \\ K_{0It} &= g_1 \epsilon_{lmn} L^l{}_0 (L^{-1})_I{}^m L^n{}_t + g_2 C_{LJK} L^L{}_0 (L^{-1})_I{}^J L^K{}_t, \end{aligned} \quad (2.15)$$

and

$$D_\mu = \partial_\mu \epsilon_A + \frac{1}{4} \omega_\mu{}^{ab} \gamma_{ab} \epsilon_A + \frac{i}{2} \left(\frac{1}{2} g_1 \epsilon^{rst} \Omega_{\mu st} + i\gamma_7 \Omega_{\mu r 0} \right) \sigma_{rAB} \epsilon^B. \quad (2.16)$$

The Pauli matrices, $\sigma^t{}_A{}^B$, satisfy the relations,

$$\sigma^t_{AB} = \sigma^{tC}{}_B \epsilon_{CA}, \quad (2.17)$$

and $\sigma^t_{AB} = \sigma^t_{(AB)}$. We also define the chirality matrix by

$$\gamma_7 = i\gamma_0\gamma_1\gamma_2\gamma_3\gamma_4\gamma_5, \quad (2.18)$$

with $\gamma_7^2 = -1$ and $\gamma_7^T = -\gamma_7$. We employ the mostly minus signature, $(+---)$.

²The numerical factors are due to the unusual convention of form fields, e.g., $\omega = \omega_{\mu\nu} dx^\mu dx^\nu$, in [22].

³There is a missing γ_7 in (4.32) and also in the m dependent term in (4.35) of [22].

2.1 $F(4)$ gauged supergravity coupled to three vector multiplets

In $F(4)$ gauged supergravity coupled to three vector multiplets, we have a scalar field from the gravity multiplet and four scalar fields from each vector multiplet: total thirteen scalar fields. The scalar fields from the gravity multiplet and the vector multiplets parametrize each factor of the coset manifold, respectively,

$$\mathcal{M} = \text{SO}(1, 1) \times \frac{\text{SO}(4, 3)}{\text{SO}(4) \times \text{SO}(3)}. \quad (2.19)$$

There are three vector fields from the gravity multiplet and three vector fields from the three vector multiplets. Two sets of three vector fields can be used to gauge $\text{SU}(2)_R \times \text{SU}(2)$ gauge group. We denote the coupling constants of two $\text{SU}(2)$ factors by g_1 and g_2 . The structure constant in (2.5) splits into

$$f_{rst} = g_1 \epsilon_{rst}, \quad f_{IJK} = g_2 C_{IJK} = g_2 \epsilon_{IJK}. \quad (2.20)$$

The generators of $\text{SO}(4)$, $\text{SU}(2)_R$, $\text{SU}(2)$, and the non-compact $\text{SO}(4, 3)$ could be represented by, respectively [25],

$$\begin{aligned} J^{\alpha\beta} &= e^{\beta, \alpha} - e^{\alpha, \beta}, & J_1^{rs} &= e^{s, r} - e^{r, s}, \\ J_2^{IJ} &= e^{J+3, I+3} - e^{I+3, J+3}, & Y_{\alpha I} &= e^{\alpha, I+3} + e^{I+3, \alpha}, \end{aligned} \quad (2.21)$$

where we define

$$(e^{\Lambda\Sigma})_{\Gamma\Pi} = \delta_{\Lambda\Gamma} \delta_{\Sigma\Pi}. \quad (2.22)$$

3 Black holes with $AdS_2 \times \Sigma_{g_1} \times \Sigma_{g_2}$ horizon

3.1 The $U(1) \times U(1)$ -invariant truncation

We truncate the theory to the $U(1) \times U(1)$ -invariant sector, which was first considered in section 3.1 of [25]. The $U(1) \times U(1)$ are generated by J_1^{12} and J_2^{12} . We find two non-compact $\text{SO}(4, 3)$ generators, Y_{03} and Y_{33} , which are invariant under the action of J_1^{12} and J_2^{12} . We exponentiate the non-compact generators and obtain the coset representative,

$$L = e^{\varphi_1 Y_{03}} e^{\varphi_2 Y_{33}}. \quad (3.1)$$

We also have two $U(1)$ gauge fields, A^3 and A^6 , and a two-form gauge potential, $B_{\mu\nu}$, in the $U(1) \times U(1)$ -invariant truncation. The Lagrangian of the truncation is given by

$$\begin{aligned} e^{-1} \mathcal{L} = & -\frac{1}{4} R + \partial_\mu \sigma \partial^\mu \sigma + \frac{1}{4} \cosh^2 \varphi_2 \partial_\mu \varphi_1 \partial^\mu \varphi_1 + \frac{1}{4} \partial_\mu \varphi_2 \partial^\mu \varphi_2 - V \\ & - \frac{1}{8} e^{-2\sigma} \cosh(2\varphi_2) F_{\mu\nu}^3 F^{3\mu\nu} - \frac{1}{8} e^{-2\sigma} (\cosh^2 \varphi_1 \cosh(2\varphi_2) + \sinh^2 \varphi_1) F_{\mu\nu}^6 F^{6\mu\nu} \\ & + \frac{1}{4} e^{-2\sigma} \cosh \varphi_1 \sinh(2\varphi_2) F_{\mu\nu}^3 F^{6\mu\nu} - \frac{1}{8} m^2 e^{-2\sigma} B_{\mu\nu} B^{\mu\nu} + \frac{3}{64} e^{4\sigma} H_{\mu\nu\rho} H^{\mu\nu\rho} \\ & - \frac{1}{64} \epsilon^{\mu\nu\rho\sigma\tau\kappa} B_{\mu\nu} \left(F_{\rho\sigma}^3 F_{\tau\kappa}^3 - F_{\rho\sigma}^6 F_{\tau\kappa}^6 + \frac{1}{3} m^2 B_{\rho\sigma} B_{\tau\kappa} \right), \end{aligned} \quad (3.2)$$

where the scalar potential is

$$V = -g_1^2 e^{2\sigma} - 4g_1 m e^{-2\sigma} \cosh \varphi_1 \cosh \varphi_2 + m^2 e^{-6\sigma} (\cosh^2 \varphi_1 + \sinh^2 \varphi_1 \cosh(2\varphi_2)). \quad (3.3)$$

3.2 The supersymmetry equations

In this section, we obtain supersymmetric AdS_6 black holes with a horizon which is a product of two Riemann surfaces. We consider the metric,

$$ds^2 = e^{2F(r)} (dt^2 - dr^2) - e^{2G_1(r)} (d\theta_1^2 + \sin^2 \theta_1 d\phi_1^2) - e^{2G_2(r)} (d\theta_2^2 + \sin^2 \theta_2 d\phi_2^2), \quad (3.4)$$

for the $S^2 \times S^2$ background, and

$$ds^2 = e^{2F(r)} (dt^2 - dr^2) - e^{2G_1(r)} (d\theta_1^2 + \sinh^2 \theta_1 d\phi_1^2) - e^{2G_2(r)} (d\theta_2^2 + \sinh^2 \theta_2 d\phi_2^2), \quad (3.5)$$

for the $H^2 \times H^2$ background. The only non-vanishing components of the non-Abelian $SU(2)$ gauge field, A_μ^Λ , $\Lambda = 0, 1, \dots, 6$, are given by

$$A^3 = -2a_1 \cos \theta_1 d\phi_1 - 2a_2 \cos \theta_2 d\phi_2, \quad A^6 = -2b_1 \cos \theta_1 d\phi_1 - 2b_2 \cos \theta_2 d\phi_2, \quad (3.6)$$

for the $S^2 \times S^2$ background and

$$A^3 = 2a_1 \cosh \theta_1 d\phi_1 + 2a_2 \cosh \theta_2 d\phi_2, \quad A^6 = 2b_1 \cosh \theta_1 d\phi_1 + 2b_2 \cosh \theta_2 d\phi_2, \quad (3.7)$$

for the $H^2 \times H^2$ background, where the magnetic charges, a_1, a_2, b_1, b_2 , are constant. In order to have equal signs for the field strengths, we set opposite signs of the gauge fields for the $S^2 \times S^2$ and $H^2 \times H^2$ backgrounds. We also have a non-trivial two-form gauge potential, $B_{\mu\nu}$, determined by solving the equations of motion,

$$B_{tr} = -\frac{1}{2m^2} (a_1 a_2 - b_1 b_2) e^{2\sigma+2F-2G_1-2G_2}. \quad (3.8)$$

The three-form field strength of the two-form gauge potential, $H_{\mu\nu\lambda}$, vanishes identically.

The supersymmetry equations are obtained by setting the supersymmetry variations of the fermionic fields to zero. From the supersymmetry variations, we obtain

$$\begin{aligned} F' e^{-F} \gamma^{\hat{r}} \epsilon_A + \frac{i}{2} (g_1 e^\sigma \cosh \varphi_2 + m e^{-3\sigma} \cosh \varphi_1) \epsilon_A - \frac{i}{2} m e^{-3\sigma} \sinh \varphi_1 \sinh \varphi_2 \gamma_7 \sigma_{AB}^3 \epsilon^B \\ + \frac{1}{4} e^{-\sigma} \sinh \varphi_1 \left(b_1 e^{-2G_1} \gamma^{\hat{\theta}_1 \hat{\phi}_1} + b_2 e^{-2G_2} \gamma^{\hat{\theta}_2 \hat{\phi}_2} \right) \gamma_7 \epsilon_A \\ - \frac{1}{4} e^{-\sigma} \cosh \varphi_2 \left(a_1 e^{-2G_1} \gamma^{\hat{\theta}_1 \hat{\phi}_1} + a_2 e^{-2G_2} \gamma^{\hat{\theta}_2 \hat{\phi}_2} \right) \sigma_{AB}^3 \epsilon^B \\ + \frac{1}{4} e^{-\sigma} \cosh \varphi_1 \sinh \varphi_2 \left(b_1 e^{-2G_1} \gamma^{\hat{\theta}_1 \hat{\phi}_1} + b_2 e^{-2G_2} \gamma^{\hat{\theta}_2 \hat{\phi}_2} \right) \sigma_{AB}^3 \epsilon^B \\ + \frac{3}{8m} (a_1 a_2 - b_1 b_2) e^{\sigma-2G_1-2G_2} \gamma^{\hat{t}\hat{r}} \gamma_7 \epsilon_A = 0, \end{aligned} \quad (3.9)$$

$$\begin{aligned} G_1' e^{-F} \gamma^{\hat{r}} \epsilon_A + \frac{i}{2} (g_1 e^\sigma \cosh \varphi_2 + m e^{-3\sigma} \cosh \varphi_1) \epsilon_A - \frac{i}{2} m e^{-3\sigma} \sinh \varphi_1 \sinh \varphi_2 \gamma_7 \sigma_{AB}^3 \epsilon^B \\ - \frac{1}{4} e^{-\sigma} \sinh \varphi_1 \left(3b_1 e^{-2G_1} \gamma^{\hat{\theta}_1 \hat{\phi}_1} - b_2 e^{-2G_2} \gamma^{\hat{\theta}_2 \hat{\phi}_2} \right) \gamma_7 \epsilon_A \\ + \frac{1}{4} e^{-\sigma} \cosh \varphi_2 \left(3a_1 e^{-2G_1} \gamma^{\hat{\theta}_1 \hat{\phi}_1} - a_2 e^{-2G_2} \gamma^{\hat{\theta}_2 \hat{\phi}_2} \right) \sigma_{AB}^3 \epsilon^B \\ - \frac{1}{4} e^{-\sigma} \cosh \varphi_1 \sinh \varphi_2 \left(3b_1 e^{-2G_1} \gamma^{\hat{\theta}_1 \hat{\phi}_1} - b_2 e^{-2G_2} \gamma^{\hat{\theta}_2 \hat{\phi}_2} \right) \sigma_{AB}^3 \epsilon^B \\ - \frac{1}{8m} (a_1 a_2 - b_1 b_2) e^{\sigma-2G_1-2G_2} \gamma^{\hat{t}\hat{r}} \gamma_7 \epsilon_A = 0, \end{aligned} \quad (3.10)$$

$$\begin{aligned}
 G'_2 e^{-F} \gamma^{\hat{r}} \epsilon_A &+ \frac{i}{2} (g_1 e^\sigma \cosh \varphi_2 + m e^{-3\sigma} \cosh \varphi_1) \epsilon_A - \frac{i}{2} m e^{-3\sigma} \sinh \varphi_1 \sinh \varphi_2 \gamma_7 \sigma_{AB}^3 \epsilon^B \\
 &- \frac{1}{4} e^{-\sigma} \sinh \varphi_1 \left(3b_2 e^{-2G_2} \gamma^{\hat{\theta}_1 \hat{\phi}_1} - b_1 e^{-2G_1} \gamma^{\hat{\theta}_2 \hat{\phi}_2} \right) \gamma_7 \epsilon_A \\
 &+ \frac{1}{4} e^{-\sigma} \cosh \varphi_2 \left(3a_2 e^{-2G_2} \gamma^{\hat{\theta}_1 \hat{\phi}_1} - a_1 e^{-2G_1} \gamma^{\hat{\theta}_2 \hat{\phi}_2} \right) \sigma_{AB}^3 \epsilon^B \\
 &- \frac{1}{4} e^{-\sigma} \cosh \varphi_1 \sinh \varphi_2 \left(3b_2 e^{-2G_2} \gamma^{\hat{\theta}_1 \hat{\phi}_1} - b_1 e^{-2G_1} \gamma^{\hat{\theta}_2 \hat{\phi}_2} \right) \sigma_{AB}^3 \epsilon^B \\
 &- \frac{1}{8m} (a_1 a_2 - b_1 b_2) e^{\sigma-2G_1-2G_2} \gamma^{\hat{t}\hat{r}} \gamma_7 \epsilon_A = 0, \tag{3.11}
 \end{aligned}$$

$$\begin{aligned}
 \sigma' e^{-F} \gamma^{\hat{r}} \epsilon_A &- \frac{i}{2} (g_1 e^\sigma \cosh \varphi_2 - 3m e^{-3\sigma} \cosh \varphi_1) \epsilon_A - \frac{3i}{2} m e^{-3\sigma} \sinh \varphi_1 \sinh \varphi_2 \gamma_7 \sigma_{AB}^3 \epsilon^B \\
 &- \frac{1}{4} e^{-\sigma} \sinh \varphi_1 \left(b_1 e^{-2G_1} \gamma^{\hat{\theta}_1 \hat{\phi}_1} + b_2 e^{-2G_2} \gamma^{\hat{\theta}_2 \hat{\phi}_2} \right) \gamma_7 \epsilon_A \\
 &+ \frac{1}{4} e^{-\sigma} \cosh \varphi_2 \left(a_1 e^{-2G_1} \gamma^{\hat{\theta}_1 \hat{\phi}_1} + a_2 e^{-2G_2} \gamma^{\hat{\theta}_2 \hat{\phi}_2} \right) \sigma_{AB}^3 \epsilon^B \\
 &- \frac{1}{4} e^{-\sigma} \cosh \varphi_1 \sinh \varphi_2 \left(b_1 e^{-2G_1} \gamma^{\hat{\theta}_1 \hat{\phi}_1} + b_2 e^{-2G_2} \gamma^{\hat{\theta}_2 \hat{\phi}_2} \right) \sigma_{AB}^3 \epsilon^B \\
 &+ \frac{1}{8m} (a_1 a_2 - b_1 b_2) e^{\sigma-2G_1-2G_2} \gamma^{\hat{t}\hat{r}} \gamma_7 \epsilon_A = 0, \tag{3.12}
 \end{aligned}$$

$$\begin{aligned}
 \varphi'_2 e^{-F} \gamma^{\hat{r}} \epsilon_A &- \varphi'_1 e^{-F} \cosh \varphi_2 \gamma_7 \gamma^{\hat{r}} \sigma_{AB}^3 \epsilon^B - 2ig_1 e^\sigma \sinh \varphi_2 \epsilon_A - 2ime^{-3\sigma} \sinh \varphi_1 \cosh \varphi_2 \gamma_7 \sigma_{AB}^3 \epsilon^B \\
 &- e^{-\sigma} \sinh \varphi_2 \left(a_1 e^{-2G_1} \gamma^{\hat{\theta}_1 \hat{\phi}_1} + a_2 e^{-2G_2} \gamma^{\hat{\theta}_2 \hat{\phi}_2} \right) \sigma_{AB}^3 \epsilon^B \\
 &+ e^{-\sigma} \cosh \varphi_1 \cosh \varphi_2 \left(b_1 e^{-2G_1} \gamma^{\hat{\theta}_1 \hat{\phi}_1} + b_2 e^{-2G_2} \gamma^{\hat{\theta}_2 \hat{\phi}_2} \right) \sigma_{AB}^3 \epsilon^B = 0, \tag{3.13}
 \end{aligned}$$

where the hatted indices are the flat indices. The t -, θ_1 -, and θ_2 -components of the gravitino variations give (3.9), (3.10), (3.11), the dilatino variation gives (3.12), and the gaugino variation gives (3.13). The ϕ_1 -, ϕ_2 -components of the gravitino variations are identical to the θ_1 -, and θ_2 -components beside few more terms,

$$\epsilon_A = -2ig_1 a_1 \gamma^{\hat{\theta}_1 \hat{\phi}_1} \sigma_{AB}^3 \epsilon^B, \quad \epsilon_A = -2ig_1 a_2 \gamma^{\hat{\theta}_2 \hat{\phi}_2} \sigma_{AB}^3 \epsilon^B. \tag{3.14}$$

We employ the projection conditions,

$$\gamma^{\hat{r}} \epsilon_A = i\epsilon_A, \quad \gamma^{\hat{\theta}_1 \hat{\phi}_1} \sigma_{AB}^3 \epsilon^B = -i\lambda \epsilon_A, \quad \gamma^{\hat{\theta}_2 \hat{\phi}_2} \sigma_{AB}^3 \epsilon^B = -i\lambda \epsilon_A, \tag{3.15}$$

where $\lambda = \pm 1$. Solutions with the projection conditions preserve 1/8 of the supersymmetries. As we check later, in order to have consistent supersymmetry equations with the equations of motion, it is required to have

$$\varphi_1 = 0. \tag{3.16}$$

Therefore, from now on, we set φ_1 to vanish.

We present the complete supersymmetry equations,

$$\begin{aligned}
 F'e^{-F} &= -\frac{1}{2}(g_1 e^\sigma \cosh \varphi_2 + m e^{-3\sigma}) - \frac{3}{8m}(a_1 a_2 - b_1 b_2) e^{\sigma-2G_1-2G_2} \\
 &\quad - \frac{\lambda}{4} e^{-\sigma} \cosh \varphi_2 (a_1 e^{-2G_1} + a_2 e^{-2G_2}) + \frac{\lambda}{4} e^{-\sigma} \sinh \varphi_2 (b_1 e^{-2G_1} + b_2 e^{-2G_2}) , \\
 G'_1 e^{-F} &= -\frac{1}{2}(g_1 e^\sigma \cosh \varphi_2 + m e^{-3\sigma}) + \frac{1}{8m}(a_1 a_2 - b_1 b_2) e^{\sigma-2G_1-2G_2} \\
 &\quad + \frac{\lambda}{4} e^{-\sigma} \cosh \varphi_2 (3a_1 e^{-2G_1} - a_2 e^{-2G_2}) - \frac{\lambda}{4} e^{-\sigma} \sinh \varphi_2 (3b_1 e^{-2G_1} - b_2 e^{-2G_2}) , \\
 G'_2 e^{-F} &= -\frac{1}{2}(g_1 e^\sigma \cosh \varphi_2 + m e^{-3\sigma}) + \frac{1}{8m}(a_1 a_2 - b_1 b_2) e^{\sigma-2G_1-2G_2} \\
 &\quad + \frac{\lambda}{4} e^{-\sigma} \cosh \varphi_2 (3a_2 e^{-2G_2} - a_1 e^{-2G_1}) - \frac{\lambda}{4} e^{-\sigma} \sinh \varphi_2 (3b_2 e^{-2G_2} - b_1 e^{-2G_1}) , \\
 \sigma' e^{-F} &= +\frac{1}{2}(g_1 e^\sigma \cosh \varphi_2 - 3m e^{-3\sigma}) - \frac{1}{8m}(a_1 a_2 - b_1 b_2) e^{\sigma-2G_1-2G_2} \\
 &\quad + \frac{\lambda}{4} e^{-\sigma} \cosh \varphi_2 (a_1 e^{-2G_1} + a_2 e^{-2G_2}) - \frac{\lambda}{4} e^{-\sigma} \sinh \varphi_2 (b_1 e^{-2G_1} + b_2 e^{-2G_2}) , \\
 \varphi'_2 e^{-F} &= +2g_1 e^\sigma \sinh \varphi_2 - \lambda e^{-\sigma} \sinh \varphi_2 (a_1 e^{-2G_1} + a_2 e^{-2G_2}) \\
 &\quad + \lambda e^{-\sigma} \cosh \varphi_2 (b_1 e^{-2G_1} + b_2 e^{-2G_2}) .
 \end{aligned} \tag{3.17}$$

We also obtain twist conditions on the magnetic charges from (3.14),

$$a_1 = -\frac{k}{2\lambda g_1}, \quad a_2 = -\frac{k}{2\lambda g_1}. \tag{3.18}$$

where $k = +1$ for the $S^2 \times S^2$ background and $k = -1$ for the $H^2 \times H^2$ background.⁴ There is no condition on b_1 and b_2 . The supersymmetry equations are consistent with the equations of motion. We present the equations of motion in appendix A.

3.3 The AdS_2 solutions

In this section, we find AdS_2 solutions of the supersymmetry equations. The solutions describe the $AdS_2 \times \Sigma_{g_1} \times \Sigma_{g_2}$ horizon of six-dimensional black holes.

Now we will consider the $\mathcal{N} = 4^+$ theory, $g_1 > 0$, $m > 0$. When $b_1 = b_2 = 0$, we find an AdS_2 fixed point solution for the $H^2 \times H^2$ background with $k = -1$,

$$e^F = \frac{1}{2^{5/4} g_1^{3/4} m^{1/4} r}, \quad e^{G_1} = e^{G_2} = \frac{1}{2^{3/4} g_1^{3/4} m^{1/4}}, \quad e^\sigma = \frac{2^{1/4} m^{1/4}}{g_1^{1/4}}, \quad e^{\varphi_2} = 1, \tag{3.19}$$

which is the AdS_2 solution first found in [11].⁵ When we consider the $S^2 \times S^2$ background with $k = +1$, AdS_2 fixed point does not exist.

⁴It is possible to have geometries like $S^2 \times H^2$ for $k_1 = +1$ and $k_2 = -1$, or vice versa. One can easily generalize our supersymmetry equations and the twist conditions to that case.

⁵In order to compare with [11], we have to reparametrize our parameters by

$$\sigma \rightarrow \frac{1}{\sqrt{2}}\phi, \quad g_1, m \rightarrow \frac{1}{2\sqrt{2}}g, \frac{1}{2\sqrt{2}}m, \quad a_1, a_2 \rightarrow \sqrt{2}a_1, \sqrt{2}a_2. \tag{3.20}$$

When we consider for non-zero b_1 and b_2 , we obtain new AdS_2 solutions,

$$\begin{aligned}
 e^F &= \frac{e^{-\sigma}}{2g_1 \cosh \varphi_2} \frac{1}{r}, \\
 e^{2G_1} &= \frac{\lambda}{2m} e^{2\sigma} (a_1 \cosh \varphi_2 - b_1 \sinh \varphi_2), \\
 e^{2G_2} &= \frac{\lambda}{2m} e^{2\sigma} (a_2 \cosh \varphi_2 - b_2 \sinh \varphi_2), \\
 e^{4\sigma} &= \frac{m}{2} \frac{2(a_1 a_2 - b_1 b_2) + (a_1 a_2 + b_1 b_2) \cosh(2\varphi_2) - (a_1 b_2 + a_2 b_1) \sinh(2\varphi_2)}{g_1 \cosh \varphi_2 (a_1 \cosh \varphi_2 - b_1 \sinh \varphi_2) (a_2 \cosh \varphi_2 - b_2 \sinh \varphi_2)}, \\
 e^{2\varphi_2} &= \frac{1}{(a_1 - b_1)(a_2 - b_2)} \left[(a_1 a_2 + a_1 b_2 + a_2 b_1 - 3b_1 b_2) - 2(\Phi/2)^{1/3} \right. \\
 &\quad \left. - \frac{2}{(\Phi/2)^{1/3}} (a_1 a_2^2 b_1 + a_1^2 a_2 b_2 - a_1 a_2 b_1 b_2 - 2a_2 b_1^2 b_2 - 2a_1 b_1 b_2^2 + 3b_1^2 b_2^2) \right], \quad (3.21)
 \end{aligned}$$

where we define

$$\begin{aligned}
 \Phi &= - (a_1 a_2 - 2b_1 b_2) (a_1^2 b_2^2 + a_2^2 b_1^2) - (a_1 b_2 + a_2 b_1) (a_1^2 a_2^2 + 10b_1^2 b_2^2 - 7a_1 a_2 b_1 b_2) \\
 &\quad - 2b_1 b_2 (a_1^2 a_2^2 - 5b_1^2 b_2^2 + a_1 a_2 b_1 b_2) \\
 &\quad + \sqrt{(a_1 - b_1)^2 (a_2 - b_2)^2 (a_1 a_2 - 2b_1 b_2) (a_1^2 b_2 + a_1 a_2 b_1 - 2b_1^2 b_2) (a_2^2 b_1 + a_1 a_2 b_2 - 2b_1 b_2^2)}. \quad (3.22)
 \end{aligned}$$

All the fields are parametrized by the magnetic charges, (a_1, a_2, b_1, b_2) . As (a_1, a_2) are fixed by the twist condition in (3.18), there are two free parameters left, (b_1, b_2) .

In order to have AdS_2 solutions, we should choose (b_1, b_2) which makes

$$e^F > 0, \quad e^{2G_1} > 0, \quad e^{2G_2} > 0, \quad e^{4\sigma} > 0, \quad e^{2\varphi_2} > 0. \quad (3.23)$$

We plot the range of (b_1, b_2) which satisfies the positivity conditions for $H^2 \times H^2$, $H^2 \times S^2$, $S^2 \times H^2$ and $S^2 \times S^2$, respectively. The positivity ranges are depicted in figure 1. We set $m = 1/2$ and $g_1 = 3m$ to have a unit radius for the AdS_6 boundary. *From the plots, we conjecture that only the $H^2 \times H^2$ background gives the AdS_2 solutions.*⁶ Even in the large region in the graph for the $H^2 \times H^2$ background, only a small part near origin yields AdS_2 solutions.

3.4 Numerical black hole solutions

Now we present the full black hole solution numerically. The full black hole solution is an interpolating geometry between the asymptotically AdS_6 boundary and the $AdS_2 \times H^2 \times H^2$ horizon. We introduce a new radial coordinate,

$$\rho = F + \sigma. \quad (3.24)$$

⁶We note that, when $G_1 \leftrightarrow G_2$, the solutions are invariant under $a_1 \leftrightarrow a_2$ and $b_1 \leftrightarrow b_2$. However, as $a_1 = a_2$ from (3.18), they are invariant under $b_1 \leftrightarrow b_2$. When we plot the positivity range for $H^2 \times S^2$ and $S^2 \times H^2$, there are small and irregular distributions of points. Most of the points do not respect the invariance under $b_1 \leftrightarrow b_2$. Even for the points invariant under $b_1 \leftrightarrow b_2$ seem not to give AdS_2 solutions. We presume that the appearance of these irregular distribution in the positivity range would be due to the complexity of the conditions.

Figure 1. Positivity range of (b_1, b_2) for the $H^2 \times H^2$, $H^2 \times S^2$, $S^2 \times H^2$ and $S^2 \times S^2$ backgrounds, from the top-left and clockwise. In the $H^2 \times H^2$ plot, the black dot at $(b_1, b_2) = (0, 0)$ corresponds to the AdS_2 solution in (3.19).

This kind of coordinate was introduced in [30]. Employing the supersymmetry equations, we obtain

$$\frac{\partial \rho}{\partial r} = F' + \sigma' = -e^F D, \quad (3.25)$$

where we define

$$D = 2me^{-3\sigma} + \frac{1}{2m} (a_1 a_2 - b_1 b_2) e^{\sigma - 2G_1 - 2G_2}. \quad (3.26)$$

Then, the supersymmetry equations are

$$\begin{aligned} -D \frac{\partial F}{\partial \rho} &= -\frac{1}{2} (g_1 e^\sigma \cosh \varphi_2 + m e^{-3\sigma}) - \frac{3}{8m} (a_1 a_2 - b_1 b_2) e^{\sigma - 2G_1 - 2G_2} \\ &\quad - \frac{\lambda}{4} e^{-\sigma} \cosh \varphi_2 (a_1 e^{-2G_1} + a_2 e^{-2G_2}) + \frac{\lambda}{4} e^{-\sigma} \sinh \varphi_2 (b_1 e^{-2G_1} + b_2 e^{-2G_2}), \\ -D \frac{\partial G_1}{\partial \rho} &= -\frac{1}{2} (g_1 e^\sigma \cosh \varphi_2 + m e^{-3\sigma}) + \frac{1}{8m} (a_1 a_2 - b_1 b_2) e^{\sigma - 2G_1 - 2G_2} \\ &\quad + \frac{\lambda}{4} e^{-\sigma} \cosh \varphi_2 (3a_1 e^{-2G_1} - a_2 e^{-2G_2}) - \frac{\lambda}{4} e^{-\sigma} \sinh \varphi_2 (3b_1 e^{-2G_1} - b_2 e^{-2G_2}), \\ -D \frac{\partial G_2}{\partial \rho} &= -\frac{1}{2} (g_1 e^\sigma \cosh \varphi_2 + m e^{-3\sigma}) + \frac{1}{8m} (a_1 a_2 - b_1 b_2) e^{\sigma - 2G_1 - 2G_2} \\ &\quad + \frac{\lambda}{4} e^{-\sigma} \cosh \varphi_2 (3a_2 e^{-2G_2} - a_1 e^{-2G_1}) - \frac{\lambda}{4} e^{-\sigma} \sinh \varphi_2 (3b_2 e^{-2G_2} - b_1 e^{-2G_1}), \end{aligned}$$

Figure 2. Numerical black hole solutions with $m = 1/2$ and $g_1 = 3m$. For the magnetic charges, (b_1, b_2) , we have $(-0.1, -0.1)$, purple, $(-0.01, -0.01)$, red, and $(-0.001, -0.001)$, orange.

$$\begin{aligned}
 -D \frac{\partial \sigma}{\partial \rho} &= + \frac{1}{2} (g_1 e^\sigma \cosh \varphi_2 - 3m e^{-3\sigma}) - \frac{1}{8m} (a_1 a_2 - b_1 b_2) e^{\sigma - 2G_1 - 2G_2} \\
 &\quad + \frac{\lambda}{4} e^{-\sigma} \cosh \varphi_2 (a_1 e^{-2G_1} + a_2 e^{-2G_2}) - \frac{\lambda}{4} e^{-\sigma} \sinh \varphi_2 (b_1 e^{-2G_1} + b_2 e^{-2G_2}) , \\
 -D \frac{\partial \varphi_2}{\partial \rho} &= + 2g_1 e^\sigma \sinh \varphi_2 - \lambda e^{-\sigma} \sinh \varphi_2 (a_1 e^{-2G_1} + a_2 e^{-2G_2}) \\
 &\quad + \lambda e^{-\sigma} \cosh \varphi_2 (b_1 e^{-2G_1} + b_2 e^{-2G_2}) .
 \end{aligned} \tag{3.27}$$

In the r -coordinate, the UV or asymptotically AdS_6 boundary is at $r = 0$, and the IR or $AdS_2 \times H^2 \times H^2$ horizon is at $r = \infty$. In this ρ -coordinate, the UV is at $\rho = +\infty$, and the IR is at $\rho = -\infty$. We present some representative plots of the full black hole solutions in figure 2.

4 Black holes with other horizons

In this section, we obtain more black hole solutions with other horizon geometries by considering D4-branes wrapped on Kähler four-cycles in Calabi-Yau fourfolds and on Cayley four-cycles in $Spin(7)$ manifolds. We believe these are all possible four-cycles on which D4-branes can wrap in $F(4)$ gauged supergravity. D4-branes on two Riemann surfaces in the previous section fall into a special case of D4-branes on Kähler four-cycles in Calabi-Yau fourfolds. The analogous solutions of M5-branes wrapped on supersymmetric four-cycles were studied in [31, 32].

4.1 Kähler four-cycles in Calabi-Yau fourfolds

We consider the $U(1) \times U(1)$ -invariant truncation presented in section 3.1. We consider the metric,

$$ds^2 = e^{2F(r)} (dt^2 - dr^2) - e^{2G(r)} ds_{M_4}^2 , \tag{4.1}$$

where M_4 is a Kähler four-cycle in Calabi-Yau fourfolds. The curved coordinates on the Kähler four-cycles will be denoted by $\{x_1, x_2, x_3, x_4\}$, and the hatted ones are the flat coordinates. For Kähler four-cycles in Calabi-Yau fourfolds, there are four directions transverse to D4-branes in the fourfolds. The normal bundle of the four-cycle has $U(2) \subset SO(4)$ structure group. We identify $U(1)$ part of the structure group with $U(1)$ gauge field from the non-Abelian $SU(2)$ gauge group, [31, 33]. The only non-vanishing components of the field strength of $SU(2)$ gauge field, A_μ^Λ , $\Lambda = 0, 1, \dots, 6$, are given by

$$\begin{aligned} F_{\hat{x}_1 \hat{x}_2}^3 &= a_1 e^{-2G}, & F_{\hat{x}_3 \hat{x}_4}^3 &= a_2 e^{-2G}, \\ F_{\hat{x}_1 \hat{x}_2}^6 &= b_1 e^{-2G}, & F_{\hat{x}_3 \hat{x}_4}^6 &= b_2 e^{-2G}, \end{aligned} \quad (4.2)$$

where the magnetic charges, a_1, a_2, b_1, b_2 , are constant. The only non-vanishing component of the two-form gauge potential is

$$B_{tr} = -\frac{1}{2m^2} (a_1 a_2 - b_1 b_2) e^{2\sigma+2F-4G}. \quad (4.3)$$

We employ the projection conditions,

$$\gamma^{\hat{r}} \epsilon_A = i \epsilon_A, \quad \gamma^{\hat{x}_1 \hat{x}_2} \sigma_{AB}^3 \epsilon^B = -i \lambda \epsilon_A, \quad \gamma^{\hat{x}_3 \hat{x}_4} \sigma_{AB}^3 \epsilon^B = -i \lambda \epsilon_A, \quad (4.4)$$

where $\lambda = \pm 1$. Solutions with the projection conditions preserve 1/8 of the supersymmetries. By employing the projection conditions, we obtain the complete supersymmetry equations,

$$\begin{aligned} F' e^{-F} &= -\frac{1}{2} (g_1 e^\sigma \cosh \varphi_2 + m e^{-3\sigma}) - \frac{\lambda}{2} e^{-\sigma-2G} (a \cosh \varphi_2 - b \sinh \varphi_2) \\ &\quad - \frac{3}{8m} (a^2 - b^2) e^{\sigma-4G} \\ G' e^{-F} &= -\frac{1}{2} (g_1 e^\sigma \cosh \varphi_2 + m e^{-3\sigma}) + \frac{\lambda}{2} e^{-\sigma-2G} (a \cosh \varphi_2 - b \sinh \varphi_2) \\ &\quad + \frac{1}{8m} (a^2 - b^2) e^{\sigma-4G} \\ \sigma' e^{-F} &= +\frac{1}{2} (g_1 e^\sigma \cosh \varphi_2 - 3m e^{-3\sigma}) + \frac{\lambda}{2} e^{-\sigma-2G} (a \cosh \varphi_2 - b \sinh \varphi_2) \\ &\quad - \frac{1}{8m} (a^2 - b^2) e^{\sigma-4G} \\ \varphi_2' e^{-F} &= +2g_1 e^\sigma \sinh \varphi_2 - 2\lambda e^{-\sigma-2G} (a \sinh \varphi_2 - b \cosh \varphi_2). \end{aligned} \quad (4.5)$$

with the twist conditions,

$$a \equiv a_1 = a_2 = -\frac{k}{2\lambda g_1}, \quad b \equiv b_1 = b_2. \quad (4.6)$$

where k determines the curvature of the Kähler four-cycles in Calabi-Yau fourfolds. There is no condition on b_1 and b_2 .

The product of two Riemann surfaces considered in the previous section is a special case of Kähler four-cycles in Calabi-Yau fourfolds. When we identify $G \equiv G_1 = G_2$ in the supersymmetry equations for D4-branes wrapped on two Riemann surfaces, (3.17), we obtain the supersymmetry equations here, (4.5). By solving the supersymmetry equations, we find the AdS_2 fixed point solutions which are identical to the ones obtained in the previous section.

4.2 Cayley four-cycles in Spin(7) manifolds

4.2.1 The SU(2)-invariant truncation

We considered matter coupled $F(4)$ gauged supergravity coupled to three vector multiplets which has $SU(2)_R \times SU(2)$ gauge symmetry. In this section we truncate the theory to the $SU(2)_{\text{diag}} \subset SU(2)_R \times SU(2)$ invariant sector, which was first considered in [23] and again in section 4.1 of [25]. There is one singlet under $SU(2)_{\text{diag}}$ which corresponds to $Y_{11} + Y_{22} + Y_{33}$ by the non-compact generators defined in (2.21). We exponentiate the non-compact generators and obtain the coset representative,

$$L = e^{\varphi(Y_{11}+Y_{22}+Y_{33})}. \quad (4.7)$$

We also have a non-Abelian $SU(2)$ gauge field and a two-form gauge potential in the $SU(2)_{\text{diag}}$ -invariant truncation. The Lagrangian of the truncation is given by

$$\begin{aligned} e^{-1}\mathcal{L} = & -\frac{1}{4}R + \partial_\mu\sigma\partial^\mu\sigma + \frac{3}{4}\partial_\mu\varphi\partial^\mu\varphi - V \\ & + \frac{1}{8}\sinh^2(2\varphi) [(g_1A_\mu^1 - g_2A_\mu^4)^2 + (g_1A_\mu^2 - g_2A_\mu^5)^2 + (g_1A_\mu^3 - g_2A_\mu^6)^2] \\ & - \frac{1}{8}e^{-2\sigma}\cosh(2\varphi) (F_{\mu\nu}^1F^{1\mu\nu} + F_{\mu\nu}^2F^{2\mu\nu} + F_{\mu\nu}^3F^{3\mu\nu} + F_{\mu\nu}^4F^{4\mu\nu} + F_{\mu\nu}^5F^{5\mu\nu} + F_{\mu\nu}^6F^{6\mu\nu}) \\ & + \frac{1}{4}e^{-2\sigma}\sinh(2\varphi) (F_{\mu\nu}^1F^{4\mu\nu} + F_{\mu\nu}^2F^{5\mu\nu} + F_{\mu\nu}^3F^{6\mu\nu}) - \frac{1}{8}m^2e^{-2\sigma}B_{\mu\nu}B^{\mu\nu} + \frac{3}{64}e^{4\sigma}H_{\mu\nu\rho}H^{\mu\nu\rho} \\ & - \frac{1}{64}\epsilon^{\mu\nu\rho\sigma\tau\kappa}B_{\mu\nu}\left(F_{\rho\sigma}^1F_{\tau\kappa}^1 + F_{\rho\sigma}^2F_{\tau\kappa}^2 + F_{\rho\sigma}^3F_{\tau\kappa}^3 - F_{\rho\sigma}^4F_{\tau\kappa}^4 - F_{\rho\sigma}^5F_{\tau\kappa}^5 - F_{\rho\sigma}^6F_{\tau\kappa}^6 + \frac{1}{3}m^2B_{\rho\sigma}B_{\tau\kappa}\right), \end{aligned} \quad (4.8)$$

where the scalar potential is

$$\begin{aligned} V = & \frac{1}{16}g_1^2e^{2\sigma}(\cosh(6\varphi) - 9\cosh(2\varphi) - 8) + \frac{1}{16}g_2^2e^{2\sigma}(\cosh(6\varphi) - 9\cosh(2\varphi) + 8) \\ & - \frac{1}{2}g_1g_2e^{2\sigma}\sinh^3(2\varphi) - 4g_1me^{-2\sigma}\cosh^3\varphi + 4g_2me^{-2\sigma}\sinh^3\varphi + m^2e^{-6\sigma}. \end{aligned} \quad (4.9)$$

4.2.2 The supersymmetry equations

We consider the metric,

$$ds^2 = e^{2F(r)}(dt^2 - dr^2) - e^{2G(r)}ds_{M_4}^2, \quad (4.10)$$

where M_4 is a Cayley four-cycle in manifolds with Spin(7) holonomy. The curved coordinates on the Cayley four-cycles will be denoted by $\{x_1, x_2, x_3, x_4\}$, and the hatted ones are the flat coordinates. In order to preserve supersymmetry for D4-branes wrapped on Cayley four-cycles in Spin(7) manifolds, we identify self-dual $SU(2)_+$ subgroup of the $SO(4)$ isometry of the four-cycle,

$$SO(4) \rightarrow SU(2)_+ \times SU(2)_-, \quad (4.11)$$

with the non-Abelian SU(2) gauge group, [31, 33]. The self-duality is defined by

$$\gamma_{\mu\nu} = \pm \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} \gamma^{\rho\sigma}, \quad (4.12)$$

and we denoted the self-duality and anti-self-duality by + and −, respectively. For the self-dual part, components are identified by

$$\gamma^{\hat{x}_1\hat{x}_2} = \gamma^{\hat{x}_3\hat{x}_4}, \quad \gamma^{\hat{x}_1\hat{x}_3} = \gamma^{\hat{x}_4\hat{x}_2}, \quad \gamma^{\hat{x}_1\hat{x}_4} = \gamma^{\hat{x}_2\hat{x}_3}. \quad (4.13)$$

The only non-vanishing components of the field strength of the SU(2) gauge field, A_μ^Λ , $\Lambda = 0, 1, \dots, 6$, are given by

$$\begin{aligned} F_{\hat{x}_1\hat{x}_2}^1 &= F_{\hat{x}_3\hat{x}_4}^1 = a_1 e^{-2G}, & F_{\hat{x}_1\hat{x}_2}^4 &= F_{\hat{x}_3\hat{x}_4}^4 = b_1 e^{-2G}, \\ F_{\hat{x}_1\hat{x}_3}^2 &= F_{\hat{x}_4\hat{x}_2}^2 = a_2 e^{-2G}, & F_{\hat{x}_1\hat{x}_3}^5 &= F_{\hat{x}_4\hat{x}_2}^5 = b_2 e^{-2G}, \\ F_{\hat{x}_1\hat{x}_4}^3 &= F_{\hat{x}_2\hat{x}_3}^3 = a_3 e^{-2G}, & F_{\hat{x}_1\hat{x}_4}^6 &= F_{\hat{x}_2\hat{x}_3}^6 = b_3 e^{-2G}, \end{aligned} \quad (4.14)$$

where the magnetic charges, $a_1, a_2, a_3, b_1, b_2, b_3$, are constant. As we have one $\text{SU}(2)_{\text{diag}} \subset \text{SU}(2)_R \times \text{SU}(2)$ non-Abelian gauge field, they are related by

$$g_1 a_1 = g_2 b_1, \quad g_1 a_2 = g_2 b_2, \quad g_1 a_3 = g_2 b_3, \quad (4.15)$$

where g_1 and g_2 are the gauge coupling constants of $\text{SU}(2)_R$ and $\text{SU}(2)$, respectively. The only non-vanishing component of the two-form gauge potential is

$$B_{tr} = -\frac{1}{2m^2} (a_1^2 + a_2^2 + a_3^2 - b_1^2 - b_2^2 - b_3^2) e^{2\sigma+2F-4G}. \quad (4.16)$$

We employ the projection conditions,

$$\begin{aligned} \gamma^{\hat{r}} \epsilon_A &= i \epsilon_A, \\ \gamma^{\hat{x}_1\hat{x}_2} \sigma_{AB}^1 \epsilon^B &= \gamma^{\hat{x}_3\hat{x}_4} \sigma_{AB}^1 \epsilon^B = -i \lambda \epsilon_A, \\ \gamma^{\hat{x}_1\hat{x}_3} \sigma_{AB}^2 \epsilon^B &= \gamma^{\hat{x}_4\hat{x}_2} \sigma_{AB}^2 \epsilon^B = -i \lambda \epsilon_A, \\ \gamma^{\hat{x}_1\hat{x}_4} \sigma_{AB}^3 \epsilon^B &= \gamma^{\hat{x}_2\hat{x}_3} \sigma_{AB}^3 \epsilon^B = -i \lambda \epsilon_A, \end{aligned} \quad (4.17)$$

where $\lambda = \pm 1$. Solutions with the projection conditions preserve 1/16 of the supersymmetries.

Employing the projection conditions, from $\delta \lambda_A^I = 0$, we obtain for $I = 1, 2, 3$, respectively,

$$\begin{aligned} \varphi' e^{-F} &= + (g_1 e^\sigma \cosh \varphi - g_2 e^\sigma \sinh \varphi) \sinh(2\varphi) - 2\lambda e^{-\sigma-2G} (a_1 \sinh \varphi - b_1 \cosh \varphi), \\ \varphi' e^{-F} &= + (g_1 e^\sigma \cosh \varphi - g_2 e^\sigma \sinh \varphi) \sinh(2\varphi) - 2\lambda e^{-\sigma-2G} (a_2 \sinh \varphi - b_2 \cosh \varphi), \\ \varphi' e^{-F} &= + (g_1 e^\sigma \cosh \varphi - g_2 e^\sigma \sinh \varphi) \sinh(2\varphi) - 2\lambda e^{-\sigma-2G} (a_3 \sinh \varphi - b_3 \cosh \varphi). \end{aligned} \quad (4.18)$$

Therefore, we conclude that the magnetic charges are

$$a \equiv a_1 = a_2 = a_3, \quad b \equiv b_1 = b_2 = b_3. \quad (4.19)$$

We present the complete supersymmetry equations,

$$\begin{aligned}
 F' e^{-F} &= -\frac{1}{2} (g_1 e^\sigma \cosh^3 \varphi - g_2 e^\sigma \sinh^3 \varphi + m e^{-3\sigma}) - \frac{3\lambda}{2} e^{-\sigma-2G} (a \cosh \varphi - b \sinh \varphi) \\
 &\quad - \frac{9}{8m} (a^2 - b^2) e^{\sigma-4G}, \\
 G' e^{-F} &= -\frac{1}{2} (g_1 e^\sigma \cosh^3 \varphi - g_2 e^\sigma \sinh^3 \varphi + m e^{-3\sigma}) + \frac{3\lambda}{2} e^{-\sigma-2G} (a \cosh \varphi - b \sinh \varphi) \\
 &\quad + \frac{3}{8m} (a^2 - b^2) e^{\sigma-4G}, \\
 \sigma' e^{-F} &= +\frac{1}{2} (g_1 e^\sigma \cosh^3 \varphi - g_2 e^\sigma \sinh^3 \varphi - 3m e^{-3\sigma}) + \frac{3\lambda}{2} e^{-\sigma-2G} (a \cosh \varphi - b \sinh \varphi) \\
 &\quad - \frac{3}{8m} (a^2 - b^2) e^{\sigma-4G}, \\
 \varphi' e^{-F} &= + (g_1 e^\sigma \cosh \varphi - g_2 e^\sigma \sinh \varphi) \sinh(2\varphi) - 2\lambda e^{-\sigma-2G} (a \sinh \varphi - b \cosh \varphi). \quad (4.20)
 \end{aligned}$$

We also obtain twist conditions on the magnetic charges,

$$a = -\frac{k}{6\lambda g_1}, \quad b = -\frac{k}{6\lambda g_2}, \quad (4.21)$$

where k determines the curvature of the Cayley four-cycles in Spin(7) manifolds. The twist condition on b comes from the $SU(2)_{\text{diag}}$ condition, (4.15).

4.2.3 The AdS_2 solutions

Now we will consider the $\mathcal{N} = 4^+$ theory, $g_1 > 0$, $m > 0$. When $b = 0$, we find an AdS_2 fixed point solution for $k = -1$,

$$e^F = \frac{3^{1/4}}{2^{3/2} g_1^{3/4} m^{1/4} r}, \quad e^G = \frac{1}{2^{1/2} 3^{1/4} g_1^{3/4} m^{1/4}}, \quad e^\sigma = \frac{2^{1/2} m^{1/4}}{3^{1/4} g_1^{1/4}}, \quad e^\varphi = 1. \quad (4.22)$$

After the reparametrization of the parameters given in (3.20), this is the AdS_2 solution first found in [11].

When we consider for non-zero b , we find new AdS_2 solutions in terms of the scalar field, φ ,

$$\begin{aligned}
 e^F &= \frac{e^{-\sigma}}{2(g_1 \cosh^3 \varphi - g_2 \sinh^3 \varphi) r}, \\
 e^{2G} &= \frac{3\lambda}{2m} e^{2\sigma} (a \cosh \varphi - b \sinh \varphi), \\
 e^{4\sigma} &= \frac{m}{6} \frac{5(a^2 - b^2) + 3(a^2 + b^2) \cosh(2\varphi) - 6ab \sinh(2\varphi)}{(g_1 \cosh^3 \varphi - g_2 \sinh^3 \varphi)(a \cosh \varphi - b \sinh \varphi)^2}. \quad (4.23)
 \end{aligned}$$

Then, the scalar field, φ , should be expressed in terms of the magnetic charges, a and b , but the expression is very unwieldy. Alternatively, we present the magnetic charge, b , in terms of the scalar field, φ ,

$$\begin{aligned}
 b &= -a \left[g_1 (1 + e^{2\varphi})^3 (1 - 6e^{2\varphi} + e^{4\varphi}) + g_2 (1 - e^{2\varphi})^3 (1 + 6e^{2\varphi} + e^{4\varphi}) \right. \\
 &\quad \left. + 4e^{2\varphi} \sqrt{g_1^2 (1 + e^{2\varphi})^4 (5 - 6e^{2\varphi} + 5e^{4\varphi}) + g_2^2 (1 - e^{2\varphi})^4 (5 + 6e^{2\varphi} + 5e^{4\varphi}) + 10g_1 g_2 (1 - e^{4\varphi})^3} \right] \\
 &\quad / \left[(1 - e^{4\varphi}) (g_1 (1 - 13e^{2\varphi} - 13e^{4\varphi} + e^{6\varphi}) + g_2 (1 - 7e^{2\varphi} + 7e^{4\varphi} - e^{6\varphi})) \right]. \quad (4.24)
 \end{aligned}$$

The solutions are parametrized by two magnetic charges, a and b . It will be interesting to have a field theory interpretation of this AdS_2 fixed point solution.

Unlike the black holes with a horizon of two Riemann surfaces, for this case, we could not devise a way to determine the positivity range for AdS_2 solutions. However, as we see in the next subsection, we obtained a number of AdS_2 solutions with negative curvature horizon, $k = -1$, numerically. On the other hand, we could not find any solutions with positive curvature horizon, $k = +1$. Thus, we will concentrate on solutions with negative curvature horizon, $k = -1$.

4.2.4 Numerical black hole solutions

Now we present the full black hole solution numerically. The full black hole solution is an interpolating geometry between the asymptotically AdS_6 boundary and the $AdS_2 \times Cayley_4$ horizon. As we explained at the end of the last subsection, we will concentrate on solutions with negative curvature horizon, $k = -1$. We introduce a new radial coordinate,

$$\rho = F + \sigma. \quad (4.25)$$

This kind of coordinate was introduced in [30]. Employing the supersymmetry equations, we obtain

$$\frac{\partial \rho}{\partial r} = F' + \sigma' = -e^F D, \quad (4.26)$$

where we define

$$D = 2me^{-3\sigma} + \frac{3}{2m} (a^2 - b^2) e^{\sigma-4G}. \quad (4.27)$$

Then, the supersymmetry equations are

$$\begin{aligned} -D \frac{\partial F}{\partial \rho} &= -\frac{1}{2} (g_1 e^\sigma \cosh^3 \varphi - g_2 e^\sigma \sinh^3 \varphi + m e^{-3\sigma}) - \frac{3\lambda}{2} e^{-\sigma-2G} (a \cosh \varphi - b \sinh \varphi) \\ &\quad - \frac{9}{8m} (a^2 - b^2) e^{\sigma-4G}, \\ -D \frac{\partial G}{\partial \rho} &= -\frac{1}{2} (g_1 e^\sigma \cosh^3 \varphi - g_2 e^\sigma \sinh^3 \varphi + m e^{-3\sigma}) + \frac{3\lambda}{2} e^{-\sigma-2G} (a \cosh \varphi - b \sinh \varphi) \\ &\quad + \frac{3}{8m} (a^2 - b^2) e^{\sigma-4G}, \\ -D \frac{\partial \sigma}{\partial \rho} &= +\frac{1}{2} (g_1 e^\sigma \cosh^3 \varphi - g_2 e^\sigma \sinh^3 \varphi - 3m e^{-3\sigma}) + \frac{3\lambda}{2} e^{-\sigma-2G} (a \cosh \varphi - b \sinh \varphi) \\ &\quad - \frac{3}{8m} (a^2 - b^2) e^{\sigma-4G}, \\ -D \frac{\partial \varphi_2}{\partial \rho} &= + (g_1 e^\sigma \cosh \varphi - g_2 e^\sigma \sinh \varphi) \sinh(2\varphi) - 2\lambda e^{-\sigma-2G} (a \sinh \varphi - b \cosh \varphi). \end{aligned} \quad (4.28)$$

In the r -coordinate, the UV or asymptotically AdS_6 boundary is at $r = 0$, and the IR or $AdS_2 \times Cayley_4$ horizon is at $r = \infty$. In this ρ -coordinate, the UV is at $\rho = +\infty$, and the IR is at $\rho = -\infty$. We set $m = 1/2$ and $g_1 = 3m$ to have a unit radius for the AdS_6 boundary. Then, a is determined by (4.21), and there is one free parameter left, b . Employing (4.23) and (4.24) to determine boundary conditions, we solve the supersymmetry

Figure 3. Numerical black hole solutions with $m = 1/2$ and $g = 3m$. We have $b = -0.1$ for purple, $b = -0.3$ for red, and $b = -0.5$ for orange.

equations numerically. With some choices of b , we present representative plots of the full black hole solutions in figure 3.

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A The equations of motion

In this appendix, we present the equations of motion for the $U(1) \times U(1)$ -invariant truncation in (4.8) with $\varphi_1 = 0$ as in (3.16),

$$\begin{aligned}
 R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = & 2 \left[Vg_{\mu\nu} + 2 \left(\partial_\mu \sigma \partial_\nu \sigma - \frac{1}{2}g_{\mu\nu}g^{\rho\sigma} \partial_\rho \sigma \partial_\sigma \sigma \right) \right. \\
 & + \frac{1}{2} \left(\partial_\mu \varphi_2 \partial_\nu \varphi_2 - \frac{1}{2}g_{\mu\nu}g^{\rho\sigma} \partial_\rho \varphi_2 \partial_\sigma \varphi_2 \right) \\
 & - \frac{1}{2}e^{-2\sigma} \cosh(2\varphi_2) \left(g^{\rho\sigma} F_{\mu\rho}^3 F_{\nu\sigma}^3 - \frac{1}{4}g_{\mu\nu} F_{\rho\sigma}^3 F^{3\rho\sigma} \right) \\
 & - \frac{1}{2}e^{-2\sigma} \cosh(2\varphi_2) \left(g^{\rho\sigma} F_{\mu\rho}^6 F_{\nu\sigma}^6 - \frac{1}{4}g_{\mu\nu} F_{\rho\sigma}^6 F^{6\rho\sigma} \right) \\
 & + e^{-2\sigma} \sinh(2\varphi_2) \left(g^{\rho\sigma} F_{\mu\rho}^3 F_{\nu\sigma}^6 - \frac{1}{4}g_{\mu\nu} F_{\rho\sigma}^3 F^{6\rho\sigma} \right) \\
 & \left. - \frac{1}{2}m^2 e^{-2\sigma} \left(g^{\rho\sigma} B_{\mu\rho} B_{\nu\sigma} - \frac{1}{4}g_{\mu\nu} B_{\rho\sigma} B^{\rho\sigma} \right) \right], \tag{A.1}
 \end{aligned}$$

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\sigma) + \frac{1}{2}\frac{\partial V}{\partial\sigma} - \frac{1}{8}e^{-2\sigma}\cosh(2\varphi_2)F_{\mu\nu}^3F^{3\mu\nu} - \frac{1}{8}e^{-2\sigma}\cosh(2\varphi_2)F_{\mu\nu}^6F^{6\mu\nu} + \frac{1}{4}e^{-2\sigma}\cosh(2\varphi_2)F_{\mu\nu}^3F^{6\mu\nu} - \frac{1}{8}m^2e^{-2\sigma}B_{\mu\nu}B^{\mu\nu} = 0, \quad (\text{A.2})$$

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\varphi_2) + 2\frac{\partial V}{\partial\varphi_2} + \frac{1}{2}e^{-2\sigma}\cosh(2\varphi_2)F_{\mu\nu}^3F^{3\mu\nu} + \frac{1}{2}e^{-2\sigma}\cosh(2\varphi_2)F_{\mu\nu}^6F^{6\mu\nu} - e^{-2\sigma}\cosh(2\varphi_2)F_{\mu\nu}^3F^{6\mu\nu} = 0. \quad (\text{A.3})$$

$$\mathcal{D}_\nu(e^{-2\sigma}F^{\Lambda\nu\mu}) = \frac{1}{24}e\epsilon^{\mu\nu\rho\sigma\tau\kappa}F_{\nu\rho}^\Lambda H_{\sigma\tau\kappa}, \quad (\text{A.4})$$

$$\mathcal{D}_\rho(e^{4\sigma}H^{\rho\mu\nu}) = -\frac{1}{16}e\epsilon^{\mu\nu\rho\sigma\tau\kappa}\eta_{\Lambda\Sigma}F_{\rho\sigma}^\Lambda F_{\tau\kappa}^\Sigma - me^{-2\sigma}\delta^{0\Lambda}F^{\Lambda\mu\nu}. \quad (\text{A.5})$$

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