Glimpses of black hole formation/evaporation in highly inelastic, ultra-planckian string collisions

Andrea Addazi,[a,b] **Massimo Bianchi**[c] **and Gabriele Veneziano**[d,e,f]

[a]*Dipartimento di Fisica, Università di L'Aquila,*
67010 Coppito, L'Aquila, Italia

[b]*LNGS, Laboratori Nazionali del Gran Sasso,*
67010 Assergi, Italia

[c]*Dipartimento di Fisica, Università di Roma Tor Vergata and*
I.N.F.N. Sezione di Roma Tor Vergata,
Via della Ricerca Scientifica 1, 00133 Roma, Italia

[d]*Collége de France,*
11 place M. Berthelot, 75005 Paris, France

[e]*Theory Division, CERN,*
CH-1211 Geneva 23, Switzerland

[f]*Dipartimento di Fisica, Università di Roma La Sapienza,*
00185 Roma, Italia

E-mail: andrea.addazi@infn.lngs.it, massimo.bianchi@roma2.infn.it, gabriele.veneziano@cern.ch

Abstract: We revisit possible glimpses of black-hole formation by looking at ultra-planckian string-string collisions at very high final-state multiplicity. We compare, in particular, previous results using the optical theorem, the resummation of ladder diagrams at arbitrary loop order, and the AGK cutting rules, with the more recent study of $2 \to N$ scattering at $N \sim sM_P^{-2} \gg 1$. We argue that some apparent tension between the two approaches disappears once a reinterpretation of the latter's results in terms of suitably defined infrared-safe cross sections is adopted. Under that assumption, the typical final state produced in an ultra-planckian collision does indeed appear to share some properties with those expected from the evaporation of a black hole of mass $\sqrt{s}$, although no sign of thermalization is seen to emerge at this level of approximation.

Contents

1   Introduction

Ultra-planckian string-string collisions represent a perfect *gedanken experiment* where one can address fundamental issues on the merging of gravitational and quantum physics within a consistent framework. One of the main aims of such a program is to understand whether and how quantum information is recovered in a process which, classically, would lead to black-hole formation [1–4] and, semi classically, to its Hawking evaporation [5]. This program has been carried out since about thirty years along two different lines: Gross and Mende (later joined by Ooguri) [6, 7] computed the high-energy, fixed angle behavior of string scattering amplitudes at arbitrary genus. Since higher and higher genus contributions were found to be more and more important in that kinematic regime, they concluded that the string loop expansion diverges. The physical reason for such a result is clear: order by order the fixed-angle string scattering amplitude is exponentially suppressed, while physically it should be sizeable owing to Einstein's gravitational deflection formula. Unfortunately, a Borel resummation of the divergent series [8] can only be justified in a

region of parameters where the process is classically forbidden and, consequently, the cross section is still exponentially small.

A very different approach was taken by Amati, Ciafaloni and one of us [9–13] (hereafter referred to as ACV). One starts by working in energy or equivalently gravitational radius

$$R_S = 2G\sqrt{s}$$

and impact parameter b space attempting an all-loop resummation. This is possible at arbitrarily high energy provided the impact parameter is also correspondingly high. In this region classical gravitational deflection as well as tidal effects [10, 14] due to the string's finite size, are effectively recovered within a unitary S-matrix framework provided one is far away from the expected gravitational-collapse region $b \sim R_S$ and as long as the tide-excited states are included in the Hilbert space (see [15] for a detailed study of that unitary S-matrix). The regime of classical gravitational collapse can be approached — but unfortunately not (yet) entered — from two different directions in parameter space (see figure 1):

- By letting b/R_S approach a critical value of $\mathcal{O}(1)$ while keeping both b and R_S much larger than the string length ℓ_s. This turns out to be quite difficult, although some interesting progress has been made over the past ten years [16–23]. We note, in particular, a recent result [24–26] on the form of gravitational brems-strahlung in the regime of small deflection angles, suggesting the emergence of a typical energy scale for the emitted gravitons of order the Hawking temperature

$$T \sim T_H = \hbar/R_S \,.$$

- By approaching the limit $R_S \to \ell_s$ from below. In this case life is easier since one can justify the validity of a (string corrected) leading eikonal approximation not suffering from the nasty classical corrections that make things complicated in the previous case. Here one can make contact with the GMO regime (finding perfect agreement with their Borel resummation) but can also try to go further [10, 27, 28] towards the expected black-hole formation regime $b < \ell_s, R_S \to \ell_s$ [29–35] while keeping some control over unitarity. It was found, in particular, that, by taking into account the opening of new channels corresponding to the imaginary part of graviton exchange in string theory, it was possible to obtain a unitary S-matrix within a Hilbert space containing, besides the tide-excited states, also those responsible for the above mentioned imaginary part (see section 2.2 for details).

In an independent study Dvali, Gomez, Isermann, Lüst, and Stieberger (DGILS henceforth) [36] succeeded in carrying out a calculation of tree-level high-energy large multiplicity scattering amplitudes both in quantum field and in quantum string theory. The claim is that their results support the validity of the idea, proposed by Dvali and Gomez [37, 38], according to which BH's can be portrayed as Bose-Einstein condensates at criticality.

One can find some tension between the results in [10] and those in [36] since in the former (ACV) approach loop corrections are crucial in restoring unitarity through an in-
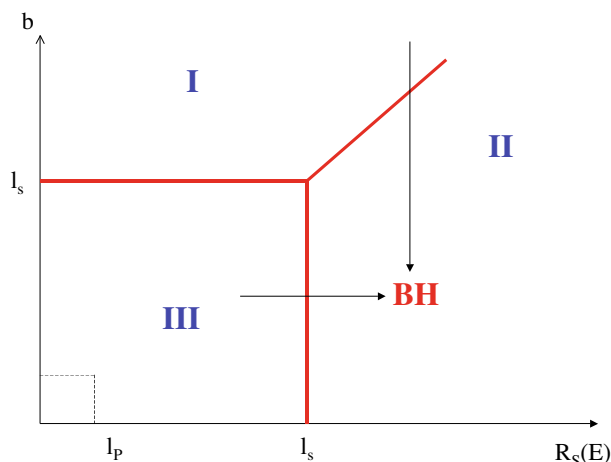
Figure 1. Rough phase diagram for transplanckian string-string collisions. In Region I and III calculations are essentially under control. They both border with region II where BH formation is expected on the ground of classical collapse criteria [1–4].

terplay of real production and virtual corrections controlled by the AGK cutting rules [39], while in the latter (DGILS) virtual corrections are largely ignored.

The main purpose of this paper is to try and understand the origin of this tension and to offer a solution of it through a reinterpretation of the result of [36].

The plan of the paper is as follows.

In section 2 we will briefly review the results of ACV for ultra-Planckian scattering in String Theory, first in the weak-gravity regime and then in the so-called string-gravity regime where a string-corrected eikonal approximation can be justified all the way up into the expected threshold for classical black-hole formation. In section 3, we review the results obtained in [36] for the $2 \to N$ scattering in string and field theory at very high energy and multiplicity. In section 4, after pointing out a tension between the two approaches, we discuss the effect of adding to the calculation in [36] both real and virtual soft gravitons, following Weinberg's classic treatment. In section 5, we reinterpret the results of [36] claiming that one can eventually reconcile the two methods. In section 6, we conclude and draw directions for further investigation. In appendix A we review the AGK formalism [39, 40], and in appendix B we discuss more details about Weinberg's soft B-factor [41] in the case of massless external particles.

2   The ACV approach: a reminder

For completeness we briefly review, in this section, some material that can be found in [10, 27, 28].

2.1   Different regimes in b, R_S, ℓ_s parameter space

The main physical idea of the ACV approach is that, as long as ultra-planckian gravitational scattering is considered at sufficiently large impact parameter (in particular $b \gg R_S$), it is dominated by soft processes in which a very large number of nearly on-shell gravitons are

exchanged between the two energetic particles. Such huge number of soft processes can build up a collision that we would otherwise call hard (at least in the language of QCD) because it corresponds to fixed angle high energy scattering.

At very small (but finite) deflection angle the leading diagrams are simply s-channel ladder diagrams whose elementary rung is nothing but the tree-level graviton-graviton scattering amplitude given by[1]

$$\mathcal{M}_0(12 \to 34) = \text{Tr}(h_1 h_4)\text{Tr}(h_2 h_3)\,\mathcal{A}_0(s,t)\,, \tag{2.1}$$

where h_i denote graviton polarisation tensors. In the following we will set $\alpha' = \ell_s^2 = 1/M_s^2 = 2$ (unless explicitly shown for clarity). For generic s and t ($s+t+u=0$) the amplitude in $D=4$ reads

$$\mathcal{M}_0(12 \to 34) = \frac{2g_s^2 \mathcal{R}^4}{stu}\frac{\Gamma(1-s/2)\Gamma(1-t/2)\Gamma(1-u/2)}{\Gamma(1+s/2)\Gamma(1+t/2)\Gamma(1+u/2)}\,,$$

where $\mathcal{R}^4$ denotes the contraction of 4 linearised Riemann tensors $\mathcal{R}_{\mu\nu\rho\sigma} = k_{[\mu}h_{\nu][\rho}k_{\sigma]}$. Taking the large s limit and relying on Stirling formula the amplitude Reggeizes[2]

$$\mathcal{A}_0(s,t) \simeq 4g_s^2 \frac{\Gamma(-t/2)}{\Gamma(1+t/2)}\left(\frac{s}{2}\right)^{\alpha(t)} e^{-i\frac{\pi}{2}t}\,, \tag{2.2}$$

i.e. the scattering process proceeds through the exchange of the gravi-Reggeon trajectory with $\alpha(t) = 2 + \alpha't/2 = 2+t$. Notice that while the real part of $\mathcal{A}_0(s,t)$ exposes the massless t-channel Coulomb pole, the imaginary part has no singularity for forward scattering. Moreover $\text{Im}\mathcal{A}_0^{FS}(s) = \lim_{t\to 0}\text{Im}\mathcal{A}(s,t)$ is related to the cross-section for production of (massive) string states at tree-level.

Clearly this amplitude is unfit to describe gravitational scattering. On one hand it is exponentially small at fixed (even small) angle while we expect a large cross section in that region from Einstein's gravitational deflection formula $\theta \sim R_S/b$. On the other hand, its Fourier transform (dominated by the fixed t Regge region) gives a partial-wave amplitude $\mathcal{A}(J=b\sqrt{s},s)$ that grows with energy thus violating unitarity bounds.[3]

It proves convenient to resort to the eikonal approximation, whereby the dominant contribution to the L-loop amplitude reads

$$\mathcal{M}_L(s,t) \approx (2\pi)^{D-2}\frac{\text{Tr}(h_1 h_4)\text{Tr}(h_2 h_3)}{(L+1)!}\frac{i^L}{(2s)^L}$$
$$\times \int\left[\prod_{i=1}^{L+1}\frac{d^{D-2}q_i}{(2\pi)^{D-2}}\mathcal{A}_0(s,-q_i^2)\right]\mathcal{V}_{L+1}^2(q_i)\,\delta^{D-2}(q-\sum_i q_i)\,, \tag{2.3}$$

---

[1]Similar calculations of string scattering from a stack of D-branes in the Regge regime were used [42] to show how Regge behavior saves string theory from possible causality violations of the kind firstly noticed in ref. [43].

[2]The choice of the phase $(-)^{t/2} = e^{-i\frac{\pi}{2}t}$ is dictated by physical considerations. Since $\text{Im}\mathcal{M}$ must be positive at $t = -q^2 = 0^-$, this is the correct choice.

[3]N.B. Given the presence of massless particles one cannot use Froissart's bound. However, partial-wave unitarity still puts the constraint $|\mathcal{A}(J,s)| \leq 1$.

where $\mathcal{V}_N(q_i)$ denotes the N gravi-reggeon vertex, which in the limit $\alpha' q_i q_j \to 0$ reduces to $\mathcal{V}_N(q_i) = 1 + \mathcal{O}\left(\sum_{i<j}(\alpha' q_i q_j)^2\right)$.

The amplitude $\mathcal{M}_L$ is a convolution in the q-space, so that it can be factorized in the dual space of impact parameter b:

$$\frac{1}{s}\mathcal{M}_L(s,t) = 4\mathrm{Tr}(h_1 h_4)\mathrm{Tr}(h_2 h_3)\int d^{D-2}b\, e^{iqb}\tilde{\mathcal{A}}_L(s,b)\,, \tag{2.4}$$

where

$$\tilde{\mathcal{A}}_L(s,b) = \frac{(2i)^{L+1}}{(L+1)!}\langle 0|\left[\hat{\delta}\left(s,b,\hat{X}^u,\hat{X}^d\right)\right]^{L+1}|0\rangle\,, \tag{2.5}$$

with $\hat{\delta}$ the 'eikonal' operator, related to the S-matrix by

$$\hat{S} = 1 + i\hat{T} = \exp 2i\hat{\delta}\quad.$$

As indicated, $\hat{\delta}$ is a functional of the closed string coordinates at equal time, and was found in [9] to take the highly suggestive form

$$\hat{\delta}\left(s,b;\hat{X}^u,\hat{X}^d\right) = \int \frac{d^{D-2}q}{(2\pi)^{D-2}}\frac{\mathcal{A}_0(s,t)}{s}\int \frac{d\sigma_u d\sigma_d}{(2\pi)^2}\, : e^{iq\left(b+\hat{X}^u(\sigma_u)-\hat{X}^d(\sigma_d)\right)}\, : \tag{2.6}$$

$$= \int \frac{d\sigma_u d\sigma_d}{(2\pi)^2}\, : \tilde{\mathcal{A}}_0\left(s,b+\hat{X}^u(\sigma_u)-\hat{X}^d(\sigma_d)\right) :\,,$$

corresponding to exchanging a gravi-reggeon between one point on one string and one on the other.

When stringy effects are negligible, one can set $\hat{X}$ to zero and (2.6) becomes an ordinary function, the eikonal phase, whose real part encodes elastic scattering (with a physically irrelevant IR divergence in $d = D - 4 = 0$), while the extra term associated with inelastic channels is finite. Setting

$$Y = \log \alpha' s\quad,$$

and following [9] (briefly reviewed in appendix A), one can perform the integral by a saddle point method for $b^2 \gg \ell_s^2 Y$ and obtain

$$\delta(b,s) = \hat{\delta}(b,s)\Big|_{\hat{X}=0} \approx \left(\frac{b_E}{b}\right)^d + i\frac{G_D s}{\ell_s^d Y^{d/2+1}}e^{-b^2/Y\ell_s^2}\,, \tag{2.7}$$

where G_D is the D-dimensional Newton's constant and

$$b_E^d(s) = \frac{s}{8\pi\Omega_d M_D^{d+2}} = \frac{g_s^2 s}{8\pi\Omega_d M_s^{d+2}}\,,$$

with $\Omega_N = 2\pi^{N/2}/\Gamma(N/2)$.

ACV distinguished several regimes in the b, R_S plane (see figure 1):

- The very large b regime, $b > b_E$, Here the massless graviton pole dominates, though distorted by a Coulomb phase in $D = 4$. One recovers here, in a saddle-point approximation, the appropriate generalization of Einstein's deflection formula.

- An intermediate regime $b_E > b > b_B, b_I$, where

$$b_I = \ell_s \sqrt{Y} = \ell_s \sqrt{\log \alpha' s}$$

is the threshold for the opening of inelastic channels and $b_B = (b_E^d R_S^2)^{\frac{1}{d+2}}$ is the threshold for gravitational radiation i.e. the onset of R_S/b corrections.[4] Here the eikonal approximation applies. In the subregion $b_E > b_t > b > b_I$, where

$$b_t^{d+2}(s) = \frac{s}{8\pi\Omega_{d+2} g_s^2 M_D^{d+4}} = \frac{g_s^2 s}{8\pi\Omega_{d+2} M_s^{d+4}}$$

determines the opening of inelastic channels, 'tidal excitations' dominate which are represented by excited string states. The diffractive b-parameter emerges by considering second-order correction to the elastic channel part of the expression (2.7) by string finite size effects that modify the S-matrix.

- The classical corrections regime $b_B > b > b_I$ where classical corrections and gravitational brems-strahlung kick in.

- Inelastic regime $b_I > b > \ell_s > \ell_P$, where inelastic channels of both classical and string absorption are opened. The relative importance of the two depends on whether R_S is larger or smaller than ℓ_s. For $R_S < \ell_s$ the situation is under control and string 'softening' effects modify General Relativity in particular the deflection angle reaches a maximum around $b = b_I$ and then decreases again towards $b \approx \ell_s$ and then $b \approx R_S$. Since this is the regime of interest here it will be discussed separately and in greater detail in the next subsection.

2.2 The string-gravity regime via the AGK cutting rules

The "string-gravity" regime of ACV is defined (up to possible logs) by the inequality $\ell_s > b, R_S$. It is believed that in this regime the so-called classical corrections (that scale as R_S^2/b^2 in $D = 4$) are tamed since they become, effectively, of order $R_S^2/\ell_s^2 \ll 1$. For this reason the string-gravity regime can be described in terms of the string-size-corrected leading eikonal approximation and, consequently, we do not expect to find here signatures of actual BH formation. The string-corrected leading eikonal was already discussed in the previous subsection and leads to a unitary S-matrix which becomes highly inelastic at $b < b_t$ i.e. when tidal excitations of the incoming strings dominates. This phenomenon will persist in the string-gravity regime; however, a new source of inelasticity takes place on top of the one due to tidal excitation.

The origin of this new source of inelasticity can be easily ascribed to the fact that in string theory the gravitons Reggeize, i.e. full Regge trajectories (starting from the graviton) are exchanged between the high-energy colliding particles (that we have taken to be

---

[4]Let us remark that $\mathcal{O}(R_S/b)$ corrections kick in much before one reaches the $b \sim R_S$ regime. The reason for this is twofold: i) the leading eikonal phase is $\delta \gg 1$, therefore corrections to the phase can be large even if R_S/b is small; ii) the leading eikonal is just a real phase while corrections contain imaginary parts. As soon as the imaginary parts become $\mathcal{O}(1)$ they drastically change the physics.

massless gravitons) [9, 10]. We will be referring to the exchanged objects, therefore, as "gravi-reggeons" (GR). That implies that the amplitude due a single GR exchange exhibits both a real and an imaginary part. The former has the Coulomb pole (as in the QFT limit of ordinary graviton exchange) and, correspondingly, has a large-b tail, while the latter is negligible at $b \gg \ell_s \sqrt{Y}$ but becomes relevant in the opposite regime $b \ll \ell_s \sqrt{Y}$ that includes the just defined string-gravity one. This imaginary part simply corresponds to the on-shell s-channel closed strings which are dual (in the old sense of DHS duality, after Dolen, Horn and Schmid) to the GR's. As in [27, 28] we will refer to such objects as "cut gravi-reggeons" (CGR). Mathematically, this implies that the eikonal operator $\hat{\delta}$ of eq. (2.6) ceases to be hermitian. In order to restore formally unitarity one needs to introduce [27, 28] new creation and destruction operators C and $C^\dagger$ for the on-shell states corresponding, in a broad sense, to a single CGR exchange.[5]

In [27, 28] it was pointed out that a formal way to recover a unitary S matrix when $\hat{\delta}$ is not hermitian consists of the replacement:

$$\exp\left(2i\hat{\delta}\right) \to \exp\left(i\left(\hat{\delta} + \hat{\delta}^\dagger\right)\right)\exp\left(i\sqrt{2i\left(\hat{\delta}^\dagger - \hat{\delta}\right)}\left(C + C^\dagger\right)\right),\qquad(2.8)$$

where the operators $\hat{\delta}$, $\hat{\delta}^\dagger$, C and $C^\dagger$ satisfy the commutation relations

$$\left[C, C^\dagger\right] = 1, \quad \left[\hat{\delta}, \hat{\delta}^\dagger\right] = \left[C, \hat{\delta}\right] = \left[C, \hat{\delta}^\dagger\right] = 0.\qquad(2.9)$$

Use of well-known harmonic-oscillator formulae leads to the more convenient form:

$$S = e^{2i\hat{\delta}} e^{i\sqrt{-2i\left(\hat{\delta} - \hat{\delta}^\dagger\right)}\, C^\dagger} e^{i\sqrt{-2i\left(\hat{\delta} - \hat{\delta}^\dagger\right)}\, C},\qquad(2.10)$$

showing that one recovers the correct elastic amplitude by taking (2.10) in the vacuum of the $C, C^\dagger$ oscillators.

Consider now the total cross section at some fixed impact parameter $b < l_s\sqrt{Y}$. By the optical theorem $\sigma_{\text{tot}}(s) = \kappa \text{Im} \mathcal{A}_{FS}(s)/s$, this will consist of a sum over all possible ways of "cutting" the ladder diagrams that build up the leading eikonal approximation.

Because of the above-mentioned nature of GRs, a ladder with n GRs, being non-planar, can be cut along any number n_c of GR with $0 \le n_c \le n$. The problem of determining the relative weights for cutting a different number n_c of GR is very similar to the one encountered in the sixties for an n-Pomerons exchange in hadronic physics and was nicely settled by the remarkably simple Abramovski-Gribov-Kancheli (AGK) cutting rules [39] (See ref. [40] for a useful review on these aspects in pQCD; a short reminder of which is given in appendix A). These rules also follow directly from (2.8) if one identifies n_c with the number operator $C^\dagger C$. We will not attempt to describe the operators C and $C^\dagger$ in details here but they may be related to the operators for higher spins originally defined by Weinberg [44–46].

---

[5]Obviously, in order to have full control of unitarity one should introduce separate, mutually commuting creation and destruction operators for each closed string contained, with a specific amplitude, in a single CGR. This remains, for the moment, an unfinished task.

The AGK rules state that the n_c-CGR contribution to the full imaginary part of the elastic (fixed b) n gravi-Reggeon exchange amplitude[6] by is given by:

$$\tilde{\sigma}^n_{n_c} = (-1)^{n-n_c} \frac{(4\mathrm{Im}\delta(s,b))^n}{n_c!(n-n_c)!} \quad \text{for} \quad 1 \le n_c \le n\,, \tag{2.11}$$

and

$$\tilde{\sigma}^n_0 = (-1)^n \frac{(4\mathrm{Im}\delta(s,b))^n}{n!} + 2\mathrm{Im}\tilde{\mathcal{A}}_n \quad \text{for} \quad n_c = 0\,, \tag{2.12}$$

where $\tilde{\mathcal{A}}_n$ is the contribution to $\tilde{\mathcal{A}}$ from the exchange of n GR. Accordingly, the sum over all contributions correctly reproduces the total imaginary part of the amplitude in agreement with the optical theorem. For partial-wave unitarity it is however more transparent to work with the full S matrix without extracting the no-transition term. In that case one should directly check that $S(s,b)S^\dagger(s,b) = 1$.

Let us check this constraint by considering the following more detailed formulation of the AGK rules: the n-GR-exchange contributions to $SS^\dagger$ can be split according to the number n_c of CGR, the number n_+ of GR in S and the number n_- of GR in $S^\dagger$ according to:

$$\left(SS^\dagger\right)_{(n)} = \sum_{n_+ + n_- + n_c = n} \frac{(2i\delta)^{n_+} \left(-2i\delta^\dagger\right)^{n_-} (4\mathrm{Im}\delta)^{n_c}}{n_c! n_+! n_-!} \ ; \ n \ge 1\,, \tag{2.13}$$

where $(SS^\dagger)_{(n)}$ corresponds to the $SS^\dagger$ element at fixed number of GR. If we now keep $n_c = N$ fixed and sum over n_+, n_- at fixed n we reproduce (2.11). Furthermore, if, for fixed N, we sum over all values of n_+, n_- we get:

$$\tilde{\sigma}_N = e^{-4\mathrm{Im}\delta} \frac{(4\mathrm{Im}\delta)^N}{N!} \Rightarrow \sum_N \tilde{\sigma}_N = 1 \ ; \ i.e. \ \ SS^\dagger = 1\,, \tag{2.14}$$

even for small N.

A simple way to understand (2.13) is as follows: the exchange of n identical bosons carries a $1/n!$ weight. Multiplying this by the number of ways we can choose, out of them, the three subsets n_+, n_-, n_c gives the combinatorial factor $(n_c! n_+! n_-!)^{-1}$. Finally, each set gives the appropriate S-matrix element or its imaginary part.[7]

It proves convenient to construct a generating function for the cross sections

$$\Sigma(z) = \sum_n^\infty \left[\tilde{\sigma}^n_0 + \sum_{m=1}^n z^m \tilde{\sigma}^n_m \right] = e^{4(z-1)\mathrm{Im}\delta}\,, \tag{2.15}$$

which is related to the S-matrix introduced in (2.8) through

$$\Sigma(z) = \langle in|S^\dagger z^{\hat{N}} S|in\rangle = \langle in|S^\dagger :e^{(z-1)\hat{N}}: S|in\rangle \ ; \ \hat{N} = C^\dagger C\,. \tag{2.16}$$

---

[6]Following [28], we denote $\tilde{\sigma} = d\sigma/d^2 b$ henceforth. The corresponding amplitude is denoted by $\tilde{\mathcal{A}}$.

[7]One may be worried about energy conservation in the AGK rule, since the total CM energy $\sqrt{s}$ should be shared among the n_c CGR, while $[\mathrm{Im}]\delta(s)$ is computed for the total s. A related issue is the (in)distinguishability of the n exchanged particles out of which only n_c are cut. Quite remarkably these two issues compensate in such a way that the AGK rules turn out to have such a simple form as in eq. (2.13).

Therefore:

$$\langle N \rangle = 4Im\delta = 4\,\frac{G_D s}{\ell_s^d Y^{d/2+1}}e^{-b^2/Y\ell_s^2}\,. \tag{2.17}$$

This result may be interpreted as the formation of a coherent state of gravi-reggeons with Poisson, rather than thermal black-body, distribution

$$\frac{d\sigma(2\to N)}{d^2b} = \frac{\langle N\rangle^N}{N!}e^{-\langle N\rangle}\,; \qquad b < \ell_s\sqrt{Y}\,. \tag{2.18}$$

An alternative proof of (2.17) is given in appendix A. One can also evaluate the average energy per cut gravi-reggeon and find

$$\langle E \rangle = \frac{\sqrt{s}}{\langle N\rangle} = \frac{M_s^2}{g_s^2\sqrt{s}}Y \sim \frac{\hbar}{R_S}Y\,. \tag{2.19}$$

Note that for $d = 0, D = 4$ the energy has an average value that coincides (modulo a $\log\alpha' s$ factor) with the Hawking temperature $T_H = \hbar/R_S$ of a would-be black hole whose temperature exceeds the Hagedorn temperature of string theory. Of course this is not the correct interpretation of the result.[8]

Rather, we can say that, as the expected threshold of BH production $R_S = \ell_s$ is approached from below, the individual CGR have still an invariant mass^2 parametrically larger (albeit just by a $\log\alpha' s$) than the string scale, justifying the use of Regge behavior.

On the other hand, as one crosses into the strong-gravity $(R_S > \ell_s, b)$ region, the expected energy of the individual CGR falls below the string scale. That means, physically, that each one of them should give rise to a bunch of massless strings. Taking seriously the logarithms, one might imagine that each CGR gives rise to $\mathcal{O}(Y)$ quanta of energy $\hbar/R_S$ for a total multiplicity of order $Gs/\hbar$ i.e. of the entropy of a "String-Hole", a BH lying just at the corresponding curve between strings and black holes [31–35].

On the other hand, this is precisely the point at which a description of the final state in terms of massless particles alone should become reliable. And this is also the lower end of the regime discussed by [36]. For consistency we would like the calculation described in this subsection and the one of [36], described in the next section, to smoothly join one another along the correspondence line.

3 The classicalization approach to high-energy, high-multiplicity gravitational scattering

Recently Dvali, Gomez and collaborators [37, 38, 47–50] proposed a quantum mechanical description of black holes as Bose-Einstein condensates of a large number of gravitons $(N \approx M_{BH}^2/M_{Pl}^2)$ that, at a critical value $\alpha_G^{\rm crit}/N \approx 1$ of the effective gravitational coupling $\alpha_G \approx Gs$, behave as Bogoliubov modes and form a BH bound state. The mechanism termed 'classicalization' provides a quantum N-picture of BH's[9] that has been tested in connection

---

[8]Eqs. (2.18), (2.19) may also be derived in the framework of coherent state formalism.

[9]More generically, 'classicalization' represents a mechanism that provides the unitarization of a UV incomplete theory by means of the resonant production of a non-perturbative classical solution. An example in the context of non-local quantum field theory was studied in ref. [51].

with ultra-planckian scattering and formation of black holes as self-critical Bose-Einstein condensate of soft gravitons in [36, 52].

In particular, in [36], relying on KLT relations (after Kawai, Lewellen, and Tye [53]) and the 'scattering equations' [54] in the Regge limit, DGILS tried to demonstrate that the perturbative exponential suppression factor e^{-N} with the number N of produced particles (gravitons)[10] is exactly compensated by the BH entropy in a self-critical phase. Let us summarise their derivation and later on comment on the issues raised by their analysis.

The starting point for computing tree-level graviton amplitudes with large multiplicity of the final states are the 'scattering equations' [54] and the KLT relations [53, 55–57], that relate closed string amplitudes on the sphere to 'squares' of open string amplitudes on the disk. In the 'field-theory' limit KLT can be used to relate non-planar graviton amplitudes to color-ordered gluon amplitudes. The relevant formula reads [56, 57]

$$
\begin{aligned}
\mathcal{M}_{\mathrm{grav}}(1,\ldots,N) = (-1)^{N-3}\kappa^{N-2} \\
\times \sum_{\gamma,\sigma\in S_{N-3}} \mathcal{A}^L_{YM}(1,\sigma,N-1,N)\mathcal{S}_{KLT}[\gamma|\sigma]\mathcal{A}^R_{YM}(1,\gamma,N-1,N) \,, \quad (3.1)
\end{aligned}
$$

where $\kappa^2 = 8\pi G = \ell_P^2$, S_{N-3} is the group of permutations of $N-3$ objects, and the KLT kernel is given by

$$
\mathcal{S}_{KLT}[i_1,\ldots,i_k|j_1,\ldots,j_k] = \prod_{t=1}^{k}\left(s_{i_t,P} + \sum_{q>t}^{k}\theta(i_t,i_q)s_{i_t,i_q}\right) \,, \quad (3.2)
$$

with $s_{ij} = 2k_ik_j$, P an arbitrary reference light-like momentum, while $\theta(i_a,i_b) = 0$ for (i_a,i_b) in the same order in $\{i_1,\ldots,i_k\}$ as in $\{j_1,\ldots,j_k\}$ and $\theta(i_a,i_b) = 1$ otherwise.

In $D = 4$ it is convenient to switch to the helicity spinor formalism, whereby light-like momenta are expressed as $k_\mu=\bar{u}(k)\sigma_\mu u(k)$ in terms of commuting Weyl spinors of opposite chirality $u_\alpha(k)$ and $\bar{u}_{\dot\alpha}(k)$. The latter are often denoted by $|k\rangle$ and $|k]$, and satisfy $u^\alpha(k)u_\alpha(k') = \epsilon_{\alpha\beta}u^\alpha(k)u^\beta(k') = \langle k,k'\rangle = -\langle k',k\rangle$ as well as $\bar{u}_{\dot\alpha}(k)\bar{u}^{\dot\alpha}(k') = \epsilon^{\dot\alpha\dot\beta}u_{\dot\alpha}(k)u_{\dot\beta}(k') = [k,k'] = -[k',k]$. As a result Mandelstam invariants can be written as $s_{ij} = \langle i,j\rangle[j,i]$, with $\langle i,j\rangle = \sqrt{2|k_ik_j|}\exp(i\phi_{ij})$ and $[j,i] = \sqrt{2|k_ik_j|}\exp(-i\phi_{ij})$, for real momenta.

For a Maximally Helicity Violating (MHV) configuration of the graviton polarizations $h_i^{\pm 2} = a_{i,L}^{\pm 1}\otimes a_{i,R}^{\pm 1}$, the relevant color-ordered YM tree-level amplitudes, coded in Parke-Taylor formula [58], read

$$
\mathcal{A}_{YM}\left(1^+,\ldots,i^-,\ldots,j^-,\ldots,N^+\right) = \frac{\langle i,j\rangle^4}{\langle 1,2\rangle\langle 2,3\rangle\ldots\langle N-1,N\rangle\langle N,1\rangle} \,, \quad (3.3)
$$

where $\pm$ denote the helicity of the gluons, all considered as incoming. Furthermore, assuming a very peculiar kinematical regime for the scattering process $1,2\to 3,\ldots,N$, *viz.* $s_{12} = s$, $t_i = s_{i(1,2)} = -s/N$, $s_{ij} = s/N^2$ with $i,j = 3,\ldots,N$, and relying on the 'scattering

---

[10]To be precise, in [36] the number of produced gravitons is $N-2$. In order to adhere to the original DGILS paper, in this section, we will follow this convention. Clearly $N-2\approx N$ for $N\gg 1$.

equations' [54] DGILS obtain $\mathcal{S}_{KLT} \sim \left(s/N^2\right)^{N-1}$ and

$$\mathcal{A}_{YM} \sim s^{(2-N)/2} f(\phi) N^N, \qquad\qquad \text{for } i^-, j^- = 1, 2 \qquad\qquad (3.4)$$
$$\mathcal{A}_{YM} \sim s^{(2-N)/2} f(\phi) N^{N-2}, \qquad\qquad \text{for } i^- = 3, \ldots, N, \ j^- = 1, 2$$
$$\mathcal{A}_{YM} \sim s^{(2-N)/2} f(\phi) N^{N-4}, \qquad\qquad \text{for } i^-, j^- = 3, \ldots, N,$$

where $f(\phi)$ is a (complicated) function of the phases $\phi = \{\phi_{ij}\}$ of the spinor bilinears. Substituting (3.4) into the KLT formula (3.1), for MHV amplitudes DGILS get

$$\mathcal{M}_{\text{grav}}\left(1^{+2}, \ldots, i^{-2}, \ldots, j^{-2}, \ldots, N^{+2}\right) \sim \kappa^N C_N s, \qquad\qquad (3.5)$$

where ± 2 denote the helicity of the gravitons, all considered as incoming. Depending on whether the two negative helicity gravitons be in the initial $(1,2)$ or final $(3,\ldots,N)$ state one has $C_N = (N+1)! N^2$ for $i^{-2}, j^{-2} = 1, 2$, $C_N = (N+1)! N^{-2}$ for $i^{-2} = 1, 2$ and $j^{-2} = 3, \ldots, N$ or *vice versa* and $C_N = (N+1)! N^{-6}$ for $i^{-2}, j^{-2} = 3, \ldots, N$. Similar results are found by DGILS for non-MHV configurations in the chosen kinematical regime, that in a sense should dominate the integral over the final phase-space. Assuming that the sum over polarisations produce a factor c_H^N with $c_H \sim \mathcal{O}(1)$, DGILS estimate the cross section for a fixed but large number N of gravitons to be of the form

$$\sigma(2 \to N-2) \sim N! \left(\frac{\ell_P^2 s}{N^2}\right)^N. \qquad\qquad (3.6)$$

Eventually, assuming self-criticality i.e. $\alpha_G = \ell_P^2 s = N = \alpha_G^{\text{crit}} \gg 1$,

$$\sigma(2 \to N-2) \sim \frac{N!}{N^N} \sim e^{-N}. \qquad\qquad (3.7)$$

This result is quite surprising for various reasons that we would like to analyze in some detail in the following. Later on we will compare the classicalization approach with the eikonal approach and propose a way to reconcile the two.

- In [36] only tree level amplitudes in a specific kinematical regime of the final gravitons (suggested by classicalization) is considered. Neither explicit integration over the final phase space nor explicit sum over all possible helicity configurations are performed for obvious technical difficulties.

- In [36] higher orders in perturbation theory, corresponding to virtual graviton exchange or 'soft' graviton radiation, are not considered and stability of the results with respect to quantum corrections is tacitly assumed. In particular, it is not clear whether an exclusive cross-section is computed, that would be infinite due to IR divergences, or some sort of inclusive cross-section.

- In [36], the criticality condition $\alpha_G = Gs = cN$ with a precise, fine-tuned proportionality constant c is assumed and eventually used in the cross section. The exponential factor e^{-N} results from such a precise fine tuning. This factor is crucial for their classicalization argument: indeed, they argue, if a resonant production of (micro)

Black Holes mediates this channel, the e^{-N} factor can be compensated by the BH entropy factor $e^{+S_{BH}} \sim e^N$. Note that, by a different choice of c, one can turn the exponential suppression into an exponential growth, reproducing the behavior of the (convergent) sum over N. In that case adding an entropy factor would overshoot unitarity.

4 A tension between the two approaches

Clearly, there appears to be some tension between the results summarized in sections 2 and 3, while one would like the two to join smoothly at the expected threshold for BH formation.

The most obvious difference is that in the ACV/AGK approach virtual corrections (corresponding to uncut GR) are essential to restore unitarity at fixed b, while no extra phase space or number of states factor is needed. By contrast, in the DGILS approach no virtual corrections are included and the S-matrix (integrated over impact parameter) satisfies unitarity bounds thanks to an extra exponential entropy factor. In the following we shall investigate the nature of soft radiative corrections to the process discussed in DGILS, while in section 5 we will offer a new interpretation of the DGILS result that appears to resolve the above mentioned tension.

To this end, we will recall Weinberg's theorem for soft gravitons [41], apply it to the tree-level amplitude computed by DGILS in the classicalization regime, and see how gravistrahlung and virtual gravitons may affect the behavior of the cross-section in different kinematical regions.

4.1 Virtual soft gravitons

Let us start by considering the effect of adding a virtual soft graviton to graviton amplitudes. Although the final state of DGILS consists of gravitons of typical energy $\sqrt{s}/N \approx M_P{}^2/\sqrt{s}$, we can always add a virtual graviton of even lower energy. In that case Weinberg has shown [41] that the single soft virtual correction amounts to multiplying the original amplitude by a factor:

$$A_{1-IR-\text{grav}} = \frac{1}{2} B \, A_0 \quad ; \qquad B = \int_\lambda^\Lambda d^4q B(q) \, ;$$

where for the sake of simplicity we have not showed the dependence of B and $B(q)$ on the external momenta p_i. Weinberg's result reads (see figure 2)

$$B(q) = \frac{-8\pi i G}{(2\pi)^4 [q^2 - i\epsilon]} \sum_{i,j} \frac{\eta_i \eta_j \left\{ (p_i \cdot p_j)^2 - \frac{1}{2} m_i^2 m_j^2 \right\}}{[p_i \cdot q - i\eta_i \epsilon][-p_j \cdot q - i\eta_j \epsilon]} \, ; \tag{4.1}$$

where λ represents an IR cutoff, Λ is an upper cutoff to be discussed later,[11] while $\eta = +1$ for outgoing particles and $\eta = -1$ for incoming particles.

---

[11]Not to be confused with some UV cutoff of Quantum Gravity!

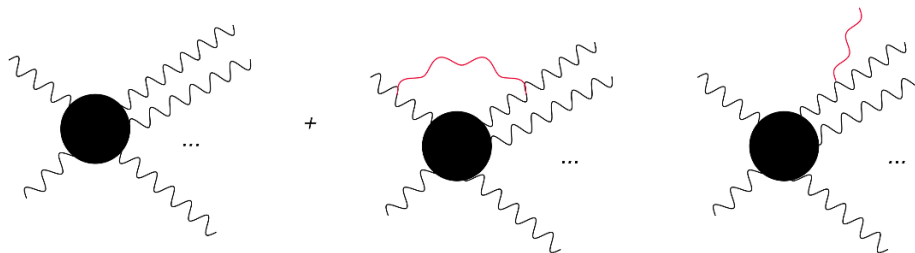**Figure 2**. $2 \to N$ graviton amplitudes with virtual soft gravitons corrections and emission of real soft gravitons from the external legs.

As shown in [41] this result can be generalized to the case of an arbitrary number of soft gravitons and the sum of all such contributions exponentiates so that:

$$S_{2 \to M} = \mathcal{C}(\lambda, \Lambda) S^0_{2 \to M} , \tag{4.2}$$

where (suppressing the p_i labels in $B(p_i, q)$)

$$\mathcal{C}(\lambda, \Lambda) = \sum_L \frac{1}{L!} \left[\frac{1}{2} \int_\lambda^\Lambda d^4q B(q) \right]^L \to \exp \left\{ \frac{1}{2} \int_\lambda^\Lambda d^4q B(q) \right\} . \tag{4.3}$$

The corresponding correction to the rate reads

$$|S_{2 \to N}|^2 = |S^0_{2 \to N}|^2 \exp \left\{ \mathrm{Re} \int_\lambda^\Lambda d^4q B(q) \right\} , \tag{4.4}$$

and depends only on the real part of the integral over the 4-momentum q of the virtual graviton, which only receives contribution from the imaginary part of the graviton propagator $i\pi\delta(q^2)$. One finally obtains:[12]

$$\mathrm{Re} \int_\lambda^\Lambda d^4q B(q) = -B_0 \log(\Lambda/\lambda) \qquad \text{with}$$

$$B_0 = \int \frac{G \, d^2\Omega}{2\pi^2} \sum_{i,j} \frac{\eta_i \eta_j \left\{ (p_i \cdot p_j)^2 - \frac{1}{2} m_i^2 m_j^2 \right\}}{[E_i - \mathbf{p}_i \cdot \mathbf{n}][E_j - \mathbf{p}_j \cdot \mathbf{n}]} = \frac{G}{2\pi} \sum_{i,j} \frac{\eta_i \eta_j m_i m_j (1 + \beta_{ij}^2)}{\beta_{ij}(1 - \beta_{ij}^2)^{1/2}} \log \frac{1 + \beta_{ij}}{1 - \beta_{ij}} \tag{4.5}$$

where

$$\beta_{ij} \equiv \left(1 - \frac{m_i^2 m_j^2}{(p_i \cdot p_j)^2} \right)^{1/2} .$$

It is straightforward to take the massless limit of (4.5) and to check that it is smooth and harmless (this is the well-known absence of collinear graviton divergences implied by the graviton's helicity ± 2). The result (that we have found nowhere in the literature) is particularly simple:

$$B_0 = \frac{2G}{\pi} \left\{ s \log \frac{s}{\mu^2} + \sum_{i=1}^N \left[t_{i1} \log \frac{-t_{i1}}{\mu^2} + t_{i2} \log \frac{-t_{i2}}{\mu^2} \right] + \sum_{i<j}^{1,N} s_{ij} \log \frac{s_{ij}}{\mu^2} \right\} , \tag{4.6}$$

---

[12]For completeness we give, in Appenxix B, a simple derivation of this result.

where $s = s_{12} = 2p_1 \cdot p_2$, $t_{i,1/2} = -2p_i \cdot p_{1/2}$, $s_{i,j} = 2p_i \cdot p_j$ and μ^2 is an arbitrary mass scale that drops out since

$$s_{12} = -\sum_i t_{i1} = -\sum_j t_{2j} = \sum_{i<j} s_{ij}$$

thanks to momentum conservation. In the case of the 4-point amplitude, choosing $\mu^2 = s = s_{12}$ for convenience, one simply finds

$$B_0 = \frac{4G}{\pi}\left(t\log\frac{-t}{s} + u\log\frac{-u}{s}\right) = -\frac{4Gs}{\pi}\left(\sin^2\frac{\theta}{2}\log\sin^2\frac{\theta}{2} + \cos^2\frac{\theta}{2}\log\cos^2\frac{\theta}{2}\right) \geq 0, \quad (4.7)$$

with a maximum $B_0 = +4Gs\log 2/\pi$ for $\theta = \pi/2$ and minima $B_0 = 0$ for $\theta = 0, \pi$. The situation is less clear for arbitrary but fixed N. In the CM frame $p_1 = E(1, \mathbf{n})$, $p_2 = E(1, -\mathbf{n})$ and $p_i = E_i(1, \mathbf{n}_i)$ with $\mathbf{n}$ and $\mathbf{n}_i$ unit vectors. Momentum conservation yields $\sum_i E_i = 2E$ and $\sum_i E_i \mathbf{n}_i = 0$, setting $w_i = E_i/2E$ one has

$$0 \leq w_i \leq 1/2 \quad, \quad \sum_i w_i = 1 \quad, \quad \sum_i w_i \mathbf{n}_i = 0,$$

and, after some algebra, one finds

$$\begin{aligned}
B_0 = \frac{4Gs}{\pi}&\left\{-\sum_i w_i\left[\sin^2\left(\frac{\theta_i}{2}\right)\log\sin^2\left(\frac{\theta_i}{2}\right) + \cos^2\left(\frac{\theta_i}{2}\right)\log\cos^2\left(\frac{\theta_i}{2}\right)\right]\right.\\
&\left.+ \sum_{i,j} w_i w_j \sin^2\left(\frac{\theta_{i,j}}{2}\right)\log\sin^2\left(\frac{\theta_{i,j}}{2}\right)\right\} = B_0^+ + B_0^-, \quad (4.8)
\end{aligned}$$

where the single sum B_0^+ is positive and smaller than $\frac{4Gs}{\pi}\log 2$ while the second term (double sum) B_0^- is negative and larger than $-\frac{4Gs}{\pi}\frac{1}{e}$. So *a priori* one may expect $\log 2 \geq \pi B_0/4Gs \geq -1/e$. While the upper bound can be reached, the lower bound cannot, due to kinematical constraints. Later on we will show that B_0 is always non negative by relating it to the integral of the square modulus of the leading soft factor.

We have systematically studied the value of B_0 in (4.6) as a function of the kinematic configuration of the N-particle final state with the following conclusions (see also appendix B):

- B_0 is zero only in very special configurations. These correspond to the forward elastic amplitude (as always) and to final states in which the above two final gravitons are replaced by an arbitrary number of strictly collinear ones. As soon as one moves away from this configuration B_0 becomes positive.

- When one goes a large amount away from the above special kinematical regions B_0 is typically of order Gs times a function of $\mathcal{O}(1)$ of the angles which does not grow with the number of final particles.
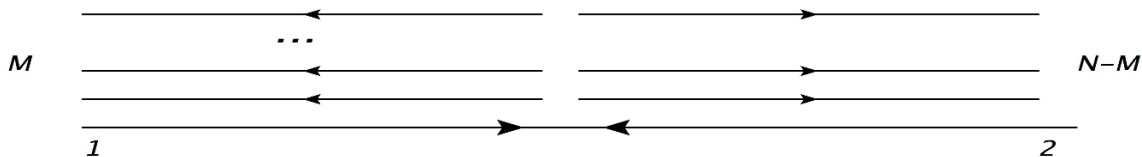
- Some examples:

Figure 3. $2 \to N$ scattering in which all the final gravitons are collinear to the initial ones. These are the only configurations with $B_0 = 0$.
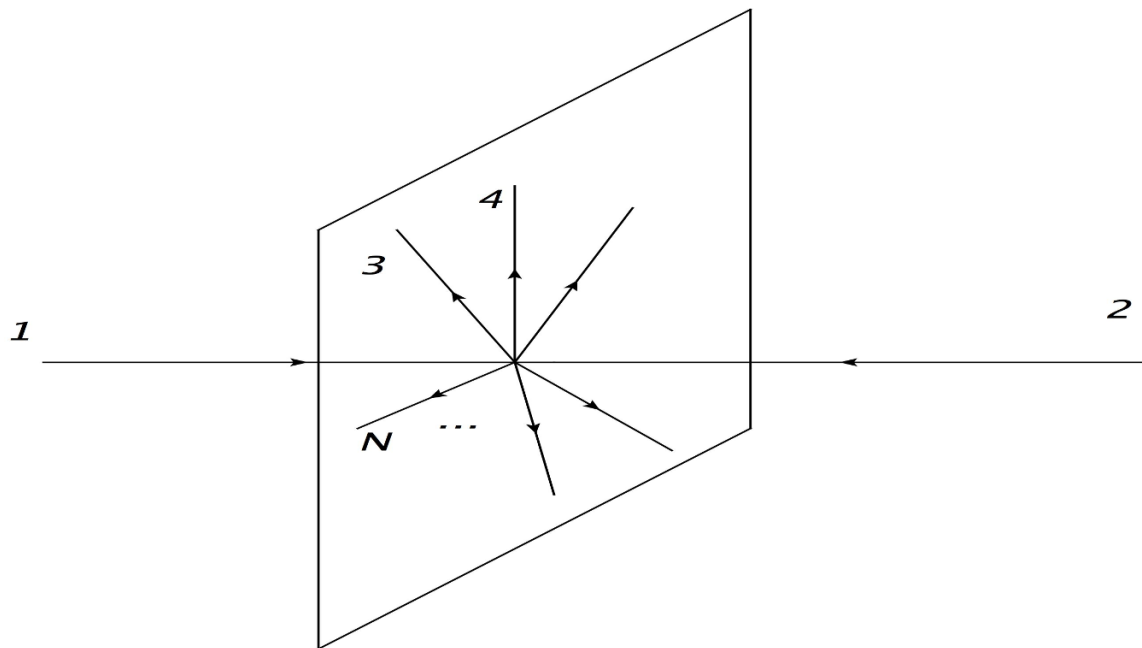

Figure 4. $2 \to N$ scattering in which all the final gravitons are emitted in a plane orthogonal to the initial momenta. Such configurations maximize B_0.

Collinear [figure 3]. For $\mathbf{n}_i = \mathbf{n}\sigma_i = 0$ with $\sigma_i = \pm 1$ such that $\sum_i w_i\sigma_i = 0$, one has

$$
B_0 = \frac{2Gs}{\pi} \left\{ \sum_{i,j} w_i w_j \frac{1-\sigma_i\sigma_j}{2} \log \frac{1-\sigma_i\sigma_j}{2} \right.
$$
$$
\left. - \sum_i w_i \left[\frac{1-\sigma_i}{2} \log \frac{1-\sigma_i}{2} + \frac{1+\sigma_i}{2} \log \frac{1+\sigma_i}{2} \right] \right\}, \qquad (4.9)
$$

which vanishes since $\frac{1-\sigma}{2} \log \frac{1-\sigma}{2} = 0$ for $\sigma = \pm 1$.

Orthogonal [figure 4]. For $\mathbf{n}_i\mathbf{n} = 0$ ($\mathbf{n}_i \perp \mathbf{n}$), one has

$$
B_0 = \frac{2Gs}{\pi} \left\{ \log 2 + \sum_{i,j} w_i w_j \frac{1-\mathbf{n}_i\mathbf{n}_j}{2} \log \frac{1-\mathbf{n}_i\mathbf{n}_j}{2} \right\} \geq \frac{2Gs}{\pi} \left[\log 2 - \frac{1}{e} \right], \quad (4.10)
$$

since $x \log x \geq -1/e$ for $0 \leq x \leq 1$, with $x = (1 - \mathbf{n}_i\mathbf{n}_j)/2 = \sin^2(\theta_{ij}/2)$.

We thus conclude that, as a consequence of soft virtual graviton corrections, the $2 \to N$ cross section goes to zero (except in a zero-measure phase space region):

$$|S_\lambda|^2 \to \left(\frac{\lambda}{\Lambda}\right)^{B_0} |S_0|^2 \sim \left(\frac{\lambda\sqrt{s}}{M_P^2}\right)^{B_0} |S_0|^2 . \tag{4.11}$$

We are of course very familiar with such a phenomenon that is counterbalanced by emission of soft radiation, i.e. brems-strahlung.

Combining the contributions from virtual soft gravitons and real ones, to be discussed momentarily, one eventually gets

$$|S_\lambda|^2 \to \left(\frac{\Delta}{\Lambda}\right)^{B_0} |S_0|^2 , \tag{4.12}$$

where Δ is the maximal energy allowed in the soft radiation (see section 5 for further details). Eq. (4.12) is nothing but the well-known cancellation between real and virtual IR divergences to the (inclusive) cross section.

4.2 Real soft gravitons

Let us now turn our attention on IR divergences arising from emission of real soft gravitons. The real soft emission (gravi-strahlung) typically contributes a factor $(\Delta E/\lambda)^{B_0}$, with the same B_0 that governs virtual IR divergences. Once more, in order to trust our treatment of soft graviton emission we have to take $E \leq M_P^2/\sqrt{s} \sim \Lambda$.

The emission of soft gravitons at tree level is governed by the universal behaviour [59–61]

$$\mathcal{M}_{N+1}(p_i; q) \approx \sum_{i=1}^{N} \left[\frac{p_i h p_i}{\eta_i q p_i} + \frac{p_i h J_i q}{\eta_i q p_i} + \frac{q J_i h J_i q}{\eta_i q p_i}\right] \mathcal{M}_N(p_i) , \tag{4.13}$$

where q^μ and $h_{\mu\nu}$ denote the momentum and the polarisation of the soft graviton, while $J_i^{\mu\nu} = p_i^\mu \partial/\partial p_\nu^i - p_i^\nu \partial/\partial p_\mu^i + S_i^{\mu\nu}$ denote the angular momentum operator acting on the momentum and polarisation of the 'hard' particles. Compared to YM and QED not only the dominant and sub-dominant terms but also the sub-sub-dominant term is universal [59–61]. This holds true whenever gravitons couple as minimally as possible i.e. in the absence of ϕR^2 interactions involving 'dilatons', while R^3 or R^4 would not spoil universality [60–67].

In this paper we limit our attention (both for real and for virtual soft gravitons) to the the dominant term in (4.13) which is universal even beyond tree-level, i.e. at any order in perturbation theory in any consistent quantum theory of gravity such as String Theory, and compute the effect of adding a soft graviton to a process

$$|\mathcal{M}_{N+1}(p_i; q)|^2 = \int \frac{d^3q}{2|q|(2\pi)^3} \sum_{s=\pm 2} \left|\sum_{i=1}^{N} \frac{p_i h_s p_i}{\eta_i q p_i}\right|^2 |\mathcal{M}_N(p_i)|^2 . \tag{4.14}$$

The sum over polarisations/helicities $s = \pm 2$, with $h_{\mu\nu}^{-s} = (h_{\mu\nu}^s)^*$, produces a transverse traceless bi-symmetric tensor

$$\sum_{s=\pm 2} h_{\mu\nu}^s h_{\rho\sigma}^{-s} = \Pi_{\mu\nu,\rho\sigma} = \frac{1}{2} \left(\Pi_{\mu\rho}\Pi_{\nu\sigma} + \Pi_{\mu\sigma}\Pi_{\nu\rho} - \Pi_{\mu\nu}\Pi_{\rho\sigma}\right) , \tag{4.15}$$

where $\Pi_{\mu\nu} = \eta_{\mu\nu} - q_\mu \bar{q}_\nu - q_\nu \bar{q}_\mu$ with $\bar{q}^2 = 0$ and $\bar{q}q = 1$. Luckily most of the terms are irrelevant thanks to momentum conservation $\sum_i \eta_i p_i = 0$ and to the mass-shell condition $p_i^2 = 0$. In fact *lo and behold*

$$\sum_{i,j} \frac{p_i^\mu p_i^\nu}{\eta_i q p_i} \Pi_{\mu\nu,\rho\sigma} \frac{p_j^\rho p_j^\sigma}{\eta_j q p_j} = \sum_{i,j} \frac{\eta_i \eta_j (p_i p_j)^2}{q p_i q p_j} \,. \qquad (4.16)$$

Integration over the soft light-like momentum $q = (|q|, \mathbf{q}) = |q|(1, \mathbf{n})$ produces

$$B_0 \log \frac{\Lambda}{\lambda} = 8\pi G \int \frac{d^3 q}{2|q|(2\pi)^3} \sum_{i,j} \frac{\eta_i \eta_j (p_i p_j)^2}{q p_i q p_j} = \frac{2G}{\pi} \log \frac{\Lambda}{\lambda} \sum_{i,j} \eta_i \eta_j (p_i p_j) \log \frac{p_i p_j}{\mu^2} \geq 0 \,,$$
$$(4.17)$$

where the $\log \mu$, which is harmless thanks to momentum conservation, arises from a logarithmic divergence over the Feynman parameter α that can be regulated by giving a small (common) mass to the 'hard' particles as shown in previous sections.

It would be very interesting to study the sub- and sub-sub-leading corrections in the soft limit to the above result that are expected to be universal in gravity, at least at tree level.

5 A reinterpretation of the DGILS result and resolution of its tension with ACV/AGK

The tree-level exclusive cross section $\sigma^{\text{tree}}(2 \to N)$ is, strictly speaking, infrared divergent (by taking, for instance, two of the final particles to be hard and the remaining $N - 2$ to be arbitrarily soft, see section 4). It is thus clear that the result of DGILS has to be reinterpreted.

We shall follow, *mutatis mutandis*, a line suggested by the classic treatment [68] of jet cross sections in QCD. In that case one has to define observables which are insensitive to both infrared and collinear singularities. In the case of gravity the latter divergences are absent (that's why we could safely take the massless limit in section 4) and therefore we shall pay attention to energies rather that to angular distributions. In the following integration over the angles has to be understood. An IR-safe quantity (carrying some analogy with the N-jet cross section in $e^+ e^- \to$ hadrons) is

$$\sigma\left(2 \to N(E_i \geq \bar{E}) + \text{soft}(\text{E}_{\text{soft}} \leq \Delta)\right); \qquad \bar{E} N < \sqrt{s} \,, \qquad (5.1)$$

in which the final state contains N gravitons of c.m. energy greater than $\bar{E}$ and any number of soft gravitons of individual energy less than $\bar{E}$ and total energy less than Δ (called ΔE in section 4), see figure 5. It is convenient, as in section 2, to introduce a generating function(al) for such cross sections:

$$\Sigma\left(z(\omega), \bar{E}, \Delta\right) = \qquad (5.2)$$
$$\sum_{N,M} \int_{\bar{E}} d\omega_1 \ldots d\omega_N z(\omega_1) \ldots z(\omega_N) \int_\lambda^{\bar{E}} d\epsilon_1 \ldots d\epsilon_M \theta(\Delta - \sum_j \epsilon_j) \frac{d^{N+M}\sigma}{d\omega_1 \ldots d\omega_N d\epsilon_1 \ldots d\epsilon_M} \,,$$

where we have denoted by ω_i, $i = 1, \ldots N$, the hard gravitons' energies and by ϵ_j, $j = 1, \ldots M$, those of the soft ones. The differential cross section carries a δ-function for energy
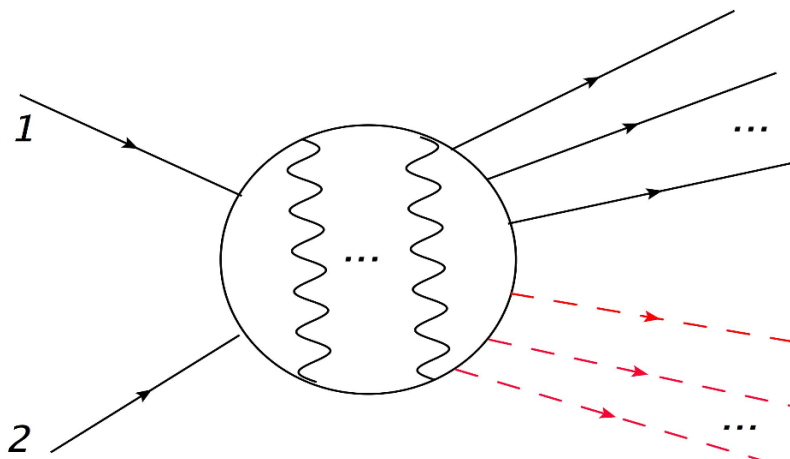
Figure 5. Representation of the $2 \to N-$jet process. The N hard $(E \geq \bar{E})$ gravitons are represented by solid outgoing lines, the IR gravitons $(E < \bar{E})$ by red dotted lines and the virtual gravitons by wiggly lines.

conservation $\delta(\sqrt{s} - \sum_i \omega_i - \sum_j \epsilon_j)$. Another δ-function comes out if we consider the derivative of Σ w.r.t. Δ.

We shall be interested in studying Σ together with some (functional) derivatives of it near $z(\omega) = 1$. This will provide information on the "total" cross-section as a function of $\bar{E}$ and Δ and on inclusive (one, two or more) hard graviton spectra. We shall carry out the first steps of this analysis below after introducing a simple ansatz for the differential cross section in (5.2). It will consist of three factors: the tree-level exclusive $2 \to N$ cross-section considered in [36], an additional factor accounting for the soft-graviton emission and, finally, a factor incorporating the effect of virtual gravitons.

Concerning the tree-level differential cross section we shall reinterpret the result of [36] as giving the pre-factor that multiplies $d\log\omega_1 \ldots d\log\omega_N$, i.e. we shall write:

$$\frac{d\sigma^{tree}}{d\omega_1 \ldots d\omega_N} \sim N! \left(c\, e^2\, \frac{Gs}{N^2} \right)^N \frac{1}{\omega_1 \ldots \omega_N} \sim \frac{1}{N!}(c\,Gs)^N \frac{1}{\omega_1 \ldots \omega_N}\,, \tag{5.3}$$

where c is some $\mathcal{O}(1)$ constant.

For the remaining two factors we simply use the results of section 4 in order to write them in terms of B_0, the quantity introduced in (4.6). In the previous section we have seen that B_0 is of order Gs unless the final particles are almost collinear with the initial ones in which case B_0 can be much smaller. In the following we will approximate B_0 with some kind of average value $\tilde{c}\,Gs$ where $\tilde{c}$ is another $\mathcal{O}(1)$ constant. This physically means that we are effectively excluding the large-b (small deflection angle) regime concentrating on the interesting one of small-b.[13] Using this approximation and the standard representation of

---

[13]Ideally one should instead project the results of [36] and of section 4 on partial waves (or fixed b) amplitudes, something non trivial and that we are deferring to further work.

the δ-functions we arrive at the following compact expression for $\partial \Sigma / \partial \Delta$:

$$\frac{\partial \Sigma}{\partial \Delta}\left(z(\omega), \bar{E}, \Delta\right) = \frac{1}{4\pi^2} \int_{-\infty}^{+\infty} d\sigma \int_{-\infty}^{+\infty} d\tau \exp\left(-i\sigma\sqrt{s} - i\tau \, \Delta\right)$$

$$\times \exp\left(cGs \int_{\bar{E}}^{\sqrt{s}} \frac{d\omega}{\omega} z(\omega) e^{i\omega\sigma} + \tilde{c}Gs \int_{\lambda}^{\bar{E}} \frac{d\epsilon}{\epsilon} e^{i\epsilon(\sigma+\tau)} - \tilde{c}Gs \int_{\lambda}^{\Lambda} \frac{d\epsilon}{\epsilon}\right), \qquad (5.4)$$

where the first term in the second exponent is from the differential cross section (5.3), the second and the third from real (contributing to energy conservation) and virtual soft gravitons, both exposing the 'averaged' Weinberg factor $B_0 = \tilde{c}Gs$. Here we have also used some large-N approximations that should not matter as far as we look at the neighborhood of $z(\omega) = 1$. Finally, we have (re)introduced the cutoff parameter Λ as an upper limit on the virtual gravitons' energy. We shall discuss its possible values below.

We will estimate (5.4) around $z = 1$ through a saddle point approximation (in σ and τ) which should be reliable in our regime $Gs \gg 1$. However, before doing that, let us note that the single (hard) graviton distribution can be formally obtained from (5.4) through an appropriate functional derivative:

$$\frac{1}{\sigma}\frac{d\sigma}{d\omega} = \left.\frac{\delta \log(\Sigma)}{\delta z(\omega)}\right|_{z=1} = \left\langle cGs \frac{e^{i\sigma\omega}}{\omega}\right\rangle, \qquad (5.5)$$

where the angled brackets mean the expectation value w.r.t. the integrals over σ and τ appearing in (5.4). In the saddle point approximation this amounts to inserting the saddle-point value of σ in (5.5).

We now look for complex saddle points for the σ and τ integrals of the form:

$$\sigma_s = ix/\sqrt{s} \quad , \quad \tau_s = iy/\Delta. \qquad (5.6)$$

Imposing stationarity of the (large) phase w.r.t. σ and τ we find:

$$x = c\,Gs(e^{-\frac{\bar{E}}{\sqrt{s}}x} - e^{-x}) + \tilde{c}\,Gs\frac{x}{(x+y\sqrt{s}/\Delta)}\left(1 - e^{-\frac{\bar{E}}{\sqrt{s}}\left(x+y\sqrt{s}/\Delta\right)}\right)$$

$$1 = \tilde{c}\,Gs\frac{\sqrt{s}/\Delta}{(x+y\sqrt{s}/\Delta\,)}\left(1 - e^{-\frac{\bar{E}}{\sqrt{s}}\left(x+y\sqrt{s}/\Delta\right)}\right), \qquad (5.7)$$

implying a condition involving just x

$$x = c\,Gs\left(e^{-\frac{\bar{E}}{\sqrt{s}}x} - e^{-x}\right) + \frac{\Delta}{\sqrt{s}}x \quad \Rightarrow xe^{\frac{\bar{E}}{\sqrt{s}}x} = c\,Gs, \qquad (5.8)$$

where in the last equation we have used $\frac{\bar{E}}{\sqrt{s}}, \frac{\Delta}{\sqrt{s}} \ll 1$. Eq. (5.8) is solved by:

$$x = \frac{\sqrt{s}}{\bar{E}}W_0(\bar{E}/T_H), \qquad (5.9)$$

where $T_H = \hbar/R_S$ is Hawking temperature and $W_0(z) = \sum_{n=1}^{\infty}(-n)^{n-1}z^n/n!$ is the (first branch of the) Lambert (or product-log) function. Inserting this result in the second

of (5.7) we find that, quite generically, $y \sim Gs$. More precisely, under the assumption that $y \gg \frac{\Delta}{\sqrt{s}}x$, (5.7) reduces to:

$$\hat{y} \equiv \frac{y}{Gs} = 1 - e^{-\gamma \hat{y}} \quad ; \quad \gamma \equiv \frac{\bar{E}}{T_H}\frac{\sqrt{s}}{\Delta} \, , \tag{5.10}$$

whose solution is again given in terms of the Lambert function:

$$\hat{y} = 1 + \frac{W_0(-\gamma e^{-\gamma})}{\gamma} \, , \tag{5.11}$$

showing that $\hat{y} = \mathcal{O}(1)$ provided $(\gamma - 1) = \mathcal{O}(1)$. Under this condition, $y \sim Gs$ and the inequality $y \gg \frac{\Delta}{\sqrt{s}}x$ can be easily checked for any value of $\bar{E}/T_H$. The above assumption on γ looks very reasonable: actually, if $\gamma < 1$, eq. (5.10) has no positive-y solution.

We also need to set a bound on Λ, the upper cutoff on virtual-graviton momenta. It seems obvious that such an upper bound should not be lower than the cutoff $\bar{\omega}$ on real (hard) gravitons. This can be easily estimated through eq. (5.5):

$$\bar{\omega} = \frac{\bar{E}}{W_0(\bar{E}/T_H)} \, . \tag{5.12}$$

Finally, let us write the result for $\Sigma(z=1)$ in a convenient form applicable to any value of $\bar{E}/T_H$):

$$\Phi \equiv \frac{\log \Sigma(z=1)}{Gs} = -\log\left(\frac{\Lambda\sqrt{s}}{T_H\Delta}\right) + \mathcal{E}_1(W_0(\bar{E}/T_H)) \, , \tag{5.13}$$

where $\mathcal{E}_1$ is the standard exponential integral function $\mathcal{E}_1(x) = \int_x^\infty dy y^{-1}\exp(-y)$.

It is amusing to see that Hawking temperature T_H emerges as a characteristic scale distinguishing two cases:

1. $\bar{\mathbf{E}} \ll \mathbf{T_H}$

In this case, using $W_0(a) \sim a$ for $a \ll 1$ and taking $\Lambda \sim \bar{\omega}$, we get:

$$\Phi < -\log\left(\frac{\sqrt{s}}{\Delta}\right) - \log\left(\frac{\bar{E}}{T_H}\right) = -\log\gamma \leq 0 \, , \tag{5.14}$$

while, applying finally (5.5), we obtain the suggestive result:

$$\frac{1}{\sigma}\frac{d\sigma}{d\omega} = \frac{Gs}{\omega}e^{-\frac{\omega}{T_H}} \, , \tag{5.15}$$

exhibiting both a brems-strahlung behavior at small ω and a Boltzmann suppression at large ω.

2. $\bar{\mathbf{E}} \gg \mathbf{T_H}$

In this case, using the large-argument limit of W_0: $W_0(a) \sim \log(a) - \log(\log(a))$ and again $\Lambda \sim \bar{\omega}$, we get

$$\Phi < -\log\left[\frac{\bar{E}}{T_H\log(\frac{\bar{E}}{T_H})}\frac{\sqrt{s}}{\Delta}\right] - \frac{T_H/\bar{E}}{\log(\bar{E}/T_H)} < -\log\left[\frac{\bar{E}}{T_H\log(\frac{\bar{E}}{T_H})}\right] < 0 \, , \tag{5.16}$$

while the analog of (5.15) for the present case becomes:

$$\frac{1}{\sigma}\frac{d\sigma}{d\omega} = \frac{Gs}{\omega}e^{-\frac{\omega}{\bar{E}}W_0(\bar{E}/T_H)} , \qquad (5.17)$$

joining smoothly with (5.15) at $\bar{E} \sim T_H$.

To summarize, under our mild assumption $\gamma - 1 \geq \mathcal{O}(1)$, we have found two regimes as a function of $\bar{E}/T_H$:

for $\frac{\bar{E}}{T_H} \gg 1$ we see from (5.16) that the multi jet cross section is exponentially suppressed as $\exp(-\frac{Gs}{\hbar}\log\frac{\bar{E}}{T_H})$ and that the hard-graviton spectrum is cut off at $\bar{\omega} \sim \frac{\bar{E}}{\log(\bar{E}/T_H)} > T_H$.

For $\frac{\bar{E}}{T_H} \ll 1$ equation (5.14) shows that the multi jet cross section can be $\mathcal{O}(1)$ and the hard-graviton cutoff is T_H independently of $\bar{E}$. To avoid an exponential suppression we need to take $\Lambda \sim T_H$ and $\gamma - 1 = \mathcal{O}(1)$ which looks physically possible. In particular, at $\bar{E} = T_H$ the fraction of energy in quanta below T_H should be of $\mathcal{O}(1)$ which is also the case for black-hole evaporation. Finally, the cutoff energy for the soft gravitons is $\frac{\Delta}{Gs}$.

A final remark concerns our choice for Λ and the virtual corrections attached to the incoming gravitons. For these corrections our estimate $\Lambda \sim \bar{\omega}$ should be revised since the soft graviton approximation can be now justified up to a scale smaller than but of order $\sqrt{s}$. These extra virtual contributions should have their own counterpart in real emission/absorption from the energetic legs. This is again what one should expect in the process of black-hole formation. In fact even before the critical impact parameter for BH formation is reached gravitational brems-strahlung takes place. While in the regime of small deflection angles this results in a small loss of energy (see [24–26] for recent classical and quantum approaches to this problem) when the gravitational collapse regime is approached only a finite fraction of the incoming energy goes (at least classically) into forming a black hole.

6 Summary and outlook

A high-energy two-body collision usually results in the production of many lower-energy quanta. This is a common phenomenon shared by essentially any realistic $4 - D$ interacting theory. A well known and much studied case is the one of strong interactions in which jets of hadrons are produced when the underlying parton shower (consisting of many final partons) hadronizes. For low momentum transfer processes the hadronic multiplicity typically grows logarithmically with energy (with the final particles filling up uniformly a rapidity plateau), while harder processes (such as $e^+e^- \to$ hadrons) lead to multiplicities that typically grow faster than a power of $\log s$ (but slower than an exponential) of $\log s$ (e.g. like $\exp(\sqrt{\log s})$).

The fact that the gravitational coupling G_N is dimensional and that the effective high-energy coupling is $\alpha_G \equiv \frac{G_N s}{\hbar}$ suggests that multiplicities in the unltraplanckian gravitational collisions should grow like a power of the center-of-mass energy $\sqrt{s}$. In particular, if $\langle n \rangle \sim \alpha_G$ we are immediately led –by energy conservation– to the simple, yet startling conclusion that a *ultra-planckian* energy collision produces *sub-planckian* final quanta of average energy $\langle E \rangle \sim \sqrt{s}/\alpha_G \sim \hbar/R_S \sim T_H$ with T_H the Hawking temperature of a black hole of mass $\sqrt{s}$.

The study of ultraplanckian-energy collisions has confirmed to a large extent the above picture through what has been termed as "fractionation". The first example [9, 10] is what we may call t-channel fractionation, i.e. the phenomenon by which a large momentum transfer is shared among $n \sim \alpha_G$ exchanged gravitons. This is the reason why a superficially hard process (fixed angle gravitational scattering at arbitrarily high energy) is actually controlled by large-distance physics with each exchanged graviton sitting very close to its mass-shell.

Within string theory t-channel fractionation has an s-channel analog [10, 27, 28]. This is because, in string theory, graviton exchange is actually "gravi-reggeon" exchange and carries an imaginary part related to the possibility of "cutting" the graviton to expose the s-channel intermediate states dual to it. As a result, in the so-called string-gravity regime, t-channel fractionation becomes, almost trivially, s-channel fractionation: the number of cut-gravi-reggeons grows with energy like α_G implying a softer and softer final state as one increases further and further $\sqrt{s}$. Unfortunately, this regime is under control only below a certain threshold energy corresponding to a Schwarzschild radius R_S of order the string length ℓ_s, i.e. to a would be Hawking temperature exceeding the Hagedorn temperature of string theory. And indeed we do not expect black-hole formation below such threshold.

In an independent development DGILS [36] have addressed the problem of s-channel fractionation (called classicalization in their context) directly in quantum field theory i.e. without the use of string theory's duality. Evidence for fractionation would lend support to a previous proposal [37, 38] of black holes as a multi graviton state near a quantum phase transition. And, indeed, the claim in [36] is that the $2 \to N$ cross section at $\alpha_G \gg 1$ is dominated by final states containing $\mathcal{O}(\alpha_G)$ quanta of energy $\mathcal{O}(T_H)$.

Taken at face value this result looks perfectly in line with the one of [10, 27, 28], actually as a smooth extension of the latter in the theoretically unaccessible region above threshold. Although one result relies on string theory while the the other does not it is conceivable that, above the above mentioned threshold energy, the final state will consist of just massless strings for which a QFT approach is already sufficient. However, at a closer scrutiny, some tension appears between the claims made within the two studies. While in [10, 27, 28] loop corrections to the $2 \to N$ process (corresponding to the possibility of cutting only a subset of the exchanged gravi-reggeons following the AGK rules) are crucial for restoring unitarity (or even just unitarity bounds), in [36] one is only considering the $2 \to N$ process at tree level.

In this paper we have tried to resolve this tension by first considering soft real and virtual corrections to the process considered in [36] and by then showing that such corrections are in principle large in spite of the fact that true IR divergences cancel by the usual Bloch-Nordsiek mechanism. We have then given a reinterpretation of the claim in [36] through the introduction of some sort of "gravitational jet cross sections", quantities that, like QCD jets, should be perturbatively calculable. They are characterized by a lower cutoff $\bar{E}$ on the energy of each jet-graviton and by an upper cutoff Δ on the total energy carried by all gravitons softer than $\bar{E}$.

So far we have only been able to estimate these jet cross section qualitatively. i.e. without control over $\mathcal{O}(1)$ parameters. This, however, is sufficient to support the conclu-

sion that, unlike QCD jets (that tend to be few and hard because of asymptotic freedom) gravitational jets tend to be many and soft, where *many* here means indeed $\mathcal{O}(\alpha_G)$ and soft means with energy of order T_H. In other words we have found that, while the jet cross section for $\bar{E} \gg T_H$ is necessarily exponentially suppressed, the one with $\bar{E} \leq T_H$ *can* be large enough to saturate unitarity. Furthermore, we have been able to study the one-jet inclusive cross section as a function of its energy. We found that, in the relevant (latter) case such a distribution is brems-strahlung-like $dN/d\omega \sim \omega^{-1}$ up to T_H and Boltzmann-like suppressed ($dN/d\omega \sim \exp(-\omega T_H^{-1})$) above. This falls short of agreeing with a Bose-Einstein thermal spectrum, for which presumably further re-interactions of the final gravitons have to be taken into account.

We should warn the reader that what was done here should only be considered as a first heuristic step into the problem of constructing a unitary gravitational S-matrix sharing some properties with those of the semiclassical original analysis by Hawking. One problem that we left unanswered is that of projecting our results on individual s-channel partial waves (equivalently on a given impact parameter for large angular momentum), as done in [10, 27, 28] but not in [36]. Such an analysis should allow to see how the final state changes progressively from one typical of a scattering process to one resembling the evaporation of a black hole. Here, in section 5, we have considered directly the cross section integrated over impact parameter and it is only at the step where we take the B_0 factor to be independent of the kinematics and of $\mathcal{O}(Gs)$ that we are implicitly excluding peripheral processes (small deflection angles) concentrating our attention on the small-b region.

We hope that the positive indications reached in this paper will motivate further work in this challenging –but hopefully highly rewarding– line of research.

A A brief review of the AGK rules

In a classic paper [39], Abramovski, Gribov and Kancheli (AGK) derived a set of "cutting rules" allowing to relate the relative contribution of the (t-channel) exchange of n Reggeons to different s-channel intermediate states. As a result, one can compute how the

total cross-section, related by the optical theorem to the discontinuity w.r.t. energy of the forward elastic amplitude, is shared among different final states. The original derivation, reviewed in [40], was in the context of hadron scattering and the Pomeron. Here, following closely [40], we argue that it applies to ultra high energy gravitational scattering (within a string-theory context where the graviton belongs to a Regge trajectory) *mutatis mutandis*.

Let us start by considering the Sommerfeld-Watson representation of the scattering amplitude for a binary $a + b \to a + b$ process (see figure 6)

$$\mathcal{A}^{ab}(s,t) = \int \frac{d\omega}{2i} \zeta(\omega) s^{1+\omega} \mathcal{F}^{ab}(\omega,t) \,, \tag{A.1}$$

where $\omega = J-1$ is the conserved quantity in complex-angular-momentum theory and the signature factor $\zeta(\omega)$ reads

$$\zeta(\omega) = \frac{\tau - e^{-i\pi\omega}}{\sin\pi\omega} = i + \frac{\tau - \cos\pi\omega}{\sin\pi\omega} \,, \tag{A.2}$$

with $\tau = \pm 1$ representing the signature. The integration contour in (A.1) is to the right of the singularities of $\mathcal{F}^{ab}$ (but to the left of those at the non-negative integer in ζ). Hereafter we shall only be interested in the case $\tau = +1$ for which, more simply:

$$\zeta(\omega) = i + \tan\left(\frac{\pi}{2}\omega\right) \,. \tag{A.3}$$

The amplitude $\mathcal{F}^{ab}(\omega,t)$ has poles and cuts in the complex (t-channel angular momentum) ω-plane. In particular, a non-planar multiple gravi-Reggeon exchange produces branch cuts, just like ordinary particles do on the complex-energy plane. In analogy with the latter case, the discontinuity of $\mathcal{F}^{ab}(\omega,t)$ across an n-Reggeon cut can be expressed as (see figure 5)

$$\mathrm{disc}_\omega^{(n)} \mathcal{F}^{ab}(\omega,t) = 2\pi i \int \frac{d\Omega_n}{n!} \Gamma_{\{\beta_j\}} \mathcal{A}_n^a(\{\mathbf{k}_j,\omega\}) \mathcal{A}_n^b(\{\mathbf{k}_j,\omega\}) \delta\left(\omega - \sum_j \beta_j\right) \,, \tag{A.4}$$

where, setting $\mathbf{q}^2 = -t$, the 'transverse' n-particle phase space reads

$$d\Omega_n = (2\pi)^2 \delta^2\left(\mathbf{q} - \sum_{j=1}^n \mathbf{k}_j\right) \prod_{j=1}^n \frac{d^2\mathbf{k}_j}{(2\pi)^2} \,, \tag{A.5}$$

with $\mathbf{k}_j$, $j = 1,\ldots n$, denoting the transverse momentum of the j-th cut gravi-Reggeon. Furthermore, the product of all signature factors gives [40]:

$$\Gamma_{\{\beta_j\}} = (-1)^{n-1} \frac{\cos\left[\frac{\pi}{2}\sum_i \beta_i\right]}{\prod_i \cos\left[\frac{\pi}{2}\beta_i\right]} \,, \tag{A.6}$$

with $\beta_j(-\mathbf{k}_j^2) = \alpha(-\mathbf{k}_j^2) - 1$ and $\alpha(t_j = -\mathbf{k}_j^2)$ the jth gravi-reggeon trajectory.

The vertex functions $\mathcal{A}_n^{a,b}$ represent the coupling of the external particles a,b to n Reggeons and depend on the process and the theory under consideration.
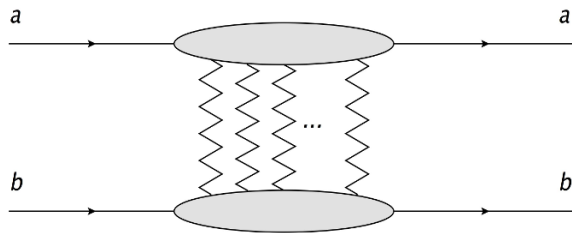
Figure 6. The $2 \to 2$ amplitude with n-gravi-reggeons exchanged is parted in two $\mathcal{A}_n^{a,b}$ subamplitudes, diagrammatically schematizing the integral (A.4).

As a result, the contribution of n gravi-Reggeons to the amplitude is given by

$$\mathcal{A}_n^{ab}(s,t) = \int \frac{d\omega}{2i} \zeta(\omega) s^{1+\omega} \mathrm{disc}_\omega^{(n)} \mathcal{F}^{ab}(\omega,t) = \pi \int \frac{d\Omega_n}{n!} \Gamma_{\{\beta_i\}} \zeta(\sum_i \beta_i) s^{1+\sum \beta_i} \mathcal{A}_n^a \mathcal{A}_n^b. \quad (A.7)$$

Let us note, at this point, that the combination $\Gamma_{\{\beta_i\}} \zeta(\sum_i \beta_i)$ appearing in (A.7) takes the simple factorized form

$$\Gamma_{\{\beta_j\}} \zeta(\sum_j \beta_j) = -i \prod_{j=1}^n \frac{-e^{-i\frac{\pi}{2}\beta_j}}{\cos(\frac{\pi}{2}\beta_j)} = -i \prod_{j=1}^n (-1 + i \tan(\frac{\pi}{2}\beta_j)). \quad (A.8)$$

In the eikonal approximation, the vertex functions $\mathcal{A}_n^{a,b}$ factorize, i.e. $\mathcal{A}_n^{a,b} = (\mathcal{A}_1^{a,b})^n$ and the expression (A.7) simplifies further if one goes over to impact parameter space by the Fourier transform:

$$\mathcal{A}_n^{ab}(s,b) = \int \frac{d^2\mathbf{q}}{(2\pi)^2} e^{-i\mathbf{b}\cdot\mathbf{q}} \frac{1}{s} \mathcal{A}_n^{ab}(s, t = -q^2). \quad (A.9)$$

We easily obtain:

$$i\mathcal{A}_n^{ab}(s,b) = \frac{1}{n!} \left(i \int \frac{d^2\mathbf{k}}{(2\pi)^2} \mathcal{A}_1^a \mathcal{A}_1^b s^{\beta(-k^2)} (i + \tan(\frac{\pi}{2}\beta(-k^2))) \right)^n. \quad (A.10)$$

Summing finally over n and adding 1 to go over to the S-matrix we recover the well-known eikonal exponentiation:

$$S^{ab}(s,b) = 1 + i \sum_{n=1}^{\infty} \mathcal{A}_n^{ab}(s,b)$$

$$= \exp\left\{ i \int \frac{d^2\mathbf{k}}{(2\pi)^2} \mathcal{A}_1^a \mathcal{A}_1^b s^{\beta(-k^2)} \left[i + \tan\left(\frac{\pi}{2}\beta(-k^2)\right) \right] \right\} \equiv e^{2i\delta(s,b)}. \quad (A.11)$$

Let's now come to the AGK cutting rules. Since $\Gamma_{\{\beta_i\}}$ is real, the full imaginary part of (A.7) is simply given by replacing ζ by 1. The AGK rules tell us how this full imaginary part is built up by individual contribution in which the imaginary part of a subset of Reggeon signature factors is taken (i.e. in which a subset of k gravi-reggeons is "cut" out of the total number n).

$$\text{disc}_s[\mathcal{A}_n^{ab}(s,t)] = \sum_{k=0}^{n} a_k^n(s,t), \tag{A.12}$$

where

$$a_k^n = 2\pi i \int d\Omega_n s^{2+\omega} (F_{AGK})_k^n \mathcal{A}_n^a \mathcal{A}_n^b, \tag{A.13}$$

with

$$(F_{AGK})_k^n = \frac{2^n}{n!}(-1)^n + \frac{1}{n!}\Gamma_{\{\beta_j\}} \qquad \text{for} \quad k = 0, \tag{A.14}$$

$$(F_{AGK})_k^n = (-1)^{n-k}\frac{2^n}{(n-k)!k!} \qquad \text{for} \quad n \geq k > 0. \tag{A.15}$$

Clearly the sum in (A.12) reproduces the total discontinuity.

Also for the AGK rules we can go over to impact parameter. Under the eikonal approximation leading to (A.10) we get,[14]

$$(SS^\dagger)_k^n = (-1)^{n-k}\frac{(4\text{Im}\delta)^n}{(n-k)!k!} \quad \text{for} \quad n \geq k > 0. \tag{A.16}$$

Keeping k fixed and summing over $n \geq k$ we get:

$$(SS^\dagger)_k = e^{-4\text{Im}\delta}\frac{(4\text{Im}\delta)^k}{k!}. \tag{A.17}$$

Finally, summing over $k \geq 1$ and adding the elastic cross section $\sigma_{el} \equiv |S_{el}|^2 = e^{-4\text{Im}\delta}$ we recover (s-channel) partial wave unitarity.

Expression (A.17) is interpreted as the probability of having k cut gravi-Reggeons at impact parameter $\mathbf{b}$. A Poisson distribution of CGR is obtained. The mean and the variance are given by

$$\langle k \rangle(s, \mathbf{b}) = \text{Var}[k(s, \mathbf{b})] = 4\text{Im}\delta. \tag{A.18}$$

B Building the B_0 factor

In this appendix, we will derive the B_0-factor in the general case of (identical) massive particles, take the massless limit and then show that it is extremized in particular kinematical regimes.

B.1 Derivation of B_0 for massive and massless particles

For massless particles, the individual integrals, for fixed i and j, are logarithmically divergent. They are given by

$$\begin{aligned} I_{\text{m-less}}(p_i, p_j) &= \int \frac{d^3q}{|q|qp_iqp_j} = \int \frac{d^3q}{|q|^3 E_i E_j (1 - \mathbf{nn}_i)(1 - \mathbf{nn}_j)} \\ &= \frac{1}{E_i E_j} \int_\lambda^\Lambda \frac{d|q|}{|q|} \int \frac{d\Omega_\mathbf{n}}{(1 - \mathbf{nn}_i)(1 - \mathbf{nn}_j)}, \end{aligned} \tag{B.1}$$

---

[14]A more intuitive direct derivation of (A.16) was given already in section 2.2 starting from a more detailed cutting formula.

where λ is an IR regulator and Λ is the upper limit for the validity of the leading soft behaviour. The angular integral is also logarithmically divergent and reads

$$
\begin{aligned}
J_{\mathrm{m-less}}(p_i, p_j) &= \int \frac{d\Omega_{\mathbf{n}}}{(1 - \mathbf{n}\mathbf{n}_i)(1 - \mathbf{n}\mathbf{n}_j)} = \int_0^1 d\alpha \int \frac{d\cos\theta d\phi}{[1 - \cos\theta|\alpha\mathbf{n}_i + (1-\alpha)\mathbf{n}_j|]^2} \\
&= \int_0^1 \frac{2\pi d\alpha}{|\alpha\mathbf{n}_i + (1-\alpha)\mathbf{n}_j|} \left[\frac{1}{1 - |\alpha\mathbf{n}_i + (1-\alpha)\mathbf{n}_j|} - \frac{1}{1 + |\alpha\mathbf{n}_i + (1-\alpha)\mathbf{n}_j|} \right] \\
&= \int_0^1 \frac{4\pi d\alpha}{1 - |\alpha\mathbf{n}_i + (1-\alpha)\mathbf{n}_j|^2} = \int_0^1 \frac{2\pi d\alpha}{\alpha(1-\alpha)(1 - \mathbf{n}_i\mathbf{n}_j)} = \frac{2\pi \log \mu}{(1 - \mathbf{n}_i\mathbf{n}_j)} .
\end{aligned}
$$

In order to regulate the logarithmic divergence of the angular integral, it is convenient to treat the 'hard' particles as massive (with a common mass m) and later on take the massless limit. In practice the only difference is that $\mathbf{n}_i$ is replaced by $\mathbf{v}_i = \mathbf{p}_i/E_i$ with $|\mathbf{v}_i| < 1$ i.e.

$$
\begin{aligned}
J_{\mathrm{m-ive}}(p_i, p_j) &= \int \frac{d\Omega_{\mathbf{n}}}{(1 - \mathbf{n}\mathbf{v}_i)(1 - \mathbf{n}\mathbf{v}_j)} = \int_{-\frac{1}{2}}^{\frac{1}{2}} d\gamma \int \frac{d\cos\theta d\phi}{[1 - \cos\theta|(\gamma + \frac{1}{2})\mathbf{v}_i + (\frac{1}{2} - \gamma)\mathbf{v}_j|]^2} \\
&= 4\pi \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{d\gamma}{1 - \frac{1}{4}(\mathbf{v}_i + \mathbf{v}_j)^2 - \gamma^2((\mathbf{v}_i - \mathbf{v}_j)^2} \\
&= \frac{4\pi}{|\mathbf{v}_i - \mathbf{v}_j|\sqrt{1 - \frac{1}{4}(\mathbf{v}_i + \mathbf{v}_j)^2}} \log \left(\frac{\sqrt{1 - \frac{1}{4}(\mathbf{v}_i + \mathbf{v}_j)^2} + \frac{1}{2}|\mathbf{v}_i - \mathbf{v}_j|}{\sqrt{1 - \frac{1}{4}(\mathbf{v}_i + \mathbf{v}_j)^2} - \frac{1}{2}|\mathbf{v}_i - \mathbf{v}_j|} \right) \\
&= \frac{2\pi}{|\mathbf{v}_i - \mathbf{v}_j|\sqrt{1 - \frac{1}{4}(\mathbf{v}_i + \mathbf{v}_j)^2}} \log \left(\frac{1 - \mathbf{v}_i \cdot \mathbf{v}_j + |\mathbf{v}_i - \mathbf{v}_j|\sqrt{1 - \frac{1}{4}(\mathbf{v}_i + \mathbf{v}_j)^2}}{1 - \mathbf{v}_i \cdot \mathbf{v}_j - |\mathbf{v}_i - \mathbf{v}_j|\sqrt{1 - \frac{1}{4}(\mathbf{v}_i + \mathbf{v}_j)^2}} \right) ,
\end{aligned}
$$

where in the last equation we have squared the argument of the log. The above result can also rewritten as:

$$
J_{\mathrm{m-ive}}(p_i, p_j) = \frac{2\pi}{|\mathbf{v}_i - \mathbf{v}_j|\sqrt{1 - \frac{1}{4}(\mathbf{v}_i + \mathbf{v}_j)^2}} \log \left(\frac{1 + \beta_{ij}}{1 - \beta_{ij}} \right) = \frac{2\pi E_i E_j}{\beta_{ij} p_i \cdot p_j} \log \left(\frac{1 + \beta_{ij}}{1 - \beta_{ij}} \right)
$$

where

$$
\beta_{ij}^2 = \frac{(\mathbf{v}_i - \mathbf{v}_j)^2(1 - \frac{1}{4}(\mathbf{v}_i + \mathbf{v}_j)^2)}{(1 - \mathbf{v}_i \cdot \mathbf{v}_j)^2} = \frac{(p_i \cdot p_j - m^2)(p_i \cdot p_j + m^2)}{(p_i \cdot p_j)^2} = \left(1 - \frac{m^4}{(p_i \cdot p_j)^2} \right)
$$

is the square of the relative velocity of the ith and jth particles. Including the overall (q-independent and thus unintegrated) factors

$$
\frac{8\pi G\eta_i\eta_j \left[(p_i \cdot p_j)^2 - \frac{1}{2}m^4 \right]}{2(2\pi)^3 E_i E_j} = \frac{G\eta_i\eta_j p_i p_j m^2(1 + \beta_{ij}^2)}{(2\pi)^2 E_i E_j (1 - \beta_{ij}^2)^{1/2}} ,
$$

and summing over i and j, leads to Weinberg's celebrated result (4.5) [41] and to its massless limit, used in the present investigation:

$$
B_0 = \frac{G}{2\pi} \sum_{i,j} \eta_i\eta_j m^2 \frac{1 + \beta_{ij}^2}{\beta_{ij}(1 - \beta_{ij}^2)^{1/2}} \log \left(\frac{1 + \beta_{ij}}{1 - \beta_{ij}} \right) \rightarrow \frac{2G}{\pi} \sum_{i,j} \eta_i\eta_j p_i p_j \log \frac{p_i p_j}{\mu^2}. \quad \text{(B.2)}
$$

B.2 Special kinematics for massless particles

Analyzing the first order constrained variation of B_0 in the massless case around collinear and orthogonal kinematical configurations, we here show that they extremize B_0.

First of all, let us rewrite the eq. (4.6) as

$$B_0 = \frac{2Gs}{\pi} \left\{ - \sum_i w_i \left[(1-\mathbf{nn}_i) \log w_i (1-\mathbf{nn}_i) + (1+\mathbf{nn}_i) \log w_i (1+\mathbf{nn}_i) \right] \right.$$
$$\left. + \frac{1}{2} \sum_{i,j} 2 w_i w_j (1-\mathbf{n}_i \mathbf{n}_j) \log 2 w_i w_j (1-\mathbf{n}_i \mathbf{n}_j) \right\} . \tag{B.3}$$

In fact, using momentum conservation, one can further simplify the above expression and arrive at

$$B_0 = \frac{2Gs}{\pi} \left\{ - \sum_i w_i \left[(1-\mathbf{nn}_i) \log \frac{1-\mathbf{nn}_i}{2} + (1+\mathbf{nn}_i) \log \frac{1+\mathbf{nn}_i}{2} \right] \right.$$
$$\left. + \sum_{i,j} w_i w_j (1-\mathbf{n}_i \mathbf{n}_j) \log \frac{1-\mathbf{n}_i \mathbf{n}_j}{2} \right\} , \tag{B.4}$$

which can be more compactly written as in eq. (4.6)

Let us then consider the first order constrained variation of the B_0-factor. In general, keeping N as well as p_1 and p_2 fixed and varying the momenta of the outgoing particles, one has $p_i \to p_i + \delta p_i$ with $p_i \cdot \delta p_i = 0$ ($p_i^2 = 0$) and $\sum_i \delta p_i = 0$. Setting $\delta p_i = (u_i, \mathbf{v}_i) = u_i(1, \mathbf{n}_i) + (0, \mathbf{q}_i) = \delta p_i^{\parallel} + \delta p_i^{\perp}$ with $u_i = \mathbf{v}_i \mathbf{n}_i$ and $\mathbf{q}_i = \mathbf{v}_i^{\perp} = \mathbf{v}_i - u_i \mathbf{n}_i$ so that $\mathbf{q}_i \mathbf{n}_i = 0$.

The (constrained) first order variation of B_0,

$$\delta B_0 = \sum_i \frac{\partial B_0}{\partial p_i} \delta p_i \Big|_{\text{constrained}}$$
$$= \frac{2G}{\pi} \left\{ - \sum_i [p_1 \delta p_i (\log p_1 p_i + 1) + p_2 \delta p_i (\log p_2 p_i + 1)] \right.$$
$$\left. + \frac{1}{2} \sum_{i,j} (p_j \delta p_i + p_i \delta p_j)(\log p_i p_j + 1) \right\} ,$$

the terms with 1 drop thanks to momentum conservation $\sum_i \delta p_i = 0$, so that

$$\delta B_0 = \frac{2G}{\pi} \left\{ - \sum_i [p_1 \delta p_i \log p_1 p_i + p_2 \delta p_i \log p_2 p_i] + \frac{1}{2} \sum_{i,j} (p_j \delta p_i + p_i \delta p_j) \log p_i p_j \right\} .$$

Near-collinear kinematics. Let us further specialise to the case of a near-collinear kinematical scattering. For a collinear configuration $\mathbf{n}_i = \sigma_i \mathbf{n}$. As a result, $\delta p_i^{\perp}$ does not contribute and one has $p_{1,2} \delta p_i = p_{1,2} \delta p_i^{\parallel} = E u_i (1-\sigma_i)$ and $p_j \delta p_i = p_j \delta p_i^{\parallel} = E_j u_i (1-\sigma_j \sigma_i)$.

Plugging into δB_0 one finds a vanishing result

$$\delta B_0 = \frac{2GE}{\pi}\left\{-\sum_i u_i\left[(1-\sigma_i)\log 2w_i\frac{1-\sigma_i}{2}+(1+\sigma_i)\log 2w_i\frac{1+\sigma_i}{2}\right]\right.$$
$$\left.+\sum_i(w_ju_i+w_iu_j)(1-\sigma_i\sigma_j)\log 4w_iw_j\frac{1-\sigma_i\sigma_j}{2}\right\}=0\,,$$

since the constant terms with $\log 2$ vanish due to the constraints $\sum_i u_i = 0 = \sum_i u_i\sigma_i$, the terms with $u_i\log w_i$ cancel each other and the terms $(1\pm\sigma)\log(1\pm\sigma/2)=0$ for $\sigma=\pm1$. This proves that collinear configurations extremize B_0. The second order variations along the longitudinal directions yield a vanishing result: the particles remain collinear and any choice of w_i and σ_i such that $\sum_i w_i = 1$ and $\sum_i w_i\sigma_i = 0$ produces $B_0 = 0$. Variations of collinear configurations along $\delta^\perp p_i$ produce a positive result: $B_0^{\text{coll}}=0$ is a local infinitely degenerate minimum and $B_0^{\text{near-coll}}>0$.

Near-orthogonal kinematics. Let us consider the case of a near-orthogonal kinematical scattering. For a configuration with two back-to-back jets perpendicular to the direction of the incoming particles $\mathbf{n}_i=\sigma_i\mathbf{m}$ with $\mathbf{nm}=0$. As before $\delta p_i = \delta p_i^\parallel + \delta p_i^\perp =$ so that $p_j\delta p_i = p_j\delta p_i^\parallel = E_ju_i(1-\sigma_j\sigma_i)$ while $p_1\delta p_i = E(u_i-\mathbf{mq}_i)$ and $p_1\delta p_i = E(u_i+\mathbf{mq}_i)$. The (constrained) first order variation of B_0,

$$\delta B_0 = \frac{2G}{\pi}\left\{-\sum_i E\log w_i[(u_i-\mathbf{mq}_i)+(u_i+\mathbf{mq}_i)\log w_i]\right.$$
$$+\sum_{i,j}(E_ju_i+E_iu_j)\frac{1-\sigma_j\sigma_i}{2}\log 4w_iw_j\frac{1-\sigma_j\sigma_i}{2}\right\}=\frac{2G}{\pi}\left\{-\sum_i 2Eu_i\log w_i\right.$$
$$\left.+\sum_{i,j}(E_ju_i+E_iu_j)\frac{1-\sigma_j\sigma_i}{2}[\log 4+\log w_i+\log w_j+\log\frac{1-\sigma_j\sigma_i}{2}\right\}=0\,,$$

since the constant terms with $\log 4$ vanish due to the constraints $\sum_i u_i = 0 = \sum_i u_i\sigma_i$, the terms with $u_i\log w_i$ cancel each other and the terms $(1\pm\sigma)\log(1\pm\sigma/2)=0$ for $\sigma=\pm1$. This proves that 'orthogonal' configurations extremize B_0 but are highly degenerate. The second order variations of the outing particles along the direction of the two jets (orthogonal to the beam-line) yield a vanishing result: and any choice of w_i and σ_i such that $\sum_i w_i = 1$ and $\sum_i w_i\sigma_i = 0$ produces $B_0 = \frac{2Gs}{\pi}\log 2$. This is to be expected, since the two jets of collinear particles behave as two particles with energy E and momentum $\mathbf{p}$ orthogonal to $\mathbf{p}_{1,2}$.

References

[1] D.M. Eardley and S.B. Giddings, *Classical black hole production in high-energy collisions*, *Phys. Rev.* **D 66** (2002) 044011 [`gr-qc/0201034`] [INSPIRE].

[2] E. Kohlprath and G. Veneziano, *Black holes from high-energy beam-beam collisions*, *JHEP* **06** (2002) 057 [`gr-qc/0203093`] [INSPIRE].

[3] H. Yoshino and Y. Nambu, *Black ring formation in particle systems*, *Phys. Rev.* **D 70** (2004) 084036 [`gr-qc/0404109`] [INSPIRE].

[4] S.B. Giddings and V.S. Rychkov, *Black holes from colliding wavepackets*, *Phys. Rev.* **D 70** (2004) 104026 [`hep-th/0409131`] [INSPIRE].

[5] S.W. Hawking, *Particle Creation by Black Holes*, *Commun. Math. Phys.* **43** (1975) 199 [*Erratum ibid.* **46** (1976) 206] [INSPIRE].

[6] D.J. Gross and P.F. Mende, *The High-Energy Behavior of String Scattering Amplitudes*, *Phys. Lett.* **B 197** (1987) 129 [INSPIRE].

[7] D.J. Gross and P.F. Mende, *String Theory Beyond the Planck Scale*, *Nucl. Phys.* **B 303** (1988) 407 [INSPIRE].

[8] P.F. Mende and H. Ooguri, *Borel Summation of String Theory for Planck Scale Scattering*, *Nucl. Phys.* **B 339** (1990) 641 [INSPIRE].

[9] D. Amati, M. Ciafaloni and G. Veneziano, *Superstring Collisions at Planckian Energies*, *Phys. Lett.* **B 197** (1987) 81 [INSPIRE].

[10] D. Amati, M. Ciafaloni and G. Veneziano, *Classical and Quantum Gravity Effects from Planckian Energy Superstring Collisions*, *Int. J. Mod. Phys.* **A 3** (1988) 1615 [INSPIRE].

[11] D. Amati, M. Ciafaloni and G. Veneziano, *Can Space-Time Be Probed Below the String Size?*, *Phys. Lett.* **B 216** (1989) 41 [INSPIRE].

[12] D. Amati, M. Ciafaloni and G. Veneziano, *Higher Order Gravitational Deflection and Soft Bremsstrahlung in Planckian Energy Superstring Collisions*, *Nucl. Phys.* **B 347** (1990) 550 [INSPIRE].

[13] D. Amati, M. Ciafaloni and G. Veneziano, *Effective action and all order gravitational eikonal at Planckian energies*, *Nucl. Phys.* **B 403** (1993) 707 [INSPIRE].

[14] S.B. Giddings, *Locality in quantum gravity and string theory*, *Phys. Rev.* **D 74** (2006) 106006 [`hep-th/0604072`] [INSPIRE].

[15] G. D'Appollonio, P. Vecchia, R. Russo and G. Veneziano, *Microscopic unitary description of tidal excitations in high-energy string-brane collisions*, *JHEP* **11** (2013) 126 [`arXiv:1310.1254`] [INSPIRE].

[16] D. Amati, M. Ciafaloni and G. Veneziano, *Towards an S-matrix description of gravitational collapse*, *JHEP* **02** (2008) 049 [`arXiv:0712.1209`] [INSPIRE].

[17] G. Marchesini and E. Onofri, *High energy gravitational scattering: A Numerical study*, *JHEP* **06** (2008) 104 [`arXiv:0803.0250`] [INSPIRE].

[18] G. Veneziano and J. Wosiek, *Exploring an S-matrix for gravitational collapse*, *JHEP* **09** (2008) 023 [`arXiv:0804.3321`] [INSPIRE].

[19] G. Veneziano and J. Wosiek, *Exploring an S-matrix for gravitational collapse. II. A Momentum space analysis*, *JHEP* **09** (2008) 024 [`arXiv:0805.2973`] [INSPIRE].

[20] M. Ciafaloni and D. Colferai, *S-matrix and Quantum Tunneling in Gravitational Collapse*, *JHEP* **11** (2008) 047 [`arXiv:0807.2117`] [ɪɴSPIRE].

[21] M. Ciafaloni and D. Colferai, *Quantum Tunneling and Unitarity Features of an S-matrix for Gravitational Collapse*, *JHEP* **12** (2009) 062 [`arXiv:0909.4523`] [ɪɴSPIRE].

[22] M. Ciafaloni, D. Colferai and G. Falcioni, *Unitarity alternatives in the reduced-action model for gravitational collapse*, *JHEP* **09** (2011) 044 [`arXiv:1106.5628`] [ɪɴSPIRE].

[23] M. Ciafaloni and D. Colferai, *Rescattering corrections and self-consistent metric in Planckian scattering*, *JHEP* **10** (2014) 85 [`arXiv:1406.6540`] [ɪɴSPIRE].

[24] A. Gruzinov and G. Veneziano, *Gravitational Radiation from Massless Particle Collisions*, *Class. Quant. Grav.* **33** (2016) 125012 [`arXiv:1409.4555`] [ɪɴSPIRE].

[25] M. Ciafaloni, D. Colferai and G. Veneziano, *Emerging Hawking-Like Radiation from Gravitational Bremsstrahlung Beyond the Planck Scale*, *Phys. Rev. Lett.* **115** (2015) 171301 [`arXiv:1505.06619`] [ɪɴSPIRE].

[26] M. Ciafaloni, D. Colferai, F. Coradeschi and G. Veneziano, *Unified limiting form of graviton radiation at extreme energies*, *Phys. Rev.* **D 93** (2016) 044052 [`arXiv:1512.00281`] [ɪɴSPIRE].

[27] G. Veneziano, *String-theoretic unitary S-matrix at the threshold of black-hole production*, *JHEP* **11** (2004) 001 [`hep-th/0410166`] [ɪɴSPIRE].

[28] G. Veneziano, *An unexpected use of the AGK rules in black-hole physics*, in proceedings of *Quarks, hadrons, and strong interactions*, memorial workshop devoted to the 75th birthday of V.N. Gribov, Budapest, Hungary, May 22-24, 2005.

[29] G. Veneziano, *A Stringy Nature Needs Just Two Constants*, *Europhys. Lett.* **2** (1986) 199 [ɪɴSPIRE].

[30] G. Veneziano, *A New approach to semiclassical gravitational scattering*, in proceedings of *Hot hadronic matter: Theory and experiment*, *NATO Advanced Research Workshop*, Divonne, France, Jun 27–Jul 1, 1994, *NATO Sci. Ser.* **B 346** (1995) 1.

[31] L. Susskind, *Some speculations about black hole entropy in string theory*, in *The black hole*, C. Teitelboim and J. Zanelli eds., World scientific Publishing Co Pte Ltd (1998), `hep-th/9309145` [ɪɴSPIRE].

[32] E. Halyo, A. Rajaraman and L. Susskind, *Braneless black holes*, *Phys. Lett.* **B 392** (1997) 319 [`hep-th/9605112`] [ɪɴSPIRE].

[33] E. Halyo, B. Kol, A. Rajaraman and L. Susskind, *Counting Schwarzschild and charged black holes*, *Phys. Lett.* **B 401** (1997) 15 [`hep-th/9609075`] [ɪɴSPIRE].

[34] G.T. Horowitz and J. Polchinski, *Selfgravitating fundamental strings*, *Phys. Rev.* **D 57** (1998) 2557 [`hep-th/9707170`] [ɪɴSPIRE].

[35] T. Damour and G. Veneziano, *Selfgravitating fundamental strings and black holes*, *Nucl. Phys.* **B 568** (2000) 93 [`hep-th/9907030`] [ɪɴSPIRE].

[36] G. Dvali, C. Gomez, R.S. Isermann, D. Lüst and S. Stieberger, *Black hole formation and classicalization in ultra-Planckian* $2 \to N$ *scattering*, *Nucl. Phys.* **B 893** (2015) 187 [`arXiv:1409.7405`] [ɪɴSPIRE].

[37] G. Dvali and C. Gomez, *Self-Completeness of Einstein Gravity*, `arXiv:1005.3497` [ɪɴSPIRE].

[38] G. Dvali and C. Gomez, *Black Hole's Quantum N-Portrait*, *Fortsch. Phys.* **61** (2013) 742 [arXiv:1112.3359] [INSPIRE].

[39] V.A. Abramovsky, V.N. Gribov and O.V. Kancheli, *Character of Inclusive Spectra and Fluctuations Produced in Inelastic Processes by Multi-Pomeron Exchange*, *Yad. Fiz.* **18** (1973) 595 [*Sov. J. Nucl. Phys.* **18** (1974) 308] [INSPIRE].

[40] J. Bartels, M. Salvadore and G.P. Vacca, *AGK cutting rules and multiple scattering in hadronic collisions*, *Eur. Phys. J.* **C 42** (2005) 53 [hep-ph/0503049] [INSPIRE].

[41] S. Weinberg, *Infrared photons and gravitons*, *Phys. Rev.* **140** (1965) B516.

[42] G. D'Appollonio, P. Di Vecchia, R. Russo and G. Veneziano, *Regge behavior saves String Theory from causality violations*, *JHEP* **05** (2015) 144 [arXiv:1502.01254] [INSPIRE].

[43] X.O. Camanho, J.D. Edelstein, J. Maldacena and A. Zhiboedov, *Causality Constraints on Corrections to the Graviton Three-Point Coupling*, *JHEP* **02** (2016) 020 [arXiv:1407.5597] [INSPIRE].

[44] S. Weinberg, *Feynman Rules for Any Spin*, *Phys. Rev.* **133** (1964) B1318.

[45] S. Weinberg, *Feynman Rules for Any Spin. 2. Massless Particles*, *Phys. Rev.* **134** (1964) B882.

[46] S. Weinberg, *Feynman rules for any spin. III*, *Phys. Rev.* **181** (1969) 1893 [INSPIRE].

[47] G. Dvali and C. Gomez, *Black Holes as Critical Point of Quantum Phase Transition*, *Eur. Phys. J.* **C 74** (2014) 2752 [arXiv:1207.4059] [INSPIRE].

[48] G. Dvali and C. Gomez, *Black Hole's 1/N Hair*, *Phys. Lett.* **B 719** (2013) 419 [arXiv:1203.6575] [INSPIRE].

[49] G. Dvali, G.F. Giudice, C. Gomez and A. Kehagias, *UV-Completion by Classicalization*, *JHEP* **08** (2011) 108 [arXiv:1010.1415] [INSPIRE].

[50] G. Dvali and D. Pirtskhalava, *Dynamics of Unitarization by Classicalization*, *Phys. Lett.* **B 699** (2011) 78 [arXiv:1011.0114] [INSPIRE].

[51] A. Addazi, *Unitarization and causalization of nonlocal quantum field theories by classicalization*, *Int. J. Mod. Phys.* **A 31** (2016) 1650009 [arXiv:1505.07357] [INSPIRE].

[52] F. Kühnel and B. Sundborg, *High-Energy Gravitational Scattering and Bose-Einstein Condensates of Gravitons*, *JHEP* **12** (2014) 016 [arXiv:1406.4147] [INSPIRE].

[53] H. Kawai, D.C. Lewellen and S.H.H. Tye, *A Relation Between Tree Amplitudes of Closed and Open Strings*, *Nucl. Phys.* **B 269** (1986) 1 [INSPIRE].

[54] F. Cachazo, S. He and E.Y. Yuan, *Scattering equations and Kawai-Lewellen-Tye orthogonality*, *Phys. Rev.* **D 90** (2014) 065001 [arXiv:1306.6575] [INSPIRE].

[55] N.E.J. Bjerrum-Bohr, P.H. Damgaard, T. Sondergaard and P. Vanhove, *The Momentum Kernel of Gauge and Gravity Theories*, *JHEP* **01** (2011) 001 [arXiv:1010.3933] [INSPIRE].

[56] S. Stieberger, *Closed superstring amplitudes, single-valued multiple zeta values and the Deligne associator*, *J. Phys.* **A 47** (2014) 155401 [arXiv:1310.3259] [INSPIRE].

[57] S. Stieberger and T.R. Taylor, *Closed String Amplitudes as Single-Valued Open String Amplitudes*, *Nucl. Phys.* **B 881** (2014) 269 [arXiv:1401.1218] [INSPIRE].

[58] S.J. Parke and T.R. Taylor, *An Amplitude for n Gluon Scattering*, *Phys. Rev. Lett.* **56** (1986) 2459 [INSPIRE].

[59] F. Cachazo and A. Strominger, *Evidence for a New Soft Graviton Theorem*, arXiv:1404.4091 [INSPIRE].

[60] M. Bianchi, S. He, Y.-t. Huang and C. Wen, *More on Soft Theorems: Trees, Loops and Strings*, *Phys. Rev.* **D 92** (2015) 065022 [arXiv:1406.5155] [INSPIRE].

[61] Z. Bern, S. Davies, P. Di Vecchia and J. Nohle, *Low-Energy Behavior of Gluons and Gravitons from Gauge Invariance*, *Phys. Rev.* **D 90** (2014) 084035 [arXiv:1406.6987] [INSPIRE].

[62] M. Bianchi and A.L. Guerrieri, *On the soft limit of closed string amplitudes with massive states*, *Nucl. Phys.* **B 905** (2016) 188 [arXiv:1512.00803] [INSPIRE].

[63] M. Bianchi and A.L. Guerrieri, *On the soft limit of open string disk amplitudes with massive states*, *JHEP* **09** (2015) 164 [arXiv:1505.05854] [INSPIRE].

[64] P. Di Vecchia, R. Marotta and M. Mojaza, *Soft theorem for the graviton, dilaton and the Kalb-Ramond field in the bosonic string*, *JHEP* **05** (2015) 137 [arXiv:1502.05258] [INSPIRE].

[65] P. Di Vecchia, R. Marotta, M. Mojaza and J. Nohle, *New soft theorems for the gravity dilaton and the Nambu-Goldstone dilaton at subsubleading order*, *Phys. Rev.* **D 93** (2016) 085015 [arXiv:1512.03316] [INSPIRE].

[66] M. Bianchi, A.L. Guerrieri, Y.-t. Huang, C.-J. Lee and C. Wen, *Exploring soft constraints on effective actions*, *JHEP* **10** (2016) 036 [arXiv:1605.08697] [INSPIRE].

[67] M. Bianchi and A.L. Guerrieri, *On the soft limit of tree-level string amplitudes*, arXiv:1601.03457 [INSPIRE].

[68] G.F. Sterman and S. Weinberg, *Jets from Quantum Chromodynamics*, *Phys. Rev. Lett.* **39** (1977) 1436 [INSPIRE].