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Supersymmetric Dirac-Born-Infeld axionic inflation and non-Gaussianity

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ABSTRACT: An analysis is given of inflation based on a supersymmetric Dirac-Born-Infeld (DBI) action in an axionic landscape. The DBI model we discuss involves a landscape of chiral superfields with one $U(1)$ shift symmetry which is broken by instanton type non-perturbative terms in the superpotential. Breaking of the shift symmetry leads to one pseudo-Nambu-Goldstone-boson which acts as the inflaton while the remaining normalized phases of the chiral fields generically labeled axions are invariant under the $U(1)$ shift symmetry. The analysis is carried out in the vacuum with stabilized saxions, which are the magnitudes of the chiral fields. Regions of the parameter space where slow-roll inflation occurs are exhibited and the spectral indices as well as the ratio of the tensor to the scalar power spectrum are computed. An interesting aspect of supersymmetric DBI models analyzed is that in most of the parameter space tensor to scalar ratio and scalar spectral index are consistent with Planck data if slow roll occurs and is not eternal. Also interesting is that the ratio of the tensor to the scalar power spectrum can be large and can lie close to the experimental upper limit and thus testable in improved experiment. Non-Gaussianity in this class of models is explored.

KEYWORDS: Supersymmetry Phenomenology

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Contents

1	Introduction	1
2	Fast-roll and slow-roll decomposition	2
3	Supersymmetric DBI action for two chiral fields	3
4	Pressure, density and inflation equations	9
5	Slow roll parameters and non-Gaussianity	10
6	Model simulation and experimental test	12
7	Conclusion	16
A	Single field supersymmetric DBI Lagrangian	17
B	Equations for F_k, F_k^* ($k = 1, 2$)	18
C	DBI scalar potential with derivative terms absent	20

1 Introduction

As is well known many of the problems associated with Big Bang cosmology which include the flatness problem, the horizon problem, and the monopole problem are resolved by inflation [1–6]. Quantum fluctuations at the time of horizon exit carry significant information regarding specifics of the inflationary model [7–12] which can be extracted from cosmic microwave background (CMB) radiation anisotropy. The data from the Planck experiment [13–15] has helped constrain inflation models excluding some and narrowing down the parameter space of others. One such model is so called natural inflation based on a $U(1)$ shift symmetry which is described by a simple potential [16, 17] $V(a) = \Lambda^4 \left(1 + \cos\left(\frac{a}{f}\right)\right)$, where a is the axion field and f is the axion decay constant. In this case consistency with Planck data requires the axion decay constant to be significantly greater than the Planck mass M_P . However, an axion decay constant larger than the Planck mass is undesirable since a global symmetry is not preserved by quantum gravity unless it has a gauge origin. Additionally string theory prefers the axion decay constant to lie below M_P [18, 19]. It turns out that the reduction of the decay constant poses a problem, and several suggestions exist regarding its resolution such as the so called alignment mechanism [20, 21].

A procedure for resolving the axion decay constant problem was proposed in [22]. One element of this analysis relies on a decomposition of the potential into a fast-roll

and a slow-roll parts where the slow roll is controlled only by the inflaton field while the remaining fields enter in fast roll and are not relevant for inflation [22] (for a review of inflation in supersymmetric theories see, e.g., [23]). An analysis within this model shows that one can obtain spectral indices as well as the ratio of the tensor to the scalar power spectrum consistent with the Planck data [13–15]. Another quantity of interest in primordial perturbations is the so-called non-Gaussianity [24–29]. It is known that models with canonical kinetic energy do not lead to non-Gaussianity and for non-Gaussianity one needs models with non-canonical kinetic energy. In this context the Dirac-Born-Infeld (DBI) models are of interest (see, e.g., [30–34]) which is the object of study in this work. Our work is focused on using shift symmetry and axions for inflation. For a partial list of other works where shift symmetry of axions is utilized in inflation see [35–41] and in the string context see [42–44]. For reviews of axionic cosmology see [45, 46].

The outline of the rest of the paper is as follows: in section 2 we give a summary of previous results on the decomposition of a landscape of axion fields which undergo shifts under a U(1) global transformation into fast-roll and slow-roll parts. This is one of the central elements in the analysis of the inflationary models we discuss later. In section 3 we give a description of the supersymmetric DBI Lagrangian in superspace for the case of two chiral fields Φ_1 and Φ_2 which are oppositely charged under a U(1) global symmetry. We then display the bosonic part of the Lagrangian after integration over the Grassmann coordinates. Here it is shown that the Lagrangian depends on the dimensionless parameters $\alpha_1, \alpha_2, \alpha_3$; and T which has the dimension of the fourth power of mass. The general case including $\alpha_1, \alpha_2, \alpha_3$ is too complicated to discuss analytically and thus here the analysis is given taking into account only the α_1 terms. In section 4 we discuss the pressure, density and the inflation equations for a generic DBI model. In section 5 we discuss the slow-roll parameters, non-Gaussianity, and the speed of sound which enters in defining non-Gaussianity. Model simulations and experimental test of the two field DBI model is discussed in section 6, and conclusions are given in section 7. Further, details of the analysis are given in appendices A, B, and C.

2 Fast-roll and slow-roll decomposition

Before discussing the supersymmetric DBI model we summarize first the slow-roll and fast-roll decomposition of the inflation potential which is one of the central components of the analysis of this paper for the DBI case. As noted above the slow-roll and fast-roll decomposition of the potential was introduced in [22]. This analysis utilizes a landscape of pairs of chiral fields which are charged under a U(1) global symmetry. Thus suppose we have a set of chiral fields Φ_i ($i = 1, \dots, n$) where Φ_i carry the same charge under the shift symmetry and the fields $\tilde{\Phi}_i$ ($i = 1, \dots, n$) carry the opposite charge. We assume that under U(1) transformations the fields transform as follows

$$\Phi_i \rightarrow e^{iq\lambda} \Phi_i, \quad \tilde{\Phi}_i \rightarrow e^{-iq\lambda} \tilde{\Phi}_i, \quad i = 1, \dots, n. \quad (2.1)$$

The superfields Φ_i have an expansion,

$$\Phi_i = \phi_i + \theta\chi_i + \theta\theta F_i, \quad (2.2)$$

where ϕ_i is a complex scalar field consisting of the saxion (the magnitude) and the axion (the normalized phase), χ_i is the axino, and F_i is an auxiliary field. Similarly the superfields $\tilde{\Phi}_i$ have an expansion: $\tilde{\Phi}_i = \tilde{\phi}_i + \tilde{\theta}\tilde{\chi}_i + \tilde{\theta}\tilde{\theta}\tilde{F}_i$. We may parametrize ϕ_i and $\tilde{\phi}_i$ so that

$$\phi_i = \frac{1}{\sqrt{2}}(f_i + \rho_i)e^{ia_i/f_i}, \quad \tilde{\phi}_i = \frac{1}{\sqrt{2}}(\tilde{f}_i + \tilde{\rho}_i)e^{i\tilde{a}_i/\tilde{f}_i}, \quad (2.3)$$

where $f_i = \langle \phi_i \rangle$, $\tilde{f}_i = \langle \tilde{\phi}_i \rangle$ and (ρ_i, a_i) and $(\tilde{\rho}_i, \tilde{a}_i)$ are the fluctuations of the quantum fields around their vacuum expectation values $f_i, \tilde{f}_i$. The above constitute $2n$ number of axionic fields $a_1, \dots, a_n$ and $\tilde{a}_1, \dots, \tilde{a}_n$. Since there is only one U(1) shift symmetry, we can pick a basis where only one linear combination of it is variant under the shift symmetry and all others are invariant. We label this new basis $a_-, a_+, b_1, b_2, \dots, b_{n-1}, \tilde{b}_1, \tilde{b}_2, \dots, \tilde{b}_{n-1}$ where only a_- is sensitive to the shift symmetry. Thus the object of central interest is the field a_- which is the pseudo-Nambu-Goldstone-Boson (pNGB) and acts as the inflaton. It can be expressed in terms of the original set of axion fields as below

$$a_- = \frac{1}{f_e} \left(\sum_{i=1}^m f_i a_i - \sum_{i=1}^m \tilde{f}_i \tilde{a}_i \right), \quad (2.4)$$

$$f_e = \sqrt{\sum_{i=1}^m f_i^2 + \sum_{i=1}^m \tilde{f}_i^2}. \quad (2.5)$$

The relation eq. (2.5) was derived in [22] (see also [47]). The result of eq. (2.5) gives $f_e = \sqrt{N}f$ for the case when $f_i = \tilde{f}_i = f$ and $N = 2m$ which is the N-flation result but derived here in a different context [48, 49].

3 Supersymmetric DBI action for two chiral fields

Supersymmetric DBI actions have been investigated by a number of authors (see, e.g., [50–59]). Here we discuss the supersymmetric DBI in the context of axion inflation. The case of a single field DBI is given in appendix A. Here we consider a pair of chiral superfields Φ_1 and Φ_2 which carry opposite charges under a global U(1) symmetry. The supersymmetric Lagrangian involving Φ_1 and Φ_2 is given by

$$\mathcal{L} = \mathcal{L}_D + \mathcal{L}_F, \quad (3.1)$$

where $\mathcal{L}_D$ is the D-part of the Lagrangian and $\mathcal{L}_F$ is the F-part. We consider the D-part consisting of a part $\mathcal{L}_D^{(1)}$ which is quadratic in the fields and a part $\mathcal{L}_D^{(2)}$ which is quartic in the fields so that

$$\mathcal{L}_D = \mathcal{L}_D^{(1)} + \mathcal{L}_D^{(2)}, \quad (3.2)$$

where $\mathcal{L}_D^{(1)}$ and $\mathcal{L}_D^{(2)}$ are invariant under the U(1) symmetry and are given by

$$\mathcal{L}_D^{(1)} = \int d^4\theta \left(\Phi_1 \Phi_1^\dagger + \Phi_2 \Phi_2^\dagger \right) \quad (3.3)$$

and

$$\mathcal{L}_D^{(2)} = \mathcal{L}_D^{(2a)} + \mathcal{L}_D^{(2b)} + \mathcal{L}_D^{(2c)} + \mathcal{L}_D^{(2d)} + \mathcal{L}_D^{(2e)}, \quad (3.4)$$

where

$$\begin{aligned} \mathcal{L}_D^{(2a)} &= \int d^4\theta \frac{\alpha_1}{16T} (D^\alpha \Phi_1 D_\alpha \Phi_1) \left(\bar{D}^{\dot{\alpha}} \Phi_1^\dagger \bar{D}_{\dot{\alpha}} \Phi_1^\dagger \right) G(\phi), \\ \mathcal{L}_D^{(2b)} &= \int d^4\theta \frac{\alpha_1}{16T} (D^\alpha \Phi_2 D_\alpha \Phi_2) \left(\bar{D}^{\dot{\alpha}} \Phi_2^\dagger \bar{D}_{\dot{\alpha}} \Phi_2^\dagger \right) G(\phi), \\ \mathcal{L}_D^{(2c)} &= \int d^4\theta \frac{\alpha_2}{16T} (D^\alpha \Phi_1 D_\alpha \Phi_1) \left(\bar{D}^{\dot{\alpha}} \Phi_2^\dagger \bar{D}_{\dot{\alpha}} \Phi_2^\dagger \right) G(\phi), \\ \mathcal{L}_D^{(2d)} &= \int d^4\theta \frac{\alpha_2}{16T} (D^\alpha \Phi_2 D_\alpha \Phi_2) \left(\bar{D}^{\dot{\alpha}} \Phi_1^\dagger \bar{D}_{\dot{\alpha}} \Phi_1^\dagger \right) G(\phi), \\ \mathcal{L}_D^{(2e)} &= \int d^4\theta \frac{\alpha_3}{16T} (D^\alpha \Phi_1 D_\alpha \Phi_2) \left(\bar{D}^{\dot{\alpha}} \Phi_1^\dagger \bar{D}_{\dot{\alpha}} \Phi_2^\dagger \right) G(\phi). \end{aligned} \quad (3.5)$$

Here

$$G(\phi) = \frac{1}{T} \frac{1}{1 + A + \sqrt{(1 + A)^2 - B}}, \quad (3.6)$$

and A and B are assumed to have the following forms

$$\begin{aligned} A &= (\partial_a \phi_1 \partial^a \phi_1^* + \partial_a \phi_2 \partial^a \phi_2^*)/T, \\ B &= \left(\alpha_1 \partial_a \phi_1 \partial^a \phi_1 \partial_b \phi_1^* \partial^b \phi_1^* + \alpha_1 \partial_a \phi_2 \partial^a \phi_2 \partial_b \phi_2^* \partial^b \phi_2^* + \alpha_2 \partial_a \phi_1 \partial^a \phi_1 \partial_b \phi_2^* \partial^b \phi_2^* \right. \\ &\quad \left. + \alpha_2 \partial_a \phi_2 \partial^a \phi_2 \partial_b \phi_1^* \partial^b \phi_1^* + \alpha_3 \partial_a \phi_1 \partial^a \phi_2 \partial_b \phi_1^* \partial^b \phi_2^* \right) / T^2, \end{aligned} \quad (3.7)$$

where the supercovariant derivatives $D^\alpha \Phi$ etc are defined in an explicit manner in appendix A. We note that the Lagrangian of eq. (3.5) is a direct generalization of the Lagrangian for the single field case (see appendix A) which can be derived from a more basic 3-brane action (see, e.g., [54, 55, 58] and the references therein). Here we simply extend the analysis to two fields in the most general supersymmetric form involving four covariant derivatives. In writing eq. (3.5) we imposed an additional constraint which is invariance under Φ_1 and Φ_2 interchange. The possible relation of this Lagrangian to an underlying string model is an open question. Here we simply treat eq. (3.5) as an effective low energy theory. Finally $\mathcal{L}_F$ is given by

$$\mathcal{L}_F = \int d^2\theta W(\Phi_1, \Phi_2) + \int d^2\bar{\theta} W^*(\Phi_1^\dagger, \Phi_2^\dagger), \quad (3.8)$$

where the superpotential W is given by

$$W = W_s + W_{sb}. \quad (3.9)$$

Here W_s is invariant under the global U(1) symmetry and is taken to be of the form

$$W_s = \mu \Phi_1 \Phi_2 + \frac{\lambda}{2} (\Phi_1 \Phi_2)^2. \quad (3.10)$$

The form eq. (3.10) is chosen so that we can stabilize the saxion VEVs. Eq. (3.10) also explains why we need a pair of chiral fields with opposite U(1) charges because with a single chiral field which is charged under a U(1) symmetry we cannot form a non-trivial W_s , needed for stabilizing the saxions, which is invariant under the U(1) symmetry. W_{sb} breaks the global U(1) symmetry and is taken to be of the form

$$W_{sb} = \sum_{k=1}^m \left(A_{1,k} \Phi_1^k + A_{2,k} \Phi_2^k \right). \quad (3.11)$$

We note in passing that the superpotential of the type eqs. (3.9)–(3.11) was considered in [22, 60]. Integration over θ' s gives for the full Lagrangian

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_D + \mathcal{L}_F = \mathcal{L}_I + \mathcal{L}_{II}, \\ \mathcal{L}_I &= T - T \sqrt{(1+A)^2 - B}, \\ \mathcal{L}_{II} &= F_1 F_1^* + F_2 F_2^* + G(\phi) \left[\alpha_1 (-2F_1 F_1^* \partial_a \phi_1 \partial^a \phi_1^* + F_1^2 F_1^{*2}) + \alpha_1 (-2F_2 F_2^* \partial_a \phi_2 \partial^a \phi_2^* \right. \\ &\quad \left. + F_2^2 F_2^{*2}) + \alpha_2 (-2F_1 F_2^* \partial_a \phi_1 \partial^a \phi_2^* + F_1^2 F_2^{*2} - 2F_2 F_1^* \partial_a \phi_2 \partial^a \phi_1^* + F_2^2 F_1^{*2}) \right. \\ &\quad \left. + \alpha_3 (-F_1 F_1^* \partial_a \phi_2 \partial^a \phi_2^* - F_2 F_2^* \partial_a \phi_1 \partial^a \phi_1^* + F_1 F_2 F_1^* F_2^*) \right] + \left(\frac{\partial W}{\partial \phi_1} F_1 + \frac{\partial W}{\partial \phi_2} F_2 + \text{h.c.} \right), \end{aligned} \quad (3.12)$$

where $\frac{\partial W}{\partial \phi_k}$ ($k = 1, 2$) are given by

$$\begin{aligned} \frac{\partial W}{\partial \phi_1} &= \mu \phi_2 + \lambda (\phi_1 \phi_2) \phi_2 + \sum_{k=1}^m k A_{1,k} \phi_1^{k-1}, \\ \frac{\partial W}{\partial \phi_2} &= \mu \phi_1 + \lambda (\phi_1 \phi_2) \phi_1 + \sum_{k=1}^m k A_{2,k} \phi_2^{k-1}. \end{aligned}$$

There are four auxiliary fields in eq. (3.12) which are F_1, F_1^*, F_2, F_2^* . The field equations obtained by varying F_1, F_1^*, F_2, F_2^* are given in appendix B. These are coupled equations involving all the F 's and F^* 's and solving them is non-trivial. Much simplicity results if we set $\alpha_2 = 0 = \alpha_3$. In this case as shown in appendix B the auxiliary fields F_k satisfy the cubic equation

$$F_k^3 + p_k F_k + q_k = 0, \quad k = 1, 2, \quad (3.13)$$

where p_k, q_k are defined by

$$\begin{aligned} p_k &= \left(\frac{\partial W}{\partial \phi_k} \right)^{-1} \frac{\partial W^*}{\partial \phi_k^*} \frac{1 - 2\alpha_1 G(\phi) \partial_\mu \phi_k \partial^\mu \phi_k}{2\alpha_1 G(\phi)}, \\ q_k &= \frac{1}{2\alpha_1 G(\phi)} \left(\frac{\partial W}{\partial \phi_k} \right)^{-1} \left(\frac{\partial W^*}{\partial \phi_k^*} \right)^2. \end{aligned} \quad (3.14)$$

Since F_k satisfies a cubic equation, there are three roots which are given by

$$F_k = \omega^j \left(-\frac{q_k}{2} + \sqrt{\left(\frac{q_k}{2}\right)^2 + \left(\frac{p_k}{3}\right)^3} \right)^{1/3} + \omega^{3-j} \left(-\frac{q_k}{2} - \sqrt{\left(\frac{q_k}{2}\right)^2 + \left(\frac{p_k}{3}\right)^3} \right)^{1/3}, \quad (3.15)$$

where ω is the cube root of unity and $j = 0, 1, 2$. Naively, it appears there are three solutions for F_k . However, as exhibited in appendix B, only $j = 0$ is a solution to the full Euler-Lagrange equations for F . Setting the derivative terms to zero, the scalar potential of the theory for this case can be computed and is exhibited in appendix C. As an expansion in $1/T$, the F_k takes the form

$$F_k = -\frac{\partial W^*}{\partial \phi_k^*} + \frac{1}{T} \left(\frac{\partial W}{\partial \phi_k} \right) \left(\frac{\partial W^*}{\partial \phi_k^*} \right)^2 - \frac{3}{T^2} \left(\frac{\partial W}{\partial \phi_k} \right)^2 \left(\frac{\partial W^*}{\partial \phi_k^*} \right)^3 + \frac{12}{T^3} \left(\frac{\partial W}{\partial \phi_k} \right)^3 \left(\frac{\partial W^*}{\partial \phi_k^*} \right)^4 - \frac{55}{T^4} \left(\frac{\partial W}{\partial \phi_k} \right)^4 \left(\frac{\partial W^*}{\partial \phi_k^*} \right)^5 + \frac{273}{T^5} \left(\frac{\partial W}{\partial \phi_k} \right)^5 \left(\frac{\partial W^*}{\partial \phi_k^*} \right)^6 + \mathcal{O}\left(\frac{1}{T^6}\right). \quad (3.16)$$

Further the scalar potential when expanded in powers of $1/T$ takes the form

$$V(\phi) = \sum_{k=1}^2 \left[\frac{\partial W}{\partial \phi_k} \frac{\partial W^*}{\partial \phi_k^*} - \frac{1}{2T} \left(\frac{\partial W}{\partial \phi_k} \frac{\partial W^*}{\partial \phi_k^*} \right)^2 + \frac{1}{T^2} \left(\frac{\partial W}{\partial \phi_k} \frac{\partial W^*}{\partial \phi_k^*} \right)^3 - \frac{3}{T^3} \left(\frac{\partial W}{\partial \phi_k} \frac{\partial W^*}{\partial \phi_k^*} \right)^4 + \frac{11}{T^4} \left(\frac{\partial W}{\partial \phi_k} \frac{\partial W^*}{\partial \phi_k^*} \right)^5 - \frac{91}{2T^5} \left(\frac{\partial W}{\partial \phi_k} \frac{\partial W^*}{\partial \phi_k^*} \right)^6 + \mathcal{O}\left(\frac{1}{T^6}\right) \right]. \quad (3.17)$$

Thus as $T \rightarrow \infty$ we recover the conventional results for F_k and $V(\phi)$. Further, the above analysis also implies stability conditions so that

$$\frac{\partial W}{\partial \phi_k} = 0 = \frac{\partial W^*}{\partial \phi_k^*}, \quad k = 1, 2. \quad (3.18)$$

We use eq. (3.18) for stabilizing the saxions. Thus we parametrize ϕ_k so that

$$\phi_k = \frac{1}{\sqrt{2}} (f_k + \rho_k) e^{ia_k/f_k}, \quad k = 1, 2, \quad (3.19)$$

where ρ_k are the saxion fields, a_k are the axions and f_k are the axion decay constants. For the case of the assumed superpotential the stabilization conditions on a CP conserving vacuum are

$$\mu f_2 + \frac{1}{2} \lambda f_1 f_2^2 + \sum_{k=1}^m \frac{1}{2^{k/2-1}} k A_{1,k} f_1^{k-1} = 0, \quad (3.20)$$

$$\mu f_1 + \frac{1}{2} \lambda f_1^2 f_2 + \sum_{k=1}^m \frac{1}{2^{k/2-1}} k A_{2,k} f_2^{k-1} = 0. \quad (3.21)$$

Eqs. (3.20) and (3.21) provide the stability conditions for the radial modes ρ_1 and ρ_2 . Here the first two terms on the left hand side of each of the above equations arise from W_s while the last term involving the sum arises from W_{sb} . For arbitrary m these equations cannot be solved in a closed form. However, we can solve for f_i in a perturbative expansion where we treat the contribution arising from W_{sb} as the perturbation. A significant simplicity results if we assume $A_k \equiv A_{1,k} = A_{2,k}$. In this case $f \equiv f_1 = f_2$ and we find that

$$f = f_0 - \frac{2}{3\lambda} \sum_k \frac{kA_k}{2^{k/2-1}} f_0^{k-3} + \dots$$

$$f_0 \equiv \pm \sqrt{\frac{-2\mu}{\lambda}}. \quad (3.22)$$

Eq. (3.22) exhibits the value of the stability point f of the radial modes in terms of the input parameters of the superpotential including W_s and W_{sb} .

For the case of two fields, we have F_1 and F_2 , which can be solved using eq. (3.15). For our choice of W , we can evaluate p_k and q_k ($k=1,2$) explicitly so that we have

$$p_1 = \frac{\mu\phi_2^* + \lambda\phi_1^*\phi_2^{*2} + \sum_{k=1}^m kA_{1,k}\phi_1^{*k-1}}{\mu\phi_2 + \lambda\phi_1\phi_2^2 + \sum_{k=1}^m kA_{1,k}\phi_1^{k-1}} \frac{1 - 2\alpha_1 G(\phi) \partial_\mu \phi_1 \partial^\mu \phi_1^*}{2\alpha_1 G(\phi)}, \quad (3.23)$$

$$p_2 = \frac{\mu\phi_1^* + \lambda\phi_1^{*2}\phi_2^* + \sum_{k=1}^m kA_{2,k}\phi_2^{*k-1}}{\mu\phi_1 + \lambda\phi_1^2\phi_2 + \sum_{k=1}^m kA_{2,k}\phi_2^{k-1}} \frac{1 - 2\alpha_1 G(\phi) \partial_\mu \phi_2 \partial^\mu \phi_2^*}{2\alpha_1 G(\phi)}, \quad (3.24)$$

$$q_1 = \frac{1}{2\alpha_1 G(\phi_1)} \frac{\left(\mu\phi_2^* + \lambda\phi_1^*\phi_2^{*2} + \sum_{k=1}^m kA_{1,k}\phi_1^{*k-1}\right)^2}{\mu\phi_2 + \lambda\phi_1\phi_2^2 + \sum_{k=1}^m kA_{1,k}\phi_1^{k-1}}, \quad (3.25)$$

$$q_2 = \frac{1}{2\alpha_1 G(\phi_2)} \frac{\left(\mu\phi_1^* + \lambda\phi_1^{*2}\phi_2^* + \sum_{k=1}^m kA_{2,k}\phi_2^{*k-1}\right)^2}{\mu\phi_1 + \lambda\phi_1^2\phi_2 + \sum_{k=1}^m kA_{2,k}\phi_2^{k-1}}. \quad (3.26)$$

In these equations we didn't impose stability conditions which must be imposed for evaluation. We now carry out a fast-roll and a slow-roll decomposition of the fields following the procedure discussed in [22] and reviewed in 2 and define

$$a_+/f_+ = \frac{1}{\sqrt{2}} (a_1/f_1 + a_2/f_2),$$

$$a_-/f_- = \frac{1}{\sqrt{2}} (a_1/f_1 - a_2/f_2). \quad (3.27)$$

Here a_+ is the field that is invariant under the shift symmetry and undergoes fast roll and a_- is the field that is sensitive to the shift symmetry and undergoes slow roll. We are interested in only the slow-roll part and thus we suppress a_+ and retain only the a_- part in the potential. This topic was dealt with in great detail in [22]. Here the more general case of m pairs of axion fields with each pair having fields with opposite U(1) charges was considered. In this case one has $2m$ number of axionic fields. After spontaneous symmetry breaking using the superpotential $W = W_s$ the potential involving the $2m$ axion fields was computed and is exhibited in eqs. (2.11) and (2.12) of [22]. This allows us to compute

the mass matrix for the $2m$ axion fields which is shown in eqs. (2.13)–(2.14) of [22]. Here one finds that $2m - 1$ linear combinations of axion fields gain mass and one can see that the masses of the axions are scaled by μ_{kl} which can be taken to the GUT scale. These $2m - 1$ linear combinations of axion fields are invariant under the $U(1)$ global symmetry. The remaining one linear combination of axion fields corresponds to the pseudo-Nambu-Goldstone bosons (pNGB) and is massless. On including W_{sb} in spontaneous breaking, the pNGB also gains a mass. However, this mass is much smaller than those of the other $2m - 1$ axions because $|W_{sb}| \ll |W_s|$. In the analysis of this paper there is just one pair of axion fields so we have only a_+ and a_- where a_+ is the axion field invariant under the $U(1)$ global symmetry and a_- is the field which is the pNGB. In [22] it was shown that a_+ rolls an order of magnitude or more faster than a_- as shown by blue lines (fast roll) vs. red lines (slow roll) in figure 1 and figure 2 of [22]. This phenomenon also holds for the current analysis presented here.

To simplify the analysis we set $A_{1,k} = A_{2,k} = A_k$ and set $f_1 = f_2 = f_+ = f_- = f$. In this case we have neglected spatial gradients:

$$A(a_-) = -\frac{\dot{a}_-^2}{2T}, \quad B(a_-) = \frac{1}{8T^2}\dot{a}_-^4, \quad (3.28)$$

$$G_i(a_-) = G(a_-) = \frac{1}{T - \dot{a}_-^2/2 + \sqrt{T^2 - T\dot{a}_-^2 + \frac{1}{8}\dot{a}_-^4(2 - \alpha_1)}}. \quad (3.29)$$

$\mathcal{L}_I$ is given by

$$\mathcal{L}_I = T \left(1 - \sqrt{1 - \frac{\dot{a}_-^2}{T} + \frac{(2 - \alpha_1)\dot{a}_-^4}{8T^2}} \right). \quad (3.30)$$

$\mathcal{L}_{II}$ is more complicated:

$$\begin{aligned} \mathcal{L}_{II} = T & \left(2\mathcal{F}_+^2 + 2\mathcal{F}_-^2 - \frac{4}{3\alpha_1} \left(\mathcal{T} + (\alpha_1 - 1) \frac{\dot{a}_-^2}{4T} \right) + 4k(\mathcal{F}_+ + \mathcal{F}_-) \right. \\ & + \frac{\alpha_1}{\mathcal{T} - \dot{a}_-^2/(4T)} \left(2 \left(\mathcal{F}_+^2 + \mathcal{F}_-^2 - \frac{2}{3\alpha_1} \left(\mathcal{T} + (\alpha_1 - 1) \frac{\dot{a}_-^2}{4T} \right) \right) \frac{\dot{a}_-^2}{4T} + \mathcal{F}_+^4 + \mathcal{F}_-^4 \right. \\ & \left. \left. + \frac{2}{3\alpha_1^2} \left(\mathcal{T} + (\alpha_1 - 1) \frac{\dot{a}_-^2}{4T} \right)^2 - \frac{4}{3\alpha_1} \left(\mathcal{T} + (\alpha_1 - 1) \frac{\dot{a}_-^2}{4T} \right) (\mathcal{F}_+^2 + \mathcal{F}_-^2) \right) \right), \end{aligned} \quad (3.31)$$

where

$$\mathcal{T} = \frac{1}{2} \left(1 + \sqrt{1 - \frac{\dot{a}_-^2}{T} + \frac{(2 - \alpha_1)\dot{a}_-^4}{8T^2}} \right), \quad (3.32)$$

$$k = \tilde{\beta} \sqrt{\sum_{m,n} mn \mathcal{G}_m \mathcal{G}_n \left(1 - \cos\left(\frac{a_- m}{\sqrt{2}f}\right) - \cos\left(\frac{a_- n}{\sqrt{2}f}\right) + \cos\left(\frac{a_- (m - n)}{\sqrt{2}f}\right) \right)}, \quad (3.33)$$

$$\mathcal{F}_{\pm} = \pm \left(\mp \frac{1}{2\alpha_1} k \left(\mathcal{T} - \frac{\dot{a}^2}{4T} \right) + \sqrt{\frac{1}{4\alpha_1^2} k^2 \left(\mathcal{T} - \frac{\dot{a}^2}{4T} \right)^2 + \frac{1}{27\alpha_1^3} \left(\mathcal{T} + (\alpha_1 - 1) \frac{\dot{a}^2}{4T} \right)^3} \right)^{1/3}, \quad (3.34)$$

$$\mathcal{G}_k = \frac{A_k 2^{1/2(1-k)}}{\tilde{\beta} \sqrt{T} f^{1-k}}. \quad (3.35)$$

Here $\tilde{\beta}$ is an arbitrary dimensionless parameter which we choose such that $\mathcal{G}_k \sim 1$, and which determines the scale of symmetry breaking terms relative to T .

Note that there are two ways here to achieve $A_k \ll \mu \approx M_{\text{P}}$, which is the condition needed for slow-roll approximation. One way is to consider small values of $\tilde{\beta}$, in which case we found DBI plays insignificant role and we reproduce the results of [22]. Another way is to consider $T \ll M_{\text{P}}$, and $\tilde{\beta} \approx M_{\text{P}}$, which is the case we will focus on for the remainder of this paper.

4 Pressure, density and inflation equations

The pressure p and density ρ are defined in terms of the stress tensor by

$$p = \frac{1}{3} \sum_{i=1}^3 T^{ii}, \quad \rho = T^{00}, \quad (4.1)$$

where $T^{\mu\nu}$ ($\mu = 0, 1, 2, 3$) is the stress tensor and is given by

$$T^{\mu\nu} = g^{\mu\nu} \mathcal{L}(\phi_k, \partial_\mu \phi_k \partial^\mu \phi_k) - 2 \frac{\delta \mathcal{L}(\phi_k, \partial_\mu \phi_k \partial^\mu \phi_k)}{\delta (\partial_\mu \phi_k \partial^\mu \phi_k)} \partial^\mu \phi_k \partial^\nu \phi_k, \quad (4.2)$$

and we use the metric $\eta^{\mu\nu} = \text{diag}(-1, 1, 1, 1)$. In the analysis we assume space to be homogeneous and isotropic, so that $\partial_i \phi_k = 0$ for $1 \leq i \leq 3$ and $\partial_0 \phi_k = \dot{\phi}_k$, one finds that eqs. (4.1) and (4.2) for pressure and density become:

$$p = \mathcal{L}(\phi_k, -\beta_k), \quad (4.3)$$

and

$$\rho = -\mathcal{L}(\phi_k, -\beta_k) + 2 \sum_k \frac{\delta \mathcal{L}(\phi_k, -\beta_k)}{\delta \beta_k} \beta_k, \quad (4.4)$$

where $\beta_k = \dot{\phi}_k^2$. These relations are valid for non-canonical kinetic terms. The Friedman equations are given by

$$\dot{\rho} = -3H(p + \rho), \quad (4.5)$$

$$3M_{\text{P}}^2 \frac{\dot{R}^2}{R^2} = \rho, \quad (4.6)$$

$$\dot{\rho} = -6H \sum_k \frac{\partial \mathcal{L}}{\partial \beta_k} \beta_k. \quad (4.7)$$

Further taking the time derivative of eq. (4.4) which along with eq. (4.7) gives

$$\sum_k \left[\left(2 \frac{\partial^2 L}{\partial \beta_k \partial \beta_k} \beta_k + \frac{\partial L}{\partial \beta_k} \right) \dot{\beta}_k - \frac{\partial L}{\partial \phi_k} \dot{\phi}_k + 6H \frac{\partial L}{\partial \beta_k} \beta_k + 2 \frac{\partial^2 L}{\partial \phi_k \partial \beta_k} \beta \dot{\phi}_k \right] = 0. \quad (4.8)$$

Next we focus on the slow-roll part where we keep only the field a_- . In this case eqs. (4.6) and (4.8) can be written in the following form

$$3M_{\text{P}}^2 \frac{\dot{R}^2}{R^2} = 2\dot{a}_-^2 \frac{\partial L}{\partial \dot{a}_-^2} - L, \quad (4.9)$$

$$2 \left[2 \frac{\partial^2 L}{\partial \dot{a}_-^2 \partial \dot{a}_-^2} \dot{a}_-^2 + \frac{\partial L}{\partial \dot{a}_-^2} \right] \ddot{a}_- - \frac{\partial L}{\partial a_-} + 6H \dot{a}_- \frac{\partial L}{\partial \dot{a}_-^2} + 2 \frac{\partial^2 L}{\partial a_- \partial \dot{a}_-^2} \dot{a}_-^2 = 0. \quad (4.10)$$

For the case of canonical kinetic energy and no dependence of the potential on time derivative of field, i.e.,

$$L = -\frac{1}{2} \partial_\mu a_- \partial a_-^\mu - V(a_-). \quad (4.11)$$

Eqs. (4.9) and (4.10) reduce to the following

$$\begin{aligned} 3M_{\text{P}}^2 \frac{\dot{R}^2}{R^2} &= \frac{1}{2} \dot{a}_-^2 + V(a_-), \\ \ddot{a}_- + 3H \dot{a}_- + V'(a_-) &= 0, \end{aligned} \quad (4.12)$$

which are correctly the relation for slow roll for the case when one has canonical kinetic energy and the potential is velocity independent. However, in our case we have more terms. Here we emphasize that while for conventional inflation models the potential plays a central role in the analysis, this is not the case for the Dirac-Born-Infeld case. Here the potential by itself is not sufficient and the entire Lagrangian enters in the analysis. The potential in isolation plays no role. This can be seen from the Friedman equations eqs. (4.9) and (4.10) above where the full Lagrangian and not just the potential enters. Only in the case when we assume canonical kinetic energy and no dependence of the potential on the derivatives of the fields that one recovers the conventional Friedman equations where the potential appears as shown in eqs. (4.12).

5 Slow roll parameters and non-Gaussianity

For non-canonical kinetic energy terms and for velocity dependent potential a quantity that enters the analysis of slow roll parameters is the speed of sound c_s defined by

$$c_s^2 = \frac{p, \beta}{\rho, \beta}. \quad (5.1)$$

c_s also enters in the analysis of non-Gaussianity to be discussed later. The speed of sound is limited by the constraint $0 < c_s^2 \leq 1$. Often a parameter γ is used which is defined by $\gamma = \frac{1}{c_s^2}$ and lies in the range $1 \leq \gamma < \infty$. For models with canonical kinetic energy $\gamma = 1$

and in this case there is no non-Gaussianity. For non-Gaussianity one requires $\gamma > 1$. For the models we consider one may write γ^2 as follows

$$\gamma^2 = 1 + 2\dot{a}_-^2 \frac{\partial^2 L}{\partial \dot{a}_-^2 \partial \ddot{a}_-^2} \bigg/ \frac{\partial L}{\partial \dot{a}_-^2} \quad (5.2)$$

and eq. (4.10) can then be written as follows

$$\frac{\partial L}{\partial (\dot{a}_-^2/2)} \ddot{a}_- - \frac{1}{\gamma^2} \frac{\partial L}{\partial a_-} + \frac{3}{\gamma^2} H \dot{a}_- \frac{\partial L}{\partial (\dot{a}_-^2/2)} + \frac{1}{\gamma^2} \frac{\partial^2 L}{\partial a_- \partial (\dot{a}_-^2/2)} \dot{a}_-^2 = 0. \quad (5.3)$$

The first three terms on the left hand side are similar to what one has normally except for γ dependence. The last term on the left hand side is new.

We define the slow roll parameters for DBI as [24–29]

$$\epsilon = -\frac{\dot{H}}{H^2}, \quad \eta = \frac{\dot{\epsilon}}{\epsilon H}, \quad s = \frac{\dot{c}_s}{c_s H}. \quad (5.4)$$

In terms of these parameters the power spectrum for the scalar perturbations P_k^ζ and the power spectrum for the tensor perturbations P_k^h are given by [61, 62]:

$$P_k^\zeta = \frac{1}{8\pi^2 M_{\text{P}}^2} \frac{H^2}{c_s \epsilon},$$

$$P_k^h = \frac{2}{3\pi^2} \frac{\rho}{M_{\text{P}}^4}. \quad (5.5)$$

Further, the spectral indices n_s and n_t in this case are given by

$$n_s = 1 - 2\epsilon - \eta - s,$$

$$n_t = -2\epsilon, \quad (5.6)$$

and the ratio r of the tensor to the scalar power spectrum is [61]

$$r = \frac{P_k^h}{P_k^\zeta} = -8c_s n_t. \quad (5.7)$$

One may compare it to the conventional slow-roll parameters ϵ_V, η_V for the case of the canonical kinetic energy term and no velocity dependence in the potential. Here one defines ϵ_V, η_V so that

$$\epsilon_V = \frac{M_{\text{P}}^2}{2} \left(\frac{V'}{V} \right)^2,$$

$$\eta_V = M_{\text{P}}^2 \frac{V''}{V}, \quad (5.8)$$

and the spectral indices and the ratio of the tensor and the scalar power spectrum in this case are given by

$$n_s = 1 - 6\epsilon_V + 2\eta_V, \quad n_t = -2\epsilon_V, \quad r = 16\epsilon_V. \quad (5.9)$$

To establish a connection between the DBI slow-roll parameters ϵ, η with the conventional slow-roll parameters we note that in the conventional slow roll one assumes dominance of the potential and one sets $c_s = 1$ and makes the following approximations

$$\dot{\phi} \simeq -\frac{V'}{3H}, \quad H^2 \simeq \frac{V}{3M_{\text{P}}^2}, \quad H' = \frac{V'}{6M_{\text{P}}^2 H}. \quad (5.10)$$

Using these one can connect ϵ, η to ϵ_V, η_V so that

$$\begin{aligned} \epsilon &= \epsilon_V, \\ \eta &= -2\eta_V + 4\epsilon_V. \end{aligned} \quad (5.11)$$

Using $c_s = 1$ and eq. (5.11) in eqs. (5.6) and (5.7) we can recover eq. (5.9).

Non-Gaussianity is defined by the three-point correlation function of perturbations involving three scalars, two scalars and a graviton, two gravitons and a scalar and three gravitons [24]. Since the work of [24] there has been a significant number of further analyses (see, e.g., [25, 27–30, 32, 34, 63–68]). The dominant non-Gaussianity arises from the correlation function of three scalar perturbations. Thus for scalar perturbation $\zeta(\vec{k})$ non-Gaussianity is defined by

$$\langle \zeta(\vec{k}_1)\zeta(\vec{k}_2)\zeta(\vec{k}_3) \rangle = (2\pi)^7 \delta^3(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) \frac{\sum_i k_i^3}{\prod_i k_i^3} \left[-\frac{3}{10} f_{\text{NL}} (P_k^\zeta)^2 \right], \quad (5.12)$$

where P_k^ζ is the scalar power spectrum and f_{NL} is a measure of non-Gaussianity. Non-Gaussianity depends strongly on shape and various types of shapes have been discussed in the literature which include local, equilibrium and orthogonal [69] (for earlier analyses see [70–72]). The non-Gaussianity we will discuss here is the one which has the shape of an equilateral triangle, when $k_1 = k_2 = k_3$ and is given by [28]

$$f_{\text{NL}}^{\text{equil}} = \frac{35}{108} \left(\frac{1}{c_s^2} - 1 \right) - \frac{5}{81} \left[\left(\frac{1}{c_s^2} - 1 - 2\frac{z_2}{z_1} \right) + (3 - 2c_1)z\frac{z_2}{z_1} \right] + \mathcal{O}(\epsilon), \quad (5.13)$$

where

$$z_1 = \beta L_{,\beta} + 2\beta^2 L_{\beta\beta}, \quad z_2 = \beta^2 L_{,\beta\beta} + \frac{2}{3}\beta^3 L_{\beta\beta\beta}, \quad z = \frac{\dot{z}_2}{z_2 H}, \quad (5.14)$$

where the superscript “equil” on f_{NL} indicates that we are specifically considering the shape to be of an equilateral triangle.

6 Model simulation and experimental test

Numerical simulations performed consist of two parts: a large 0.5×10^6 points simulation in which $\alpha_2 = \alpha_3 = 0$, and a small simulation with α_2 and α_3 perturbed around an experimentally consistent point with the largest non-Gaussianity from the first simulation. The evolution of the inflaton field a_- in the large simulation of figures 1, 2, and 4 is obtained by taking the effective axion Lagrangian eqs. (3.30) and (3.31), deriving Euler-Lagrange equation and Friedmann eq. (4.6) using computer algebra, and solving them numerically.

Figure 1. Left panel: a plot of the ratio r of the tensor to scalar power spectrum vs. the scalar spectral index n_s for the two field DBI model. The blue region enclosed by the blue line is the one allowed by experiment in the 2σ range. 85% of the simulated points are consistent with Planck constraints as shown by green points while the red points lie outside the 68% CL contour. Right panel: a plot of non-Gaussianity parameter $f_{\text{NL}}^{\text{equil}}$ as a function of α_1 for the same data set as in the left plot. As in the left panel the green dots are parameter points which satisfy the Planck constraints on r and n_s and lie in the blue region and the red dots are parameter points which are outside the experimentally allowed region on r and n_s . Points are distributed according to the following: $\alpha_1 \sim X_{0,\infty}$, $\alpha_2 = \alpha_3 = 0$, $T = 10^{-12} M_{\text{P}}^4$, $f \sim X_{0,\infty} M_{\text{P}}$, $\tilde{\beta} \sim X_{0,\infty}$, $m = 6$, $\mathcal{G}_1 = \mathcal{G}_2 = \mathcal{G}_3 = 0$, $\mathcal{G}_4 = 1$, $\mathcal{G}_5 \sim \mathcal{G}_6 \sim \mathcal{U}\{-1, 1\} \times X_{0,\infty}$, $N_{\text{pivot}} \sim \mathcal{U}(50, 60)$, $a_{-,0} \sim \mathcal{U}(0, 2\pi) \times f$, $\dot{a}_{-,0} \sim \mathcal{U}(-1, 1) \times \left(2\sqrt{T}/(\sqrt{2}\sqrt{\alpha_1} + 2)^{1/2}\right)$ where $X_{0,\infty} = \mathcal{U}\{X_{0,1}, 1/X_{0,1}\}$, $X_{0,1} = \mathcal{U}(0, 1)$, $\mathcal{U}$ refers to a uniform distribution, and the distribution of $\dot{a}_{-,0}$ is chosen such that the expression under the square root in eq. (3.30) is positive. Points are further filtered such that the mass of the inflaton $m_{a_-} < 0.1 M_{\text{P}}$.

Figure 2. Left panel: a plot of the ratio n_s vs. N_{pivot} in the range 50–60 for the same data set as in figure 1. The plot shows a mild dependence of n_s on N_{pivot} in the range indicated. Right panel: the same as the left panel except $f_{\text{NL}}^{\text{equil}}$ is plotted vs. N_{pivot} . The green and red dots are parameter points and have the same meaning as in figure 1.

Figure 3. The effect of α_2 (left panel) and α_3 (right panel) on non-Gaussianity for a parameter point taken from figure 1 which is consistent with Planck data and has the largest non-Gaussianity among the set of parameter points in figure 1. Specifically, here $f = 0.429 M_{\text{P}}$, $a_{-,0} = 0.233 M_{\text{P}}$, $\dot{a}_{-,0} = -0.118\sqrt{T}$, $m = 6$, $\mathcal{G}_1 = \mathcal{G}_2 = \mathcal{G}_3 = 0$, $\mathcal{G}_4 = 1$, $\mathcal{G}_5 = -1.53$, $\mathcal{G}_6 = 563$, $\alpha_1 = 1207$, $\tilde{\beta} = 0.802$, $T = 10^{-12} M_{\text{P}}^4$. One finds that the effect is not sufficient to obtain a significantly larger value of f_{NL} .

The integration is stopped when $\ddot{a} = 0$ where a is the scale factor and one may write the end of inflation constraint as

$$\ddot{a} = aH^2 \left(1 + \frac{\dot{H}}{H^2} \right) = 0. \quad (6.1)$$

An analytic expression similar to eq. (3.31), however, cannot be derived for non-zero α_2 and α_3 . In this case, equations for the auxiliary fields F_k are solved numerically, and the Lagrangian in terms of ϕ (eq. (3.12)) is also evaluated numerically for each set of values for $(a_-, \dot{a}_-)$. This process is significantly slower, therefore, in this case the simulations are only done for 29 sets of parameter values as can be seen in figure 3. The time of horizon exit is then found by counting N_{pivot} e-foldings back from the end of integration, where N_{pivot} is varied between 50 and 60.

Finally, we evaluate the experimental observables at the time of horizon exit using equations from section 5 and reusing Euler-Lagrange and Friedmann equations to compute higher time derivatives of the Lagrangian, pressure and density. The observables computed are the ratio r of the power spectrum of tensor to scalar perturbations, the spectral indices of scalar n_s and tensor n_t perturbations, speed of sound c_s and the non-Gaussianity amplitude f_{NL} . In the 0.5×10^6 points Monte Carlo analysis we allow α_1 to vary (full distribution is specified in figure 1) and search for solutions that satisfy the experimental constraints. Thus the current experimental limits from Planck experiment at $k_0 = 0.05 \text{ Mpc}^{-1}$ are as follows [13–15]

$$\begin{aligned} n_s &= 0.9645 \pm 0.0049 \text{ (68\%CL)}, \\ r &< 0.07 \text{ (95\%CL)}. \end{aligned} \quad (6.2)$$

and the current experimental constraint on c_s is [14]

$$c_s \geq 0.087 \quad (\text{at } 95\% \text{ CL}). \quad (6.3)$$

Figure 4. Left panel: a plot of f/M_P as a function of n_s which is in the allowed range of experiment and N_{pivot} in the range 50–60 for the same data set as in figure 1. Right panel: the same as the left panel except that the plot is of f/M_P as a function of r . We note that there are experimentally consistent points with $f/M_P \in [0.1, 1]$.

The results of the analysis for the two field supersymmetric DBI discussed above are presented in figures 1, 2, and 4. The left panel of figure 1 gives a plot of r vs. n_s . Here the region enclosed by the blue line is the one allowed by experiment in the 2σ range. The Monte Carlo analysis shows that the experimentally allowed region is well populated by the parameter points of the model. One interesting feature of the analysis is that compared to the supersymmetric axion models discussed in [22] where r was extremely small, here r has significantly larger values. Thus the largeness of r discriminates this class of supersymmetric DBI models from the models of [22]. The right panel of figure 1 gives a plot of $f_{\text{NL}}^{\text{equil}}$ vs. α_1 . We see a significant sensitivity of $f_{\text{NL}}^{\text{equil}}$ to α_1 . However, overall $f_{\text{NL}}^{\text{equil}}$ is typically small and mostly lies below 0.5. The effect of α_2 and α_3 for the point from figure 1 with largest non-Gaussianity is shown on figure 3. One can see that although non-Gaussianity f_{NL} is sensitive to α_2 and α_3 , it still lies below 0.5. This level of non-Gaussianity appears too low for observation in the near future.

The most recent analyses of non-Gaussianity based on Planck data are given in [69]. Here the analysis is done using cosmic microwave background (CMB) temperature and E-mode polarization data. The analysis using temperature alone give $f^{\text{equil}} = -16 \pm 70$ (68 % CL, statistical) and an analysis combining temperature and polarization data gives $f^{\text{equil}} = -4 \pm 43$ (68 % CL, statistical). In the left panel of figure 2 a plot of n_s vs. N_{pivot} is given. Here one finds that n_s has a mild positive slope as a function of N_{pivot} . The right panel gives a plot of $f_{\text{NL}}^{\text{equil}}$ on N_{pivot} . Here one finds the $f_{\text{NL}}^{\text{equil}}$ has a relatively small variation in the range [50, 60] for N_{pivot} . In either case one cannot draw any significant conclusion regarding the dependence of these parameters on the number of e-foldings as long as one is in the N_{pivot} range of [50, 60].

Then, in figure 4 we exhibit that the analysis contains a significant part of the parameter space where inflation consistent with the desired number of e-foldings in the range [50, 60] and n_s and r consistent with Planck data can be gotten in the axion inflation model with $f/M_P < 1$. Thus the left panel of figure 4 exhibits f/M_P as a function of n_s in the range allowed by the Planck data constraint of eq. (6.2) and the right panel of figure 4

Figure 5. Left panel: a histogram of the values of Hubble parameter at horizon exit for the same data set as in figure 1 for points that are consistent (green) and not consistent (red) with experimental constraints on r and n_s . Right panel: the same as the left panel except the histogram of the inflaton a_- mass is shown instead.

gives f/M_P as a function of r containing the range in r consistent with the Planck data constraint of eq. (6.2).

Finally, we demonstrate in figure 5 that both the Hubble parameter at horizon exit, and mass of the inflaton a_- are much smaller than M_P , which are necessary and sufficient conditions for the slow-roll approximation to hold.

7 Conclusion

In this work we have analyzed inflation in a supersymmetric Dirac-Born-Infeld action with a $U(1)$ symmetry. Specifically we have carried out a detailed analysis of a pair of chiral DBI fields which possess opposite charges under the $U(1)$ symmetry. A transformation is then made to go to the co-ordinate frame where a linear combination of the axion fields is invariant under the global $U(1)$ transformations and an orthogonal combination is variant which acts as the inflaton. The $U(1)$ shift symmetry is broken by instanton type non-perturbative terms in the superpotential. The analysis is done in the vacuum state with stabilized saxions and the scalar potential can be decomposed into a fast-roll and a slow-roll part where the slow-roll part of the potential is now velocity dependent. This velocity dependence is a direct consequence of the DBI form of the action. In the analysis for the case $\alpha_2 = 0 = \alpha_3$ we have obtained an explicit form for the auxiliary fields F_k which satisfy a cubic equation in terms of the inflation field. We have analyzed the scalar and tensor power spectrum and computed the spectral indices. It is shown that a significant part of the parameter space exists which supports inflation consistent with the current experimental constraints on the ratio of the tensor to the scalar power spectrum and the spectral indices. A remarkable aspect of the proposed supersymmetric DBI model is that the model supports an observable value of the tensor to the scalar power spectrum and consequently also a significant value for the spectral index n_t . This is in contrast to a supersymmetric non-DBI model which typically has a suppressed value of r and of n_t . An analysis of non-Gaussianity in the model was also carried out. It is found that for the model

parameters that support inflation consistent with the Planck experimental values on r and n_s , the non-Gaussianity is typically small. This holds true even when the parameters α_2 and α_3 along with α_1 are included in the analysis. It is of interest to achieve this class of models in string theory using moduli stabilization of the type used in KKLT [73] or the Large Volume Scenario [74].

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A Single field supersymmetric DBI Lagrangian

To define notation and to discuss the technique used in the analysis of this work we consider the case here of a single superfield. Thus we consider a supersymmetric DBI Lagrangian of the form [54, 55, 58]

$$\mathcal{L}_{\text{DBI}} = \int d^4\theta \left(\Phi\Phi^\dagger + \frac{1}{16} (D^\alpha\Phi D_\alpha\Phi) (\bar{D}^{\dot{\alpha}}\Phi^\dagger \bar{D}_{\dot{\alpha}}\Phi^\dagger) G(\Phi) \right), \quad (\text{A.1})$$

where Φ and $\Phi^\dagger$ are the chiral and anti-chiral superfields, D_α and $\bar{D}_{\dot{\alpha}}$ are the supercovariant derivatives, and $G(\Phi)$ is given by

$$G(\Phi) = \frac{1}{T} \frac{1}{1 + A(\Phi) + \sqrt{(1 + A(\Phi))^2 - B(\Phi)}}. \quad (\text{A.2})$$

Here T is a parameter of the dimension of $(\text{mass})^4$ and is related to the warp factor from the point of view of reduction of a ten dimensional theory to 4 dimensions. A and B are given by

$$A(\Phi) = \frac{\partial_\mu\Phi\partial^\mu\Phi^\dagger}{T}, \quad B(\Phi) = \frac{\partial_\mu\Phi\partial^\mu\Phi\partial_\nu\Phi^\dagger\partial^\nu\Phi^\dagger}{T^2}. \quad (\text{A.3})$$

Ignoring fermions, the superfields have the expansion:

$$\begin{aligned} \Phi(x, \theta, \bar{\theta}) &= \Phi_L(x, \theta, \bar{\theta}) \\ &= \phi(x) + \theta^\alpha\theta_\alpha F(x) + i\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu\phi(x) + \frac{1}{4}\theta^\alpha\theta_\alpha\bar{\theta}_{\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial^\mu\partial_\mu\phi(x), \end{aligned} \quad (\text{A.4})$$

$$\begin{aligned} \Phi^\dagger(x, \theta, \bar{\theta}) &= \Phi_R(x, \theta, \bar{\theta}) \\ &= \phi^*(x) + \bar{\theta}_{\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}F^*(x) - i\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu\phi^*(x) + \frac{1}{4}\theta^\alpha\theta_\alpha\bar{\theta}_{\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial^\mu\partial_\mu\phi^*(x). \end{aligned} \quad (\text{A.5})$$

$\Phi\Phi^\dagger$ can be written in the form

$$\begin{aligned} \Phi\Phi^\dagger &= \phi\phi^* + \theta^2\phi^*F + \bar{\theta}^2\phi F^* + i\theta\sigma^\mu\bar{\theta}(\phi^*\partial_\mu\phi - \phi\partial_\mu\phi^*) \\ &\quad + \theta^2\bar{\theta}^2\left(FF^* + \frac{1}{4}\phi\partial^\mu\partial_\mu\phi^* + \frac{1}{4}\phi^*\partial^\mu\partial_\mu\phi - \frac{1}{2}\partial^\mu\phi^*\partial_\mu\phi\right). \end{aligned} \quad (\text{A.6})$$

Further, since $\int d^4\theta \Phi \Phi^\dagger$ is the additive term in the Lagrangian, we can integrate by parts and get

$$\int d^2\theta d^2\bar{\theta} \Phi \Phi^\dagger = FF^* - \partial^\mu \phi^* \partial_\mu \phi. \quad (\text{A.7})$$

Next we note that

$$D_\beta \Phi = \left(\partial_\beta + i\sigma^\mu_{\beta\dot{\beta}} \bar{\theta}^{\dot{\beta}} \partial_\mu \right) \Phi. \quad (\text{A.8})$$

This leads to

$$D_\beta \Phi = -2\theta_\beta F(x) + 2i\sigma^\kappa_{\beta\dot{\gamma}} \bar{\theta}^{\dot{\gamma}} \partial_\kappa \phi(x) + i\theta^\gamma \theta_\gamma \sigma^\mu_{\beta\dot{\beta}} \bar{\theta}^{\dot{\beta}} \partial_\mu F(x). \quad (\text{A.9})$$

Using the above we can compute $D^\alpha \Phi D_\alpha \Phi$ and the computation gives

$$D^\alpha \Phi D_\alpha \Phi = 4F^2 \theta^2 - 4\partial^\kappa \phi \partial_\kappa \phi \bar{\theta}^2 - 8iF \partial_\kappa \phi \theta \sigma^\kappa \bar{\theta} - 4\partial^\mu F \partial_\mu \phi \theta^2 \bar{\theta}^2. \quad (\text{A.10})$$

The conjugate of eq. (A.10) can be computed as follows:

$$\bar{D}^{\dot{\alpha}} \Phi^\dagger \bar{D}_{\dot{\alpha}} \Phi^\dagger = -\bar{D}_{\dot{\alpha}} \Phi^\dagger \bar{D}^{\dot{\alpha}} \Phi^\dagger = -(D^\alpha \Phi D_\alpha \Phi)^\dagger, \quad (\text{A.11})$$

which gives

$$\bar{D}^{\dot{\alpha}} \Phi^\dagger \bar{D}_{\dot{\alpha}} \Phi^\dagger = 4F^{*2} \bar{\theta}^2 - 4\partial^\kappa \phi^* \partial_\kappa \phi^* \bar{\theta}^2 + 8iF^* \partial_\kappa \phi^* \theta \sigma^\kappa \bar{\theta} - 4\partial^\mu F^* \partial_\mu \phi^* \theta^2 \bar{\theta}^2. \quad (\text{A.12})$$

The product $(D^\alpha \Phi D_\alpha \Phi) (\bar{D}^{\dot{\alpha}} \Phi^\dagger \bar{D}_{\dot{\alpha}} \Phi^\dagger)$ in terms of component fields is given by

$$(D^\alpha \Phi D_\alpha \Phi) (\bar{D}^{\dot{\alpha}} \Phi^\dagger \bar{D}_{\dot{\alpha}} \Phi^\dagger) = 16 \left(F^2 F^{*2} + \partial^\kappa \phi \partial_\kappa \phi \partial^\mu \phi^* \partial_\mu \phi^* - 2FF^* \partial^\mu \phi \partial_\mu \phi^* \right) \theta^2 \bar{\theta}^2. \quad (\text{A.13})$$

Note that $(D^\alpha \Phi D_\alpha \Phi) (\bar{D}^{\dot{\alpha}} \Phi^\dagger \bar{D}_{\dot{\alpha}} \Phi^\dagger)$ already contains the highest possible power of the Grassmann numbers, therefore, the factor $G(\Phi)$ multiplying it in the case of the Lagrangian eq. (A.1) can simply be replaced with $G(\phi)$.

Combining terms we get the following expression for the Lagrangian:

$$\mathcal{L}_{\text{DBI}} = FF^* + G(\phi) \left(-2FF^* \partial^\mu \phi \partial_\mu \phi^* + F^2 F^{*2} \right) - TA(\phi) + G(\phi) B(\phi) T^2. \quad (\text{A.14})$$

We can further simplify and write the DBI Lagrangian in the form

$$\begin{aligned} \mathcal{L}_{\text{DBI}} = & -T \sqrt{1 + 2T^{-1} \partial_\mu \phi \partial^\mu \phi^* + T^{-2} (\partial_\mu \phi \partial^\mu \phi^*)^2 - T^{-2} (\partial_\mu \phi \partial^\mu \phi) (\partial_\nu \phi^* \partial^\nu \phi^*)} \\ & + T + FF^* + G(\phi) \left(-2FF^* \partial^\mu \phi \partial_\mu \phi^* + F^2 F^{*2} \right). \end{aligned} \quad (\text{A.15})$$

B Equations for F_k, F_k^* ($k = 1, 2$)

We need to eliminate F_k and F_k^* from the Lagrangian of eq. (3.12) to construct an equation of motion for ϕ_k only. In order to do that, we first vary with respect to F_k and F_k^* to obtain

the following equations for F_k and F_k^* . Here variations with respect to F_1^* and F_1 give

$$\begin{aligned}
 F_1 + \frac{\partial W^*}{\partial \phi_1^*} + G(\phi) \left[\alpha_1 (-2F_1 \partial_a \phi_1 \partial^a \phi_1^* + 2F_1^2 F_1^*) + \alpha_2 (-2F_2 \partial_a \phi_2 \partial^a \phi_1^* + 2F_2^2 F_1^*) \right. \\
 \left. + \alpha_3 (-F_1 \partial_a \phi_2 \partial^a \phi_2^* + F_1 F_2 F_2^*) \right] = 0, \\
 F_1^* + \frac{\partial W}{\partial \phi_1} + G(\phi) \left[\alpha_1 (-2F_1^* \partial_a \phi_1 \partial^a \phi_1^* + 2F_1 F_1^{*2}) + \alpha_2 (-2F_2^* \partial_a \phi_2 \partial^a \phi_1 + 2F_2^2 F_1) \right. \\
 \left. + \alpha_3 (-F_1^* \partial_a \phi_2 \partial^a \phi_2 + F_1^* F_2 F_2^*) \right] = 0.
 \end{aligned} \tag{B.1}$$

Similarly variations with respect to F_2^* and F_2 give

$$\begin{aligned}
 F_2 + \frac{\partial W^*}{\partial \phi_2^*} + G(\phi) \left[\alpha_1 (-2F_2 \partial_a \phi_2 \partial^a \phi_2^* + 2F_2^2 F_2^*) + \alpha_2 (-2F_1 \partial_a \phi_1 \partial^a \phi_2^* + 2F_1^2 F_2^*) \right. \\
 \left. + \alpha_3 (-F_2 \partial_a \phi_1 \partial^a \phi_1^* + F_2 F_1 F_1^*) \right] = 0, \\
 F_2^* + \frac{\partial W}{\partial \phi_2} + G(\phi) \left[\alpha_1 (-2F_2^* \partial_a \phi_2 \partial^a \phi_2^* + 2F_2 F_2^{*2}) + \alpha_2 (-2F_1^* \partial_a \phi_1 \partial^a \phi_2 + 2F_1^2 F_2) \right. \\
 \left. + \alpha_3 (-F_2^* \partial_a \phi_1 \partial^a \phi_1 + F_2^* F_1 F_1^*) \right] = 0.
 \end{aligned} \tag{B.2}$$

These give rise to a set of four coupled equations for F_1, F_1^*, F_2, F_2^* which are difficult to solve analytically. To keep the analysis under control we set $\alpha_2 = 0 = \alpha_3$. In this case the equations for F_1, F_1^* become decoupled from those for F_2, F_2^* and we get

$$F_k^* + \frac{\partial W}{\partial \phi_k} + \alpha_1 G(\phi) \left(-2F_k^* \partial^\mu \phi_k \partial_\mu \phi_k^* + 2F_k F_k^{*2} \right) = 0, \quad k = 1, 2, \tag{B.3}$$

$$F_k + \frac{\partial W^*}{\partial \phi_k^*} + \alpha_1 G(\phi) \left(-2F_k \partial^\mu \phi_k \partial_\mu \phi_k^* + 2F_k^2 F_k^* \right) = 0, \quad k = 1, 2. \tag{B.4}$$

We multiply eq. (B.3) by F_k and eq. (B.4) by F_k^* , and subtract one from another from which we can extract an equation for F_k^* in terms of F_k :

$$F_k^* = \left(\frac{\partial W}{\partial \phi_k} \Big/ \frac{\partial W^*}{\partial \phi_k^*} \right) F_k. \tag{B.5}$$

Substitution back in eq. (B.4) gives an equation for F_k :

$$2\alpha_1 G(\phi) \frac{\partial W}{\partial \phi_k} F_k^3 + \frac{\partial W^*}{\partial \phi_k^*} (1 - 2\alpha_1 G(\phi) \partial^\mu \phi_k \partial_\mu \phi_k^*) F_k + \left(\frac{\partial W^*}{\partial \phi_k^*} \right)^2 = 0. \tag{B.6}$$

To simplify this equation we define p and q so that

$$p_k = \left(\frac{\partial W}{\partial \phi_k} \right)^{-1} \frac{\partial W^*}{\partial \phi_k^*} \frac{1 - 2\alpha_1 G \partial^\mu \phi_k \partial_\mu \phi_k^*}{2\alpha_1 G}, \tag{B.7}$$

$$q_k = \frac{1}{2\alpha_1 G} \left(\frac{\partial W}{\partial \phi_k} \right)^{-1} \left(\frac{\partial W^*}{\partial \phi_k^*} \right)^2. \tag{B.8}$$

Substitution of $\partial^\mu \phi_k \partial_\mu \phi_k^*$ in terms of p_k using eq. (B.7) in eq. (B.6) and using eq. (B.8) to eliminate $\left(\frac{\partial W^*}{\partial \phi_k^*}\right)^2$ in terms of q_k in eq. (B.6) gives

$$F_k^3 + p_k F_k + q_k = 0, \quad (\text{B.9})$$

where we cancelled a common factor of $2G\frac{\partial W}{\partial \phi_k}$. The equation above is cubic in F_k and it might appear that there are three consistent solutions. To see if this is the case we substitute eq. (B.5) back into eq. (3.15) to obtain a solution for F_k^* . Upon simplification, one gets

$$\begin{aligned} F_k^* = & \omega^j \left(-\frac{q_k^*}{2} + \sqrt{\left(\frac{q_k^*}{2}\right)^2 + \left(\frac{p_k^*}{3}\right)^3} \right)^{1/3} \\ & + \omega^{3-j} \left(-\frac{q_k^*}{2} - \sqrt{\left(\frac{q_k^*}{2}\right)^2 + \left(\frac{p_k^*}{3}\right)^3} \right)^{1/3}. \end{aligned} \quad (\text{B.10})$$

Note that this expression is only a complex conjugate of eq. (3.15) if $j = 0$ (ω^j and ω^{3-j} will have to be interchanged for it to be a complex conjugate in the case of $j = 1$ and $j = 2$), therefore only $j = 0$ corresponds to a solution for the auxiliary fields F_k .

C DBI scalar potential with derivative terms absent

To discuss the limit of the DBI potential to the standard supersymmetric potential we need to drop the derivative terms on ϕ in the potential. In this case the full form of the scalar potential looks very different from the usual supersymmetric potential. To keep the expressions as simple as possible we consider the case of just one scalar field although extension to more fields is straightforward. In this case we have

$$\begin{aligned} G &= \frac{1}{2T}, \\ p &= T \left(\frac{\partial W}{\partial \phi} \right)^{-1} \frac{\partial W^*}{\partial \phi^*}, \\ q &= T \left(\frac{\partial W}{\partial \phi} \right)^{-1} \left(\frac{\partial W^*}{\partial \phi^*} \right)^2, \end{aligned} \quad (\text{C.1})$$

F is given by

$$\begin{aligned} F &= T^{1/2} \\ &\times \left(\frac{\partial W}{\partial \phi} \right)^{-1/3} \left(\frac{\partial W^*}{\partial \phi^*} \right)^{1/2} \left(\left(-\frac{1}{T^{1/2}} \sqrt{\frac{1}{4} \frac{\partial W^*}{\partial \phi^*}} + \sqrt{\frac{1}{27} \left(\frac{\partial W}{\partial \phi} \right)^{-1} + \frac{1}{4T} \frac{\partial W^*}{\partial \phi^*}} \right)^{1/3} \right. \\ &\quad \left. + \left(-\frac{1}{T^{1/2}} \sqrt{\frac{1}{4} \frac{\partial W^*}{\partial \phi^*}} - \sqrt{\frac{1}{27} \left(\frac{\partial W}{\partial \phi} \right)^{-1} + \frac{1}{4T} \frac{\partial W^*}{\partial \phi^*}} \right)^{1/3} \right), \end{aligned} \quad (\text{C.2})$$

and the scalar potential is

$$V(\phi) = - \left(F^* F + \frac{\partial W}{\partial \phi} F + \frac{\partial W^*}{\partial \phi} F^* + \frac{1}{2T} F^2 F^{*2} \right). \quad (\text{C.3})$$

Further, an explicit form of the potential can be gotten by using eq. (C.2) back into the potential eq. (C.3) which gives

$$V(\phi) = V_1(\phi) + V_2(\phi) + V_3(\phi) + V_4(\phi), \quad (\text{C.4})$$

where

$$V_1(\phi) = -T \left(\frac{\partial W}{\partial \phi} \frac{\partial W^*}{\partial \phi^*} \right)^{1/6} (Q_+ + Q_-) (Q_+^* + Q_-^*), \quad (\text{C.5})$$

$$V_2(\phi) = -T^{1/2} \left(\frac{\partial W}{\partial \phi} \right)^{2/3} \left(\frac{\partial W^*}{\partial \phi^*} \right)^{1/2} (Q_+ + Q_-), \quad (\text{C.6})$$

$$V_3(\phi) = V_2^*(\phi), \quad (\text{C.7})$$

$$V_4(\phi) = -\frac{1}{2} T \left(\frac{\partial W}{\partial \phi} \frac{\partial W^*}{\partial \phi^*} \right)^{1/3} (Q_+ + Q_-)^2 (Q_+^* + Q_-^*)^2, \quad (\text{C.8})$$

and

$$Q_{\pm} = \left(-\frac{1}{T^{1/2}} \sqrt{\frac{1}{4} \frac{\partial W^*}{\partial \phi^*}} \pm \sqrt{\frac{1}{27} \left(\frac{\partial W}{\partial \phi} \right)^{-1} + \frac{1}{4T} \frac{\partial W^*}{\partial \phi^*}} \right)^{1/3}. \quad (\text{C.9})$$

We can expand F given by eq. (C.2) in powers of $1/T$ and this expansion is exhibited in eq. (3.16). Similarly we can expand V given by eq. (C.3) in powers of $1/T$ and this expansion is given in eq. (3.17). One can see that the lowest terms in the expansion for both F and V give the standard result.

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