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Deformations of vector-scalar models

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Abstract: Abelian vector fields non-minimally coupled to uncharged scalar fields arise in many contexts. We investigate here through algebraic methods their consistent deformations ("gaugings"), i.e., the deformations that preserve the number (but not necessarily the form or the algebra) of the gauge symmetries. Infinitesimal consistent deformations are given by the BRST cohomology classes at ghost number zero. We parametrize explicitly these classes in terms of various types of global symmetries and corresponding Noether currents through the characteristic cohomology related to antifields and equations of motion. The analysis applies to all ghost numbers and not just ghost number zero. We also provide a systematic discussion of the linear and quadratic constraints on these parameters that follow from higher-order consistency. Our work is relevant to the gaugings of extended supergravities.

Keywords: BRST Quantization, Gauge Symmetry, Global Symmetries, Supergravity Models

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1 Introduction

Our paper is devoted to a systematic study of the consistent deformations of the gauge invariant actions of the form

$$S_0[A^I_\mu, \phi^i] = \int d^4x\, \mathcal{L}_0, \tag{1.1}$$

depending on n_s uncharged scalar fields ϕ^i and n_v abelian vector fields A^I_μ. We assume that the only gauge symmetries of (1.1) are the standard U(1) gauge transformations for each vector field, so that the gauge algebra is abelian and given by n_v copies of $\mathfrak{u}(1)$. A generating set of gauge invariances can be taken to be

$$\delta A^I_\mu = \partial_\mu \epsilon^I, \quad \delta\phi^i = 0. \tag{1.2}$$

The Lagrangian takes the form

$$\mathcal{L}_0 = \mathcal{L}_S[\phi^i] + \mathcal{L}_V[A^I_\mu, \phi^i], \tag{1.3}$$

where $\mathcal{L}_V$ is a function that depends on the vector fields through the abelian curvatures $F^I_{\mu\nu} = \partial_\mu A^I_\nu - \partial_\nu A^I_\mu$ only, and which can also involve the scalar fields ϕ^i. Derivatives of these variables are in principle allowed in the general analysis carried out below, but actually do not occur in the explicit Lagrangians discussed in more detail. The scalar fields can occur non linearly, e.g. terms of the form $\mathcal{I}_{IJ}(\phi)F^I_{\mu\nu}F^{J\mu\nu}$ where $\mathcal{I}_{IJ}(\phi)$ are some functions of the ϕ^i's are allowed. Similarly, the scalar Lagrangian need not be quadratic. More on this in subsection 3.1.

The gauge transformations (1.2) are sometimes called "free abelian gauge transformations" to emphasize that the scalar fields are uncharged and do not transform under them. This does not mean that the abelian vector fields themselves are free since non linear terms (non minimal couplings) are allowed in (1.3).

This class of models contains the vector-scalar sectors of "ungauged" extended supergravities, of which $\mathcal{N}=4$ [1–3] and $\mathcal{N}=8$ [4, 5] supergravities offer prime examples. These will be considered in detail in sections 5 and 6. Born-Infeld type generalizations [6] are also covered together with first order manifestly duality invariant formulations [7–9], which fall into this class when reformulated with suitable additional scalar fields [10].

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Consistent deformations of a gauge invariant action are deformations that preserve the number (but not necessarily the form or the algebra) of the gauge symmetries. In the supergravity context, these are called "gaugings", and the deformed theories are called "gauged supergravities", even though the undeformed theories possess already a gauge freedom. We shall often adopt this terminology here. We shall consider only local deformations, i.e., deformations of the Lagrangian by functions of the fields and their derivatives up to some finite (but unspecified) order.

Gaugings in extended supergravities have a long history that goes back to [11, 12]. For maximal supergravity, the first gauging has been performed in [13] in the Lagrangian formulation of [5], which involves a specific choice of so-called "duality frame" (a choice of "electric" directions among a set of electric-magnetic pairs). More recent gaugings involving a change of the duality frame have been constructed in [14]. All these are reviewed in [15].

These works consider from the very beginning deformations in which the vector fields become Yang-Mills connections for a non-abelian deformation of the original abelian gauge algebra. The corresponding couplings are induced through the replacement of the abelian curvatures by non-abelian ones and the ordinary derivatives by covariant ones, plus possible additional couplings necessary for consistency. One natural question to be asked is whether this embraces all possible consistent deformations. There exist of course theorems establishing the uniqueness of the Yang-Mills coupling under general conditions (see e.g. [16, 17]), but couplings to nonlinear scalar fields were not considered in these early works, which focused furthermore on algebra-deforming deformations.

The gaugings of supergravities have revealed the importance of the choice of duality frame, in the sense that the space of consistent deformations depends on that choice (see [18, 19] and the recent analysis in [20, 21]). In order to take this feature into account, a formalism has been developed in [22–25], called the "embedding tensor" formalism. It is reviewed in [15]. In this formalism, additional fields are introduced besides those appearing in (1.1), which are magnetic vector potentials and 2-form auxiliary gauge fields. The theory possesses also additional gauge symmetries. The choice of duality frame is implicitly encoded in the "embedding" tensor, which is subject to a number of constraints. It was shown in [26] that the space of consistent deformations in the "embedding" formalism is isomorphic to the space of consistent deformations for the action (1.1) written in the duality frame picked by the choice of embedding tensor. For that reason, one can investigate the question of gaugings by taking (1.1) as starting point of the deformation procedure, provided one allows the scalar field dependence in the vector piece of the Lagrangian to cover all possible choices of duality frame. It is this task which is carried out here. By doing so, one does not miss any of the gaugings available in the embedding tensor formalism.

One systematic way to explore deformations of theories with a gauge freedom is provided by the BV-BRST formalism [27]. In the BRST approach, inequivalent infinitesimal local gaugings correspond to BRST cohomology classes in ghost number zero computed in the space of local functionals. In this work, we completely characterize the BRST cohomology for the theories defined by (1.1), i.e., we completely characterize, in four spacetime dimensions, the deformations of abelian vector fields coupled non-minimally to scalar chargeless fields with a possibly non polynomial dependence on the (undifferentiated) scalar fields.

In particular, we show that besides the obvious deformations that consist in adding gauge invariant terms to the Lagrangian without changing the gauge symmetries, the gaugings can be related to the global symmetries of the action (1.1). These gaugings modify the form of the gauge transformations.

The global symmetries can be classified into two different types: (i) global symmetries with covariantizable Noether currents, where by "covariantizable", we mean that one can choose the ambiguities in the Noether currents so as to take them gauge invariant (V-type symmetries); (ii) global symmetries with non-covariantizable Noether currents. Only the first type directly gives rise to an infinitesimal consistent deformation through minimal coupling of the corresponding current to the vector potentials.

The gaugings associated with the other type of global symmetries need to satisfy additional constraints. This second type of global symmetries, in turn, can be subdivided into two subtypes: (a) global symmetries with non-covariantizable Noether currents that lead to a deformation that does not modify the gauge algebra (W-type symmetries); (b) global symmetries with non-covariantizable Noether currents that lead to a deformation that does modify also the gauge algebra (U-type symmetries). The global symmetries of type (a) contain in their Noether current non-gauge invariant Chern-Simons terms that cannot be removed by suitably adjusting trivial contributions. The global symmetries of type (b) are associated with ordinary "free" abelian gauge symmetries with co-dimension 2 conservation laws (see e.g. [28] for an early discussion). The divergence of a current of type (a) is itself gauge invariant, while the divergence of a current of type (b) is not. Yang-Mills gaugings are associated with currents of type (b) and are hence of U-type. Topological couplings [29] are associated with non-covariantizable Noether currents of either type (a) or (b). "Charging deformations" (if available), in which the scalar fields become charged but the gauge transformations of the vector fields are not modified and remain therefore abelian, are of V- or W-type.

The BRST deformation procedure applies not only to the consistent first order deformations, but also to higher orders where one might encounter obstructions. That procedure provides a natural deformation-theoretic interpretation of quadratic constraints and higher order constraints in terms of what is called the antibracket map.

After establishing general theorems on the BRST cohomology valid without assuming a specific form of the Lagrangian or the rigid symmetries, including the above classification of the deformations and useful triangular properties of their algebra, we turn to various models that have been considered in the literature, for which we completely compute the deformations of U and W-types.

Our paper is organized as follows. In section 2, we provide a brief survey of the BRST deformation procedure. We then compute in section 3 the local BRST cohomology of the models described by the action (1.1). This is done by following the method of [30, 31] where the BRST cohomology was computed for arbitrary compact — in fact reductive — gauge group. The difficulty in the computation comes from the free abelian factors, where by "free abelian factors", we mean abelian factors of the gauge algebra such that all matter fields are uncharged, i.e., invariant under the associated gauge transformations. This is precisely the case relevant to the action (1.1), which needs thus special care. The method of [30, 31] is based on an expansion according to the antifield number. It makes direct contact with symmetries and conservation laws through the lowest antifield number piece of the BRST differential, called the "Koszul-Tate" differential, which involves the equations of motion [32, 33]. The Noether charges appear through the "characteristic" cohomology, given by the local cohomology of the "Koszul-Tate" differential [30].

We then discuss in section 4 the structure of the antibracket map, which is relevant for the consistency of the deformation at second order and the possible appearance of obstructions, and provide information on the structure of the global symmetry algebra.

The method of [30, 31] provides the general structure of the BRST cocycles in terms of conserved currents. In order to reach more complete results, one must use additional information specific to each model. We therefore specify further the models in section 5, where we concentrate on scalar-coupled second order Lagrangians that are quadratic in the vector fields and their derivatives. These specialized models still cover the scalar-vector sectors of extended supergravities. Explicit examples are treated in detail to illustrate the method in section 6, where complete results for the local BRST cohomology, up to the determination of V-type symmetries, are worked out. In section 7, we then illustrate our techniques in the case of the manifestly duality-symmetric first order action of [9], in the formulation of [10], which is adapted to the direct use of the methods developed here.

The last section (section 8) summarizes our results and recapitulates the structure of the local BRST cohomology. Two appendices complete our work by respectively displaying our notations and conventions on exterior forms and their duals (appendix A) and discussing further properties of the antibracket map (appendix B). Appendix C is devoted to the detailed analysis of the W-component of the commutator of two U-type transformations.

2 BRST deformation theory: a quick survey

2.1 Batalin-Vilkovisky antifield formalism

In order to systematically construct consistent interactions in gauge theories, it is useful to reformulate the problem in the context of algebraic deformation theory [34–37]. The appropriate framework is provided by the Batalin-Vilkovisky antifield formalism [27, 38–40].

The structure of an irreducible gauge system, i.e., the Lagrangian $\mathcal{L}_0$ with field content φ^a, generating set of gauge symmetries[1] $\delta_\epsilon \varphi^a = R^a{}_\alpha[\varphi^b]\,(\epsilon^\alpha)$ and their algebra, is captured by the Batalin-Vilkovisky (BV) master action S (see e.g. [41, 42] for reviews). The master action is a ghost number 0 functional

$$S = \int d^n x\, \mathcal{L} = \int d^n x \left[\mathcal{L}_0 + \varphi^*_a R^a{}_\alpha\,(C^\alpha) + \frac{1}{2} C^*_\alpha f^\alpha{}_{\beta\gamma}\,(C^\beta, C^\gamma) + \dots \right], \tag{2.1}$$

that satisfies what is called the master equation

$$\frac{1}{2}(S,S) = 0. \tag{2.2}$$

[1]We use the condensed De Witt notation.

In this equation, the BV antibracket is the odd graded Lie bracket defined by

$$(X,Y) = \int d^n x \left[\frac{\delta^R X}{\delta \Phi^A(x)} \frac{\delta^L Y}{\delta \Phi^*_A(x)} - \frac{\delta^R X}{\delta \Phi^*_A(x)} \frac{\delta^L Y}{\delta \Phi^A(x)}\right] \tag{2.3}$$

on the extended space $\Phi^A = (\varphi^a, C^\alpha, \dots)$ of original fields and ghosts (and ghosts for ghosts in the case of reducible gauge theories) and their antifields Φ^*_A. The ghost numbers of φ^a, C^α are $0, 1$, while $\mathrm{gh}(\Phi^*_A) = -\mathrm{gh}(\Phi^A) - 1$. The Lagrangian, gauge variations and structure functions of the gauge algebra are contained in the first, second and third term of the master action (2.1) respectively.

For the deformation problem, one assumes the existence of an undeformed theory described by $S^{(0)}$ satisfying the master equation $\frac{1}{2}(S^{(0)}, S^{(0)}) = 0$ and one analyzes the conditions coming from the requirement that, in a suitable expansion, the deformed theory

$$S = S^{(0)} + S^{(1)} + S^{(2)} + \dots, \tag{2.4}$$

satisfies the master equation (2.2). The deformed Lagrangian, gauge symmetries and structure functions can then be read off from the deformed master action (2.4).

The first condition on the “infinitesimal” deformation $S^{(1)}$ is

$$(S^{(0)}, S^{(1)}) = 0. \tag{2.5}$$

This equation admits solutions $S^{(1)} = (S^{(0)}, \Xi)$, for all Ξ of ghost number -1. Such deformations can be shown to be trivial in the sense that they can be absorbed by (anticanonical) field-antifield redefinitions. Moreover, trivial deformations in that sense are always of the form $S^{(1)} = (S^{(0)}, \Xi)$ for some local Ξ. It thus follows that equivalence classes of deformations up to trivial ones are classified by $H^0(s)$, the ghost number zero cohomology of the antifield dependent BRST differential $s = (S^{(0)}, \cdot)$ of the undeformed theory,

$$[S^{(1)}] \in H^0(s). \tag{2.6}$$

For our problem of determining the most general deformation, we start by computing $H^0(s)$ and couple its elements with independent parameters to the starting point action to obtain $S^{(0)}+S^{(1)}$. The parameters thus play the role of generalized coupling constants. In a second step, we determine the constraints on these coupling constants coming from the existence of a completion such that (2.2) holds. The expansion is then in terms of homogeneity in these generalized coupling constants and not, as often done, in homogeneity of fields (in which case $S^{(0)}$ corresponds to an action quadratic in the fields). In particular, this approach treats the different types of symmetries involved in the determination of $H^0(s)$ on the same footing.

In the standard field theoretic setting, one insists on spacetime locality which implies that the cohomology is computed in the space of local functionals in the fields and antifields. In turn, this can be shown to be equivalent to the cohomology of s in the space of local functions up to total derivatives or, in form notation, to the cohomology of s in top form degree n, up to the horizontal differential of an $n-1$ form. Local functions are functions that depend on the spacetime coordinates, the fields and a finite number of

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derivatives. The horizontal differential is not the de Rham differential but is instead given by $d = dx^\mu \partial_\mu$, where $\partial_\mu = \frac{\partial}{\partial x^\mu} + \partial_\mu z^\Sigma \frac{\partial}{\partial z^\Sigma} + \dots$ is the total derivative. Here, fields and antifields are collectively denoted by $z^\Sigma = (\Phi^A, \Phi^*_A)$. More explicitly, the ghost number g cohomology $H^g(s)$ of the antifield dependent BRST differential s computed in the space of local functionals is isomorphic to $H^{g,n}(s|d)$, where the latter group is defined by

$$sa^{g,n} + da^{g+1,n-1} = 0, \quad a^{g,n} \sim a^{g,n} + sb^{g-1,n} + db^{g,n-1}, \tag{2.7}$$

the first superscript referring to ghost number and the second to form degree. The BRST differential is defined on the undifferentiated fields and antifields by $s\Phi^A = -\frac{\delta^R \mathcal{L}}{\delta \Phi^*_A}$, $s\Phi^*_A = \frac{\delta^R \mathcal{L}}{\delta \Phi^A}$. It is extended to the derivatives through $[s, \partial_\mu] = 0$ resulting in $\{s, d\} = 0$. This reformulation allows one to use systematic homological techniques ("descent equations") for the computation of these classes (see e.g. [43]).

At second order, the condition on the infinitesimal deformation $S^{(1)}$ is

$$\frac{1}{2}(S^{(1)}, S^{(1)}) + (S^{(0)}, S^{(2)}) = 0. \tag{2.8}$$

The antibracket gives rise to a well defined map in cohomology,

$$(\cdot, \cdot) : H^{g_1}(s|d) \otimes H^{g_2}(s|d) \longrightarrow H^{g_1+g_2+1}(s|d). \tag{2.9}$$

For cocycles C_i with $[C_i] \in H^{g_i}(s|d)$, it is explicitly given by

$$([C_1], [C_2]) = [(C_1, C_2)] \in H^{g_1+g_2+1}(s|d). \tag{2.10}$$

Condition (2.8) constrains the infinitesimal deformation $S^{(1)}$ to satisfy

$$\frac{1}{2}([S^{(1)}], [S^{(1)}]) = [0] \in H^1(s|d). \tag{2.11}$$

If this is the case, $S^{(2)}$ in (2.8) is defined up to a cocycle in ghost number 0. Higher order brackets and constraints can be analyzed in a similar way, see e.g. [44, 45].

Besides the group $H^0(s|d)$ that describes infinitesimal deformations, and $H^1(s|d)$ that controls the obstructions to extending these to finite deformations, one can furthermore show [30] that $H^g(s|d) \simeq H^{n+g}_{\text{char}}(d)$ for $g \leq -1$. The latter "characteristic" cohomology groups are defined by forms ω in the original fields φ^a such that

$$d\omega^{n+g} \approx 0, \quad \omega^{n+g} \sim \omega^{n+g} + d\eta^{n+g-1} + t^{n+g}, \tag{2.12}$$

with $t^{n+g} \approx 0$ and where ≈ 0 denote terms that vanish on all solutions to the Euler-Lagrange equations of motion. In particular, these groups can be shown to vanish for $g \leq -3$ in irreducible gauge theories [30, 46]. The group $H^{-2}(s|d)$ describes equivalence classes of "global" reducibility parameters, i.e., particular local functions f^α such that $R^a{}_\alpha(f^\alpha) \approx 0$ where $f^\alpha \sim f^\alpha + t^\alpha$ with $t^\alpha \approx 0$. This terminology reflects the fact that this cohomology may be non trivial even for (locally) "irreducible" gauge systems, in other words in the absence of p-form gauge fields with higher p. This will become clear momentarily and is crucial in this paper. These classes correspond to global symmetries of the master action rather than of the original action alone [47, 48]. The associated characteristic cohomology $H^{n-2}_{\text{char}}(d)$ captures non-trivial (flux) conservation laws. More generally in the case of free abelian p-form gauge symmetry it was shown in [28] that one can generalize the first Noether theorem ($p = 0$) and deduce by a similar formula a class of $H^{n-p-1}_{\text{char}}(d)$ generalizing the electric flux which corresponds to the case $p = 1$, i.e., to ordinary gauge invariance. The groups $H^{-1-p}(s|d)$ appear for p-form gauge theories and vanish for $p \geq 2$ in the irreducible case [49]. The group $H^{-1}(s|d)$ describes and generates the inequivalent global symmetries, with $H^{n-1}_{\text{char}}(d)$ encoding the associated inequivalent Noether currents.[2] We mention these groups here since they play an important role in the determination of $H^0(s|d)$ as it will be seen in section 3 below.

When $g_1 = -1 = g_2$, $(\cdot,\cdot) : H^{-1} \otimes H^{-1} \to H^{-1}$; in this case the antibracket map encodes the Lie algebra structure of the inequivalent global symmetries [50]. More generally, it follows from $(\cdot,\cdot) : H^{-1} \otimes H^g \to H^g$ that, for any ghost number g, the BRST cohomology classes form a representation of the Lie algebra of inequivalent global symmetries. As a side-remark, let us also mention that in the context of perturbative quantum field theory, $H^1(s|d)$ classifies potential gauge anomalies while $H^0(s|d)$ classifies counterterms.

For notational simplicity, we will drop the square brackets when computing the antibracket map below, but keep in mind that it involves classes and not their representatives.

2.2 Depth of an element

With any cocycle $\omega^{g,k}$ of the local BRST cohomology is associated a (s,d)-descent

$$s\omega_l^{g,k} + d\omega_l^{g+1,k-1} = 0, \quad s\omega_l^{g+1,k-1} + d\omega_l^{g+2,k-2} = 0, \; \dots \;, \; s\omega_l^{g+l,k-l} = 0, \tag{2.13}$$

that stops at some BRST cocycle $\omega_l^{g+l,k-l}$. The length l of the shortest non trivial descent is called the "depth" of $[\omega^{g,k}] \in H^{g,k}(s|d)$. The last element $\omega_l^{g+l,k-l}$ is then non trivial in $H^{g+l,k-l}(s)$. The usefulness of the depth in analyzing the BRST cohomology is particularly transparent in [43, 51, 52].

Local BRST cohomology classes $[\omega^{g,k}] \in H^{g,k}(s|d)$ are thus characterized, besides ghost number g and form degree k, by the depth l. In appendix B, we work out how the antibracket map behaves with respect to the depth of its elements.

3 Abelian vector-scalar models in 4 dimensions

3.1 Structure of the models

We now apply the formalism to the scalar-vector models described by the action (1.1). We write $\mathcal{L}_0 = \mathcal{L}_S[\phi^i] + \mathcal{L}_V[A^I_\mu, \phi^i]$. In four spacetime dimensions, there is no Chern-Simons term in the Lagrangian, which can be assumed to be strictly gauge invariant and not just invariant up to a total derivative. Gauge invariant functions are functions that depend on $F^I_{\mu\nu} = \partial_\mu A^I_\nu - \partial_\nu A^I_\mu$, ϕ^i and their derivatives, but not on $A^I_\mu, \partial_{(\nu} A^I_{\mu)}, \partial_{(\nu_1}\partial_{\nu_2} A^I_{\mu)}$, etc. Thus

[2] This is the "first" theorem by E. Noether and its converse. More details can be found in section 6.1 of [46].

$\mathcal{L}_V[A^I_\mu, \phi^i]$ depends on the vector potentials A^I_μ only through $F^I_{\mu\nu} = \partial_\mu A^I_\nu - \partial_\nu A^I_\mu$ and their derivatives.

We define

$$\frac{\delta \mathcal{L}_V}{\delta F^I_{\mu\nu}} = \frac{1}{2}(\star G_I)^{\mu\nu} \tag{3.1}$$

where the $(\star G_I)^{\mu\nu}$ are also manifestly gauge invariant functions. The equations of motion for the vector fields can be written as

$$\frac{\delta \mathcal{L}_0}{\delta A^I_\mu} = \partial_\nu (\star G_I)^{\mu\nu} \tag{3.2}$$

and the Lagrangian can be taken to be

$$\mathcal{L}_0 = \mathcal{L}_S[\phi^i] + \mathcal{L}_V[A^I_\mu, \phi^i], \quad d^4x\, \mathcal{L}_V = \int_0^1 \frac{dt}{t}[G_I F^I][tA^I_\mu, \phi^i]. \tag{3.3}$$

The associated solution to the BV master equation is given by

$$S^{(0)} = S_0 + \int d^4x\, A^{*\mu}_I \partial_\mu C^I. \tag{3.4}$$

The ghost number of the various fields and antifields is

	ϕ^i	A^I_μ	C^I	ϕ^*_i	$A^{*\mu}_I$	C^*_I
gh	0	0	1	-1	-1	-2

and the action of the BRST differential is given by

$$\begin{aligned} s\phi^i &= 0, & sA^I_\mu &= \partial_\mu C^I, & sC^I &= 0, \\ s\phi^*_i &= \frac{\delta \mathcal{L}_0}{\delta \phi^i}, & sA^{*\mu}_I &= \partial_\nu (\star G_I)^{\mu\nu}, & sC^*_I &= -\partial_\mu A^{*\mu}. \end{aligned} \tag{3.5}$$

It is useful to introduce the antifield number

	ϕ^i	A^I_μ	C^I	ϕ^*_i	$A^{*\mu}_I$	C^*_I
afd	0	0	0	1	1	2

and the pure ghost number

	ϕ^i	A^I_μ	C^I	ϕ^*_i	$A^{*\mu}_I$	C^*_I
pgh	0	0	1	0	0	0

so that the ghost number is the difference between the pure ghost number and the antifield number.

The BRST differential s splits according to antifield number as

$$s = \delta + \gamma \tag{3.6}$$

where δ is the "Koszul-Tate differential" [32, 41] and has antifield number -1. The differential γ has antifield number equal to zero. One has

$$\delta^2 = 0, \quad \delta\gamma + \gamma\delta = 0, \quad \gamma^2 = 0. \tag{3.7}$$

The action of δ and γ are respectively given by

$$\begin{aligned} &\delta\phi^i = 0, && \delta A^I_\mu = 0, && \delta C^I = 0, \\ &\delta\phi^*_i = \frac{\delta\mathcal{L}_0}{\delta\phi^i}, && \delta A^{*\mu}_I = \partial_\nu(\star G_I)^{\mu\nu}, && \delta C^*_I = -\partial_\mu A^{*\mu}, \end{aligned} \tag{3.8}$$

and

$$\begin{aligned} &\gamma\phi^i = 0, && \gamma A^I_\mu = \partial_\mu C^I, && \gamma C^I = 0, \\ &\gamma\phi^*_i = 0, && \gamma A^{*\mu}_I = 0, && \gamma C^*_I = 0. \end{aligned} \tag{3.9}$$

In terms of the Koszul-Tate differential, the cocycle condition for m in characteristic cohomology takes the form $dm + \delta n = 0$. This equation is the same as the (co)cycle condition for n in the local (co)homology of δ, which is indeed $\delta n + dm = 0$. Using this observation, and vanishing theorems for $H(d)$ and $H(\delta)$ in relevant degrees, one can establish isomorphisms between the characteristic cohomology and $H(\delta|d)$ [30]. For example, the characteristic cohomology $H^{n-2}_{\text{char}}(d)$ is given by the 2-forms $\mu^I G_I$, while $H^n_2(\delta|d)$ (where the superscript refers to form degree and the subscript to antifield number) is given by the 4-forms $d^4x\,\mu^I C^*_I$. The isomorphism is realized through the (δ, d)-descent

$$\delta\, d^4x\, C^*_I + d\star A^*_I = 0, \quad \delta \star A^*_I + dG_I = 0, \tag{3.10}$$

where $A^*_I = dx^\mu A^*_{I\mu}$.

3.2 Consistent deformations

One can characterize the BRST cohomological classes with non trivial antifield dependence in terms of conserved currents and rigid symmetries for all values of the ghost number. For definiteness, we illustrate explicitly the procedure for $H^0(s|d)$ in maximum form degree, which defines the local consistent deformations. We consider next the case of general ghost number.

The main equation to be solved for a is

$$sa + db = 0, \tag{3.11}$$

where a has form degree 4 and ghost number 0. To solve it, we expand the cocycle a according to the antifield number,

$$a = a_0 + a_1 + a_2. \tag{3.12}$$

Because a has total ghost number zero, each term a_n has antifield number n and pure ghost number (degree in the ghosts) n as well. As shown in [31], the expansion stops at most at antifield number 2. The term a_0 is the (first order) deformation of the Lagrangian. A non-vanishing a_1 corresponds to a deformation of the gauge variations, while a non-vanishing a_2 corresponds to a deformation of the gauge algebra. All three terms are related by the cocycle condition (3.11).

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3.2.1 Solutions of U-type (a_2 non trivial)

The first case to consider is when a_2 is non-trivial. This defines "class I" solutions in the terminology of [31], which we call here "U-type" solutions to comply with the general terminology introduced below. One has from the general theorems of [30, 31] on the invariant characteristic cohomology that

$$a_2 = d^4x\, C^*_I \Theta^I \tag{3.13}$$

with

$$\Theta^I = \frac{1}{2!} f^I{}_{J_1 J_2} C^{J_1} C^{J_2}. \tag{3.14}$$

Here $f^I{}_{J_1 J_2}$ are some constants, antisymmetric in J_1, J_2. The reason why the coefficient $d^4x\, C^*_I$ of the ghosts in a_2 is determined by the characteristic cohomology follows from the equation $\delta a_2 + \gamma a_1 + db_1 = 0$ that a_2 must fulfill in order for a to be a cocycle of $H(s|d)$. Given that a_2 has antifield number equal to 2, it is the characteristic cohomology in form degree $n-2=2$ that is relevant.[3] We refer the reader to [30, 31] for the details. The emergence of the characteristic cohomology in the computation of $H(s|d)$ will be observed again for a_1 below, where it will be the conserved currents that appear. This central feature follows from the fact that the Koszul-Tate differential, which encapsulates the equations of motion, is an essential building block of the BRST differential. We must now find the lower terms $a_1 + a_0$ and relate them as expected to Noether currents that correspond to $H^4_1(\delta|d)$.

By the argument of [31] (section 8) suitably generalized (section 12), the term a_1 is then found to be

$$a_1 = \star A^*_I A^K \partial_K \Theta^I + m_1 \tag{3.15}$$

where $\gamma m_1 = 0$ and $\partial_K = \frac{\partial}{\partial C^K}$. The term m_1 (to be determined by the next equation) is linear in C^I and can be taken to be linear in the undifferentiated antifields A^*_I and ϕ^*_i since derivatives of these antifields, which can occur only linearly, can be redefined away through trivial terms. We thus write

$$m_1 = \hat{K} = \star A^*_I \hat{g}^I - \star \phi^*_i \hat{\Phi}^i \tag{3.16}$$

with

$$\hat{g}^I = dx^\mu g^I_{\mu K} C^K\,, \qquad \hat{\Phi}^i = \Phi^i_K C^K. \tag{3.17}$$

Here $g^I_{\mu K}$ and Φ^i_K are gauge invariant functions, arbitrary at this stage, but which will be constrained by the requirement that a_0 exists.

We must now consider the equation $\delta a_1 + \gamma a_0 + db_0 = 0$ that determines a_0 up to a solution of $\gamma a'_0 + db'_0 = 0$. This equation is equivalent to

$$\left(\frac{\delta \mathcal{L}_0}{\delta A^I_\mu} \delta_K A^I_\mu + \frac{\delta \mathcal{L}_0}{\delta \phi^i} \delta_K \phi^i \right) C^K + \gamma \alpha_0 + \partial_\mu \beta^\mu_0 = 0, \tag{3.18}$$

[3] The precise way to express the relation between the local cohomology of δ and the highest term of the equation obeyed by a is given in section 7 of [31]: a_2 must be a non trivial representative of $H_{inv}(\delta|d)$, more precisely it must come from $H^4_{inv,2}(\delta|d)$ in ghost number zero. This relates the U-type deformations to the free abelian factors of the undeformed gauge group.

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where we have passed to dual notations ($a_0 = d^4x\, \alpha_0$, $db_0 = d^4x\, \partial_\mu \beta_0^\mu$) and where we have set

$$\delta_K A^I_\mu = A^J_\mu f^I{}_{JK} + g^I_{\mu K}, \quad \delta_K \phi^i = \Phi^i_K. \tag{3.19}$$

Writing $\beta^\mu_0 = j^\mu_K C^K +$ "terms containing derivatives of the ghosts", we read from (3.18), by comparing the coefficients of the undifferentiated ghosts, that

$$\frac{\delta \mathcal{L}_0}{\delta A^I_\mu} \delta_K A^I_\mu + \frac{\delta \mathcal{L}_0}{\delta \phi^i} \delta_K \phi^i + \partial_\mu j^\mu_K = 0. \tag{3.20}$$

A necessary condition for a_0 (and thus a) to exist is therefore that $\delta_K A^I_\mu$ and $\delta_K \phi^i$ define symmetries.

To proceed further and determine a_0, we observe that the non-gauge invariant term $\frac{\delta \mathcal{L}_0}{\delta A^I_\mu} A^J_\mu f^I{}_{JK}$ in $\frac{\delta \mathcal{L}_0}{\delta A^I_\mu} \delta_K A^I_\mu$ can be written as $\partial_\mu \left(\star G^{\nu\mu}_I A^J_\nu f^I{}_{JK} \right)$ plus a gauge invariant term, so that $j^\mu_K - \star G^{\mu\nu}_I A^J_\nu f^I{}_{JK}$ has a gauge invariant divergence. Results on the invariant cohomology of d [53, 54] imply then that the non-gauge invariant part of such an object can only be a Chern-Simons form, i.e. $j^\mu_K - \star G^{\mu\nu}_I A^J_\nu f^I{}_{JK} = J^\mu_K + \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} A^I_\nu F^J_{\rho\sigma} h_{I|JK}$, or

$$j^\mu_K = J^\mu_K + \star G^{\mu\nu}_I A^J_\nu f^I{}_{JK} + \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} A^I_\nu F^J_{\rho\sigma} h_{I|JK} \tag{3.21}$$

where J^μ_K is gauge invariant and where the symmetries of the constants $h_{I|JK}$ will be discussed in a moment. It is useful to point out that one can switch the indices I and J modulo a trivial term.

The equation (3.18) becomes $-(\partial_\mu j^\mu_K)\, C^K + \gamma \alpha_0 + \partial_\mu \beta^\mu_0 = 0$, i.e., $j^\mu_K\, (\gamma A^K_\mu) + \gamma \alpha_0 + \partial_\mu \beta'^\mu_0 = 0$. The first two terms in the current yield manifestly γ-exact terms,

$$J^\mu_K\, (\gamma A^K_\mu) = \gamma (J^\mu_K\, A^K_\mu), \quad \star G^{\mu\nu}_I A^J_\nu f^I{}_{JK}\, (\gamma A^K_\mu) = \frac{1}{2} \gamma (\star G^{\mu\nu}_I A^J_\nu f^I{}_{JK}\, A^K_\mu) \tag{3.22}$$

and so $h_{I|JK}$ must be such that the term $A^I F^J dC^K h_{I|JK}$ is by itself γ-exact modulo d. This is a problem that has been much studied in the literature through descent equations (see e.g. [52]). It has been shown that $h_{I|JK}$ must be antisymmetric in J, K and should have vanishing totally antisymmetric part in order to be "liftable" to a_0 and non-trivial,

$$h_{I|JK} = h_{I|[JK]}, \quad h_{[I|JK]} = 0. \tag{3.23}$$

Putting things together, one finds for a_0

$$a_0 = A^I \partial_I \hat{J} + \frac{1}{2} G_I A^K A^L \partial_L \partial_K \Theta^I + \frac{1}{2} F^I A^K A^L \partial_L \partial_K \Theta'_I \tag{3.24}$$

where

$$\hat{J} = \star dx^\mu J_{\mu K} C^K\,, \quad \Theta'_I = \frac{1}{2} h_{I|J_1 J_2} C^{J_1} C^{J_2}. \tag{3.25}$$

A non-trivial U-solution modifies the gauge algebra. Deformations of the Yang-Mills type belong to this class. A U-solution is characterized by constants $f^I{}_{J_1 J_2}$ which are antisymmetric in J_1, J_2. These constants must be such that there exist gauge invariant functions $g^I_{\mu K}$ and Φ^i_K such that $\delta_K A^I_\mu$ and $\delta_K \phi^i$ define symmetries of the undeformed

Lagrangian. Here $\delta_K A^I_\mu$ and $\delta_K \phi^i$ are given by (3.19). Furthermore, the h-term in the corresponding conserved current (if any) must fulfill (3.23). The deformation a_0 of the Lagrangian takes the Noether-like form.

Given the "head" a_2 of a U-type solution, characterized by a set of $f^I{}_{J_1 J_2}$'s, the lower terms a_1 and a_0, and in particular the h-piece, are not uniquely determined. One can always add solutions of W, V or I-types described below, which have the property that they have no a_2-piece. Hence one may require that the completion of the "head" a_2 of a U-type solution should be chosen to vanish when a_2 itself vanishes. But this leaves some freedom in the completion of a_2, since for instance any W-type solution multiplied by a component of $f^I{}_{J_1 J_2}$ will vanish when the $f^I{}_{J_1 J_2}$'s are set to zero. The situation has a triangular nature since two U-type solutions with the same a_2 differ by solutions of "lower" types, for which there might not be a canonical choice.

Note that further constraints on $f^I{}_{J_1 J_2}$ (notably the Jacobi identity) arise at second order in the deformation parameter.

3.2.2 Solutions of W and V-type (vanishing a_2 but a_1 non trivial)

These solutions are called "class II" solutions in [31].

We now have

$$a = a_0 + a_1 \tag{3.26}$$

and a_1 can be taken to be gauge invariant, i.e., annihilated by γ [31]. We thus have

$$a_1 = \hat{K} = \star A^*_I \hat{g}^I - \star \phi^*_i \hat{\Phi}^i \tag{3.27}$$

with

$$\hat{g}^I = dx^\mu g^I_{\mu K} C^K \,, \qquad \hat{\Phi}^i = \Phi^i_K C^K \,. \tag{3.28}$$

Here $g^I_{\mu K}$ and Φ^i_K are again gauge invariant functions, which we still denote by the same letters as above, although they are independent from the similar functions related to the constants $f^I{}_{J_1 J_2}$. We also set

$$\delta_K A^I_\mu = g^I_{\mu K}, \quad \delta_K \phi^i = \Phi^i_K. \tag{3.29}$$

The equation $\delta a_1 + \gamma a_0 + d b_0 = 0$ implies then, as above,

$$\frac{\delta \mathcal{L}_0}{\delta A^I_\mu} \delta_K A^I_\mu + \frac{\delta \mathcal{L}_0}{\delta \phi^i} \delta_K \phi^i + \partial_\mu j^\mu_K = 0 \,. \tag{3.30}$$

A necessary condition for a_0 (and thus a) to exist is therefore that $\delta_K A^I_\mu$ and $\delta_K \phi^i$ given by (3.29) define symmetries. Equation (3.30) take the same form as eq. (3.20), but there is one important difference: the divergence of the current j^μ_K is now gauge invariant, while it is *not* in (3.20), due to the contribution coming from a_2.

The current takes this time the form

$$j^\mu_K = J^\mu_K + \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} A^I_\nu F^J_{\rho\sigma} h_{I|JK}, \tag{3.31}$$

(with $h_{I|JK}$ fulfilling the above symmetry properties) yielding

$$a_0 = A^I \partial_I \hat{J} + \frac{1}{2} F^I A^K A^L \partial_L \partial_K \Theta'_I \tag{3.32}$$

where still

$$\hat{J} = \star dx^\mu J_{\mu K} C^K \,, \quad \Theta'_I = \frac{1}{2} h_{I|J_1 J_2} C^{J_1} C^{J_2}. \tag{3.33}$$

We define W-type solutions to have $h_{I|JK} \neq 0$, while V-type have $h_{I|JK} = 0$. Both these types deform the gauge transformations but not their algebra (to first order in the deformation). They are determined by rigid symmetries of the undeformed Lagrangian with gauge invariant variations (3.29). The V-type have gauge invariant currents, while the currents of the W-type contain a non-gauge invariant piece.

Note that again, the solutions of W and V-types are determined up to a solution of lower type with no a_1-"head", and that there might not be a canonical choice. In fact one may require similarly that W-type transformations become trivial when $h_{I|JK}$ tends to zero.

3.2.3 Solutions of I-type (vanishing a_2 and a_1)

In that case,

$$a = a_0 \tag{3.34}$$

with $\gamma a_0 + d b_0 = 0$.

Since there is no Chern-Simons term in four dimensions, one can assume that $b_0 = 0$. The deformation b_0 is therefore a gauge invariant function, i.e., a function of the abelian curvatures $F^I_{\mu\nu}$, the scalar fields, and their derivatives. The I-type deformations neither deform the gauge transformations nor (a fortiori) the gauge algebra. Born-Infeld deformations belong to this type. They are called "class III" solutions in [31].

3.3 Local BRST cohomology at other ghost numbers

3.3.1 h-terms

The previous discussion can be repeated straightforwardly at all ghost numbers. The analysis proceeds as above. The tools necessary to handle the "h-term" in the non gauge invariant "currents" have been generalized to higher ghost numbers through familiar means and can be found in [43, 51, 52].

The h-terms belong to the "small" or "universal" algebra involving only the 1-forms A^I, the 2-forms $F^I = dA^I$, the ghosts C^I and their exterior derivative. The product is the exterior product. One describes the h-term through a (γ, d)-descent equation and what is called the "bottom" of that descent, which is annihilated by γ and has form degree < 4 in four dimensions. The only possibilities in the free abelian case are the 2-forms

$$\frac{1}{m} h_{I|J_1 \cdots J_m} F^I C^{J_1} \cdots C^{J_m} \tag{3.35}$$

where

$$h_{I|J_1 \cdots J_m} = h_{I|[J_1 \cdots J_m]}. \tag{3.36}$$

One can assume $h_{[I|J_1\cdots J_m]}=0$ since the totally antisymmetric part gives a trivial bottom. The lift of this bottom goes two steps, up to the 4-form

$$h_{I|J_1J_2\cdots J_m}F^IF^{J_1}C^{J_2}\cdots C^{J_m} \tag{3.37}$$

producing along the way a 3-form

$$h_{I|J_1J_2\cdots J_m}F^IA^{J_1}C^{J_2}\cdots C^{J_m} \tag{3.38}$$

which has the property of not being gauge (BRST) invariant although its exterior derivative is (modulo trivial terms).

3.3.2 Explicit description of cohomology

By applying the above method, one finds that the local BRST cohomology of the models of section 3.1 can be described along exactly the same lines as given below. Note that the cohomology at negative ghost numbers reflect general properties of the characteristic cohomology that go beyond the mere models considered here [30].

(i) $H^g(s|d)$ is empty for $g\leqslant -3$.

(ii) $H^{-2}(s|d)$ is represented by the 4-forms

$$U^{-2}=\mu^I d^4x\, C^*_I. \tag{3.39}$$

If $A^*_I=dx^\mu A^*_{I\mu}$, the associated descent equations are

$$s\, d^4x\, C^*_I+d\star A^*_I=0,\quad s\star A^*_I+dG_I=0,\quad sG_I=0. \tag{3.40}$$

Characteristic cohomology $H^{n-2}_{\text{char}}(d)$ is then represented by the 2-forms μ^IG_I.

(iii) Several types of cohomology classes in ghost numbers $g\geqslant -1$, which we call U, V and W-type, can be described by constants $f^I{}_{JK_1\ldots K_{g+1}}$ which are antisymmetric in the last $g+2$ indices,

$$f^I{}_{JK_1\ldots K_{g+1}}=f^I{}_{[JK_1\ldots K_{g+1}]}, \tag{3.41}$$

and constants $h_{I|JK_1\ldots K_{g+1}}$ that are antisymmetric in the last $g+2$ indices but without any totally antisymmetric part,[4]

$$h_{I|JK_1\ldots K_{g+1}}=h_{I|[JK_1\ldots K_{g+1}]},\quad h_{[I|JK_1\ldots K_{g+1}]}=0, \tag{3.42}$$

together with gauge invariant functions $g^I_{\mu K_1\ldots K_{g+1}}$, $\Phi^i_{K_1\ldots K_{g+1}}$ that are antisymmetric in the last $g+1$ indices. They are constrained by the requirement that the transformations

$$\delta_{K_1\ldots K_{g+1}}A^I_\mu=A^J_\mu f^I{}_{JK_1\ldots K_{g+1}}+g^I_{\mu K_1\ldots K_{g+1}},\quad \delta_{K_1\ldots K_{g+1}}\phi^i=\Phi^i_{K_1\ldots K_{g+1}}, \tag{3.43}$$

[4] We write $h_{IJ}:=h_{I|J}$ for $g=-1$.

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define symmetries of the action in the sense that

$$\frac{\delta \mathcal{L}_0}{\delta A^I_\mu}\delta_{K_1\dots K_{g+1}}A^I_\mu + \frac{\delta \mathcal{L}_0}{\delta \phi^i}\delta_{K_1\dots K_{g+1}}\phi^i + \partial_\mu j^\mu_{K_1\dots K_{g+1}} = 0, \tag{3.44}$$

with currents $j^\mu_{K_1\dots K_{g+1}}$ that are antisymmetric in the last $g+1$ indices. This can be made more precise by making the gauge (non-)invariance properties of these currents manifest. One finds

$$j^\mu_{K_1\dots K_{g+1}} = J^\mu_{K_1\dots K_{g+1}} + \star G^{\mu\nu}_I A^J_\nu f^I{}_{JK_1\dots K_{g+1}} + \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}A^I_\nu F^J_{\rho\sigma}h_{I|JK_1\dots K_{g+1}}, \tag{3.45}$$

where $J^\mu_{K_1\dots K_{g+1}}$ is gauge invariant and antisymmetric in the lower $g+1$ indices. When taking into account that

$$G_I F^J = d(G_I A^J + \star A^*_I C^J) + s(\star A^*_I A^J + d^4x C^*_I C^J), \quad F^I F^J = d(A^I F^J), \tag{3.46}$$

and defining $C^{K_1\dots K_g} = C^{K_1}\dots C^{K_g}$,

$$\begin{aligned} \Theta^I &= \frac{1}{(g+2)!} f^I{}_{J_1\dots J_{g+2}} C^{J_1\dots J_{g+2}}, & \Theta'_I &= \frac{1}{(g+2)!} h_{I|J_1\dots J_{g+2}} C^{J_1\dots J_{g+2}}, \\ \hat{J} &= \star dx^\mu J_{\mu K_1\dots K_{g+1}} \frac{1}{(g+1)!} C^{K_1\dots K_{g+1}}, & \hat{K} &= (\star A^*_I \hat{g}^I - \star \phi^*_i \hat{\Phi}^i), \\ \hat{g}^I &= \frac{1}{(g+1)!} dx^\mu g^I_{\mu K_1\dots K_{g+1}} C^{K_1\dots K_{g+1}}, & \hat{\Phi}^i &= \frac{1}{(g+1)!} \Phi^i_{K_1\dots K_{g+1}} C^{K_1\dots K_{g+1}}, \end{aligned} \tag{3.47}$$

the "global symmetry" condition (3.44) is equivalent to a (s,d)-obstruction equation,

$$G_I F^J \partial_J \Theta^I + F^I F^J \partial_J \Theta'_I + s(\hat{K} + A^I \partial_I \hat{J}) + d\hat{J} = 0, \tag{3.48}$$

with $\partial_I = \frac{\partial}{\partial C^I}$. Note that the last two terms combine into

$$d[\star dx^\mu J_{\mu K_1\dots K_{g+1}}]\frac{1}{(g+1)!}C^{K_1}\dots C^{K_{g+1}},$$

so that this equation involves gauge invariant quantities only. It is this form that arises in a systematic analysis of the descent equations. One can now distinguish the three types of solutions.

a) U-type corresponds to solutions with non vanishing $f^I{}_{JK_1\dots K_{g+1}}$ and particular ${}^U h_{I|JK_1\dots K_{g+1}}$, ${}^U g^I_{\mu K_1\dots K_{g+1}}$, ${}^U\Phi^i_{K_1\dots K_{g+1}}$, ${}^U J_{\mu K_1\dots K_{g+1}}$ that vanish when the f's vanish (and that may be vanishing even when the f's do not). As we explained above, different choices of the particular completion ${}^U h_{I|JK_1\dots K_{g+1}}$, ${}^U g^I_{\mu K_1\dots K_{g+1}}$, ${}^U\Phi^i_{K_1\dots K_{g+1}}$, ${}^U J_{\mu K_1\dots K_{g+1}}$ of a_2 exist and there might not be a canonical one, but a completion exists if the U-type solution is indeed a solution. Similar ambiguity holds for the solutions of W and V-types described below. A U-type solution is trivial if and only if it vanishes. Denoting by $\hat{K}_U, \hat{J}_U, (\Theta'_U)_I$, the expressions as in (3.47) but involving the particular solutions, the associated BRST cohomology classes are represented by

$$U = \left(d^4x C_I^* + \star A_I^* A^K \partial_K + \frac{1}{2} G_I A^K A^L \partial_L \partial_K \right) \Theta^I + \hat{K}_U + \frac{1}{2} F^I A^K A^L \partial_L \partial_K (\Theta'_U)_I + A^I \partial_I \hat{J}_U, \tag{3.49}$$

with $sU + d(\star A_I^* \Theta^I + G_I A^J \partial_J \Theta^I + F^I A^J \partial_J (\Theta'_U)_I + \hat{J}_U) = 0$;

b) W-type corresponds to solutions with vanishing f's but non vanishing $h_{I|JK_1\dots K_{g+1}}$ and particular ${}^W g^I_{\mu K_1\dots K_{g+1}}$, ${}^W \Phi^i_{K_1\dots K_{g+1}}$, ${}^W J_{\mu K_1\dots K_{g+1}}$ that may be chosen to vanish when the h's vanish. Such solutions are trivial when the h's vanish. With the obvious notation, the associated BRST cohomology classes are represented by

$$W = \hat{K}_W + \frac{1}{2} F^I A^K A^L \partial_L \partial_K \Theta'_I + A^I \partial_I \hat{J}_W, \tag{3.50}$$

with $sW + d(F^I A^J \partial_J \Theta'_I + \hat{J}_W) = 0$;

c) V-type corresponds to solutions with vanishing f's and h's. They are represented by

$$V = \hat{K}_V + A^I \partial_I \hat{J}_V, \tag{3.51}$$

with $sV + d\hat{J}_V = 0$ and $s\hat{J}_V = 0$. V and its descent have depth 1.

(iv) Lastly, I-type cohomology classes exist in ghost numbers $g \geqslant 0$ and are described by

$$\hat{I} = d^4x \, \frac{1}{g!} I_{K_1\dots K_g} C^{K_1} \dots C^{K_g} \tag{3.52}$$

with $s\hat{I} = 0$, i.e., gauge invariant $I_{K_1\dots K_g}$ that are completely antisymmetric in the K indices. Such classes are to be considered trivial if the $I_{K_1\dots K_s}$ vanish on-shell up to a total derivative. This can again be made more precise by making the gauge (non-)invariance properties manifest: an element of type I is trivial if and only if

$$d^4x \, I_{K_1\dots K_g} \approx dJ_{K_1\dots K_g} + m^I{}_{JK_1\dots K_g} G_I F^J + \frac{1}{2} F^I F^J m'_{IJK_1\dots K_g}, \tag{3.53}$$

where $J_{K_1\dots K_g}$ are gauge invariant 3 forms that are completely antisymmetric in the K indices, while $m^I{}_{JK_1\dots K_g}, m'_{IJK_1\dots K_g}$ are constants that are completely antisymmetric in the last $g+1$ indices. Note also that the on-shell vanishing terms in (3.53) need to be gauge invariant. When there are suitable restrictions on the space of gauge invariant functions (such as for instance x^μ independent, Lorentz invariant polynomials with power counting restrictions) one may sometimes construct an explicit basis of non-trivial gauge invariant 4 forms, in the sense that if $d^4xI \approx \rho^{\mathcal{A}} I_{\mathcal{A}} + d\omega^3$ and $\rho^{\mathcal{A}} I_{\mathcal{A}} \approx d\omega^3$, then $\rho^{\mathcal{A}} = 0$. The associated BRST cohomology classes are then parametrized by constants $\rho^{\mathcal{A}}{}_{K_1\dots K_g}$.

At a given ghost number $g \geqslant -1$, the cohomology is the direct sum of elements of type U, W, V and also I when $g \geqslant 0$.

This completes our general discussion of the local BRST cohomology. Reference [31] also considered simple factors in addition to the abelian factors, as well as any spacetime dimension ≥ 3. One can extend the above results to cover these cases. In a separate publication [55], the computation of the local BRST cohomology $H^{*,*}(s|d)$ for gauge models involving general reductive gauge algebras will be carried out by following the different route adopted in [46], which did not consider free abelian factors in full generality. As requested by the analysis of the deformations of the action (1.1), reference [55] generalizes Theorem 11.1 of [46] to arbitrary reductive Lie algebras that include also (free) abelian factors (and in any spacetime dimension ≥ 3).

3.3.3 Depth of solutions

The depth of the various BRST cocycles plays a key role in the analysis of the higher-order consistency condition. It is given here.

The U-type and W-type solutions have depth 2 because they involve $A_\mu j^\mu$ with a non-gauge invariant current. The V-type solutions have depth 1 because the Noether term $A_\mu j^\mu$ involves for them a gauge invariant current. Finally, I-type solutions clearly have depth 0.

4 Antibracket map and structure of symmetries

4.1 Antibracket map in cohomology

We now investigate the antibracket map $H^g \otimes H^{g'} \to H^{g+g'+1}$ for the different types of cohomology classes described above. It follows from the detailed discussion of the cohomology in section 3.3 that the shortest non trivial length of descents, the "depth", of elements of type U, W, V, I is 2, 2, 1, 0. In particular, the antibracket map is sensitive to the depth of its arguments: the depth of the map is less than or equal to the depth of its most shallow element, see appendix B.

The antibracket map involving $U^{-2} = \mu^I d^4x\, C^*_I$ in H^{-2} is given by

$$(\cdot, U^{-2}) : H^g \to H^{g-1}, \quad \omega^{g,n} \mapsto \frac{\delta^R \omega^{g,n}}{\delta C^I}\mu^I. \tag{4.1}$$

More explicitly, it is trivial for $g = -2$. It is also trivial for $g = -1$ except for U-type where it is described by $f^I{}_J \mapsto f^I{}_J \mu^J$. For $g > 0$, it is described by $\rho^{\mathcal{A}}_{K_1 \dots K_g} \mapsto \rho^{\mathcal{A}}_{K_1 \dots K_g} \mu^{K_g}$ for I-type, $k^{v_1}{}_{K_1 \dots K_{g+1}} \mapsto k^{v_1}{}_{K_1 \dots K_{g+1}} \mu^{K_{g+1}}$ for V-type, $h_{IJK_1 \dots K_{g+1}} \mapsto h_{IJK_1 \dots K_{g+1}} \mu^{K_{g+1}}$ and $f^I{}_{JK_1 \dots K_{g+1}} \mapsto f^I{}_{JK_1 \dots K_{g+1}} \mu^{K_{g+1}}$ for U- and W-type.

The antibracket map for $g, g' \geqslant -1$ has the following triangular structure:

$(\cdot,\cdot)$	U	W	V	I
U	$U \oplus W \oplus V \oplus I$	$W \oplus V \oplus I$	$V \oplus I$	I
W	$W \oplus V \oplus I$	$W \oplus V \oplus I$	$V \oplus I$	I
V	$V \oplus I$	$V \oplus I$	$V \oplus I$	I
I	I	I	I	0

(4.2)

Indeed, $(\hat{I}, \hat{I}') = 0$ because I-type cocycles can be chosen to be antifield independent. For all other brackets involving I-type cocycles, it follows from appendix B that the result must have depth 0 and the only such classes are of I type. Alternatively, since all cocycles can be chosen to be at most linear in antifields, the result will be a cocycle that is antifield independent and only classes of I-type have trivial antifield dependence. It thus follows that I-type cohomology forms an abelian ideal.

According to appendix B, the depth of the antibracket map of V-type cohomology with V, W, U-type is less or equal to 1, so it must be of V- or I-type.

Finally, the remaining structure follows from the fact that only brackets of U-type cocycles with themselves may give rise to terms that involve C^*_I's.

4.2 Structure of the global symmetry algebra

Let us now concentrate on brackets between two elements that have both ghost number -1, i.e., on the detailed structure of the Lie algebra of inequivalent global symmetries when taking into account their different types.

In this case, one may use the table above supplemented by the fact that $I^{-1} = 0$. Let then

$$U_u, \quad W_w, \quad V_v, \tag{4.3}$$

be bases of symmetries of U, W, V-type.[5] At ghost number $g = -1$, equations (3.49), (3.50), (3.51) give

$$V_v = K_v, \quad W_w = K_w, \quad U_u = (f_u)^I{}_J[d^4x\, C^*_I C^J + \star A^*_I A^J] + K_u. \tag{4.4}$$

It follows from (4.2) that V-type symmetries and the direct sum of V and W-type symmetries form ideals in the Lie algebra of inequivalent global symmetries.

The symmetry algebra $\mathfrak{g}_U$ is defined as the quotient of all inequivalent global symmetries by the ideal of $V \oplus W$-type symmetries. In particular, if U-type symmetries form a sub-algebra, it is isomorphic to $\mathfrak{g}_U$.

First, V-type symmetries are parametrized by constants k^v, $V^{-1} = k^v V_v$. The gauge invariant symmetry transformation on the original fields then are

$$\delta_v A^I_\mu = -(V_v, A^I_\mu) = g_{v\mu}{}^I, \quad \delta_v \phi^i = -(V_v, \phi^i) = \Phi^i_v. \tag{4.5}$$

Furthermore, there exist constants $C^{v_3}{}_{v_1 v_2}$ such that

$$([V_{v_1}], [V_{v_2}]) = -C^{v_3}{}_{v_1 v_2}[V_{v_3}] \tag{4.6}$$

holds for the cohomology classes. We choose the minus sign because

$$(V_{v_1}, V_{v_2}) = -d^4x(A^{*\mu}_I[\delta_{v_1}, \delta_{v_2}]A^I_\mu + \phi^*_i[\delta_{v_1}, \delta_{v_2}]\phi^i), \tag{4.7}$$

[5] These are bases in the cohomological sense, i.e., $\sum_u \lambda^u [U_u] = [0] \Rightarrow \lambda^u = 0$ (and similarly for W_w and V_v). In terms of the representatives, this becomes $\sum_u \lambda^u U_u = sa + db \Rightarrow \lambda^u = 0$.

so that the $C^{v_3}{}_{v_1v_2}$ are the structure constants of the commutator algebra of the V-type symmetries, $[\delta_{v_1}, \delta_{v_2}] = C^{v_3}{}_{v_1v_2}\delta_{v_3}$. For the functions $g_{v\mu}{}^I$ and Φ^i_v, this gives

$$\begin{aligned} \delta_{v_1} g_{v_2\mu}{}^I - \delta_{v_2} g_{v_1\mu}{}^I &= C^{v_3}{}_{v_1v_2} g_{v_3\mu}{}^I + (\text{trivial}) \\ \delta_{v_1}\Phi^i_{v_2} - \delta_{v_2}\Phi^i_{v_1} &= C^{v_3}{}_{v_1v_2}\Phi^i_{v_3} + (\text{trivial}). \end{aligned} \tag{4.8}$$

The "trivial" terms on the right hand side take the form "(gauge transformation) + (antisymmetric combination of the equations of motion)" which is the usual ambiguity in the form of global symmetries, see e.g. section 6 of [46]. They come from the fact that equation (4.6) holds for classes: for the representatives V_v themselves, (4.6) is $(V_{v_1}, V_{v_2}) = -C^{v_3}{}_{v_1v_2}V_{v_3} + sa + db$. The trivial terms in (4.8) are then the symmetries generated by the extra term $sa + db$, which is zero in cohomology. The graded Jacobi identity for the antibracket map implies the ordinary Jacobi identity for these structure constants,

$$C^{v_1}{}_{v_2[v_3}C^{v_2}{}_{v_4v_5]} = 0. \tag{4.9}$$

Next, W-type symmetries are parametrized by constants k^w, $W^{-1} = k^w W_w$ and encode the gauge invariant symmetry transformations

$$\delta_w A^I_\mu = -(W_w, A^I_\mu) = g_{w\mu}{}^I, \quad \delta_w\phi^i = -(W_w, \phi^i) = \Phi^i_w \tag{4.10}$$

with associated Noether 3 forms $j_W = k^w (h_w)_{IJ} F^{(I}A^{J)} + k^w J_{Ww}$. There then exist $C^{v_2}{}_{wv_1}$, $C^{w_3}{}_{w_1w_2}$, $C^v{}_{w_1w_2}$ such that

$$\begin{aligned} ([W_w], [V_v]) &= -C^{v_2}{}_{wv}[V_{v_2}], \\ ([W_{w_1}], [W_{w_2}]) &= -C^{w_3}{}_{w_1w_2}[W_{w_3}] - C^v{}_{w_1w_2}[V_v], \end{aligned} \tag{4.11}$$

with associated Jacobi identities that we do not spell out. For the functions $g_{w\mu}{}^I$ and Φ^i_w, this implies

$$\delta_w g_{v\mu}{}^I - \delta_v g_{w\mu}{}^I = C^{v_2}{}_{wv} g_{v_2\mu}{}^I, \quad \delta_w\Phi^i_v - \delta_v\Phi^i_w = C^{v_2}{}_{wv}\Phi^i_{v_2}, \tag{4.12}$$

$$\delta_{w_1} g_{w_2\mu}{}^I - \delta_{w_2} g_{w_1\mu}{}^I = C^{w_3}{}_{w_1w_2} g_{w_3\mu}{}^I + C^v{}_{w_1w_2} g_{v\mu}{}^I, \tag{4.13}$$

$$\delta_{w_1}\Phi^i_{w_2} - \delta_{w_2}\Phi^i_{w_1} = C^{w_3}{}_{w_1w_2}\Phi^i_{w_3} + C^v{}_{w_1w_2}\Phi^i_v, \tag{4.14}$$

up to trivial terms, see the discussion below (4.8).

Finally, U-type symmetries are parametrized by k^u, $U^{-1} = k^u U_u$ and encode the symmetry transformations

$$\begin{aligned} \delta_u A^I_\mu &= -(U_u, A^I_\mu) = (f_u)^I{}_J A^J_\mu + g_{u\mu}{}^I, & \delta_u\phi^i &= -(U_u, \phi^i) = \Phi^i_u, \\ \delta_u A^{*\mu}_I &= -(f_u)^K{}_I A^{*\mu}_K - \frac{\delta}{\delta A^I_\mu}(A^{*\nu}_K g_{u\nu}{}^K + \phi^*_i\Phi^i_u), & \delta_u\phi^*_i &= -\frac{\delta}{\delta\phi^i}(A^{*\nu}_K g_{u\nu}{}^K + \phi^*_j\Phi^j_u), \\ \delta_u C^I &= (f_u)^I{}_J C^J, & \delta_u C^*_I &= -(f_u)^K{}_I C^*_K. \end{aligned} \tag{4.15}$$

Again, there exist constants C with various types of indices such that

$$([U_u], [V_v]) = -C^{v_2}{}_{uv}[V_{v_2}], \tag{4.16}$$

$$([U_u], [W_w]) = -C^{w_2}{}_{uw}[W_{w_2}] - C^v{}_{uw}[V_v], \tag{4.17}$$

$$([U_{u_1}], [U_{u_2}]) = -C^v{}_{u_1u_2}[V_v] - C^w{}_{u_1u_2}[W_w] - C^{u_3}{}_{u_1u_2}[U_{u_3}], \tag{4.18}$$

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with associated Jacobi identities. Working out the term proportional to C^*_I in (U_{u_1}, U_{u_2}) gives the commutation relations for the $(f_u)^I{}_J$ matrices,

$$[f_{u_1}, f_{u_2}] = -C^{u_3}{}_{u_1 u_2} f_{u_3}. \tag{4.19}$$

In turn, this implies Jacobi identities for this type of structure constants alone:

$$C^{u_1}{}_{u_2[u_3} C^{u_2}{}_{u_4 u_5]} = 0. \tag{4.20}$$

The $C^{u_3}{}_{u_1 u_2}$ are the structure constants of $\mathfrak{g}_U$.

From equation (4.16), we get the identities

$$\delta_u g_{v\mu}{}^I - \delta_v g_{u\mu}{}^I - (f_u)^I{}_J g_{v\mu}{}^J = C^{v_2}{}_{uv} g_{v_2\mu}{}^I, \quad \delta_u \Phi^i_v - \delta_v \Phi^i_u = C^{v_2}{}_{uv} \Phi^i_{v_2}. \tag{4.21}$$

Equation (4.17) gives the same identities with the right-hand side replaced by the appropriate sum, as in (4.13)–(4.14). The last relation (4.18) gives

$$\begin{aligned} \delta_{u_1} g_{u_2\mu}{}^I - (f_{u_1})^I{}_J g_{u_2\mu}{}^J - (u_1 \leftrightarrow u_2) &= C^{u_3}{}_{u_1u_2} g_{u_3\mu}{}^I + C^{w}{}_{u_1u_2} g_{w\mu}{}^I + C^{v}{}_{u_1u_2} g_{v\mu}{}^I, \\ \delta_{u_1} \Phi^i_{u_2} - \delta_{u_2} \Phi^i_{u_1} &= C^{u_3}{}_{u_1u_2} \Phi^i_{u_3} + C^{w}{}_{u_1u_2} \Phi^i_w + C^{v}{}_{u_1u_2} \Phi^i_v. \end{aligned} \tag{4.22}$$

Equations (4.21) and (4.22) are again valid only up to trivial symmetries.

Let us now concentrate on identities containing the h_{IJ}, which appear in the currents of U and W-type. We first consider (U_u, W_w) projected to W-type. As in appendix B, we have $s(U_u, W_w)_{\text{alt}} = -d(U_u, (h_w)_{IJ} F^I A^J + J_w)_{\text{alt}} = d\{(h_w)_{IJ}[(f_u)^I{}_K F^K A^J + F^I (f_u)^J{}_K A^K] + \text{invariant}\}$. When comparing this to s applied to the right hand side of (4.17) and using the fact that W-type cohomology is characterized by the Chern-Simons term in its Noether current, we get

$$(h_w)_{IN} (f_u)^I{}_M + (h_w)_{MI} (f_u)^I{}_N = C^{w_2}{}_{uw} (h_{w_2})_{MN}. \tag{4.23}$$

This computation amounts to identifying the Chern-Simons term in the U-variation $\delta_u j_w$ of a current of W-type. The same computation applied to (W_{w_1}, W_{w_2}) shows that $C^{w_3}{}_{w_1 w_2} (h_{w_3})_{MN} = 0$, which implies

$$C^{w_3}{}_{w_1 w_2} = 0 \tag{4.24}$$

since the matrices h_w are linearly independent (otherwise, the W_w would not form a basis). In other words, the W-variation $\delta_{w_1} j_{w_2}$ of a current of W-type is gauge invariant up to trivial terms, i.e., is of V-type.

In order to work out (U_{u_1}, U_{u_2}) projected to W-type, a slightly involved reasoning gives

$$\delta_u G_I + (f_u)^J{}_I G_J \approx -2(h_u)_{IJ} F^J + \lambda^w_u (h_w)_{IJ} F^J + d(\text{invariant}) \tag{4.25}$$

for some constants λ^w_u. This is proved in appendix C in the case where G_I does not depend on derivatives of F^I (but can have otherwise arbitrary dependence of F^I). We were not able to find the analog of (4.25) in the higher derivative case.

Applying then $(U_{u_1},\cdot)_{\rm alt}$ to the chain of descent equations for U_{u_2} and adding the chain of descent equations for $C^{u_3}_{u_1u_2}U_{u_3}$ yields

$$\begin{aligned}(h_{u_2})_{IN}(f_{u_1})^I{}_M + (h_{u_2})_{MI}(f_{u_1})^I{}_N - (h_{u_1})_{IN}(f_{u_2})^I{}_M - (h_{u_1})_{MI}(f_{u_2})^I{}_N \\ + \frac{1}{2}\left[(h_w)_{IN}(f_{u_2})^I{}_M + (h_w)_{IM}(f_{u_2})^I{}_N\right]\lambda^w_{u_1} \\ = C^{u_3}{}_{u_1u_2}(h_{u_3})_{MN} + C^w{}_{u_1u_2}(h_w)_{MN}.\end{aligned} \tag{4.26}$$

Again, this amounts to identifying the Chern-Simons terms in the U-variation $\delta_{u_1}j_{u_2}$ of a U-type current. Equation (4.25) is crucial for this computation since U-type currents contain G_I. Using (4.23), this becomes

$$\begin{aligned}(h_{u_2})_{IN}(f_{u_1})^I{}_M + (h_{u_2})_{MI}(f_{u_1})^I{}_N - (h_{u_1})_{IN}(f_{u_2})^I{}_M - (h_{u_1})_{MI}(f_{u_2})^I{}_N \\ = C^{u_3}{}_{u_1u_2}(h_{u_3})_{MN} + \left[C^w{}_{u_1u_2} - \frac{1}{2}C^w{}_{u_2w_2}\lambda^{w_2}_{u_1}\right](h_w)_{MN}.\end{aligned} \tag{4.27}$$

We see that the effect of the λ^w_u is to shift the structure constants of type $C^w{}_{u_1u_2}$. The constants λ^w_u vanish for the explicit models considered below; it would be interesting to find an explicit example where this is not the case. As a last comment, we note that antisymmetry of equation (4.27) in u_1 and u_2 imposes the constraint

$$C^w{}_{u_2w_2}\lambda^{w_2}_{u_1} + C^w{}_{u_1w_2}\lambda^{w_2}_{u_2} = 0 \tag{4.28}$$

on the constants λ^w_u.

4.3 Parametrization through symmetries

It follows from the discussion of the antibracket map involving H^{-2} after (4.1) that cohomologies of U, W, V-type in ghost numbers $g \geqslant 0$ can be parametrized by symmetries of the corresponding type with suitably constrained coefficients

$$k^u{}_{K_1\dots K_{g+1}}, \quad k^v{}_{K_1\dots K_{g+1}}, \quad k^w{}_{K_1\dots K_{g+1}}. \tag{4.29}$$

In this way, for $g=0$, the problem of finding all infinitesimal gaugings can be reformulated as the question of which of these symmetries can be gauged.

In order to do this, it is useful to first rewrite the $h_{I|JK_1\dots K_{g+1}}$ appearing in the cohomology classes of U and W-types in the equivalent symmetric convention

$$X_{IJ,K_1\dots K_{g+1}} := h_{(I|J)K_1\dots K_{g+1}} \iff h_{I|JK_1\dots K_{g+1}} = \frac{2(g+2)}{g+3}X_{I[J,K_1\dots K_{g+1}]} \tag{4.30}$$

where (3.42) is now replaced by

$$X_{IJ,K_1\dots K_{g+1}} = X_{(IJ),[K_1\dots K_{g+1}]}, \quad X_{(IJ,K_1)K_2\dots K_{g+1}} = 0\,. \tag{4.31}$$

Note that for $g=-1$, $h_{IJ} = X_{IJ}$.

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For cohomology classes of U, W-type, we can write

$$f^I{}_{JK_1\ldots K_{g+1}} = (f_u)^I{}_J \, k^u{}_{K_1\ldots K_{g+1}}, \tag{4.32}$$

$$^U X_{IJ,K_1\ldots K_{g+1}} = (h_u)_{IJ} \, k^u{}_{K_1\ldots K_{g+1}}, \tag{4.33}$$

$$X_{IJ,K_1\ldots K_{g+1}} = (h_w)_{IJ} \, k^w{}_{K_1\ldots K_{g+1}}, \tag{4.34}$$

where $(f_u)^I{}_J$, $(h_u)_{IJ}$ and $(h_w)_{IJ}$ appear in the basis elements U_u and W_w. (One has similar parametrizations for the quantities $g^I_{\mu K_1\ldots K_{g+1}}$, $\Phi^i_{K_1\ldots K_{g+1}}$, $J_{\mu K_1\ldots K_{g+1}}$ in the cohomology classes of the various types.) This guarantees that condition (3.44) (or (3.48)) is automatically satisfied.

However, the symmetry properties (3.41) and (4.31) imply the following linear constraints on the parameters:

$$(f_u)^I{}_{(J} \, k^u{}_{K_1)K_2\ldots K_{g+1}} = 0, \tag{4.35}$$

$$(h_u)_{(IJ} \, k^u{}_{K_1)K_2\ldots K_{g+1}} = 0, \tag{4.36}$$

$$(h_w)_{(IJ} \, k^w{}_{K_1)K_2\ldots K_{g+1}} = 0. \tag{4.37}$$

From the discussion of the cohomology, it also follows that V-type cohomology classes are entirely determined by V-type symmetries in terms of $k^v{}_{K_1\ldots K_{g+1}}$ without any additional constraints.

4.4 2nd order constraints on deformations and gauge algebra

The most general infinitesimal gauging is given by $S^{(1)} = \int (U^0 + W^0 + V^0 + I^0)$. We have

$$\frac{1}{2}(S^{(1)}, S^{(1)}) = \int \left(U^1 + W^1 + V^1 + I^1\right). \tag{4.38}$$

The infinitesimal deformation $S^{(1)}$ can be extended to second order whenever the right hand side vanishes in cohomology, resulting in quadratic constraints on the constants $k^{u_1}{}_K$, $k^{w_1}{}_K$, $k^{v_1}{}_K$ and $\rho^{\mathcal{A}}$. Working all of them out explicitly requires computing all brackets between U^0, W^0, V^0 and I^0.

However, it follows from the previous section that the only contribution to U^1 comes from $\frac{1}{2}(U^0, U^0)$. The vanishing of the terms containing the antighosts C^*_I requires

$$f^I{}_{J[K_1} f^J{}_{K_2K_3]} = 0, \tag{4.39}$$

i.e., the Jacobi identity for the $f^I{}_{JK}$. The associated n_v-dimensional Lie algebra is the gauge algebra and is denoted by $\mathfrak{g}_g$.

Using $f^I{}_{JK} = (f_{u_1})^I{}_J k^{u_1}{}_K$ and equation (4.19), the Jacobi identity reduces to the following quadratic constraint on $k^{u_1}{}_K$:

$$k^{u_1}{}_I k^{u_2}{}_J C^{u_3}{}_{u_1u_2} - (f_{u_4})^K{}_I k^{u_4}{}_J k^{u_3}{}_K = 0. \tag{4.40}$$

Note that the antisymmetry in IJ of the second term is guaranteed by the linear constraint (4.35). The terms at antifield number 1 give the constraints

$$\delta_I g^K_J + f^K{}_{MJ} g^M_I - (I \leftrightarrow J) = f^L{}_{IJ} g^K_L \tag{4.41}$$

$$\delta_I \Phi^i_J - (I \leftrightarrow J) = f^L{}_{IJ} \Phi^i_L. \tag{4.42}$$

Expressed with k's, this gives

$$k^{\Gamma}{}_I k^{\Delta}{}_J C^{\Sigma}{}_{\Gamma\Delta} - (f_u)^K{}_I k^u{}_J k^{\Sigma}{}_K = 0, \tag{4.43}$$

where the capital Greek indices take all values u, w, v. This gives three constraints, according to the type of the free index Σ. When $\Sigma = u$, we get the constraint (4.40), because the only non-vanishing structure constants with an upper u index are the $C^{u_3}{}_{u_1u_2}$. When $\Sigma = w$, the possible structure constants are $C^{w_3}{}_{w_1w_2}$, $C^{w_3}{}_{u_1w_2} = -C^{w_3}{}_{w_2u_1}$ and $C^{w_3}{}_{u_1u_2}$, giving the constraint

$$k^{w_1}{}_I k^{w_2}{}_J C^{w_3}{}_{w_1w_2} + 2k^{u_1}{}_{[I} k^{w_2}{}_{J]} C^{w_3}{}_{u_1w_2} + k^{u_1}{}_I k^{u_2}{}_J C^{w_3}{}_{u_1u_2} - (f_{u_4})^K{}_I k^{u_4}{}_J k^{w_3}{}_K = 0. \tag{4.44}$$

When the free index Σ is of type v, one gets a similar identity with all possible types of values in the lower indices of the structure constants,

$$\begin{aligned} k^{v_1}{}_I k^{v_2}{}_J C^{v_3}{}_{v_1v_2} + 2k^{w_1}{}_{[I} k^{v_2}{}_{J]} C^{v_3}{}_{w_1v_2} + k^{w_1}{}_I k^{w_2}{}_J C^{v_3}{}_{w_1w_2} + 2k^{u_1}{}_{[I} k^{v_2}{}_{J]} C^{v_3}{}_{u_1v_2} \\ + 2k^{u_1}{}_{[I} k^{w_2}{}_{J]} C^{v_3}{}_{u_1w_2} + k^{u_1}{}_I k^{u_2}{}_J C^{v_3}{}_{u_1u_2} - (f_{u_4})^K{}_I k^{u_4}{}_J k^{v_3}{}_K = 0. \end{aligned} \tag{4.45}$$

5 Quadratic vector models

5.1 Description of the model

To go further, one needs to specialize the form of the Lagrangian, which has been assumed to be quite general so far. In this section, we focus on second order Lagrangians arising in the context of supergravities that contain n_s scalar fields and depend quadratically on n_v abelian vector fields, non-minimally coupled to each other, in four space-time dimensions.

More specifically, we take $\mathcal{L} = \mathcal{L}_S + \mathcal{L}_V$, where

$$\mathcal{L}_V = -\frac{1}{4}\,\mathcal{I}_{IJ}(\phi) F^I_{\mu\nu} F^{J\mu\nu} + \frac{1}{8}\,\mathcal{R}_{IJ}(\phi)\,\varepsilon^{\mu\nu\rho\sigma} F^I_{\mu\nu} F^J_{\rho\sigma} \tag{5.1}$$

and the scalar Lagrangian is of the sigma model form

$$\mathcal{L}_S = -\frac{1}{2} g_{ij}(\phi)\partial_\mu\phi^i\partial^\mu\phi^j - V(\phi) \tag{5.2}$$

where g_{ij} is symmetric and invertible. Both g_{ij} and V depend only on undifferentiated scalar fields. Neglecting gravity, this is the generic bosonic sector of ungauged supergravity. The symmetric matrices $\mathcal{I}$ and $\mathcal{R}$, with $\mathcal{I}$ invertible, depend only on undifferentiated scalar fields and encode the non-minimal couplings between the scalars and the abelian vectors. The Bianchi identities and equations of motion for the vector fields are given by

$$\partial_\mu(\star F^I)^{\mu\nu} = 0, \qquad \partial_\nu(\star G_I)^{\mu\nu} \approx 0. \tag{5.3}$$

The Lagrangian (5.1) falls into the general class of models described previously, with the gauge invariant two-form $G_I = \mathcal{I}_{IJ} \star F^J + \mathcal{R}_{IJ} F^J$ and $d^4x\, \mathcal{L}_V = \frac{1}{2} G_I F^I$.

We assume

$$\mathcal{R}_{IJ}(0) = 0. \tag{5.4}$$

Note that a constant part in $\mathcal{R}_{IJ}$ can be put to zero without loss of generality since the associated term in the Lagrangian is a total derivative. In most cases, we also take $V = 0$ or assume (writing $\partial_i = \frac{\partial}{\partial\phi^i}$) that

$$(\partial_i V)(0) = 0. \tag{5.5}$$

5.2 Constraints on U, W-type symmetries

We assume here and in the examples below that there is no explicit x^μ-dependence in the space of local functions in order to constrain U and W-type symmetries. For simplicity, we also assume that the potential vanishes, $V = 0$. In section 6, these constraints will allow us to determine all symmetries of U and W type for specific models.

We need the scalar field equations, which are encoded in

$$s \star \phi^*_i + d(g_{ij} \star d\phi^j) = - \star \partial_i(\mathcal{L}_S + \mathcal{L}_V), \tag{5.6}$$

where $\partial_i = \frac{\partial}{\partial \phi^i}$. For $g = -1$, equation (3.48) becomes

$$G_I F^J f^I{}_J + F^I F^J h_{IJ} + dI^{n-1} - dG_I\, g^I - [d(g_{ij} \star d\phi^j) + \star\partial_i(\mathcal{L}_S + \mathcal{L}_V)]\Phi^i = 0. \tag{5.7}$$

When putting all derivatives of $F^I_{\mu\nu}, \phi^i$ to zero, one remains with

$$G_I F^J f^I{}_J + F^I F^J h_{IJ} - \star\partial_i \mathcal{L}_V\, \Phi^i|_{\text{der}=0} = 0. \tag{5.8}$$

It is here that the assumption that there is no explicit x^μ dependence in the gauge invariant functions $g^{I\alpha}, \Phi^{i\alpha}$ is used. Using $-\partial_i \star \mathcal{L}_V = \frac{1}{2}\partial_i G_I\, F^I$, and the decomposition $\Phi^i|_{\text{der}=0} = \Phi^i_0 + \Phi^i_1 + \dots$, where the Φ^i_n depend on undifferentiated scalar fields and are homogeneous of degree n in $F^I_{\mu\nu}$, the equation implies that

$$\frac{1}{2} M_{IJ}(\phi) \star F^I F^J + \frac{1}{2} N_{IJ}(\phi) F^I F^J = 0, \tag{5.9}$$

where

$$M_{IJ} = 2\mathcal{I}_{K(I} f^K{}_{J)} + \partial_i \mathcal{I}_{IJ} \Phi^i_0, \tag{5.10}$$

$$N_{IJ} = 2\mathcal{R}_{K(I} f^K{}_{J)} + 2h_{IJ} + \partial_i \mathcal{R}_{IJ} \Phi^i_0, \tag{5.11}$$

by using that $h_{IJ} = h_{JI}$ on account of (3.42). When taking an Euler-Lagrange derivative of (5.9) with respect to A^I_μ, one concludes that both terms have to vanish separately,

$$M_{IJ} = 0, \quad N_{IJ} = 0. \tag{5.12}$$

Setting $\phi^i = 0$ and using (5.4) then gives

$$f^{(\mathcal{I}(0))}_{IJ} + f^{(\mathcal{I}(0))}_{JI} = -(\partial_i \mathcal{I}_{IJ})(0)\Phi^i_0(0), \quad 2h_{IJ} = -(\partial_i \mathcal{R}_{IJ})(0)\Phi^i_0(0). \tag{5.13}$$

where the abelian index is lowered and raised with $\mathcal{I}_{IJ}(0)$ and its inverse. Note that completely skew-symmetric $f^{(\mathcal{I}(0))}_{IJ}$ solve the equations with $\Phi^i_0(0) = 0$, $h_{IJ} = 0$. More conditions are obtained by expanding equations (5.12) in terms of power series in ϕ^i.

In all examples considered below, the algebra $\mathfrak{g}_U$ and the W-type symmetries can be entirely determined from the analysis of this subsection.

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5.3 Electric symmetry algebra

An important result of our general analysis is that the symmetries of the action that can lead to consistent gaugings may have a term that is not gauge invariant. This term is present only in the variation of the vector potential and is restricted to be linear in the undifferentiated vector potential, i.e., $\delta A^I_\mu = f^I{}_J A^J_\mu + g^I_\mu$, $\delta\phi^i = \Phi^i$. Here $f^I{}_J$ are constants, and g^I_μ and Φ^i are gauge invariant functions. The symbol δ represents the variation of the fields and is of course not the Koszul-Tate differential. No confusion should arise as the context is clear.

It is of interest to investigate a subalgebra of the gaugeable symmetries, obtained by restricting oneself from the outset to transformations of the gauge potentials that are linear and homogeneous in the undifferentiated potentials and to transformations of the scalars that depend on undifferentiated scalars alone,

$$\delta A^I_\mu = f^I{}_J A^J_\mu, \quad \delta\phi^i = \Phi^i(\phi). \tag{5.14}$$

This means that one takes $g^I_\mu = 0$ and that the functions Φ^i only depend on the undifferentiated scalar fields. These symmetries form a sub-algebra $\mathfrak{g}_e$ that includes the symmetries usually considered in the supergravity literature and which is, in this context, called the "electric symmetry algebra" (in the given duality frame) [15]. It can be shown to be a subgroup of the duality group $G \subset \mathrm{Sp}(2n_v, \mathbb{R})$ [56]. Although our Lagrangians are not necessarily connected with supergravity, we shall nevertherless call the symmetries of the form (5.14) "electric symmetries" and the subalgebra $\mathfrak{g}_e$ the "electric algebra". It need not be a subalgebra of $\mathrm{Sp}(2n_v, \mathbb{R})$. It generically does not exhaust all symmetries and does not contain for example the conformal symmetries of free electromagnetism.

The transformations of the form (5.14) are symmetries of the action (5.1) + (5.2) if and only if the scalar variations leave the scalar action invariant separately, and $f^I{}_J, \Phi^i(\phi)$ satisfy

$$\frac{\partial\mathcal{I}}{\partial\phi^i}\Phi^i = -f^T\mathcal{I} - \mathcal{I}f, \tag{5.15}$$

$$\frac{\partial\mathcal{R}}{\partial\phi^i}\Phi^i = -f^T\mathcal{R} - \mathcal{R}f - 2h, \tag{5.16}$$

where the h are constant symmetric matrices. In particular, when the scalar Lagrangian is given by $\mathcal{L}_S = \frac{1}{2}g_{ij}(\phi)\partial_\mu\phi^i\partial^\mu\phi^j$, the first condition means that Φ^i must be a Killing vector of the metric g_{ij}. If U and W-type symmetries are of electric type, the electric symmetry algebra contains in addition only V-type symmetries of electric type, i.e., transformations among the undifferentiated scalars alone that leave invariant both the scalar action and the matrices $\mathcal{I}, \mathcal{R}$ (i.e., that satisfy $\delta S_S = 0$ and (5.15), (5.16) with 0's on the right hand sides). This will be the case in all examples below. In particular, the f's, and thus also the gauge algebra, will be the same for $\mathfrak{g}_U$ and $\mathfrak{g}_e$. The h matrix is determined by the transformation parameters Φ^i and the parity-odd term $\mathcal{R}$ of the action via (5.16).

We then suppose that we have a basis of symmetries of the action of this form,

$$\delta_\Gamma A^I_\mu = (f_\Gamma)^I{}_J A^J_\mu, \tag{5.17}$$

$$\delta_\Gamma\phi^i = \Phi^i_\Gamma(\phi). \tag{5.18}$$

When compared to the previous sections, the index Γ can take u, v or w values. Only the f_u matrices are non-zero. The h_Γ matrices are non-vanishing only for $\Gamma = u$ or w. When $h_\Gamma \neq 0$, the Lagrangian is only invariant up to a total derivative.

Closure of symmetries of this form then implies

$$[f_\Delta, f_\Gamma] = -C^\Sigma{}_{\Delta\Gamma} f_\Sigma, \tag{5.19}$$

$$f_\Gamma^T h_\Delta - f_\Delta^T h_\Gamma + h_\Delta f_\Gamma - h_\Gamma f_\Delta = -C^\Sigma{}_{\Delta\Gamma}\, h_\Sigma, \tag{5.20}$$

$$\frac{\partial \Phi^i_\Delta}{\partial \phi^j}\, \Phi^j_\Gamma - \frac{\partial \Phi^i_\Gamma}{\partial \phi^j}\, \Phi^j_\Delta = -C^\Sigma{}_{\Delta\Gamma}\, \Phi^i_\Sigma. \tag{5.21}$$

Decomposing the indices into U, W and V type, this is consistent with the relations of section 4.2 with $\lambda_u^w = 0$. Let us note that (5.16) expresses the surface term in the variation of the action. Equation (5.20) follows by commutation.

5.4 Restricted first order deformations

We now limit ourselves to first order deformations of the master action with the condition that all infinitesimal gaugings come from symmetries that belong to the electric symmetry algebra above. In order to simplify formulas, we will no longer make the distinction between U-, W- and V-type which can easily be recovered.

According to section 4.3, the deformations are parametrized through electric symmetries by a matrix k^Γ_I, with

$$f^I{}_{JK} = (f_\Gamma)^I{}_J k^\Gamma_K, \tag{5.22}$$

$$\Phi^i_I(\phi) = \Phi^i_\Gamma(\phi) k^\Gamma_I, \tag{5.23}$$

$$X_{IJ,K} = (h_\Gamma)_{IJ} k^\Gamma_K. \tag{5.24}$$

The linear constraints (4.35) – (4.37) on the matrix k^Γ_K become

$$(f_\Gamma)^I{}_J k^\Gamma_K + (f_\Gamma)^I{}_K k^\Gamma_J = 0, \tag{5.25}$$

$$h_{\Gamma\,(IJ} k^\Gamma_{K)} = 0. \tag{5.26}$$

They guarantee that the first order deformation of the master action is given by

$$S^{(1)} = \int d^4x\, (a_2 + a_1 + a_0)\,, \tag{5.27}$$

where

$$a_2 = \frac{1}{2} C^*_I f^I{}_{JK}\, C^J C^K \tag{5.28}$$

encodes the first order deformation of the gauge algebra and

$$a_1 = A^{*\mu}_I f^I{}_{JK}\, A^J_\mu C^K + \phi^*_i \Phi^i_K C^K \tag{5.29}$$

encodes the first order deformation of the gauge symmetries. When taking (5.22) and (5.23) into account, this deformation of the gauge symmetries corresponds to gauging the underlying global symmetries by using local parameters $\eta^\Gamma(x) = k^\Gamma_I \epsilon^I(x)$. The deformation a_0

of the Lagrangian is given by the sum of three terms:

$$a_0^{(\mathrm{YM})} = \frac{1}{2}(\star G_I)^{\mu\nu} f^I{}_{JK}\, A^J_\mu A^K_\nu, \tag{5.30}$$

$$a_0^{(\mathrm{CD})} = J^\mu_K A^K_\mu, \tag{5.31}$$

$$a_0^{(\mathrm{CS})} = \frac{1}{3} X_{IJ,K} \epsilon^{\mu\nu\rho\sigma} F^I_{\mu\nu} A^J_\rho A^K_\sigma. \tag{5.32}$$

The terms $a^{(\mathrm{YM})}$ and $a^{(\mathrm{CD})}$ are exactly those necessary to complete the abelian field strengths and ordinary derivatives of the scalars into covariant quantities. The term $a^{(\mathrm{CD})}$ is responsible for charging the matter fields. The Chern-Simons term $a_0^{(\mathrm{CS})}$ appears when $h_\Gamma \neq 0$: its role is to cancel the variation

$$\delta\mathcal{L} = -\frac{1}{4}\eta^\Gamma h_{\Gamma\, IJ}\, \varepsilon^{\mu\nu\rho\sigma} F^I_{\mu\nu} F^J_{\rho\sigma} \tag{5.33}$$

that is no longer a total derivative when $\eta^\Gamma = k^\Gamma_I \epsilon^I(x)$ [29, 57].

5.5 Complete restricted deformations

The second order deformation $S^{(2)}$ to the master action is then determined by the first order deformation through equation (2.8). As discussed in section 4.4, the existence of $S^{(2)}$ imposes additional quadratic constraints on the matrix k^Γ_I,

$$k^\Gamma{}_I k^\Delta{}_J C^\Sigma{}_{\Gamma\Delta} - (f_\Gamma)^K{}_I k^\Gamma{}_J k^\Sigma{}_K = 0. \tag{5.34}$$

Explicit computation shows that $S^{(2)}$ can be chosen such that there is no further deformation of the gauge symmetries or of their algebra. The second order terms in the Lagrangian are exactly those necessary to complete abelian field strengths $F^I_{\mu\nu} = \partial_\mu A^I_\nu - \partial_\nu A^I_\mu$ and ordinary derivatives of the scalars to non-abelian field strengths and covariant derivatives,

$$\mathcal{F}^I_{\mu\nu} = \partial_\mu A^I_\nu - \partial_\nu A^I_\mu + f^I{}_{JK}\, A^J_\mu A^K_\nu, \tag{5.35}$$

$$D_\mu \phi^i = \partial_\mu \phi^i - \Phi^i_I(\phi) A^I_\mu. \tag{5.36}$$

One also finds a non-abelian completion of the Chern-Simons term $a_0^{(\mathrm{CS})}$. Putting everything together, the Lagrangian after adding the second order deformation is

$$\begin{aligned}\mathcal{L} = \mathcal{L}_S(\phi^i, D_\mu\phi^i) - \frac{1}{4}\mathcal{I}_{IJ}(\phi)\mathcal{F}^I_{\mu\nu}\mathcal{F}^{J\mu\nu} + \frac{1}{8}\mathcal{R}_{IJ}(\phi)\, \varepsilon^{\mu\nu\rho\sigma}\mathcal{F}^I_{\mu\nu}\mathcal{F}^J_{\rho\sigma} \\ + \frac{2}{3} X_{IJ,K}\, \varepsilon^{\mu\nu\rho\sigma} A^J_\mu A^K_\nu \left(\partial_\rho A^I_\sigma + \frac{3}{8} f^I{}_{LM}\, A^L_\rho A^M_\sigma\right).\end{aligned} \tag{5.37}$$

The associated action can be checked to be invariant under the gauge transformations

$$\delta A^I_\mu = \partial_\mu \epsilon^I + f^I{}_{JK}\, A^J_\mu \epsilon^K, \tag{5.38}$$

$$\delta\phi^i = \epsilon^I \Phi^i_I(\phi). \tag{5.39}$$

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This is equivalent to the fact that the deformation stops at second order, i.e, that $S = S^{(0)} + S^{(1)} + S^{(2)}$ gives a solution to the master equation $(S, S) = 0$.

Checking directly the invariance of this action under (5.38) – (5.39) without first parametrizing ${f^I}_{JK}$, $\Phi^i_I(\phi)$ and X_{IJK} through symmetries requires the use of the linear identities

$$ {f^I}_{JK} = {f^I}_{[JK]}, \quad X_{(IJ,K)} = 0 \tag{5.40} $$

and of the quadratic ones

$$ {f^I}_{J[K_1} {f^J}_{K_2K_3]} = 0, \tag{5.41} $$

$$ {f^K}_{I[L} X_{M]J,K} + {f^K}_{J[L} X_{M]I,K} - \frac{1}{2} X_{IJ,K} {f^K}_{LM} = 0, \tag{5.42} $$

$$ \frac{\partial \Phi^i_I}{\partial \phi^j} \Phi^j_J - \frac{\partial \Phi^i_J}{\partial \phi^j} \Phi^j_I + {f^K}_{IJ} \Phi^i_K = 0. \tag{5.43} $$

In terms of k^Γ_I, these three quadratic identities all come from the single quadratic constraint (5.34) once the algebra of global symmetries (5.19) – (5.21) is taken into account.

5.6 Remarks on $\mathrm{GL}(n_v)$ transformations

Consider a linear field redefinition of the abelian vector potentials, $A^I_\mu = {M^I}_J {A'}^J_\mu$ with $M \in \mathrm{GL}(n_v)$. Such a transformation gives rise to a trivial infinitesimal gauging which corresponds to the antifield independent part of the trivial ghost number 0 cocycle

$$ S^{(1)}_{\text{triv.}} = (S^{(0)}, \Xi_s), \quad \Xi_s = {f_s^I}_J [d^4x\, C^*_I C^J + \star A^*_I A^J], \tag{5.44} $$

with $f_s \in \mathfrak{gl}(n_v, \mathbb{R})$.

Two remarks are in order.

The first concerns the relation to the algebra $\mathfrak{g}_U$ defined in section 4.2. It can also be defined as the largest sub-algebra of $\mathfrak{gl}(n_v, \mathbb{R})$ that can be turned into symmetries of the theory by adding suitable gauge invariant transformations of the vector and scalar fields, or in other words, for which there exists a gauge invariant K_u of ghost number -1 such that $(S^{(0)}, \Xi_u + K_u) = 0$.

In particular, (4.32) and (4.35) for $g = 0$, as well as (4.40), can be summarized as follows: non-trivial U-type gaugings require the existence of a map (described by k^u_K) from the defining representation of the symmetry algebra $\mathfrak{g}_U \subset \mathfrak{gl}(n_v, \mathbb{R})$ into the adjoint representation of the n_v-dimensional gauge algebra $\mathfrak{g}_g$.

The second remark is about families of Lagrangians related by linear transformations of the vector potentials among themselves. It is sometimes useful not to work with fixed (canonical) values for various $\mathrm{GL}(n_v)$ tensors that appear in the action. Instead, one considers the deformation problem for sets of Lagrangians parametrized by arbitrary $\mathrm{GL}(n_v)$ tensors, for instance generic non-degenerate symmetric $\mathcal{I}_{MN}$ and symmetric $\mathcal{R}_{MN}$ that vanish at the origin of the scalar field space.

If the tensors of two such Lagrangians are related by a $\mathrm{GL}(n_v)$ transformation, they should be considered as equivalent. Indeed, the local BRST cohomology for all members of such an equivalence class are isomorphic and related by the above anti-canonical field redefinition. In particular, all members of the same equivalence class have isomorphic gaugings.

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All general considerations and results on local BRST cohomology above apply in a unified way to all equivalence classes. When one explicitly solves the obstruction equation (3.48) (for instance at $g = -1$ in order to determine the symmetries), the results on local BRST cohomologies do depend on the various equivalence classes.

5.7 Comparison with the embedding tensor constraints

In the embedding tensor formalism [15, 22–25],[6] the possible gaugings are described by the embedding tensor $\Theta_M{}^\alpha = (\Theta_I{}^\alpha, \Theta^{I\alpha})$ with electric and magnetic components, which satisfies a number of linear and quadratic constraints. In this notation, the index I runs from 1 to n_v, while α runs from 1 to the dimension of the group G of invariances of the equations of motion of the initial Lagrangian (5.1). More precisely, G is defined only as the group of transformations that act linearly on the field strengths F^I and their "magnetic duals" G_I, and whose action on the scalars contains no derivatives. This coincides with the group of symmetries of the first order Lagrangian discussed in [26] which are of the restricted form (5.14).

As explained in section 3 of [15], one can always go to a duality frame in which the magnetic components of the embedding tensor vanishes, $\Theta^{I\alpha} = 0$. Moreover, only the components $\Theta_I{}^\Gamma$ survive, where Γ runs over the generators of the electric subgroup $G_e \subset G$ that act as local symmetries of the Lagrangian in that frame. Then, the gauged Lagrangian in the electric frame is exactly the Lagrangian (5.37), where the matrix k is identified with the remaining electric components of the embedding tensor, $k_I^\Gamma = \Theta_I{}^\Gamma$ (or $\Theta_{\hat{I}}{}^\Gamma$ in the notation of [15]). The linear and quadratic constraints on the embedding tensor then agree with the constraints on k. More precisely, the constraints (3.11), (3.12) and (3.39) of [15] in the electric frame correspond to our (5.34), (5.25) and (5.26) respectively. As explained in sections 4.3 and 4.4, the constraints can be refined using the split corresponding to the various (U, W, V) types of symmetry.

It was shown in [26] that the embedding tensor formalism does not allow for more general deformations than those of the Lagrangian (5.1) studied in this paper. Indeed, their BRST cohomologies are isomorphic even though the field content and gauge transformations are different. Conversely, as long as one restricts the attention to the symmetries of (5.1) that are of the electric type (5.14), we showed that the embedding tensor formalism captures all consistent deformations that deform the gauge transformations of the fields.

6 Applications

6.1 Abelian gauge fields: U-type gauging

As a first example, let us consider the case where we have no scalars, $\mathcal{I}_{IJ} = \delta_{IJ}$, $\mathcal{R}_{IJ} = 0$. The Lagrangian is then simply

$$\mathcal{L} = -\frac{1}{4}\delta_{IJ} F^I_{\mu\nu} F^{J\mu\nu}. \tag{6.1}$$

[6] See also [69], where a relation between the embedding tensor formalism and the BRST-BV antifield formalism has been considered with a different purpose.

From (5.13), it can be shown that U-type symmetries are of electric form. Furthermore, there are no W-type symmetries. We have in this case $\mathfrak{g}_U = \mathfrak{g}_e = \mathfrak{so}(n_v)$.

The vector fields transform in the fundamental representation of $\mathfrak{so}(n_v)$. A basis of the Lie algebra $\mathfrak{so}(n_v)$ may be labeled by an antisymmetric pair of indices $[LM]$ that now plays the role of the index u,

$$\delta_{[LM]} A^I_\mu = (f_{[LM]})^I{}_J A^J_\mu, \qquad (f_{[LM]})^I{}_J = \frac{1}{2}(\delta^I_L \delta_{JM} - \delta^I_M \delta_{JL}). \tag{6.2}$$

Concerning the associated gaugings, the matrices $f_{[LM]IJ} = \delta_{II'}(f_{[LM]})^{I'}{}_J$ are antisymmetric in I, J; therefore, the structure constants of the gauge group

$$f^{(\delta)}_{IJK} = (f_{[LM]})_{IJ} k^{LM}_K \tag{6.3}$$

are automatically antisymmetric in their first two indices. The constraints on k^{LM}_K ensure antisymmetry in the last two indices (which in turn implies total antisymmetry) and the Jacobi identity. Moreover, any set of totally antisymmetric structure constants can be obtained in this way by taking $k^{LM}_K = f^{LM}{}_K$, as can be easily seen using the expression for $f_{[LM]}$ given above.

We thereby recover the result of [17, 70] stating that the most general deformation of the free Lagrangian (6.1) that is not of V or I-type is given by the Yang-Mills Lagrangian with a compact gauge group of dimension equal to the number of vector fields.

Remark. Note that Poincaré (conformal) symmetries (for $n = 4$) are of V-type if one allows for x^μ-dependent local functions. If such a dependence is allowed for U, W-type symmetries and gaugings as well, results can be very different. For instance, as shown by equation (13.21) of [46], if $n \neq 4$, there are additional U-type symmetries described by the cohomology class

$$U^{-1} = d^n x f_{(IJ)} \left[C^{*I} C^J + A^{*\mu I} A^J_\mu + \frac{2}{n-4} F^I_{\mu\nu} x^\mu A^{*\nu J} \right], \tag{6.4}$$

where indices $I, J, \dots$ are raised and lowered with the Kronecker delta. The associated Noether current can be obtained by working out the descent equation following (3.49), $sU^{-1} + d(f_{(IJ)}[\star A^{*I} C^J + \star F^I A^J + J_U{}^{IJ}]) = 0$, where

$$J_U{}^{IJ} = \frac{2}{n-4} (T_{\mu\nu})^{IJ} x^\nu \star dx^\mu, \quad (T^\mu{}_\nu)^{IJ} = F^{(I|\mu\rho} F^{|J)}_{\rho\nu} + \frac{1}{4} F^{I\alpha\beta} F^J_{\alpha\beta} \delta^\mu_\nu. \tag{6.5}$$

In other words, $\mathfrak{g}_U = \mathfrak{gl}(n_v)$. Note also that these U-type symmetries involve a non-vanishing ${}^U g^I_\mu$. It has furthermore been shown in section 13.2.2 of [46] that there are associated U-type gaugings and cohomology classes in higher ghost numbers. In the present context, they are obtained as follows: the role of u for the additional symmetries is played by a symmetric pair of indices (LM),

$$\delta_{(LM)} A^I_\mu = (f_{(LM)})^I{}_J A^J_\mu, \quad (f_{(LM)})^I{}_J = \frac{1}{2}(\delta^I_L \delta_{JM} + \delta^I_M \delta_{JL}). \tag{6.6}$$

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Once the linear constraints (4.35) on $k^{(LM)}_{K_1\ldots K_{g+1}}$ are fulfilled, the associated U-type gaugings and higher cohomology classes can be read off from equation (3.49) when taking (4.32).

After multiplying (6.4) by $n-4$, it represents for $n=4$ the V-type symmetry associated with the dilatation of the conformal group. The associated cubic and higher order vertices for the full conformal group have been studied in detail in [58].

6.2 Abelian gauge fields with uncoupled scalars: U, V-type gaugings

We now take the case

$$\mathcal{L} = \mathcal{L}_S(\phi^i, \partial_\mu \phi^i) - \frac{1}{4}\delta_{IJ} F^I_{\mu\nu} F^{J\mu\nu}, \tag{6.7}$$

where there is no interaction between the scalars and the vector fields. The $\mathfrak{g}_U$ algebra is again $\mathfrak{so}(n_v)$ and there are no W-type symmetries.

The electric symmetry algebra is the direct sum of $\mathfrak{so}(n_v)$ with the electric V-type symmetry algebra $\mathfrak{g}_s$ of the scalar Lagrangian. The matrices f_Γ split into two groups and are given by

$$(f_\alpha)^I{}_J = 0, \qquad (f_{[LM]})^I{}_J = \frac{1}{2}(\delta^I_L \delta_{JM} - \delta^I_M \delta_{JL}) \tag{6.8}$$

where $\alpha = 1, \ldots, \dim \mathfrak{g}_s$ labels the G_s generators and the antisymmetric pair $[LM]$ labels the $\mathrm{SO}(n_v)$ generators as before. The matrix k^Γ_I accordingly splits in two components k^{LM}_I and k^α_I. The constraints on k^{LM}_I again amount to the fact that the quantities $f^I{}_{JK} = (f_{[LM]})^I{}_J k^{LM}_K$ are the structure constants of a compact Lie group. The constraint on k^α_I tells us that the gauge variations

$$\delta\phi^i = \epsilon^I k^\alpha_I \Phi^i_\alpha(\phi) \tag{6.9}$$

close according to the structure constants $f^I{}_{JK}$. In the case where these variations are linear, $\Phi^i_\alpha(\phi) = (t_\alpha)^i{}_j \phi^j$, the constraint is that the matrices $T_I = k^\alpha_I t_\alpha$ form a representation of the gauge group, $[T_I, T_J] = f^K{}_{IJ} T_K$.

6.3 Bosonic sector of $\mathcal{N}=4$ supergravity

Neglecting gravity, the bosonic sector of $\mathcal{N}=4$ supergravity is given by two scalar fields parametrizing the coset $\mathrm{SL}(2,\mathbb{R})/\mathrm{SO}(2)$ along with $n_v = 6$ vector fields [1–3, 5].[7] We study three formulations of this model, where we determine the symmetry algebras $\mathfrak{g}_U$ and $\mathfrak{g}_e$ and the allowed gaugings. In all formulations, the scalar Lagrangian is determined by

$$\phi^i = (\phi, \chi), \quad g_{ij} = \mathrm{diag}(1, e^{2\phi}), \quad V = 0. \tag{6.10}$$

They differ by the form of the matrices $\mathcal{I}$ and $\mathcal{R}$.

[7] For generalisations of the models treated in this section, see [59] for the couplings of $\mathcal{N}=4$ supergravity to an arbitrary number of vector supermultiplets.

SO(6) formulation. The vector Lagrangian (5.1) is determined by

$$\mathcal{I}_{IJ} = e^{-\phi}\delta_{IJ}, \quad \mathcal{R}_{IJ} = \chi\delta_{IJ}. \tag{6.11}$$

When $f^{(\delta)}_{IJ}$ is antisymmetric, the transformations $\delta A^I_\mu = f^I{}_J A^J_\mu$ define an $\mathfrak{so}(6)$ sub-algebra of U-type symmetries on their own. Note also that we can assume h_{IJ} to be symmetric. Equations (5.13) then imply that the traceless parts of $f^{(\delta)}_{IJ}, h_{IJ}$ have to vanish.

If $f^{(\delta)}_{IJ} = \delta_{IJ}\eta^0$, equations (5.13) are solved with $h_{IJ} = 0$, $\Phi^\phi_0(0) = 2\eta^0, \Phi^\chi_0(0) = 0$. It then follows that (5.12) are solved with

$$\Phi^\phi_0 = 2\eta^0, \quad \Phi^\chi_0 = -2\eta^0\chi. \tag{6.12}$$

Equation (5.7) is then also solved with $g^I = 0$ and $I^3 = 2\eta^0(e^{2\phi} \star d\chi\chi - \star d\phi)$. According to equation (3.49), the associated cohomology class is given by

$$\omega^{-1,4} = \eta^0[d^4x C^*_I C^I + \star A^*_I A^I + 2(\star\phi^* - \star\chi^*\chi)], \tag{6.13}$$

with $s\omega^{-1,4} + d[\eta^0(\star A^*_I C^I + G_I A^I) + I^3] = 0$. This cohomology class encodes the symmetry

$$\delta A^I_\mu = \eta^0 A^I_\mu, \quad \delta\phi = 2\eta^0, \quad \delta\chi = -2\eta^0\chi, \tag{6.14}$$

with $\delta\mathcal{L}_0 = 0$. The associated Noether current is given by

$$j^\mu = \left[-\left(e^{-\phi}F^{\mu\lambda}_I - \frac{1}{2}\chi\epsilon^{\mu\lambda\rho\sigma}F_{I\rho\sigma}\right)A^I_\lambda - 2\partial^\mu\phi + 2\chi e^{2\phi}\partial^\mu\chi\right]. \tag{6.15}$$

It cannot be made gauge invariant through allowed redefinitions.

For $f_{IJ} = 0$, $h_{IJ} = \eta^+\delta_{IJ}$, $\Phi^\chi_0 = -2\eta^+, \Phi^\phi_0 = 0$ is a solution to the full problem (5.7) since $\frac{1}{2}F^I F_I = s\star\chi^* + d(e^{2\phi}\star d\chi)$. This gives then the only class of W-type, which is also of restricted type. More explicitly, $W^{-1} = \star\chi^*$ with $s\star\chi^* + d(-\frac{1}{2}A^I F_I + e^{2\phi}\star d\chi) = 0$. The symmetry it describes is $\delta\chi = \eta^+$ with the associated Noether current given above that can again not be made gauge invariant.

The algebra $\mathfrak{g}_U$ is therefore isomorphic to $\mathfrak{so}(6)\oplus\mathfrak{h}$, where $\mathfrak{h}$ is the sub-algebra of $\mathfrak{sl}(2,\mathbb{R})$ generated by diagonal traceless matrices. It is a sub-algebra of the electric symmetry algebra $\mathfrak{g}_e = \mathfrak{so}(6) \oplus \mathfrak{b}^+$, where $\mathfrak{b}^+$ corresponds to the sub-algebra of $\mathfrak{sl}(2,\mathbb{R})$ of upper triangular matrices. The electric algebra acts as

$$\delta\phi = 2\eta^0, \quad \delta\chi = -2\eta^0\chi + \eta^+, \quad \delta A^I_\mu = \eta^0 A^I_\mu + \eta^{LM}(f_{[LM]})^I{}_J A^J_\mu. \tag{6.16}$$

Accordingly $f_\Gamma = (f_0, f_+, f_{[LM]})$ where $f_{[LM]}$ are given in (6.2), while

$$(f_0)^I{}_J = \delta^I_J, \qquad (f_+)^I{}_J = 0. \tag{6.17}$$

The matrix $\mathcal{R}_{IJ} = \chi\delta_{IJ}$ transforms as

$$\delta\mathcal{R}_{IJ} = -2\eta^0\mathcal{R}_{IJ} + \eta^+\delta_{IJ}. \tag{6.18}$$

Therefore, contrary to the previous examples, the tensor $h_{\Gamma IJ}$ has a non-vanishing component $h_{+IJ} = -\frac{1}{2}\delta_{IJ}$.

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The generalized structure constants $f^I{}_{JK_1\dots K_{g+1}} = (f_\Gamma)^I{}_J k^\Gamma_{K_1\dots K_{g+1}}$ are then

$$f^I{}_{JK_1\dots K_{g+1}} = \delta^I_J k^0_{K_1\dots K_{g+1}} + \frac{1}{2}(f_{[LM]})^I{}_J k^{LM}_{K_1\dots K_{g+1}}. \tag{6.19}$$

The linear constraint (4.35) now implies that $k^0_{K_1\dots K_{g+1}} = 0$ as can be seen by taking the first three indices equal and using the antisymmetry of the matrices $f_{[LM]}$, while $k^{(\delta)}_{LMK_1\dots K_{g+1}}$ is restricted to be completed skew-symmetric in all indices. In the same way, the linear constraint (4.37) implies that $k^+_{K_1\dots K_{g+1}} = 0$. Indeed, it reduces to

$$\delta_{(IJ}k^+_{K_1)\dots K_{g+1}} = 0, \tag{6.20}$$

from which we deduce $k^+_{K_1\dots K_{g+1}} = 0$ by taking the first three indices equal.

It follows that there are no cohomology classes of W-type when $g \geqslant 0$ and that the only cohomology classes of U-type when $g \geqslant 0$ are given by

$$[d^4x C^{*I}\partial_I + \star A^{*I}A^J\partial_J\partial_I + \frac{1}{2}G^I A^J A^K \partial_K\partial_J\partial_I]\Theta, \tag{6.21}$$

with Θ a polynomial in C^I of ghost number $\geqslant 1$.

In particular, the symmetries of $\mathfrak{b}^+$ cannot be gauged and the gauge algebra is given by a compact sub-algebra of $\mathfrak{so}(6)$. The gauged Lagrangian is the original one, except that the abelian field strengths are replaced by non-abelian ones.

Dual SO(6) formulation. We now have

$$\mathcal{I}_{IJ} = \frac{1}{e^{-\phi} + \chi^2 e^\phi}\delta_{IJ}, \quad \mathcal{R}_{IJ} = -\frac{\chi e^\phi}{e^{-\phi} + \chi^2 e^\phi}\delta_{IJ}, \tag{6.22}$$

and the same analysis gives similar conclusions:

1. We still have the cohomology classes (6.21) since we still have that $\mathcal{I}_{IJ}$ and $\mathcal{R}_{IJ}$ are proportional to δ_{IJ}.

2. There are no additional gaugings or cohomology classes in ghost number higher than 0 of U or W-type.

3. The only additional non-covariantizable characteristic cohomology comes from two additional solutions to (5.13).

The first of these additional solutions is of electric U-type and comes from $f^{(\delta)}_{IJ} = \tilde{\eta}^0\delta_{IJ}$, $h_{IJ} = 0$, with $\Phi^\phi_0(0) = -2\tilde{\eta}^0$, $\Phi^\chi_0(0) = 0$. Equation (5.12) reduces to

$$2\tilde{\eta}^0\mathcal{I}_{IJ} + \partial_i\mathcal{I}_{IJ}\Phi^i_0 = 0, \quad 2\tilde{\eta}^0\mathcal{R}_{IJ} + 2h_{IJ}\delta_{IJ} + \partial_i\mathcal{R}_{IJ}\Phi^i_0 = 0, \tag{6.23}$$

and is solved by

$$h_{IJ} = 0 \quad \Phi^\phi_0 = -2\tilde{\eta}^0, \quad \Phi^\chi_0 = 2\tilde{\eta}^0\chi. \tag{6.24}$$

This gives also a solution to the full problem since this transformation leaves the scalar field Lagrangian invariant. According to equation (3.49), the associated cohomology class is given by

$$\omega^{-1,4} = \tilde{\eta}^0[d^4x C_I^* C^I + \star A_I^* A^I - 2(\star\phi^* - \star\chi^*\chi)]. \tag{6.25}$$

The second is of restricted W-type and comes from $f^{(\delta)}_{IJ} = 0$ while $h_{IJ} = \tilde{\eta}^+\delta_{IJ}$ with $\Phi_0^\phi(0) = 0$, $\Phi_0^\chi(0) = 2\tilde{\eta}^+$. Equation (5.12) reduces to

$$\partial_i \mathcal{I}_{IJ}\Phi_0^i = 0, \quad 2\tilde{\eta}^+\delta_{IJ} + \partial_i \mathcal{R}_{IJ}\Phi_0^i = 0, \tag{6.26}$$

and is solved by

$$\Phi_0^\chi = 2\tilde{\eta}^+(e^{-2\phi} - \chi^2), \quad \Phi_0^\phi = 4\tilde{\eta}^+\chi, \tag{6.27}$$

This is also a solution to the full problem since these transformations leave the scalar field Lagrangian invariant. The associated cohomology class is given by

$$2\tilde{\eta}^+[\star\phi^* 2\chi + \star\chi^*(e^{-2\phi} - \chi^2)]. \tag{6.28}$$

In this case, we therefore have $\mathfrak{g}_U = \mathfrak{so}(6) \oplus \mathfrak{h} \subset \mathfrak{g}_e = \mathfrak{so}(6) \oplus \mathfrak{b}^-$, where $\mathfrak{b}^-$ is the sub-algebra of $\mathfrak{sl}(2,\mathbb{R})$ of lower triangular matrices.

Again, the symmetries of $\mathfrak{b}^+$ cannot be gauged and the gauge algebra is given by a compact sub-algebra of $\mathfrak{so}(6)$.

SO(3) × SO(3) formulation. The indices split as $I = (A, A')$, where $A, A' = 1, 2, 3$, and we have

$$\begin{aligned}
&\mathcal{I}_{IJ} = \mathrm{diag}(\mathcal{I}_{AB}, \mathcal{I}_{A'B'}), \qquad && \mathcal{R}_{IJ} = \mathrm{diag}(\mathcal{R}_{AB}, \mathcal{R}_{A'B'}), \\
&\mathcal{I}_{AB} = e^{-\phi}\delta_{AB}, && \mathcal{I}_{A'B'} = \frac{1}{e^{-\phi} + \chi^2 e^{\phi}}\delta_{A'B'}, \\
&\mathcal{R}_{AB} = \chi\delta_{AB}, && \mathcal{R}_{A'B'} = -\frac{\chi e^{\phi}}{e^{-\phi} + \chi^2 e^{\phi}}\delta_{A'B'}.
\end{aligned} \tag{6.29}$$

Spelling out equation (5.12) gives

$$\begin{aligned}
\mathcal{I}_{IL}f^L{}_J + \mathcal{I}_{LJ}f^L{}_I + \partial_i \mathcal{I}_{IJ}\Phi_0^i = 0, \\
\mathcal{R}_{IL}f^L{}_J + \mathcal{R}_{JL}f^L{}_I + h_{IJ} + h_{JI} + \partial_i \mathcal{R}_{IJ}\Phi_0^i = 0.
\end{aligned} \tag{6.30}$$

Choosing $I = A$, $J = A'$, the first equation reduces to

$$e^{-\phi} f^{(\delta)}_{AA'} + \frac{1}{e^{-\phi} + \chi^2 e^{\phi}} f^{(\delta)}_{A'A} = 0. \tag{6.31}$$

Putting $\phi = 0 = \chi$ and taking the derivative with respect to ϕ before putting $\phi = 0 = \chi$, implies that $f^{(\delta)}_{AA'} = 0 = f^{(\delta)}_{A'A}$. When combined with the linear constraint (4.35), this implies that the $f^I{}_{JK_1\ldots K_{g+1}}$'s have to vanish unless all indices are of A, or of A', type respectively, which is thus a necessary condition to have non trivial U-type solutions.

When the f's vanish, the second equation for $I = A$, $J = A'$ gives $h_{AA'} + h_{A'A} = 0$. When combined with the linear constraint (4.37), this implies that for non-trivial solutions of W-type associated to $X_{IJ,K_1\ldots K_{g+1}}$ one again needs all indices to be either of A or of A' type.

The discussion then reduces to the one we had before in each of the sectors. For U-type solutions, this gives in a first stage the symmetries, gaugings and higher ghost cohomology classes associated with each of the SO(3) rotations separately. There are again no additional solutions of U or W type when $g \geqslant 0$.

Only the remaining non-covariantizable symmetries, i.e., solutions of type U and W at $g = -1$ that correspond to $\mathfrak{b}^{\pm}$, remain to be discussed. For the U type solutions, one finds in the first sector that $f_{AB} = \eta^0 \delta_{AB}$ with (6.12) holding, while for the second sector $f_{A'B'} = \tilde{\eta}^0 \delta_{A'B'}$ with (6.24) holding. This gives a solution to the full problem if and only if $\tilde{\eta}^0 = -\eta^0$. Hence $\mathfrak{g}_U = \mathfrak{so}(3) \oplus \mathfrak{so}(3) \oplus \mathfrak{h}$. On the other hand the solutions of W type for both sectors are solutions to the full problem if and only if $\eta^+ = \tilde{\eta}^+ = 0$ so that there is no surviving W-type symmetry. In particular $\mathfrak{g}_e = \mathfrak{g}_U$. The symmetry of $\mathfrak{h}$ cannot be gauged, and the gauge algebra is a compact sub-algebra of $\mathfrak{so}(3) \oplus \mathfrak{so}(3)$.

This concludes the discussion with the expected results (see [60–62]).

6.4 Axion models: W-type gaugings and anomalies

We now give examples of gaugings and anomalies where the generalized Chern-Simons term appears. They involve several axions and correspond to the examples given in [46], equations (12.4) and (12.6).

The initial Lagrangian is

$$\mathcal{L}_0 = -\frac{1}{4} F^I_{\mu\nu} F^{J\mu\nu} \delta_{IJ} - \frac{1}{2} \partial_\mu \phi^I \partial^\mu \phi^J \delta_{IJ} + \frac{1}{4} \varepsilon^{\mu\nu\rho\sigma} F^I_{\mu\nu} F^J_{\rho\sigma} \phi_I V_J \,, \tag{6.32}$$

where V^I is a vector of dimension n_v of unit norm and there are n_v scalar fields ϕ^I whose indices are raised and lowered with the Kronecker symbol. With respect to the general Lagrangian (5.1) and (5.2), we have here $g_{ij} = \delta_{IJ}$, $V = 0$, $\mathcal{I}_{IJ} = \delta_{IJ}$ and $\mathcal{R}_{IJ} = 2\phi_{(I} V_{J)}$. The example in [46], equation (12.4), corresponds to the case $n_v = 2$, with $V_J = \delta^2_J$.

By using equations (5.13), (5.11), one finds $\mathfrak{g}_U = \mathfrak{so}(n_v - 1)$. The symmetries act like $\delta_u A^I = f^I{}_J \, A^J$ and $\delta_u \phi^I = f^I{}_J \, \phi^J$ for an antisymmetric symbol f_{IJ} that is transverse to the vector V, $V_I f^I{}_J = 0$.

To determine the W-type symmetries, one needs in particular to solve equation (5.13) with $f^I{}_J = 0$. This is done through $\delta\phi^I = \Phi^I = \eta^I$ for independent constants η^I with $(h_K)_{IJ} = -\delta_{K(I} V_{J)}$. There are thus n_v independent such symmetries. It then follows that a basis of W-type symmetries is given by $\delta_K \phi^I = \delta^I_K$, $\delta_K A^I_\mu = 0$, where K plays the role of the index w. Furthermore, there are no V-type symmetries of electric type so that $\mathfrak{g}_e = \mathfrak{so}(n_v - 1) \oplus \mathfrak{u}(1)^{n_v}$.

The linear constraints (4.35) require the $k^u_{K_1 \dots K_{g+1}}$ to be transverse, $k^u_{K_1 \dots K_{g+1}} V^{K_1} = 0$. For the associated gaugings, the rest of the discussion then follows the one in subsection 6.1. For W-type cohomology, the linear constraints (4.37) imply that

$$V_{(I}\, k_{JK_1) \dots K_{g+1}} = 0 \,, \quad k_{IK_1 \dots K_{g+1}} := \delta_{IJ}\, k^J{}_{K_1 J_2 \dots K_{g+1}} \,, \tag{6.33}$$

which is solved if and only if k is a totally antisymmetric rank-$(g+2)$ symbol, $k_{IK_1 \dots K_{g+1}} = k_{[IK_1 \dots K_{g+1}]}$.

For W-type gaugings in particular, $g = 0$ and one has an antisymmetric $n_v \times n_v$ matrix. There are no extra quadratic constraints. The gauge transformations, covariant derivatives and generalized Chern-Simons term for the deformed theory are then

$$\begin{aligned} \delta A^I_\mu &= \partial_\mu \epsilon^I, \quad \delta\phi_I = -k_{[IJ]}\epsilon^J, \quad D_\mu\phi_I = \partial_\mu\phi_I + k_{[IJ]}A^J_\mu, \\ \mathcal{L}^{(\mathrm{CS})} &= -\frac{1}{3}(V_K k_{IJ} + V_I k_{KJ})\,\varepsilon^{\mu\nu\rho\sigma} A^I_\mu A^J_\nu F^K_{\rho\sigma}\,. \end{aligned} \tag{6.34}$$

For a W-type anomaly example, let $n_s = n_v = 3$ and consider $V_K = \delta_{K\,3}$. The associated anomaly candidate is then

$$W^{1,4} = d^4x\,\epsilon_{IJK}\left[\frac{1}{4}\,\epsilon^{\mu\nu\rho\sigma}\,C^I\,A^J_\mu\,A^K_\nu\,F^3_{\rho\sigma} + C^I\,A^J_\mu\,\partial^\mu\phi^K - \frac{1}{2}\,C^I\,C^J\,\phi^{*K}\right]. \tag{6.35}$$

7 First order manifestly duality-invariant actions

7.1 Non-minimal version with covariant gauge structure

We now investigate the first order formulation [9, 26] of the models discussed previously. Those models are interesting because they contain more symmetries and therefore potentially more gaugings. In the original, minimal version, they are given by the action

$$S = \int d^4x\left(\frac{1}{2}\Omega_{MN}B^{Mi}\dot{A}^N_i - \frac{1}{2}\mathcal{M}_{MN}(\phi)B^M_iB^{Ni}\right), \tag{7.1}$$

where the potentials are packed into a vector

$$(A^M) = (A^I, Z_I), \quad M = 1, \ldots, 2n_v, \tag{7.2}$$

and the magnetic fields are

$$B^{Mi} = \epsilon^{ijk}\partial_j A^M_k. \tag{7.3}$$

The matrices Ω and $\mathcal{M}(\phi)$ are the $2n_v \times 2n_v$ matrices

$$\Omega = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}, \qquad \mathcal{M} = \begin{pmatrix} \mathcal{I} + \mathcal{R}\mathcal{I}^{-1}\mathcal{R} & -\mathcal{R}\mathcal{I}^{-1} \\ -\mathcal{I}^{-1}\mathcal{R} & \mathcal{I}^{-1} \end{pmatrix}, \tag{7.4}$$

each block being $n_v \times n_v$. The matrix $\mathcal{N} = \Omega^{-1}\mathcal{M}$ is symplectic, $\mathcal{N}^T\Omega\mathcal{N} = \Omega$.

Local BRST cohomology and gaugings for this class of models with non-covariant gauge symmetries $\delta A^M_i = \partial_i\epsilon^M$ could then be discussed by generalizing the results of [63] in the presence of coupled scalars.

However, in order to be able to directly use the discussion of local BRST cohomology developed for the second order covariant Lagrangian in the case of the first order manifestly duality invariant formulation, we consider a modification of the non-minimal variant [10] with additional scalar potentials for the longitudinal parts of electric and magnetic fields. More precisely, we now take instead of (7.1) the action

$$S[A^M_\mu, D^M, \pi_M, \phi^i] = S_S[\phi] + S_{DP}, \tag{7.5}$$

with

$$\mathcal{L}_{DP} = \frac{1}{2}[\Omega_{MN}(\mathcal{B}^{Mi} + \partial^i D^M)(\partial_0 A_i^N - \partial_i A_0^N) - \mathcal{B}^{Mi}\mathcal{M}_{MN}(\phi)\mathcal{B}_i^N]$$
$$+ \pi_M \partial_0 D^M - \frac{1}{2}\pi_M (\mathcal{M}^{-1})^{MN}\pi_N - \mathcal{V}(\phi, D). \tag{7.6}$$

Here

$$\mathcal{B}^{Mi} = \epsilon^{ijk}\partial_j A_k^M + \partial^i D^M, \tag{7.7}$$

spatial indices $i, j, k, \ldots$ are raised and lowered with δ_{ij} and its inverse, with Ω_{MN} the symplectic matrix, $\mathcal{M}_{MN}$ symmetric and invertible and

$$(\partial_i \mathcal{V})(0,0) = 0 = (\partial_M \mathcal{V})(0,0), \quad \mathcal{V}(\phi, 0) = 0. \tag{7.8}$$

The modification with respect to [10] consists in the addition of the kinetic and potential terms for the longitudinal electric and magnetic potentials in the last line of (7.6). Defining

$$\mathcal{F}^M_{\mu\nu} = \partial_\mu A^M_\nu - \partial_\nu A^M_\mu + \star S^M_{\mu\nu}, \quad \star S^M_{0i} = \Delta^{-1}(\Omega^{-1})^{MN}\partial_0\partial_i\pi_N, \quad \star S^M_{ij} = \epsilon_{ijk}\partial^k D^M, \tag{7.9}$$

we have $\mathcal{B}^{Mi} = \frac{1}{2}\epsilon^{ijk}\mathcal{F}_{jk}$ and can write

$$S_{DP} = \int d^4x \frac{1}{4}[\Omega_{MN}\epsilon^{ijk}(\mathcal{F}^M_{jk} + \star S^M_{jk})\mathcal{F}^N_{0i} - \mathcal{F}^M_{ij}\mathcal{M}_{MN}\mathcal{F}^{Nij} - 2\pi_M(\mathcal{M}^{-1})^{MN}\pi_N - 4\mathcal{V}], \tag{7.10}$$

where a total derivative has been dropped.

The gauge invariances are then doubled but still of the same covariant form as in the second order Lagrangian case,

$$\delta A^M_\mu = \partial_\mu \epsilon^M, \quad \delta D^M = 0, \quad \delta \pi_M = 0, \quad \delta\phi^i = 0. \tag{7.11}$$

The equations of motion for the gauge and scalar potentials are determined by the vanishing of

$$\begin{aligned}
\frac{\delta \mathcal{L}_{DP}}{\delta \pi_M} &= -(\mathcal{M}^{-1})^{MN}(\pi_N - \mathcal{M}_{NL}\partial_0 D^L),\\
\frac{\delta \mathcal{L}_{DP}}{\delta D^M} &= \Omega_{MN}(\Delta A^N_0 - \partial_0\partial^i A^N_i) + \partial^i(\mathcal{M}_{MN}\mathcal{B}^N_i) - \partial_0\pi_M - \frac{\partial \mathcal{V}}{\partial D^M},\\
\frac{\delta \mathcal{L}_{DP}}{\delta A^M_0} &= -\Omega_{MN}\Delta D^N = -\frac{1}{2}\Omega_{MN}\epsilon^{ijk}\partial_i\mathcal{F}^N_{jk},\\
\frac{\delta \mathcal{L}_{DP}}{\delta A^M_i} &= \Omega_{MN}\partial_0\mathcal{B}^{Ni} - \epsilon^{ijk}\partial_j(\mathcal{M}_{MN}\mathcal{B}^N_k) = \frac{1}{2}\Omega_{MN}\epsilon^{ijk}\partial_0\mathcal{F}^N_{jk} - \partial_j(\mathcal{M}_{MN}\mathcal{F}^{Nij}).
\end{aligned} \tag{7.12}$$

The first set of equations then allows one to eliminate the momenta π_M by their own equations of motion. When Δ is invertible, the second and third set of equations allow one to solve D^M and A^M_0 by their own equations of motion in the action, which yields (7.1). It is in this sense that these variants of the double potential formalism are equivalent, but of course not locally so. The third and fourth set of equations can be written as

$$\frac{\delta \mathcal{L}_{DP}}{\delta A^M_\mu} = \partial_\nu \star G^{\mu\nu}_M, \tag{7.13}$$

when defining

$$\star G^{i0}_M = \frac{1}{2}\Omega_{MN}\epsilon^{ijk}\mathcal{F}^N_{jk}, \quad \star G^{ij}_M = -\mathcal{M}_{MN}\mathcal{F}^{Nij}. \tag{7.14}$$

This definition implies that the components of $G_M = \frac{1}{2}G_{Mjk}dx^j dx^k + G_{Mi0}dx^i dx^0$ are explicitly given by

$$G_{Mjk} = -\Omega_{MN}\mathcal{F}^N_{jk}, \quad G_{Mi0} = \frac{1}{2}\epsilon_{ijk}\mathcal{M}_{MN}\mathcal{F}^{Njk}. \tag{7.15}$$

After elimination of the π_M, the action of the theory can then also be written as the integral of $\mathcal{L}_0 = \mathcal{L}_{ES} + \mathcal{L}_V$ with

$$\mathcal{L}_{ES} = \mathcal{L}_S - \frac{1}{2}\partial_\mu D^M \mathcal{M}_{MN}\partial^\mu D^N - \mathcal{V}, \quad d^4x\mathcal{L}_V = \int_0^1 \frac{dt}{t}[G_M F^M][tA^M, D^M, \phi^i]. \tag{7.16}$$

so that the scalar sector has been enlarged to $\phi^m = (\phi^i, D^M)$ and the scalar metric and potential are now $(g_{ij}, \mathcal{M}_{MN})$, respectively $(V, \mathcal{V})$. It is thus a particular case of the actions of the form (3.3) studied in section 3.

7.2 Local BRST cohomology

The master action is given by

$$S = \int d^4x \, [\mathcal{L}_0 + A^{*\mu}_M \partial_\mu C^M] \tag{7.17}$$

with an antifield and ghost sector that is doubled as compared to the second order covariant formulation.

We then can copy previous results:

(i) $H^g(s) = 0$ for $g \leq -3$.

(ii) $H^{-2}(s)$ is doubled: $U^{-2} = \mu^M d^4x C^*_M$ with descent equation

$$s \, d^4x \, C^*_M + d \star A^*_M = 0, \quad s \star A^*_M + dG_M = 0, \quad sG_M = 0. \tag{7.18}$$

Characteristic cohomology $H^{n-2}_W(d)$ is then represented by the 2-forms $\mu^M G_M$.

For $g \geq -1$, the discussion in terms of U, W, V types is the same as before with indices $I, J, K, \cdots \to M, N, O, \ldots$ on vector potentials, ghosts and their antifields, and $i, j, k \cdots \to m, n, o \ldots$ on scalar fields.

The obstruction equation for symmetries, equation (5.7), becomes

$$\begin{aligned} &G_M F^N f^M{}_N + F^M F^N h_{MN} + dI^{n-1} \\ &\quad - \left(dG_M g^M + [d(g_{ij} \star d\phi^j) + \star\partial_i(\mathcal{L}_{ES} + \mathcal{L}_V)]\Phi^i + \frac{\delta \mathcal{L}_0}{\delta D^M}\Phi^M \right) = 0. \end{aligned} \tag{7.19}$$

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7.3 Constraints on W, U-type cohomology

When there is no explicit x^μ dependence and $V = 0 = \mathcal{V}$, putting all derivatives of $F^M_{\mu\nu}, \phi^i, D^M$ to zero, one remains with

$$G_M|_{\text{der}=0} F^N f^M{}_N + F^M F^N h_{MN} - \star\partial_m \mathcal{L}_V \Phi^m|_{\text{der}=0} = 0, \tag{7.20}$$

where $G_M|_{\text{der}=0}$ amounts to replacing $\mathcal{F}^M_{\mu\nu}$ by $F^M_{\mu\nu}$ in (7.15).

Using $-\partial_m \star \mathcal{L}_V = \delta^i_m \frac{1}{2}\partial_i G_M|_{\text{der}=0} F^M$, and the decomposition $\Phi^m|_{\text{der}=0} = \Phi^m_0 + \Phi^m_1 + \dots$, where the Φ^m_n depend on undifferentiated scalar fields and are homogeneous of degree n in $F^M_{\mu\nu}$, the equation implies

$$G_M|_{\text{der}=0} F^N f^M{}_N + F^M F^N h_{MN} + \frac{1}{2}\partial_i G_M|_{\text{der}=0} F^M \Phi^i_0 = 0. \tag{7.21}$$

When taking account that

$$G_M|_{\text{der}=0} F^N = d^4x \frac{1}{2}[\Omega_{OM}\epsilon^{ijk} F^O_{jk} F^N_{0i} - \mathcal{M}_{MO} F^O_{jk} F^{Njk}], \tag{7.22}$$

this gives an equation of the type

$$\frac{1}{4} d^4x [\mathcal{O}_{MN}(\phi)\epsilon^{ijk} F^M_{jk} F^N_{0i} - \mathcal{P}_{MN}(\phi) F^M_{jk} F^{Njk}] = 0, \tag{7.23}$$

where

$$\mathcal{P}_{MN} = 2\mathcal{M}_{O(M} f^O{}_{N)} + \partial_i \mathcal{M}_{MN}\Phi^i_0, \tag{7.24}$$

$$\mathcal{O}_{MN} = 2\Omega_{MO} f^O{}_N + 2h_{MN}, \tag{7.25}$$

and $h_{MN} = h_{NM}$ on account of (3.42). Note that there is one less term as compared to (5.11) since the kinetic term does not depend on the scalars and also that $\mathcal{O}_{MN}$ is not symmetric.

Now both terms have to vanish separately because they involve different field strengths,

$$\mathcal{P}_{MN} = 0, \quad \mathcal{O}_{MN} = 0. \tag{7.26}$$

Setting $\phi^i = 0 = D^M$ then gives

$$\begin{aligned} f^{(\mathcal{M}(0))}_{MN} + f^{(\mathcal{M}(0))}_{NM} + (\partial_i \mathcal{M}_{MN})(0)\Phi^i_0(0) = 0, \\ f^{(\Omega)}_{MN} = -h_{MN}, \end{aligned} \tag{7.27}$$

with $f^{(\Omega)}_{MN} = \Omega_{MO} f^O{}_N$. Consider first symmetries of W-type, i.e., take the case when the f's vanish. The first equation is then satisfied with $\Phi^i_0(0) = 0$, while the second equation then requires h_{MN} to vanish. This implies:

> *There are neither W-type symmetries nor W-type cohomology in ghost numbers $g \geq 0$ for the first order model.*

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As a consequence, $h_{MN} = h_{uMN}$, and the second of equation (7.27) is equivalent to

$$f_{u}{}^{(\Omega)}_{[MN]} = 0, \quad f_{u}{}^{(\Omega)}_{(MN)} = -h_{uMN}. \tag{7.28}$$

It follows that:

The algebra $\mathfrak{g}_U$ is the largest sub-algebra of $\mathfrak{sp}(2n_v, \mathbb{R})$ that can be turned into symmetries of the full theory. All non-trivial U-type symmetries require a non-vanishing h_{uMN} and thus involve a Chern-Simons term in their Noether currents.

On its own, the first equation of (7.27) is solved for skew-symmetric $f^{(\mathcal{M}(0))}_{MN}$ with vanishing $\Phi^i(0)$. Symmetric $f^{(\mathcal{M}(0))}_{MN}$ needs a non trivial scalar symmetry.

For U-type cohomologies in higher ghost number $g \geq 0$, the $k^u_{O_1 \dots O_{g+1}}$ tensor has to satisfy (4.35), which becomes

$$f_{u}{}^{(\Omega)}_{M(N} k^u_{O_1)\dots O_{g+1}} = 0. \tag{7.29}$$

The object $D_{MO_1NO_2\dots O_{g+1}} = f_{u}{}^{(\Omega)}_{MN} k^u_{O_1\dots O_{g+1}}$ is then symmetric in the first and third indices because $f_{u}{}^{(\Omega)}_{MN}$ is symmetric, and antisymmetric in the second and third indices on account of (7.29). It thus has to vanish,

$$\begin{aligned} D_{MO_1NO_2\dots O_{g+1}} &= D_{NO_1MO_2\dots O_{g+1}} = -D_{NMO_1O_2\dots O_{g+1}} = -D_{O_1MNO_2\dots O_{g+1}} \\ &= D_{O_1NMO_2\dots O_{g+1}} = D_{MNO_1O_2\dots O_{g+1}} = -D_{MO_1NO_2\dots O_{g+1}}. \end{aligned} \tag{7.30}$$

It follows that $k^u_{O_1\dots O_{g+1}} = 0$:

There are no U-type cohomology classes in ghost number $g \geq 0$.

In particular, there are no U-type gaugings even though there are U-type symmetries. We thus recover the results on gaugings of [8] from the current perspective.

7.4 Remarks on GL($2n_v$) transformations

The two remarks on linear changes of variables from section 5.6 also apply in the first order case. More precisely, the second remark can be rephrased as follows.

The general discussion of the structure of the BRST cohomology of the first order model in sections 7.2 and 7.3 goes through unchanged for arbitrary skew-symmetric non-degenerate Ω_{MN} and symmetric non-degenerate $\mathcal{M}_{MN}$. The local BRST cohomology for sets of Ω_{MN}, $\mathcal{M}_{MN}$ related by GL($2n_v, \mathbb{R}$) transformations will be isomorphic, whereas explicit results for the local BRST cohomology do depend on the equivalence classes. For instance for the symmetries, this is the case when explicitly solving the obstruction equation (7.19). As concerns Ω_{MN}, there is just one equivalence class since all such matrices are related to a canonical Ω_{MN}, say $\Omega_{MN} = \delta_{IJ}\epsilon_{ab}$, by a GL($2n_v, \mathbb{R}$) transformation. Hence, one can restrict oneself to equivalence classes of Ω_{MN}, $\mathcal{M}_{MN}$ with canonical Ω_{MN}, and $\mathcal{M}_{MN}$'s related by Sp($2n_v, \mathbb{R}$) changes of variables.

The first remark of section 5.6 then boils down to the statement that the algebra $\mathfrak{g}_U$ is the largest sub-algebra of $\mathfrak{sp}(2n_v, \mathbb{R})$ that can be turned into symmetries of the full theory, in agreement with the discussion of the previous section. In addition we have recovered there the result that the gauge algebra remains abelian.

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7.5 Application to the bosonic sector of $\mathcal{N} = 4$ supergravity

For definiteness, let us again concentrate on the bosonic sector of four dimensional supergravity, without gravity. As in section 6.3, we use the standard second order formulation for the $\mathrm{SL}(2,\mathbb{R})/\mathrm{SO}(2)$ sigma model. Alternatively, one could use a first order formulation in terms of fields parametrizing $\mathrm{SL}(2,\mathbb{R})$, with a first class constraint eliminating the field for the $\mathrm{SO}(2)$ subgroup. It would provide a first order formulation for all fields and make all global symmetries manifest.

To this scalar action, we first couple one vector field, i.e. add the action associated to (7.6) where $\mathcal{V} = 0$, the indices M, N take two values a, b, $\Omega_{ab} = \epsilon_{ab}$, and $\mathcal{M}_{ab} = M^{-1}_{ab}$. The matrix M and its inverse are given by

$$M = \begin{pmatrix} e^{\phi} & \chi e^{\phi} \\ \chi e^{\phi} & \chi^2 e^{\phi} + e^{-\phi} \end{pmatrix}, \quad M^{-1} = \begin{pmatrix} \chi^2 e^{\phi} + e^{-\phi} & -\chi e^{\phi} \\ -\chi e^{\phi} & e^{\phi} \end{pmatrix} \tag{7.31}$$

and are such that M transforms as $M \to g^T M g$ under an $\mathrm{SL}(2,\mathbb{R})$ transformation. The model is invariant under $\mathrm{SL}(2,\mathbb{R})$ if the other fields transform as $A^a \to (g^T A)^a$, $D^a \to (g^T D)^a$, $\pi_a \to (g^{-1}\pi)_a$ because $\mathrm{SL}(2,\mathbb{R})$ transformations are symplectic, $g\epsilon g^T = \epsilon$.

For the U-type symmetries, equation (7.28) requires $f^{(\epsilon)}_{ab} = \epsilon_{ac} f^c{}_b$ to be symmetric, so there are at most 3 linearly independent solutions. According to the above discussion, all of these give rise to symmetries, which need h_{uab} and also Φ^i_u. The U-type symmetries constitute the $\mathfrak{sl}(2,\mathbb{R})$ electric symmetry algebra.

We now consider the coupling to six vector fields in the different formulations of section 6.3. For the $\mathrm{SO}(6)$ invariant model, $M = (I, a)$, $\Omega_{MN} = \delta_{IJ}\epsilon_{ab}$ and $\mathcal{M}_{MN} = \delta_{IJ} M^{-1}{}_{ab}$, while the dual formulation corresponds to $\mathcal{M}_{MN} = \delta_{IJ} M_{ab}$. Finally, in the $\mathrm{SO}(3) \times \mathrm{SO}(3)$ formulation $\mathcal{M}_{MN} = (M^{-1}_{ab}\delta_{AB}, M_{ab}\delta_{A'B'})$.

It then follows from (7.27) that both in the $\mathrm{SO}(6)$ invariant formulation and in the dual formulation, the electric symmetry algebra is $\mathfrak{sl}(2,\mathbb{R}) \oplus \mathfrak{so}(6)$, where the $\mathfrak{sl}(2,\mathbb{R})$ transformations on the vectors $A^{(I,a)}$ and on $D^{(I,a)}, \pi_{(J,b)}$ in the dual formulation corresponds to the infinitesimal version of the above transformations where $g^T \to g^{-1}$.

Finally, in the $\mathrm{SO}(3) \times \mathrm{SO}(3)$ formulation, the electric symmetry algebra is also $\mathfrak{sl}(2,\mathbb{R}) \oplus \mathfrak{so}(6)$. This is so because the $\mathrm{SL}(2,\mathbb{R})$ element ϵ is such that $M = \epsilon^T M^{-1} \epsilon$.

8 Conclusions and comments

In this work we have systematically analyzed gaugings of vector-scalar models through a standard deformation theoretic approach. In the case of gauge systems, this is most naturally done in the BV-BRST antifield formalism. We have shown that different types of symmetries behave differently when one tries to gauge them. The method allows one to find all the infinitesimal gaugings and higher order cohomology classes once all symmetries are known.

The symmetries are classified into U, W and V-types. Only U-type symmetries give rise to gaugings that deform the abelian gauge algebra. They contain the standard "Yang-Mills" deformations. The W-type symmetries contain the topological gaugings of [29].

The Noether currents of both these types of symmetries are the only ones that cannot be redefined so as to be gauge invariant. We have treated explicit examples, for which all symmetries of U, W-types have been computed (in the x^μ independent case considered here).

For the models explicitly considered in the article, we have found that the only possible gaugings of U and W-types are the ones previously considered in the literature, namely Yang-Mills and topological couplings among the gauge fields, with minimal couplings of the scalars.

In order to achieve complete results, one should also compute the V-type symmetries which admit gauge invariant Noether currents. This is very much a model-dependent question that requires the use of more standard symmetry techniques (see e.g., [64, 65]). However, we have shown in section 4 that given the graded structure of the antibracket map, the leading obstruction to extend first order deformations of U-type to second order, leading to the Jacobi identity for the structure constants, cannot be eliminated by adding V-terms.

Furthermore, in some cases, for instance when one imposes Poincaré invariance as relevant to relativistic theories, the V-type symmetries can be shown to be absent [66]. It turns out that the effect of coupling the models to Einstein gravity justifies this assumption [67] and simplifies the problem. It would be interesting to see if it also justifies the ansatz for the electric symmetry algebra.

We have analyzed the problem in the second order Lagrangian and in the first order manifestly duality invariant formulation, both of which are non-locally related in space (but not in time). The results are very different: whereas the former formulation allows for standard gaugings, the latter formulation allows for more (generalized) symmetries of U-type, but none of those can be gauged. This is because the analysis is performed in each formulation by insisting on space-time locality. To go beyond such no-go results, one should presumably try to work in a controlled way with deformations that are spatially non-local.

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A Conventions and notation

The components of the Minkowski metric are given, in inertial coordinates in which we work, by the mostly plus expression $\eta_{\mu\nu} = \mathrm{diag}(-1, +1, \ldots, +1)$. The symbol $\epsilon_{\mu_1 \ldots \mu_n}$ denotes the completely antisymmetric Levi-Civita density with the convention that $\epsilon^{01\ldots n-1} = 1$ so that $\epsilon_{01\ldots n-1} = -1$. A local basis of anticommuting exterior differential

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1-forms is given by the family $(dx^\mu)_{\mu=0,\ldots,n-1}$. The wedge product symbol $\wedge$ will always be omitted.

We will sometimes use the notation $(d^{n-p}x)_{\mu_{n-p+1}\ldots\mu_n} := -\frac{1}{p!(n-p)!} dx^{\mu_1} \ldots dx^{\mu_{n-p}} \epsilon_{\mu_1\ldots\mu_n}$ for $1 \leqslant p \leqslant n$, and $d^n x := dx^0 \ldots dx^{n-1}$. The Hodge dual of a differential p-form $\omega^p \equiv \frac{1}{p!} dx^{\mu_1} \ldots dx^{\mu_p} \omega_{\mu_1\ldots\mu_p}$, is the $n-p$-form given, in our convention, by

$$\begin{aligned}
\star\,\omega^p &= \frac{1}{p!(n-p)!}\, dx^{\nu_1} \ldots dx^{\nu_{n-p}} \epsilon_{\nu_1\ldots\nu_{n-p}\mu_{n-p+1}\ldots\mu_n} \omega^{\mu_{n-p+1}\ldots\mu_n} \\
&= -(d^{n-p}x)_{\mu_{n-p+1}\ldots\mu_n} \omega^{\mu_{n-p+1}\ldots\mu_n}\,.
\end{aligned}$$

As a consequence, the exterior differential of the dual of a p-form reads

$$d \star \omega^p = -(-)^{n-p} (d^{n-p+1}x)_{\nu_1\ldots\nu_{p-1}}\, \partial_\mu \omega^{\mu\nu_1\ldots\nu_{p-1}}\,. \tag{A.1}$$

B Antibracket maps and descents

As discussed in section 2, the first obstruction to extending infinitesimal deformations to finite ones is controlled by the antibracket map. We show here how the antibracket map behaves with respect to the length of shortest non trivial descent, i.e., the "depth".

Since covariantizable and non-covariantizable currents as elements of $H^{-1,n}(s|d)$, and the associated infinitesimal deformations as elements of $H^{0,n}(s|d)$ are distinguished by the property that the depth is 1, respectively deeper than one, the following will be relevant when studying the obstruction to infinitesimal deformations.

Proposition. *The depth of an image of the antibracket map is less or equal to the depth of its most shallow argument.*

Proof. Consider $[\omega^{g_1,n}_{l_1}], [\omega^{g_2,n}_{l_2}] \in H^{*,n}(s|d)$, where we can assume without loss of generality that $l_1 \geqslant l_2$. For the antibracket, let us not choose the expression with Euler-Lagrange derivatives on the left and right that is graded antisymmetric without boundary terms, but rather the one that satisfies a graded Leibniz rule on the right

$$(\omega^{g,n}, \cdot)_{\rm alt} = \partial_{(\nu)} \frac{\delta^R(-\star\omega^{g,n})}{\delta\phi^A} \frac{\partial^L \cdot}{\partial\partial_{(\nu)}\phi^*_A} - (\phi^A \leftrightarrow \phi^*_A), \tag{B.1}$$

and the following version of the graded Jacobi identity without boundary terms,

$$(\omega^{g_1,n},(\omega^{g_2,n},\cdot)_{\rm alt})_{\rm alt} = ((\omega^{g_1,n},\omega^{g_2,n})_{\rm alt},\cdot)_{\rm alt} + (-)^{(g_1+1)(g_2+1)}(\omega^{g_2,n},(\omega^{g_1,n},\cdot)_{\rm alt})_{\rm alt} \tag{B.2}$$

(see appendix B of [50] for details and a proof). Furthermore,

$$(\omega^{g,n}, d(\cdot))_{\rm alt} = (-)^{g+1} d((\omega^{g,n},\cdot)_{\rm alt}), \quad (d\omega^{g+1,n-1},\cdot)_{\rm alt} = 0. \tag{B.3}$$

Let $S = \int(-\star\mathcal{L})$ be the BV master action. We have $s\cdot = (-\star\mathcal{L},\cdot)_{\rm alt}$. Using these properties, we get

$$s(\omega^{g_1,n}_{l_1},\omega^{g_2,n}_{l_2})_{\rm alt} + d((\omega^{g_1,n}_{l_1},\omega^{g_2+1,n-1}_{l_2})_{\rm alt}) = 0, \ldots, s(\omega^{g_1,n}_{l_1},\omega^{g_2+l_2,n-l_2}_{l_2})_{\rm alt} = 0, \tag{B.4}$$

which proves the proposition. □

By using $[(\omega^{-1,n}, \omega^{g,n})] \in H^{g,n}(s|d)$, it follows that:

(i) Characteristic cohomology in degree $n-1$ described by $H^{-1,n}(s|d)$ is a Lie algebra. It is the Lie algebra of non trivial global symmetries. It also describes the Dirac or Dickey bracket algebra of non trivial conserved currents (up to constants or more generally topological classes),

(ii) $H^{-2,n}(s|d)$ is a module thereof (module structure of flux charges — Gauss or ADM type surface charges — under global symmetries). The proposition gives rise for instance to the following refinements:

Corollary. Covariantizable characteristic cohomology in form degree $n-1$ forms an ideal in the Lie algebra of characteristic cohomology in form degree $n-1$. The module action of covariantizable characteristic cohomology of degree $n-1$ on characteristic cohomology in degree $n-2$ is trivial.

Similar results hold for the associated infinitesimal deformations.

C Derivation of equation (4.25)

In this appendix, we derive formula (4.25) for the variation $\delta_u G_I = -(U_u, G_I)$ of the two-form G_I under a U-type symmetry. This is done in two steps:

1. First, we show that

$$\delta_u G_I + (f_u)^J{}_I G_J \approx c_{IJ} F^J + d(\text{invariant}) \tag{C.1}$$

for some constants c_{IJ}.

2. Then, we prove that the c_{IJ} take the form

$$c_{IJ} = -2(h_u)_{IJ} + \lambda_u^w (h_w)_{IJ}, \tag{C.2}$$

where the constants $(h_u)_{IJ}$ and $(h_w)_{IJ}$ are those appearing in the currents associated with U_u and W_w respectively.

The proof is given in the case where the Lagrangian (or, equivalently, G_I) does not depend on the derivatives of $F^I_{\mu\nu}$.

A lemma. The proof of the above steps uses the following result on the W-type cohomology classes (with $g = -1$):

$$t_{IJ} F^I F^J \approx d(\text{invariant}) \;\Rightarrow\; t_{IJ} = \sum_w \lambda^w (h_w)_{IJ} \;\text{ for some } \lambda^w. \tag{C.3}$$

This is proven as follows: $t_{IJ} F^I F^J \approx d(\text{invariant})$ implies that

$$t_{IJ} F^I F^J + dI + \delta k = 0 \tag{C.4}$$

for some gauge invariant I and some k of antifield number 1, where δ is here the Koszul-Tate differential. Now, it is proven in [46] that k must be gauge invariant; hence, it can be written as

$$k = \hat{K} + dR, \quad \hat{K} = d^4x[A_I^{*\mu} g_\mu^I + \phi_i^* \Phi^i] \tag{C.5}$$

for some gauge invariant R, g_μ^I and Φ^i. Indeed, derivatives acting on the antifields contained in k are pushed to the term dR by integration by parts, leaving the form (C.5) where $\hat{K}$ contains only the undifferentiated antifields. Putting this back in (C.4) and using the fact that $\delta\hat{K} = s\hat{K}$ because $\hat{K}$ is gauge invariant, we get

$$s\hat{K} + d\left(t_{IJ} A^I F^J + J\right) = 0 \tag{C.6}$$

for some gauge invariant $J = I - \delta R$. This shows that $\hat{K}$ is a W-type cohomology class: we can therefore expand $\hat{K}$ in the W_w basis as $\hat{K} = \sum \lambda^w W_w$. In particular, this implies that $t_{IJ} = \sum \lambda^w (h_w)_{IJ}$, which proves the lemma.

First step. We start from the chain of descent equations involving G_I,

$$s\, d^4x\, C_I^* + d \star A_I^* = 0, \quad s \star A_I^* + dG_I = 0, \quad sG_I = 0. \tag{C.7}$$

Applying $(U_u, \cdot)_{\text{alt}}$ to this chain, we get

$$s\left[d^4x\, (f_u)^J{}_I C_J^*\right] + d\left[(f_u)^J{}_I \star A_J^* + \frac{\delta K_u}{\delta A^I}\right] = 0, \tag{C.8}$$

$$s\left[(f_u)^J{}_I \star A_J^* + \frac{\delta K_u}{\delta A^I}\right] + d\left[-\delta_u G_I\right] = 0, \tag{C.9}$$

$$s\left[-\delta_u G_I\right] = 0, \tag{C.10}$$

which can be simplified to

$$d\left(\frac{\delta K_u}{\delta A^I}\right) = 0, \tag{C.11}$$

$$s\left(\frac{\delta K_u}{\delta A^I}\right) + d\left(-\delta_u G_I - (f_u)^J{}_I G_J\right) = 0, \tag{C.12}$$

$$s\left(-\delta_u G_I\right) = 0, \tag{C.13}$$

using equations (C.7) again. Equation (C.11) implies that

$$\frac{\delta K_u}{\delta A^I} = d\eta^{-1,2} \tag{C.14}$$

for some $\eta^{-1,2}$ of ghost number -1 and form degree 2. Because the left-hand side is gauge invariant and $\eta^{-1,2}$ is of form degree two, $\eta^{-1,2}$ must also be gauge invariant. This follows from theorems on the invariant cohomology of d in form degree 2 [53, 54]. Equation (C.12) implies then

$$d\left(\delta_u G_I + (f_u)^J{}_I G_J + s\eta^{-1,2}\right) = 0, \tag{C.15}$$

i.e.

$$\delta_u G_I + (f_u)^J{}_I G_J + s\eta^{-1,2} = d\eta^{0,1} \tag{C.16}$$

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for some $\eta^{0,1}$ of ghost number 0 and form degree 1. Again, the left-hand side of this equation is gauge invariant: results on the invariant cohomology of d in form degree 1 [53, 54] now imply that the non-gauge invariant part of $\eta^{0,1}$ can only be a linear combination of the one-forms A^I,

$$\eta^{0,1} = c_{IJ} A^J + (\text{gauge invariant}). \tag{C.17}$$

Plugging this back in equation (C.16) and using the fact that $s\eta^{-1,2} \approx 0$ (since $\eta^{-1,2}$ is gauge invariant), we recover equation (C.1). This concludes the first step of the proof.

Second step. For the second step, we introduce

$$N = -\int d^4x \, (C^*_I C^I + A^{*\mu}_I A^I_\mu), \quad \hat{N} = (N, \cdot)_{\text{alt}}. \tag{C.18}$$

The operator $\hat{N}$ counts the number of A^I's and C^I's minus the number of A^*_I's and C^*_I's. Because it carries ghost number -1, it commutes with the exterior derivative, $\hat{N} d = d\hat{N}$. Applying this operator to the equation

$$sU_u + d\left[(f_u)^I{}_J (\star A^*_I C^J + G_I A^J) + (h_u)_{IJ} F^I A^J + J_u\right] = 0 \tag{C.19}$$

gives

$$\left(\int G_I F^I, U_u\right)_{\text{alt}} + d\left[(f_u)^I{}_J (\hat{N}+1)(G_I) A^J + 2(h_u)_{IJ} F^I A^J + \hat{N}(J_u)\right] \approx 0. \tag{C.20}$$

The second term is evident. The first term is

$$\hat{N}(sU_u) = (N, (S, U_u)_{\text{alt}})_{\text{alt}} = ((N,S), U_u)_{\text{alt}} + (S, (N, U_u)_{\text{alt}})_{\text{alt}} \tag{C.21}$$

according to the graded Jacobi identity. The counting operator $\hat{N}$ kills the $A^{*\mu}_I \partial_\mu C^I$ term in the master action S, which implies

$$(N, S) = \int d^4x \, A^I_\mu \frac{\delta \mathcal{L}_V}{\delta A^I_\mu} = \int d^4x \, A^I_\mu \partial_\nu (\star G_I)^{\mu\nu} = \int G_I F^I. \tag{C.22}$$

Similarly, $\hat{N}$ kills the first two terms of U_u, leaving $\hat{N} U_u = \hat{N} K_u$ which is gauge invariant. This implies $(S, (N, U_u)_{\text{alt}})_{\text{alt}} = s(\hat{N} U_u) \approx 0$. Therefore, we have indeed

$$\hat{N}(sU_u) \approx \left(\int G_I F^I, U_u\right)_{\text{alt}} \tag{C.23}$$

which proves equation (C.20).

We now compute $(\int G_I F^I, U_u)_{\text{alt}}$ using the result of the first step. We have

$$\left(\int G_I F^I, U_u\right)_{\text{alt}} = \frac{\delta(G_K F^K)}{\delta A^I_\mu} \delta_u A^I_\mu + \frac{\delta(G_K F^K)}{\delta \phi^i} \delta_u \phi^i. \tag{C.24}$$

This looks like the U-variation $\delta_u(G_I F^I)$, but it is not because there are Euler-Lagrange derivatives. For a top form ω, the general rule is [68]

$$\delta_Q \omega = Q^a \frac{\delta \omega}{\delta z^a} + d\rho, \quad \rho = \partial_{(\nu)}\left[Q^a \frac{\delta}{\delta z^a_{(\nu)\rho}} \frac{\partial \omega}{\partial dx^\rho}\right]. \tag{C.25}$$

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In our case, this becomes

$$\delta_u(G_I F^I) = \frac{\delta(G_K F^K)}{\delta A^I_\mu}\,\delta_u A^I_\mu + \frac{\delta(G_K F^K)}{\delta\phi^i}\,\delta_u\phi^i + d\rho_A + d(\mathrm{inv}), \tag{C.26}$$

$$\rho_A = \partial_{(\nu)}\left((f_u)^I{}_J A^J_\mu \frac{\delta}{\delta A^I_{\mu,(\nu)\rho}} \frac{\partial(G_K F^K)}{\partial dx^\rho}\right). \tag{C.27}$$

Using property (C.1) and putting together the terms of the form d(invariant), we get then from (C.20)

$$(c_{IJ} + 2(h_u)_{IJ})F^I F^J + d\left[(f_u)^I{}_J A^J(\hat{N}+1)(G_I) - \rho_A\right] + d(\mathrm{inv}) \approx 0. \tag{C.28}$$

Now, it is sufficient to prove that

$$d\left[(f_u)^I{}_J A^J(\hat{N}+1)(G_I) - \rho_A\right] \approx d(\mathrm{inv}). \tag{C.29}$$

Indeed, this implies $(c_{IJ} + 2(h_u)_{IJ})F^I F^J \approx d(\mathrm{inv})$, which in turn gives

$$c_{IJ} = -2(h_u)_{IJ} + \lambda^w_u (h_w)_{IJ} \tag{C.30}$$

for some constants λ^w_u using property (C.3) of the W-type cohomology classes.

Proof of (C.29). We will actually prove the stronger equation

$$\rho_A = (f_u)^I{}_J A^J(\hat{N}+1)(G_I) \tag{C.31}$$

in the case where G_I depends on F but not on its derivatives.

To do this, we can assume that G_I a homogeneous function of degree n in A^I, i.e. $\hat{N}(G_I) = nG_I$. If it is not, we can separate it into a sum of homogenous parts; the result then still holds because equation (C.31) is linear in G_I.

In components, equation (C.31) is

$$\frac{1}{2}\partial_{(\nu)}\left((f_u)^I{}_J A^J_\mu \frac{\delta}{\delta A^I_{\mu,(\nu)\rho}} G_{K\sigma\tau}F^K_{\lambda\gamma}\varepsilon^{\sigma\tau\lambda\gamma}\right) = (n+1)(f_u)^I{}_J A^J_\lambda G_{I\sigma\tau}\varepsilon^{\rho\lambda\sigma\tau}. \tag{C.32}$$

Under the homogeneity assumption $\hat{N}(G_I) = nG_I$, we have

$$G_{K\sigma\tau}F^K_{\lambda\gamma}\varepsilon^{\sigma\tau\lambda\gamma} = 4(n+1)\mathcal{L}_V. \tag{C.33}$$

Equation (C.31) now becomes

$$\frac{1}{2}\partial_{(\nu)}\left((f_u)^I{}_J A^J_\mu \frac{\delta\mathcal{L}_V}{\delta A^I_{\mu,(\nu)\rho}}\right) = \frac{1}{4}(f_u)^I{}_J A^J_\lambda G_{I\sigma\tau}\varepsilon^{\rho\lambda\sigma\tau}. \tag{C.34}$$

We now use the fact that G_I does not depend on derivatives of F, which implies that the higher order derivatives $\partial_{(\nu)}$ are not present and that the Euler-Lagrange derivatives are only partial derivatives. We then have

$$\frac{1}{2}\frac{\delta\mathcal{L}_V}{\delta A^I_{\mu,\rho}} = \frac{\delta\mathcal{L}_V}{\delta F^I_{\rho\mu}} = \frac{1}{4}\varepsilon^{\rho\mu\sigma\tau}G_{I\sigma\tau} \tag{C.35}$$

(see (3.1)), which proves (C.34) in this case.

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