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Type IIB supergravity solutions with AdS_5 from Abelian and non-Abelian T dualities

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Abstract: We present a large class of new backgrounds that are solutions of type IIB supergravity with a warped AdS_5 factor, non-trivial axion-dilaton, B-field and three-form Ramond-Ramond flux but yet have no five-form flux. We obtain these solutions and many of their variations by judiciously applying non-Abelian and Abelian T-dualities, as well as coordinate shifts to $\mathrm{AdS}_5 \times X_5$ IIB supergravity solutions with $X_5 = S^5, T^{1,1}, Y^{p,q}$. We address a number of issues pertaining to charge quantization in the context of non-Abelian T-duality. We comment on some properties of the expected dual super conformal field theories by studying their CFT central charge holographically. We also use the structure of the supergravity Page charges, central charges and some probe branes to infer aspects of the dual super conformal field theories.

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Contents

1 Introduction

In its most precise formulation, the AdS/CFT correspondence conjectures an equivalence between string theory in $\mathrm{AdS}_5 \times S^5$ with N units of Ramond-Ramond five-form flux and $\mathcal{N} = 4$ supersymmetric Yang Mills with SU(N) gauge group [1–4]. There are many versions of the correspondence which extend to string theory on other manifolds and, respectively, other field theories. One particularly intuitive entry in the AdS/CFT dictionary is how conformal invariance in the corresponding field theory is translated into an isometry of the metric in $\mathrm{AdS}_5 \times X_5$: a rescaling of the radial direction in the AdS_5 component of the metric corresponds to change in the energy scale of the field theory. This gravitational isometry gives a geometrical perspective to the idea that in conformal field theories, re-scaling of the coordinates and the energy scale together leave the theory invariant.

The central role that the AdS_5 component of space-time plays is that its SO(4, 2) isometry will dictate that a (super)conformal field theory will be its dual. This has prompted the search for solutions in IIB superstrings and M-theory that contain AdS_5 as a space-time factor. Some fairly systematic attacks have been launched in such search. For example, in [5], a search in the context of M-theory yielded many interesting new backgrounds. Similar methods, based on a combination of supersymmetry and other endemic symmetries, were extended to type IIB [6], an interesting subcase relevant to this manuscript was presented in [7]. As in the references above, the search for gravity solutions in general has historically been based on symmetries [8]. There is although a relatively new conceptual strategy, distinct from merely exploiting symmetries to identify solutions. It is based on the use of symmetry transformations to generate solutions. Indeed, solution-generating techniques have already been successfully applied for black hole and intersecting brane solutions in string theory; for a review see [9]. This particular strategy has also been used in

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the context of the AdS/CFT correspondence, where a $U(1) \times U(1)$ isometry was exploited to generate the Lunin-Maldacena backgrounds [10].

A generalization of T-duality, called non-Abelian T-Duality (NATD), was suggested some time ago [11–13]. Many investigations of this possibly new symmetry followed that focused on transformations of chiral σ models [14–20]; for a review see [21, 22]. However the question of how this symmetry would manifest itself in the Ramond-Ramond sector remained unanswered. Recently, there has been a revival of NATD and in particular, the crucial extension to the Ramond-Ramond sector has been proposed [23, 24]. This resurrected symmetry has already been used to generate solutions from various seed backgrounds in the context of the AdS/CFT correspondence [25–36]. Several investigations about the interplay of NATD and physical properties of holographically interesting backgrounds were discussed in [25, 26, 28, 31, 32, 36, 37].

The main goal of this paper is to further use T-duality and NATD to construct supergravity backgrounds that contain an AdS_5 factor in the metric. Furthermore, we will also investigate some of the salient properties of the dual super-conformal field theories. Our strategy for the construction of such backgrounds relies on starting with seed backgrounds of the form $AdS_5 \times X_5$ with $X_5 = S^5, T^{1,1}, Y^{p,q}$ and applying a set of Abelian and non-Abelian T-dualities on the X_5 factor. The dual SCFT to $AdS_5 \times X_5$ is well understood and we use this information to deduce some of the properties of the dual SCFT for the cases of T-dualized backgrounds.

The paper is organized as follows. In section 2 we start with revisiting NATD for $AdS_5 \times X_5$ where $X_5 = S^5, T^{1,1}, Y^{p,q}$. This analysis has already been presented in the literature [25, 28, 38]. We pay, however, special attention to normalization factors, including powers of α' as this will be important in subsequently interpreting the gravity results from the field theory point of view. Furthermore we lift these solutions to eleven dimensions in section 3 in order to probe their structure in M-theory. Whilst the present work can be understood strictly in the context of constructing supergravity backgrounds; our ultimate motivation is to investigate the dual field theories that arise through the AdS/CFT correspondence. In section 4, we examine the structure of Page charges in supergravity and its implications on the field theory, we also compute the central charge of the dual field theories.

In section 5, we present one of the main results of this work. There, we apply another T-duality to the backgrounds discussed in section 2 which leads to interesting backgrounds for IIB. In the work of Lunin and Maldacena [10], they generated a plethora of interesting solutions by performing a T-duality, followed by a shift in one of the coordinates, and then followed by yet another T-duality (TsT) on gravity theories with $U(1) \times U(1)$ isometry. Motivated by this procedure, we consider an NATD-s-T transformation as our backgrounds have $SU(2) \times U(1)$ isometry. Thus, in section 5, we also present a sample of the one-parameter family of solutions that contain an AdS_5 factor. It should be mentioned that some of these backgrounds have singularities in the Ricci scalar.

In section 6, we show that the solutions we constructed using NATD followed by a T-duality (but without the introduction of a free parameter) are supersymmetric and also explicitly show their G-structure. We discuss the interpretation of the dual field theories in section 7 and conclude in section 8.

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For the sake of coherence as well as a sense of completeness, we relegate a number of notational and technical issues to the appendices. Appendix A, for example, describes our prescriptions for Abelian and Non-Abelian T-dualities. In the appendices, we also present various new one-parameter family of solutions obtained by considering different actions of Abelian and Non-Abelian T-dualities on the $\mathrm{AdS}_5 \times S^5$ background with various shifts of U(1) isometries. In applying NATD to backgrounds with SU(2) isometry, there is a three-dimensional space of parameters that can be introduced via gauge fixing. Although formally NATD allows for the introduction of three parameters, we explicitly classify a sample of twenty of the most obvious possibilities and determine whether their volume form can lead to consistent non-degenerate backgrounds. We further discuss the gauge ambiguity that might arise in NATD and show how the previously mentioned solutions are parameterized in appendix B.2.

2 Non-Abelian T-duality for IIB Freund-Rubin backgrounds

In this section we review the results of applying non-Abelian T-dualities to the $AdS_5 \times S^5$, $AdS_5 \times T^{1,1}$ and $AdS_5 \times Y^{p,q}$ backgrounds. It is common throughout the literature to work in units where $\alpha' = 1$. But in order to make clear some aspects of the field theory dual to the backgrounds we will obtain, we find it useful to focus on the normalization and the factors of α'. Therefore, in this section we present the results with the appropriate factors of L and α' restored for the benefit of the reader. However, we do not pay attention to factors of g_s, thus we set $g_s = 1$ throughout the paper. In appendix A, we review the Büscher rules for Abelian T-duality and their extension to Non-Abelian T-duality, with the proper factors of α'. Below, we summarize the results for the NATD applied to the backgrounds of the form $AdS_5 \times X^5$ mentioned above.

2.1 $AdS_5 \times S^5$

We start with our conventions for the metric on $AdS_5 \times S^5$,

$$ds^2 = 4ds^2(AdS_5) + ds^2(S^5), \tag{2.1}$$

where

$$ds^2(AdS_5) = \left(\frac{R^2(dx_{1,3}^2)}{L^2} + \frac{L^2 dR^2}{R^2}\right), \quad ds^2(S^5) = L^2(4(d\alpha^2 + \sin^2\alpha d\theta^2) + \cos^2\alpha ds^2(S^3)). \tag{2.2}$$

The metric on S^3 is defined as

$$ds^2(S^3) = d\beta^2 + d\phi^2 + d\psi^2 + 2\cos\beta d\psi d\phi. \tag{2.3}$$

Here, and throughout this paper, we take $0 \le \beta \le \pi$, $0 \le \phi \le 2\pi$, $0 \le \psi \le 4\pi$. The attentive reader will notice that we have introduced non-standard factors of 4. This was driven by demanding that the the S^3 needed for NATD has a simple form (this just

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means that its radius is $R_{S^3} = 2$). The above geometry is supported by the self-dual RR 5-form flux,

$$F_5 = \frac{4}{L}(d\mathrm{Vol}(AdS_5) - d\mathrm{Vol}(S^5)). \tag{2.4}$$

We will pay particular attention to normalizations, as they will be relevant for our discussion of properties of the dual field theory. In particular, for the RR 5-form flux we take

$$\frac{1}{(4\pi\alpha')^2}\int_{S^5} F_5 = N, \tag{2.5}$$

which leads to the result $L^4 = \frac{1}{4}\pi N\alpha'^2$. Note that, using the normalization above, this is consistent with the usual result, $R^4 = 4\pi N\alpha'^2$.

2.1.1 NATD of $AdS_5 \times S^5$

We present now the results of a NATD transformation on the S^3 displayed eq. (2.2). These were originally presented in [23]. The gauge fixing we use is $(v_1, v_2, v_3) \to (\rho, \chi, \xi)$. That is, the Lagrange multipliers introduced in NATD (see appendix A for details) are written in spherical polar coordinates, where $v_1 = \rho\cos\xi\sin\chi$, $v_2 = \rho\sin\chi\sin\xi$, $v_3 = \rho\cos\chi$. The range of the angles are $0 < \chi < \pi$ and $0 < \xi < 2\pi$. We will discuss general gauge fixing procedures and present the results of alternate gauge fixings in appendix B.2. We have included the correct factors of α', appearing from the duality transformation, to emphasize that the dual coordinates (ρ, χ, ξ) remain dimensionless,

$$\begin{aligned}
\hat{ds}^2 &= 4ds^2(AdS_5) + 4L^2(d\alpha^2 + \sin^2\alpha d\theta^2) + \frac{\alpha'^2 d\rho^2}{L^2\cos^2\alpha} + \frac{\alpha'^2 L^2\rho^2\cos^2\alpha\left(d\xi^2\sin^2\chi + d\chi^2\right)}{\alpha'^2\rho^2 + L^4\cos^4\alpha},\\
\hat{B} &= \frac{\alpha'^3\rho^3\sin\chi d\xi\wedge d\chi}{\alpha'^2\rho^2 + L^4\cos^4\alpha}, \qquad e^{-2\hat{\Phi}} = L^2\cos^2\alpha\left(\frac{L^4\cos^4\alpha + \alpha'^2\rho^2}{\alpha'^3}\right).
\end{aligned} \tag{2.6}$$

Notice that the dilaton has a singularity at $\alpha = \pi/2$. Indeed, this is a curvature singularity as can be seen from the 10D string frame Ricci Scalar,

$$\hat{\mathcal{R}} = \frac{3\sec^2\alpha + \frac{4\left(-7\rho^2\alpha'^2 + 3L^4\cos 2\alpha + 3L^4\right)}{\rho^2\alpha'^2 + L^4\cos^4\alpha} + \frac{28\rho^4\alpha'^4}{(\rho^2\alpha'^2 + L^4\cos^4\alpha)^2} - 6}{2L^2}. \tag{2.7}$$

The singularity appears because we are dualising on a manifold that shrinks to zero-size at $\alpha = \pi/2$ — see eq. (2.2). The non-trivial dual RR fluxes are given by,

$$\begin{aligned}
\hat{F}_2 &= \frac{8L^4\sin\alpha\cos^3\alpha d\alpha\wedge d\theta}{\alpha'^{3/2}},\\
\hat{F}_4 &= \frac{8L^4\rho^3\alpha'^{3/2}\sin\alpha\cos^3\alpha\sin\chi d\alpha\wedge d\theta\wedge d\xi\wedge d\chi}{\rho^2\alpha'^2 + L^4\cos^4\alpha},\\
\hat{F}_6 &= \frac{2\sqrt{\alpha'}\rho d\rho\wedge d\mathrm{Vol}(AdS_5)}{L},\\
\hat{F}_8 &= \frac{2L^3\rho^2\alpha'^{3/2}\cos^4\alpha\sin\chi d\xi\wedge d\rho\wedge d\chi\wedge d\mathrm{Vol}(AdS_5)}{\rho^2\alpha'^2 + L^4\cos^4\alpha},
\end{aligned} \tag{2.8}$$

satisfying $\star\hat{F}_2 = \hat{F}_8$, $\star\hat{F}_4 = -\hat{F}_6$. Unless stated otherwise the RR p-forms quoted in the text are those that appear in the equations of motions: $\hat{F}_4 = dC_3 - C_1\wedge H_3$.

Let us now move to present the results for the minimally SUSY background $AdS_5\times T^{1,1}$.

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2.2 $AdS_5 \times T^{1,1}$

In this section we discuss the background originally presented by Klebanov and Witten [39]. The field theory dual to this supergravity background has played an important role in the understanding of the AdS/CFT correspondence beyond the maximally supersymmetric context, see [40] for a review.

The metric is given by,

$$\begin{aligned} ds^2 &= ds^2(AdS_5) + L^2 ds^2_{T^{1,1}}, \\ ds^2_{T^{1,1}} &= \lambda_1^2(\sigma_{\hat{1}}^2 + \sigma_{\hat{2}}^2) + \lambda_2^2(\sigma_1^2 + \sigma_2^2) + \lambda^2(\sigma_3 + \cos\theta_1 d\phi_1)^2, \end{aligned} \tag{2.9}$$

where $\lambda^2 = \frac{1}{9}$, $\lambda_1^2 = \lambda_2^2 = \frac{1}{6}$ and

$$\begin{aligned} \sigma_{\hat{1}} &= \sin\theta_1 d\phi_1, \qquad \sigma_{\hat{2}} = d\theta_1, \\ \sigma_1 &= \cos\psi\, \sin\theta_2 d\phi_2 - \sin\psi\, d\theta_2, \quad \sigma_2 = \sin\psi\, \sin\theta_2\, d\phi_2 + \cos\psi\, d\theta_2, \\ \sigma_3 &= d\psi + \cos\theta_2\, d\phi_2. \end{aligned} \tag{2.10}$$

The metric of AdS_5 was written in eq. (2.2). The full background includes a self-dual RR five-form,

$$F_5 = \frac{4}{L}(d\mathrm{Vol}(AdS_5) - d\mathrm{Vol}(T^{1,1})). \tag{2.11}$$

We use the normalization

$$\frac{1}{(4\pi\alpha')^2}\int_{T^{1,1}} F_5 = N_{D3}. \tag{2.12}$$

The Einstein equations of motion then lead to

$$L^4 = \frac{27}{4}\pi\alpha'^2\, N_{D3}. \tag{2.13}$$

Let us study the action of NATD on one of the SU(2) isometries displayed by the background in eq. (2.9).

2.2.1 NATD of $AdS_5 \times T^{1,1}$

We now consider the NATD of the Klebanov-Witten background. The result was originally presented in [25, 28]. Unlike in those works, we will choose a gauge where $(v_1, v_2, v_3) \to (\rho, \chi, \xi)$. As above, the Lagrange multipliers will be written in spherical polar coordinates with the angles varying as, $0 \le \chi \le \pi$, $0 \le \xi \le 2\pi$. We start by presenting the expressions for the NS fields,

$$\begin{aligned} d\hat{s}^2 &= \frac{r^2}{L^2}dx_{1,3}^2 + \frac{L^2}{r^2}dr^2 + L^2\lambda_1^2(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2) \\ &\quad + \frac{\alpha'^2}{QL^2}\Big[(\lambda_2^4 L^4(d\rho\cos\chi - \rho d\chi\sin\chi)^2 + \lambda^2\lambda_2^2 L^4\left(\rho^2 d\chi^2\cos^2\chi + \rho d\rho d\chi\sin 2\chi\right. \\ &\quad \left. + \sin^2\chi\left(d\rho^2 + \rho^2\left(d\xi + d\phi_1\cos\theta_1\right)^2\right)\right) + \rho^2\alpha'^2 d\rho^2\Big], \\ QL^2\hat{B}_2 &= \frac{1}{2}\rho^2\alpha'^3\sin\chi\left(\left(\lambda^2 - \lambda_2^2\right)\sin 2\chi d\xi\wedge d\rho + 2\rho N d\xi\wedge d\chi\right) \\ &\quad -\lambda^2\alpha'\cos\theta_1\left(\cos\chi\left(\rho^2\alpha'^2 + \lambda_2^4 L^4\right)d\rho\wedge d\phi_1 - \lambda_2^4 L^4\rho\sin\chi d\chi\wedge d\phi_1\right), \\ e^{-2\hat{\Phi}} &= \frac{QL^2}{\alpha'^3}, \end{aligned} \tag{2.14}$$

with,

$$Q = \left(\lambda^2\lambda_2^4 L^4 + \rho^2\alpha'^2 N\right), \quad N = (\lambda^2\cos^2\chi + \lambda_2^2\sin^2\chi). \tag{2.15}$$

Although the range of the coordinate ρ has not been established yet, we notice that if it were compact — we will later argue that $0 \le \rho \le \pi$ — the quantity Q would then be bounded, leading to a completely smooth background. The RR fields are given by

$$\begin{aligned}
\hat{F}_2 &= \frac{4\lambda\lambda_1^2\lambda_2^2 L^4 \sin\theta_1 d\theta_1 \wedge d\phi_1}{\alpha'^{3/2}}, \\
Q\hat{F}_4 &= 2\lambda\lambda_1^2\lambda_2^2 L^2\rho^2\alpha'^{3/2}\sin\chi\sin\theta_1 d\theta_1 \wedge d\phi \wedge \Big[\left(\lambda^2 - \lambda_2^2\right)\sin 2\chi d\xi \wedge d\rho \\
&\quad + 2\rho N d\xi \wedge d\chi_1\Big], \\
\hat{F}_6 &= \frac{4\rho\sqrt{\alpha'}d\mathrm{Vol}(AdS_5) \wedge d\rho}{L}, \\
QL\hat{F}_8 &= 4\lambda^2\lambda_2^4\rho^2\alpha'^{3/2}\sin\chi d\mathrm{Vol}(AdS_5) \wedge d\rho \wedge d\chi \wedge (\cos\theta_1 d\phi_1 + d\xi).
\end{aligned} \tag{2.16}$$

The coordinate ξ, plays the role of the R-symmetry after NATD. Note, again, that we write the shifted F_p, in particular, $\hat{F}_4 = dC_3 - C_1 \wedge H_3$. Let us now move into our last case study.

2.3 $AdS_5 \times Y^{p,q}$

We will study here the action of NATD on the geometry $AdS_5 \times Y^{p,q}$. We will follow the conventions of [5, 41, 42]. We start by presenting the background,

$$\begin{aligned}
ds^2 &= ds^2(AdS_5) + L^2 ds^2(Y^{p,q}), \\
ds^2(Y^{p,q}) &= \frac{1-y}{6}(\sigma_1^2 + \sigma_2^2) + \frac{1}{wy}dy^2 + \frac{v}{9}\sigma_3^2 + w(d\alpha + f\sigma_3)^2,
\end{aligned} \tag{2.17}$$

where the σ's were defined above, and

$$w = \frac{2(b-y^2)}{1-y}, \quad v = \frac{b - 3y^2 + 2y^3}{b - y^2}, \quad f = \frac{v - 2y + y^2}{6(b-y^2)}, \tag{2.18}$$

with the quantity b given by,

$$b = \frac{1}{2} - \frac{(p^2 - 3q^2)}{4p^3}\sqrt{4p^2 - 3q^2}. \tag{2.19}$$

The ranges of α and y are $0 \le \alpha \le 2\pi l$, $y_1 \le y \le y_2$, where the numbers l, y_1, y_2 are,

$$y_1 = \frac{1}{4p}\left(2p - 3q - \sqrt{4p^2 - 3q^2}\right), \tag{2.20}$$

$$\begin{aligned}
y_2 &= \frac{1}{4p}\left(2p + 3q - \sqrt{4p^2 - 3q^2}\right), \\
l &= \frac{q}{3q^2 - 2p^2 + p\sqrt{4p^2 - 3q^2}}.
\end{aligned} \tag{2.21}$$

The self-dual RR flux is,

$$F_5 = \frac{4}{L}(d\mathrm{Vol}(AdS_5) - d\mathrm{Vol}(Y^{p,q})). \tag{2.22}$$

Again, we normalize the F_5

$$\frac{1}{(4\pi\alpha')^2}\int_{S^5} F_5 = N, \tag{2.23}$$

which leads to the relation between L and N, $L^4 = \frac{9\pi g_s N\alpha'^2}{2l(y_2^2-y_1^2+2(y_1-y_2))}$. We now proceed to apply a NATD on the SU(2) isometry parametrised by σ_i's.

2.3.1 NATD of $AdS_5 \times Y^{p,q}$

If we choose a gauge fixing such that we keep all Lagrange multipliers $(v_1,\ v_2,\ v_3)$ and, as we did above, we change coordinates to spherical polar coordinates $(\rho,\ \chi,\ \xi)$; where $v_1 = \rho\sin\chi\cos\xi,\ v_2 = \rho\sin\chi\sin\xi,\ v_3 = \rho\cos\chi$. We obtain,

$$\begin{aligned}
d\hat{s}^2 &= ds^2(AdS_5) + \frac{L^2}{vw}dy^2 + L^2k^2d\alpha^2 + \frac{\alpha'^2}{\Upsilon}\Big[6L^2\rho^2 m\sin^2\chi(h\,d\alpha + \sqrt{g}d\xi)^2 \\
&\quad + \left(\frac{36\alpha'^2}{L^2}\rho^2 + L^2m^2\cos^2\chi\right)d\rho^2 + L^2\rho m^2\sin\chi(\rho\sin\chi d\chi^2 - 2\cos\chi d\rho d\chi) \\
&\quad + 6L^2gm(\sin\chi d\rho + \rho\cos\chi d\chi)^2\Big], \\
\Upsilon\hat{B} &= \alpha'\Big[\sqrt{g}h\cos\chi(36\alpha'^2\rho^2 + L^4m^2)d\alpha\wedge d\rho - L^4\sqrt{g}hm^2\rho\sin\chi d\alpha\wedge d\chi \\
&\quad + \alpha'^2\rho^2\sin\chi\big(6(6g-m)\cos\chi\sin\chi d\xi\wedge d\rho + 6\rho(m\sin^2\chi + 6g\cos^2\chi)d\xi\wedge d\chi\big)\Big], \\
e^{-2\hat{\Phi}} &= \frac{L^2}{36\alpha'^3}\Upsilon.
\end{aligned} \tag{2.24}$$

Where $\Upsilon = g(36\alpha'^2\rho^2\cos^2\chi + L^4m^2) + 6\alpha'^2\rho^2 m\sin^2\chi$, and

$$g = \frac{v}{9} + wf^2, \quad h = \frac{wf}{\sqrt{g}}, \quad k = \sqrt{\frac{vw}{9g}}, \quad m = 1-y. \tag{2.25}$$

The RR fields will read,

$$\begin{aligned}
\hat{F}_2 &= \frac{2L^4m}{9\alpha'^{3/2}}d\alpha\wedge dy, \\
3\Upsilon\hat{F}_4 &= 4L^4\alpha'^{3/2}\rho^2 m\sin\chi dy\wedge d\alpha\wedge d\xi\wedge\Big[\cos\chi\sin\chi(m-6g)d\rho \\
&\quad -\rho(6g\cos^2\chi + m\sin^2\chi)d\chi\Big].
\end{aligned} \tag{2.26}$$

Had we chosen a gauge such that the Lagrange multipliers $(v_1,\ v_2,\ v_3)$ are changed to cylindrical polar coordinates $(\rho,\ \xi,\ x)$ with, $v_1 = \rho\sin\xi,\ v_2 = \rho\cos\xi$, we would have obtained the recent result of [38]. This completes the presentation of the three backgrounds and their NATD's. Below we will work with these new IIA backgrounds. Let us close the section with some general comments. First of all (as it is obvious), we have checked that all equations of motion (Einstein, Dilaton, Maxwell and Bianchi) are satisfied[1] The generated solutions are non-singular in the cases of the NATD of $AdS_5 \times T^{1,1}$ and $AdS_5 \times Y^{p,q}$.

Finally, as shown in [23], the lift to M-theory of the solution described around eq. (2.6) gives an eleven-dimensional background quite similar to the Gaiotto-Maldacena geometries [44]. The M-theory lift of the geometries presented in section 2.2.1 is in the same way, resemblant of the $\mathcal{N} = 1$ version of the Gaiotto T_N theories (as shown in [25, 28]). Both dual field theories can be thought to be connected by an RG flow induced by a relevant operator. It would be interesting to understand if there is a way to connect these with the solution presented in section 2.3.1. It would also be of interest to connect the Type IIA geometries with the solutions discussed by [45] and [46].

3 M-theory lifts of NATD of Freund-Rubin solutions

In this section we consider the M-theory lift of the solutions generated in the previous section. Given a IIA background composed of a metric in string frame ds^2_{10} and with potentials $C_{(1)}, B_{(2)}$ and $C_{(3)}$ one constructs the corresponding M-theory solution composed of ds^2_{11} and $C^M_{(3)}$ as follows [47]:

$$\begin{aligned} ds^2_{11} &= e^{-\frac{2}{3}\hat{\Phi}} ds^2_{10} + e^{\frac{4}{3}\hat{\Phi}} (dy + C_{(1)})^2, \\ C^M_{(3)} &= C^{IIA}_{(3)} + B_{(2)} \wedge dy, \qquad \text{or} \qquad F^M_{(4)} = F^{IIA}_{(4)} + H_{(3)} \wedge dy, \end{aligned}$$

Note that $F^{IIA}_{(4)}$ differs from the value quoted in previous sections precisely because it is the closed part of $F^{IIA}_{(4)} = dC_3$. In the previous section we wrote $\hat{F}_4 = F^{IIA}_{(4)} - C_1 \wedge H_3$ which is different from the one needed in this section.

One important aspect of any M-theory lifting is the fate of the M-theory circle as it geometrizes the coupling in the IIA frame. As can be seen above, the radius is proportional to $e^{4\hat{\Phi}/3}$. Since we are dealing with dimensionful quantities, we must introduce the 11D y coordinate with a length scale. In this section we recall that the natural scale is the 11-d Planck length, l_P, which is related to the string theory scale as $l_P = g_s^{1/3}\sqrt{\alpha'}$.

The history of solutions in M-theory containing an AdS_5 factor dates back to more than a decade ago starting with the work [48]. Interpretations in the context of wrapped M5 branes were subsequently systematically studied in [49]. More recently, attention to this type of solutions has surged as potential gravity dual to Giaotto's theories as discussed in [44]. Other aspects of these solutions including their origins as holographic RG flows has recently been presented in [50]. A more systematic approach to the construction of wrapped M5-branes with an AdS_5 factor in M-theory has been presented recently in [51]. We should also mention the works [52] and [53]–though the comparison with our backgrounds is difficult.

We hope that some of the solutions we present in this section might ultimately find a place in this bigger picture.

[1]This check is actually superfluous for the NATD of $AdS_5 \times S^5$, as it is proved that all equations of motion and Bianchi identities are implied when one dualises any solution with an SO(4) isometry on one of its SU(2) subgroups in [43].

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3.1 M-theory lift of NATD of $AdS_5 \times S^5$

Let us first consider the M-theory lift of the solution obtained by applying NATD to $AdS_5 \times S^5$. The resulting M-theory background is [23],

$$
\begin{aligned}
d\hat{s}^2_{11} &= e^{-\frac{2}{3}\hat{\Phi}} ds^2(AdS_5) + e^{\frac{4}{3}\hat{\Phi}}(dy - 2\frac{L^4\cos^4\alpha}{\alpha'^{3/2}}d\theta)^2 \\
&\quad + e^{-\frac{2}{3}\hat{\Phi}}\left(4L^2(d\alpha^2 + \sin^2\alpha d\theta^2) + \frac{\alpha'^2 d\rho^2}{L^2\cos^2\alpha} + \frac{\alpha'^2 L^2\rho^2\cos^2\alpha\left(d\xi^2\sin^2\chi + d\chi^2\right)}{\alpha'^2\rho^2 + L^4\cos^4\alpha}\right) \\
B_2 &= \frac{\alpha'^3\rho^3\sin\chi d\xi\wedge d\chi}{\alpha'^2\rho^2 + L^4\cos^4\alpha}, \quad e^{-2\hat{\Phi}} = L^2\cos^2\alpha\left(\frac{L^4\cos^4\alpha + \alpha'^2\rho^2}{\alpha'^3}\right), \\
F^M_{(4)} &= F^{IIA}_{(4)} + H_{(3)}\wedge dy, \quad H_{(3)} = dB_2 \\
F^{IIA}_{(4)} &= \frac{2L\alpha'^{3/2}\rho^2\sin\chi}{(\alpha'^2\rho^2 + L^4\cos^4\alpha)^2}\left(4L^3\alpha'^{3/2}\rho\left(L + \alpha'^{1/2}\rho^2\right)\cos^3\alpha\sin\alpha d\alpha\wedge d\theta\wedge d\xi\wedge d\chi\right. \\
&\quad \left. + \left(\alpha'^{3/2} - L^3\cos^4\alpha\right)\left(\alpha'^2\rho^2 + 3L^4\cos^4\alpha\right) d\theta\wedge d\xi\wedge d\rho\wedge d\chi\right)
\end{aligned}
\tag{3.1}
$$

The M-theory radius in this case, $e^{4\hat{\Phi}/3}$ above, behaves like a trumpet starting out at non-vanishing size and blowing up when $\cos\alpha = 0$. This singularity was already present in the IIA picture where it manifested itself as a curvature singularity.

3.2 M-theory lift of NATD of $AdS_5 \times T^{1,1}$

In this subsection we present the M-theory lift of the background obtained from applying NATD to $AdS_5 \times T^{1,1}$; the result is

$$
\begin{aligned}
\hat{ds}^2 &= e^{-\frac{2}{3}\hat{\Phi}}\left[\frac{r^2}{L^2}dx^2_{1,3} + \frac{L^2}{r^2}dr^2 + L^2\lambda_1^2(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2)\right. \\
&\quad + \frac{\alpha'^2}{QL^2}\left(\lambda_2^4 L^4(d\rho\cos\chi - \rho d\chi\sin\chi)^2 + \lambda^2\lambda_2^2 L^4\left(\rho^2 d\chi^2\cos^2\chi + \rho d\rho d\chi\sin 2\chi\right.\right. \\
&\quad \left.\left.\left. + \sin^2\chi(d\rho^2 + \rho^2(d\xi + d\phi_1\cos\theta_1)^2)\right) + \rho^2\alpha'^2 d\rho^2\right)\right] \\
&\quad + e^{\frac{4}{3}\hat{\Phi}}\left(dy - \frac{4\lambda\lambda_1^2\lambda_2^2 L^4}{\alpha'^{3/2}}\cos\theta_1 d\phi_1\right)^2, \\
QL^2\hat{B}_2 &= \frac{1}{2}\rho^2\alpha'^3\sin\chi\left((\lambda^2 - \lambda_2^2)\sin 2\chi d\xi\wedge d\rho + 2\rho N d\xi\wedge d\chi\right) \\
&\quad - \lambda^2\alpha'\cos\theta_1\left(\cos\chi\left(\rho^2\alpha'^2 + \lambda_2^4 L^4\right) d\rho\wedge d\phi_1 - \lambda_2^4 L^4\rho\sin\chi d\chi\wedge d\phi_1\right), \\
e^{-2\hat{\Phi}} &= \frac{QL^2}{\alpha'^3}.
\end{aligned}
\tag{3.2}
$$

The fluxes are:

$$
\begin{aligned}
F^M_{(4)} &= F^{IIA}_{(4)} + H_{(3)}\wedge dy, \quad H_{(3)} = dB_2 \\
QF^{IIA}_{(4)} &= 4L^4\alpha'^{3/2}\lambda\rho^2\cos\chi\sin\theta_1\sin^2\chi\lambda_1^2\lambda_2^2\left(\lambda^2 - \lambda_2^2\right) d\theta_1\wedge d\xi\wedge d\rho\wedge d\phi_1 \\
&\quad - 4L^4\alpha'^{3/2}\lambda\rho^3\sin\theta_1\sin\chi\lambda_1^2\lambda_2^2\left(\lambda^2\cos^2\chi + \sin^2\chi\lambda_2^2\right) d\theta_1\wedge d\xi\wedge d\phi_1\wedge d\chi
\end{aligned}
$$

$$
-2L^4\alpha'^{3/2}\lambda^3\rho^2\cos\theta_1\sin\chi\lambda_1^2\lambda_2^2 d\xi\wedge d\rho\wedge d\phi_1\wedge d\chi\Big[-2\alpha'^2\lambda^2\rho^2\cos^2\chi
+\alpha'^2\rho^2(3+\cos 2\chi)\lambda_2^2+2L^4\lambda_2^4(\lambda^2+2\lambda_2^2)\Big] \tag{3.3}
$$

Let us pay particular attention to the M-theory radius

$$
R_{11}=e^{\frac{2}{3}\hat{\Phi}}\sim\left(\lambda^2\lambda_2^4+\rho^2\alpha'^2(\lambda^2\cos^2\chi+\lambda_2^2\sin^2\chi\right)^{-1/3}. \tag{3.4}
$$

One really remarkable aspect of this solution is the fact that the M-theory radius is bounded above and below. The means that the solutions is a completely smooth 11d supergravity background. It would be interesting to study this background in more detail and its field theory dual.

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3.3 M-theory lift of NATD of $AdS_5\times Y^{p,q}$

In this subsection we denote $\tilde{y}$ the M-theory circle to avoid confusion with the y-coordinate originally defined in $Y^{p,q}$. The M-theory lifts reads [38],

$$
\begin{aligned}
ds_{11}^2 &= e^{-\frac{2}{3}\hat{\Phi}}ds_{10}^2+e^{\frac{4}{3}\hat{\Phi}}\left(d\tilde{y}-\frac{2L^4}{9\alpha'^{3/2}}y\,d\alpha\right)^2,\\
ds_{10}^2 &= ds^2(AdS_5)+\frac{L^2}{vw}dy^2+\frac{1}{\Sigma}\bigg(\frac{\alpha'^2}{L^2}(36x^2\alpha'^2+L^4m^2)dx^2+L^2(6\alpha'^2\rho^2h^2+\Sigma k^2)d\alpha^2\\
&\quad -12L^2\alpha'^2\rho^2\sqrt{g}hm\,d\alpha\, d\xi+\frac{6\alpha'^2}{L^2}\left(6\alpha'^2\rho^2d\rho^2+L^4gm(\rho^2d\xi^2+d\rho^2)+\frac{12\alpha'^2}{L^2}x\rho\, dx\, d\rho\right)\bigg),\\
B_2 &= \frac{\alpha'}{\Sigma}(6\alpha'^2\rho^2m\,d\xi\wedge dx+\sqrt{g}h((36x^2\alpha'^2+L^4m^2)d\alpha\wedge dx+36x\alpha'^2\rho\, d\alpha\wedge d\rho)\\
&\quad +36x\alpha'^2\rho g\, d\rho\wedge d\xi),\\
e^{-2\hat{\Phi}} &= \frac{L^2}{36\alpha'^3}\Sigma
\end{aligned} \tag{3.5}
$$

where $\Sigma=6\alpha'^2\rho^2m+g(36\alpha'^2x^2+L^4m^2)$. The 4-form field strength is given by

$$
\begin{aligned}
F_{(4)}^M &= F_{(4)}^{IIA}+H_{(3)}\wedge dy,\quad H_{(3)}=dB_2\\
3F_{(4)}^{IIA} &= -\frac{4L^4\alpha'^{3/2}\rho^2gm^2\sin\chi\, d\alpha\wedge d\xi\wedge d\rho\wedge d\chi}{3\left(g\left(36\alpha'^2\rho^2\cos^2\chi+L^4m^2\right)+6\alpha'^2\rho^2m\sin^2\chi\right)^2}\left(9\alpha'^2\rho^2(3+\cos 2\chi)m\right.\\
&\quad \left.+L^4m^3+3gg\left(-36\alpha'^2\rho^2\cos^2\chi+L^4m^2\right)\right)\\
&\quad +\frac{2L^4\alpha'^{3/2}\rho^3m\sin\chi\, dy\wedge d\alpha\wedge d\xi\wedge d\chi}{\left(3\left(g\left(36\alpha'^2\rho^2\cos^2\chi+L^4m^2\right)+6\alpha'^2\rho^2m\sin^2\chi\right)^2\right)}\left(432\alpha'^2\rho^2\cos^4\chi g^2\right.\\
&\quad \left.+12\alpha'^2\rho^2m^2\sin^4\chi+g\left(L^4m^3\sin^2\chi+36\alpha'^2\rho^2m\sin^2 2\chi\right)+L^4m^4g'\sin^2\chi\right)\\
&\quad -\frac{2L^4\alpha'^{3/2}\rho^2\cos\chi m\sin^2\chi\, dy\wedge d\alpha\wedge d\xi\wedge d\rho}{3\left(g\left(36\alpha'^2\rho^2\cos^2\chi+L^4m^2\right)+6\alpha'^2\rho^2m\sin^2\chi\right)^2}\left(-432\alpha'^2\rho^2\cos^2\chi g^2\right.\\
&\quad \left.+gm\left(36\alpha'^2\rho^2(1+2\cos 2\chi)+L^4m^2\right)+m^2\left(12\alpha'^2\rho^2\sin^2\chi+g'\left(36\alpha'^2\rho^2+L^4m^2\right)\right)\right)
\end{aligned}
$$

One important aspect of this background is its M-theory radius, given roughly by $\Sigma^{-1/3}$; this quantity is well-behaved leading to a potential interpretation in the context of M-theory. The M-theory backgrounds presented in this section have been previously presented in [23, 25, 38].

4 Page charges in gravity and field theory central charge

In this section, we will study a proposal to determine the range of the ρ-coordinate after NATD. Also, we will discuss two quantities of physical importance, Page charges and central charge. We will apply our results to the three backgrounds in section 2

4.1 Global properties and Page charges

Non-Abelian T-duality, as well as regular Abelian T-duality, is an intrinsically local transformation. As mentioned, we have checked explicitly in all the backgrounds presented in this paper that the equations of motion are satisfied but we have no rule or intuition for determining the range of coordinates in the backgrounds.

In this subsection we discuss a global issue that has haunted non-Abelian T-duality for some time (see [54] for early studies and [31, 36] for recent discussions). One of the difficulties with the interpretation of the backgrounds is the lack of knowledge of the range of the coordinates after NATD. The prescription we adopt is *the same* as the one presented in [36], but we are applying it in a background without singularities, making the procedure more trustable.[2] Hence this is another example of the prescription provided in [36].

We impose bounds on the integral via, $4\pi^2\alpha' b_0 = \int_{\Sigma_2} B_2 \in [0,1]$, where Σ_2 is a suitably chosen two-manifold. As we will discuss in our examples, this condition will imply a bound on the range of the 'radial' coordinate ρ. The internal space after the NATD will be compact. When restricting the B_2-field to the manifold Σ_2 and computing the quantity b_0, it will be periodically identified as $b_0 \sim b_0+n$ when we perform a large gauge transformation

$$B_2 \to B_2 + n\pi\alpha'\Omega_2. \tag{4.1}$$

Here Ω_2 is a closed two-form non-vanishing asymptotically. This condition will imply that ρ varies in $[n\pi, \pi(n+1)]$. The range of the radial coordinate is 'quantised'. Let us present various motivations for this condition.

- String theory has the power to quantize certain symmetries, while supergravity generically lacks such power. The prototypical example is $\mathrm{SL}(2,\mathbb{Z})$ versus $\mathrm{SL}(2,\mathbb{R})$. We are using this when imposing that $b_0 \sim b_0 + 1$.
- The condition $\frac{1}{4\pi^2\alpha'}\int_{\Sigma_2} B_2 \in (0,1)$ comes also from the quantization of the string action, $\exp\left(\frac{i}{4\pi^2\alpha'}\int_{\Sigma_2} B_2\right)$, as part of the string path integral. This is similar to what happens in quantum mechanics when coupling particles to a gauge field A_μ.

[2] We thank Yolanda Lozano for various discussions on this point.

- For the case of $AdS_5 \times T^{1,1}$, in the dual field theory, this condition is typically related to a linear combination of gauge couplings. Therefore, we are imposing that they remain well defined under certain transformations of the rank of various gauge groups.

We could, ultimately, disregard the string-theoretic motivations and accept the prescription as a way of completing the supergravity background. Note, and this is crucial, that we do not require a stringy object to form part of our background which remains strictly a supergravity one; we merely use string intuition to propose a way to impose global information on the local solution provided by NATD.

To gain further intuition into the implications of this condition and to relate it to the Page charges of the background, we can compare it with a somewhat similar situation taking place in the *cascade* of the Klebanov-Tseytlin-Strassler system [55–57].[3] Let us recall some aspects of the Klebanov-Tseytlin-Strassler [55, 56] pair, a quiver field theory with gauge group $\mathrm{SU}(kM) \times \mathrm{SU}(kM + M)$ and its dual Type IIB solution. In that background, one computes the Page charges for D3 and D5 branes and obtains,

$$Q_{P,D3} = 0, \quad Q_{P,D5} = M. \tag{4.2}$$

Restricting the B_2 field to a two-cycle and performing a large gauge transformation of the B_2 field, one obtains [58] the effect equivalent to a Seiberg duality when flowing to the IR. The change in the Page charges is:

$$\Delta Q_{P,D5} = 0, \quad \Delta Q_{P,D3} = -M. \tag{4.3}$$

One of the roles of the Page charges in the context of the AdS/CFT is to encode the information about Seiberg dualities.

In summary, we will impose that under large gauge transformations of the B_2-field, $b_0 \sim b_0 + 1$. This will imply the quantisation of changes in the ρ-coordinate. The Page charges will change accordingly, hence suggesting a form of Seiberg duality in our conformal field theories. We will study this in each of the backgrounds presented in section 2. It is worth pointing out that a sigma model approach to this problem was pursued recently in [59], where it was shown for a particular example that coordinates after NATD are indeed compact.

It is worthwhile to mention another viewpoint one can adopt. We may think that translations in the ρ-coordinate that increase $b_0 \to b_0 + n$ can be 'undone' by a large gauge transformation of the B_2-field. This mixing between metric and B-field points to a possible understanding in terms of non-geometric backgrounds [60].

4.1.1 Page charges for NATD of $AdS_5 \times S^5$

In the following we will use,

$$2\kappa_{10}^2 = (2\pi)^7 \alpha'^4, \quad T_{Dp} = \frac{1}{(2\pi)^p \alpha'^{\frac{p+1}{2}}}.$$

[3] In the cases analyzed in this paper, we do not have a flow in energies, we are moving in the ρ direction. We are proposing that motions in the ρ-coordinate correspond to Seiberg dualities between CFT's, all equivalent to each other. We will return to some aspects of the field theory dual in section 7.

For the NATD dual of $AdS_5 \times S^5$ described in section 2.1.1, we compute the Page charges using the definitions,

$$Q_{P,D6} = \frac{1}{2\kappa_{10}^2 T_{D6}} \int_{\Sigma_2} (F_2 - B_2 F_0) = N_{D6},$$
$$Q_{P,D4} = \frac{1}{2\kappa_{10}^2 T_{D4}} \int_{\Sigma_4} (F_4 - B_2 \wedge F_2) = N_{D4} = 0. \tag{4.4}$$

Since $F_0 = 0$, the Page charge involving quantization of the number of D6-branes, N_{D6}, amounts to quantizing the F_2, leading to a constraint from the SUGRA equations that determines the radius of the space after NATD. Flux quantisation imposes, after NATD, the relation

$$L^4 = \frac{1}{2} N_{D6} \alpha'^2 \Rightarrow Q_{D6} = N_{D6}. \tag{4.5}$$

We then consider the S^2 spanned by χ, ξ as this is where B_2 has legs. If we further restrict to $\alpha = \frac{\pi}{2}$ the NS two form reduces to

$$B_2 = \alpha' \rho Vol(S^2). \tag{4.6}$$

If we examine the space spanned by (α, χ, ξ) close to $\alpha = \pi/2$ we find that is conformally a singular cone with boundary S^2. This is reminiscent of what was found for the NATD of $AdS_4 \times \mathbb{CP}^3$ in [36] and shows that S^2 is indeed a cycle. Now consider a large gauge transformation in B_2 of the form:

$$B_2 \to B_2 + n\pi\alpha' \sin\chi d\xi \wedge d\chi, \tag{4.7}$$

and we calculate the change in the Page charges to be,

$$\Delta Q_{D6} = 0, \quad \Delta Q_{D4} = -nN_{D6},$$

where we have used the relation $L^4 = \frac{1}{2} N_{D6} \alpha'^2$ found above. Hence, a large gauge transformation leaves untouched the number of D6's charge, but changes the charge associated with D4-branes. Note the striking similarity with the case of the KS-cascade, summarized around eqs. (4.2)–(4.3), in the case $n = 1$ for the gauge transformation in eq. (4.7).

Integrating B_2 with $\alpha = \frac{\pi}{2}$ gives,

$$b_0 = \frac{1}{4\pi^2\alpha'} \int_{S^2} B_2 = \frac{\rho}{\pi}. \tag{4.8}$$

The periodic identification of b_0 would imply that the dual QFT should be identified as we change $\rho \sim \rho + \pi$. Coming back to the ideas exposed above, we could associate translations in the ρ-coordinate with an operation similar to Seiberg duality — in the conformal case for the example at hand. It may seem strange the presence of dualities between CFTs that change the CFT and some of its observables. As a toy example we may think of $\mathcal{N} = 1$ SQCD for $N_f = 2N_c + \epsilon$. the theory is not self dual, both electric and magnetic theories are conformal in the IR.

We will study below similar structures in the case of $AdS_5 \times T^{1,1}$ and $AdS_5 \times Y^{p,q}$.

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4.1.2 Page charges for NATD of $AdS_5 \times T^{1,1}$

Let us apply the previous analysis of Page charges and their changes to our background. The Page charge for D6 and D4 branes and their respective changes under a large gauge transformation in the B_2-field, as indicated in eq. (4.1). For the NATD applied to the Klebanov-Witten background, we have,

$$F_2 - F_0 B_2 = \frac{4L^4 \lambda_1^4 \lambda}{g_s \alpha'^{3/2}} \sin\theta_1 d\theta_1 \wedge d\phi_1, \quad F_4 = B_2 \wedge F_2. \tag{4.9}$$

This result leads to the following values for the Page charges,

$$Q_{P,D6} = N_{D6}, \quad Q_{P,D4} = 0. \tag{4.10}$$

Just like in the case of S^5, imposing the quantization of the Page charge of D6-branes implies for the radius of the space

$$L^4 = \frac{27}{2} N_{D6} \alpha'^2. \tag{4.11}$$

We will choose a two-submanifold,[4] $\Sigma_2 = [\phi_1 = 2\pi - \xi, \chi]$, for constant values of ρ and $\theta_1 = 0$. We evaluate the B_2 field on the sub-manifold, i.e

$$B_2|_{\Sigma_2} = \alpha' \rho \sin\chi d\chi \wedge d\xi. \tag{4.12}$$

Like above, we perform a large gauge transformation,

$$\Delta B_2 = -n\pi\alpha' \sin\chi d\chi \wedge d\xi. \tag{4.13}$$

Using this, we find that under large gauge transformations of the B_2-field, the Page charges change according to,

$$\Delta Q_{P,D6} = 0, \quad \Delta Q_{P,D4} = -nN_{D6}. \tag{4.14}$$

Here again, large gauge transformations are linked with an operation similar to Seiberg duality in the conformal field theory dual to our background.

Finally, imposing the identification of the quantity b_0, under the same gauge transformations, we find that

$$b_0 = \frac{1}{4\pi^2\alpha'} \int_{\Sigma_2} B_2 = \frac{\rho}{\pi}, \tag{4.15}$$

which again suggest that the field theory description should change — with a Seiberg duality; for example — every time we change the coordinate $\rho \to \rho + \pi$ in the dual background. Here again, a motion in ρ could be undone by a suitable large gauge transformation, suggesting a non-geometric interpretation of the background. Let us briefly summarize the same calculations for the case of the NATD of $AdS_5 \times Y^{p,q}$.

[4] It would be interesting to have a criterium to select this particular manifold, like the one discussed above eq. (4.7)– see [36].

4.1.3 Page charges for NATD of $AdS_5 \times Y^{p,q}$

We will be quite brief here, as the results are very similar to those discussed above. In this case, the two cycle of interest is $\Sigma_2 = [y, \alpha]$ with $\alpha = 2\pi - \xi$. As noted in [38], we must also take $y = y_0$, where y_0 is a solution of $h = \sqrt{g}$, with h and g functions defined in eq. (2.25).

The quantization of the Page charge of D6 branes implies a relation,

$$L^4 = \frac{9}{2l(y_2^2 - y_1^2 + 2(y_1 - y_2))} N_{D6}\alpha'^2 \Rightarrow Q_{D6} = N_{D6}, \quad Q_{D4} = 0. \tag{4.16}$$

Just like above, applying a large gauge transformation, like the one of eq. (4.7), implies that

$$\Delta Q_{D6} = 0, \quad \Delta Q_{D4} = -nN_{D6}. \tag{4.17}$$

Similarly, periodic identification on b_0 implies that the ρ-coordinate is divided in 'domains' and should be identified $\rho \sim \rho + \pi$.

The structure we have identified in these three examples is quite similar. The Page charge of D6-branes is quantized. The Page charge of D4-branes is zero. It is possible to identify a two dimensional submanifold, where the B_2 field takes a simple form. Large gauge transformations change the Page charge of D4 branes in a multiple of the original D6 charge. This suggests a form of duality between CFTs when moving in ρ. Besides, imposing the identification $b_0 \sim b_0 + 1$, the characteristic of large gauge transformations also implies that the ρ-coordinate is also identified[5] $\rho \sim \rho + \pi$. This also suggest that under changes in the ρ direction, the field theory undergoes a form of Seiberg-like transformation. The bounds on the ρ coordinate are quite welcomed. Indeed, a KK reduction to five dimensions would lead to a continuous spectrum of operators if ρ had infinite range.

We will now move to the study of another observable, quite important in the understanding of the dual conformal field theory; the central charge. Relations with the Entanglement Entropy will also be discussed.

4.2 On central charge and entanglement entropy

The fate of some physical observables under solution generating techniques is an important question. Quantities such as temperature and entropy have been shown to be frame-invariant under solution generating techniques applied to black holes and black branes [9]. In the context of the AdS/CFT, where geometric backgrounds encode defining properties of the field theory dual, the question of invariance of observables under frame changing transformations becomes an important one. In this section we will focus on the behavior of the central charge and we will also comment on the expressions defining the holographic entanglement entropy under Abelian and Non-Abelian T-dualities.

The general prescription for the calculation of the field theory central charge was introduced by Henningson and Skenderis in [61]. More directly related to our context are [6, 62] and the pedagogically lucid account of [63]. Although completely consistent with the various presentations mentioned above, in this manuscript we will follow, in particular,

[5] We are not saying that the background is periodic in the coordinate ρ, but that every time that we pass the position $\rho = n\pi$ in the String Theory we should change the CFT description.

a slightly more general analysis due to [64]. Our goal is to compute simultaneously the central charge of the dual field theory and the entanglement entropy of a slab region.

Let us first summarize briefly the treatment of [64]. These authors considered a generic metric in type II string theory dual to a putative QFT in $(d+1)$-dimensions. In string frame, this reads

$$ds^2 = a dz_{1,d}^2 + ab dr^2 + g_{ij} d\theta^i d\theta^j. \tag{4.18}$$

In general, there is a dilaton Φ as part of the background. The functions a, b are typically functions of the radial coordinate r, but this is not necessarily the case and it will not always be the case for us. The paper [64] defines,

$$\hat{H} = e^{-4\Phi} V_{\text{int}}^2 a^{d/2}, \qquad V_{\text{int}} = \int d\theta^i \sqrt{\det g_{ij}}. \tag{4.19}$$

With these definitions the integral defining the EE (this is the area of an eight-manifold) that includes the internal space, the d z-coordinates and where r is a function of one of the z-coordinates (that denotes the separation between the entangled regions).

We introduce a sensible modification to the prescription of [64]. Namely, it may be the case that the function a does depend on the internal coordinates $\vec{\theta^i}$; this possibility was not considered in [64]. In that case, we define

$$\hat{V}_{\text{int}} = \int d\vec{\theta} \sqrt{e^{-4\Phi} \det[g_{\text{int}}] a^d}, \tag{4.20}$$

so that the function $\hat{H}$ is in general given by,

$$\hat{H} = \hat{V}_{\text{int}}^2. \tag{4.21}$$

Then, the central charge for a QFT in $(d+1)$ spacetime dimensions is defined to be [64]:

$$c = d^d \frac{b^{d/2} \hat{H}^{(2d+1)/2}}{G_N (\hat{H}')^d}. \tag{4.22}$$

where $G_N = (l_p)^{D-1} = \alpha'^{\frac{D-1}{2}}$ for D space-time dimensions. The G_N factor is needed to cancel the length dimensions in $\hat{H}$. We now apply these definitions to the calculation of the functions $a, b, \hat{H}$ for the different backgrounds discussed in section 2. Importantly, we will require that integrals over the internal ρ-coordinate after NATD is bounded between $[0, \pi]$. We will present details of the calculations for the S^5 case, and simply report the results in the T^{11} and $Y^{p,q}$ cases.

The point we want to make with these results is that in all cases, the central charge before and after the NATD, behaves as N_c^2, where N_c is the number of relevant branes before and after the duality (that is $N_c = N_{D3}$ before and $N_c = N_{D6}$ after the NATD). The numerical coefficients are changed by the duality. This we interpret as a change in the field theory dual to each background.

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4.2.1 Central charge for $AdS_5 \times S^5$ and its NATD

In this case, the functions referred to in eq. (4.18) characterizing the system are,

$$\alpha = \frac{4r^2}{L^2}, \quad b = \frac{L^4}{r^4}, \quad d = 3. \tag{4.23}$$

A straightforward calculation leads to

$$\hat{H} = (16L)^4 \pi^6 r^6, \qquad c = \frac{32\pi^3 L^8}{\alpha'^4} = 2\pi^5 N_{D3}^2 \tag{4.24}$$

After the duality, we have the same a, b, d as in eq. (4.23). The quantity $\hat{V}_{\text{int}}$ can then be computed to be

$$\hat{V}_{\text{int}} = \int \sqrt{e^{-4\Phi} \det(g) a^3} = \frac{64}{3} \pi^5 L^2 r^3, \tag{4.25}$$

where we remind the reader that the domain of ρ is $[0, \pi]$.

Using this and the relation between L and N_{D6}, we find that after the NATD,

$$\hat{H} = \frac{1}{9}(8L)^4 \pi^{10} r^6, \qquad c = \frac{8\pi^5 L^8}{3\alpha'^4} = \frac{2}{3}\pi^5 N_{D6}^2. \tag{4.26}$$

We observe the usual gauge-theoretic dependence with the number of degrees of freedom, but we also point out that the coefficient is different, suggesting that the dual field theory has changed by the effect of the NATD.

4.2.2 Central charge for $AdS_5 \times T^{1,1}$ and its NATD

In this case, before and after the duality the functions and parameter a, b, d are the same as those written in eq. (4.23). Before the duality, we find,

$$\sqrt{e^{-4\hat{\Phi}} \det(g) a^3} = L^2 r^3 \lambda \lambda_1^2 \lambda_2^2 \sin\theta_1 \sin\theta_2, \tag{4.27}$$

after straightforward operations we obtain

$$c_{KW} = \frac{\pi^3 L^8}{27\alpha'^4} = \frac{27}{8}\pi^5 N_{D3}^2. \tag{4.28}$$

After the NATD, we find

$$\sqrt{e^{-4\hat{\Phi}} \det(g) a^3} = L^2 r^3 \rho^2 \lambda \lambda_1^2 \lambda_2^2 \sin\theta_1 \sin\chi. \tag{4.29}$$

and performing integrals and straightforward algebra, we find

$$c_{\text{NATDKW}} = \frac{2L^8 \pi^5 \lambda \lambda_1^2 \lambda_2^2}{3\alpha'^4} = \frac{9}{8}\pi^5 N_{D6}^2. \tag{4.30}$$

Again, emphasizing on the point that the coefficient differences between eqs. (4.28) and (4.30) can be understood as NATD changing the dual QFT.

4.2.3 Central charge for M-theory lift of $AdS_5 \times S^5$

For the case of the M-theory lift of $AdS_5 \times S^5$, we compute the Page charge of F_4 found in eq. (3.1),

$$Q_{P,M5} = \frac{1}{2\kappa_{11}^2 T_{M5}} \int_{\Sigma_4} F_4 = N_{M5}, \tag{4.31}$$

where here, $\kappa_{11} = (2\pi)^4 \alpha'^9$ and $T_{M5} = \frac{1}{(2\pi)^5 \alpha'^3}$. We consider the submanifold, Σ_4, with θ a constant and α is suitably chosen after integration. (Note that α cannot be set to $\frac{\pi}{2}$ or the volume will vanish.) We also exploit an ambiguity in the uplifting procedure in which we introduce the y coordinate with a scaling factor of $(\frac{L^2}{\alpha'})^\gamma \sqrt{\alpha'}$ (instead of just $\sqrt{\alpha'}$, or l_P). Then, after imposing the charge quantization above, we find that

$$L^4 = 2^{\frac{8}{\gamma}} (N_{M5})^{\frac{2}{\gamma}} \alpha'^2. \tag{4.32}$$

In order to compute the central charge one needs the functions $a = e^{-\frac{2}{3}\hat{\Phi}} L^2 \frac{R^2}{L^4}$ and $b = \frac{L^4}{R^4}$. The determinant factor of the internal metric is,

$$\sqrt{\det(g_{\text{int}}) a^3} = 32 L^{2(1+\gamma)} \alpha'^{\frac{1}{2}-\gamma} R^3 \rho^2 \cos^3\alpha \sin\alpha \sin\chi \tag{4.33}$$

Then one has

$$V_{\text{int}} = \frac{128}{3} \pi^6 R^3 L^{2(1+\gamma)} \alpha'^{\frac{1}{2}-\gamma}.$$

where we have assumed the range of y to be $0 \leq y \leq 2\pi$. Using $G_{11} = \alpha'^{\frac{9}{2}}$, we compute the central charge to therefore be,

$$c = \frac{16\pi^6}{3} \left(\frac{L^2}{\alpha'}\right)^{4+\gamma}. \tag{4.34}$$

Now using the condition on L from above we find that

$$c = \frac{2^{8\left(1+\frac{2}{\gamma}\right)}}{3} \pi^6 (N_{M5})^{1+\frac{4}{\gamma}}. \tag{4.35}$$

For $\gamma = 4$, $c \sim (N_{M5})^2$, while for $\gamma = 2$, $c \sim (N_{M5})^3$. It is interesting that the $\gamma = 4$ scaling leaves the central charge invariant after the lift while $\gamma = 2$ takes the system into a field theory that is similar to what is expected of the Gaiotto type theories.

4.2.4 Brief comments on Entanglement Entropy

The Holographic Entanglement Entropy is a very interesting observable. It can be calculated by solving a minimization problem for an eight-manifold that hangs from radial infinity. There are many analogies and important differences with the calculation for Wilson loops [65]. After the usual manipulations with a Hamiltonian system we obtain two formulas for the Entanglement Entropy and the separation between the two entangled regions. They can be written in terms of r_*, the minimal radial position of the hanging eight-manifold. They read,

$$L_{EE}(r_*) = 2\sqrt{\hat{H}(r_*)}\int_{r_*}^{\infty} \frac{\sqrt{\beta(r)}}{\sqrt{\hat{H}(r)-\hat{H}(r_*)}}dr, \tag{4.36}$$

$$\frac{2G_{10}}{V_3}S_{EE}(r_*) = \int_{r_*}^{r_{UV}} \frac{\sqrt{\beta(r)}H(r)}{\sqrt{\hat{H}(r)-\hat{H}(r_*)}}. \tag{4.37}$$

There is, as in the case of Wilson loops, a substraction procedure. This motivated the upper limit r_{UV} in the integral defining the Entanglement Entropy.

As observed above, in all of our examples, before and after the NATD the functions $\alpha(r), \beta(r)$ are the same. The changes occur in the internal volume $\hat{V}_{\rm int}$ and consequently in $\hat{H}$.

It is clear that the dependence of S_{EE} on the separation L_{EE} will be the same and driven by conformal invariance. The differences will be in coefficients appearing in the function $\hat{H}(r)$, due to differences in the volume of the internal manifold. Compare for example the function $\hat{H}(r)$ for the case of $AdS_5 \times S^5$, before and after the NATD, as calculated in eqs. (4.24)–(4.26).

4.3 Quasi frame independence of the central charge volume form

After having considered the examples most relevant to this manuscript, we pose the question pertaining to the invariance of the functional form of quantities such as the central charge and Entanglement Entropy under Abelian and Non-Abelian T-dualities.

The question naturally arises in the context of solution-generating techniques applied to backgrounds describing black holes and intersecting branes. In fact, in this section we borrow heavily from an analysis of Horowitz and Welch [66]. We consider the effects of performing Abelian and Non-Abelian T-duality on the field theory central charge. Let us start with the definition of central charge presented in eq. (4.22). The key functions to focus on are,

$$H = \hat{V}_{\rm int}^2, \qquad \hat{V}_{\rm int} = \int d\vec{\theta} e^{-2\Phi} \det \sqrt{g_{\rm int}} a^{3/2}. \tag{4.38}$$

It is easy to show that this combination is invariant under Abelian T-duality. Namely, recall that under Abelian T-duality, the Büscher rule imply,

$$\tilde{g}_{xx} = 1/g_{xx}, \quad \tilde{\phi} = \phi - \frac{1}{2}\ln g_{xx}. \tag{4.39}$$

Therefore,

$$e^{-2\tilde{\phi}}\sqrt{\tilde{g}_{xx}} = e^{-2\phi + \ln g_{xx}}\sqrt{\frac{1}{g_{xx}}} = e^{-2\phi}\sqrt{g_{xx}}. \tag{4.40}$$

The above argument helps us establish that the *central charge volume form* is invariant under Abelian T-duality. It still leaves us with the daunting question of what is the range of integration. We need to access global information in the form of range of coordinates to be able to conclusively establish that the central charge of the field theory is frame independent.

In the NATD case a similar argument can be constructed albeit with more complicated expressions. Note, for example that the dilaton transforms as,

$$\hat{\Phi} = \Phi - \frac{1}{2}\ln\left(\frac{\det M}{\alpha'^3}\right). \tag{4.41}$$

The complete form of the NATD transformation is given in appendix A. One can verify that the central charge volume form, that is, the un-integrated expression for $d\hat{V}_{\rm int}$ is not invariant under NATD. But there is in all cases a very interesting cancellation between the terms $e^{-4\Phi}$ and $\det[g_{\rm int}]$. Indeed, this was also observed in the case in which we flow away from the fixed point in [25]. A general proof of this fact requires certain identities of the seed B-field which was zero in our cases. Let us move now to a different, more geometrical aspect of our study.

5 New solutions in IIB via NATD and T-duality

In the next two sections we will switch our focus a bit. Indeed, we will move into a more geometrical part of our paper. We will present *new* solutions of Type IIB Supergravity. These solutions as we anticipated in the Introduction, will be *singular*. In some of the cases discussed below, they will be SUSY preserving, in some other cases families of solutions will be presented, but in all cases our new solutions will present an AdS_5 factor and will avoid presently known classifications [6].

The guiding logic will be the following: we will start with the NATD of the $AdS_5 \times X^5$ backgrounds discussed in detail in section 2. It was shown that they are SUSY preserving and in most cases non-singular. We will then apply a T duality (or first shift one of the coordinates with a parameter γ and then apply a T-duality). Our procedure is guided by the Lunin-Maldacena T-s-T transformations [10]. We have checked in sections 5.1–5.3 and section 5.6 below, that the Einstein, Maxwell, dilaton and Bianchi equations are satisfied.

One point of interest for the solutions presented in the following section will be to understand the field theory meaning of this class of backgrounds, where the five-form flux vanishes but that still contain and AdS_5 factor. The natural tendency would be to interpret these backgrounds as in the class of wrapped D5 branes but we will argue that the answer is more subtle. We present other Type IIA solutions in appendix B.2 where we extend these type of backgrounds by keeping some of the parameters involved in the procedure of NATD.

The figure 1 summarizes the idea of the procedure advocated above. Let us present below our new backgrounds.

5.1 T-dual on the ξ-direction of the NATD of $AdS_5 \times S^5$

In this section we present the results of performing an Abelian T-duality on the ξ angle of the background in section 2.1.1, carrying through explicitly the appropriate powers of α'

$$
\begin{array}{ccccc}
AdS_5 \times X_5 & \xrightarrow{NATD} & AdS_5 \tilde{\times} Y_5 & \xrightarrow{T-duality} & AdS_5 \tilde{\times} W_5 \\[2ex]
F_5 & \xrightarrow{NATD} & (F_2, F_4) & \xrightarrow{T-duality} & \begin{pmatrix} F_3; F_1 = 0,\ F_5 = 0 \\ F_1,\ F_3; F_5 = 0 \end{pmatrix} \\[2ex]
Q_{D3} = N & \xrightarrow{NATD} & (Q_{D6} = N, Q_{D4} = 0) & \xrightarrow{T-duality} & \begin{pmatrix} Q_{D7} = 0,\ Q_{D5} = N,\ Q_{D3} = 0 \\ Q_{D7} = N,\ Q_{D5} = 0,\ Q_{D3} = 0 \end{pmatrix} \\[2ex]
\Delta Q_{D3} = 0 & \xrightarrow{NATD} & (\Delta Q_{D6} = 0, \Delta Q_{D4} = nN) & \xrightarrow{T-duality} & \begin{pmatrix} \Delta Q_{D5} = 0,\ \Delta Q_{D3} = 0 \\ \Delta Q_{D5} = nN,\ \Delta Q_{D3} = 0 \end{pmatrix}
\end{array}
$$

Figure 1. A schematic description of the supergravity solutions discussed in this manuscript and the properties of RR fluxes that are relevant for a field theory interpretation. The expression between parenthesis corresponds to the Page charges and their changes under a large gauge transformation of the B-field.

in the Büscher rules:

$$
\begin{aligned}
d\tilde{\hat{s}}^2 &= ds^2(AdS_5) + 4L^2(d\alpha^2 + \sin^2\alpha d\theta^2) + \frac{\alpha'^2 d\rho^2}{L^2\cos^2\alpha} \\
&\quad + \frac{\rho^4\alpha'^2 d\chi^2 - 2\rho^3\alpha'^2 d\xi d\chi \csc\chi + d\xi^2 \csc^2\chi \left(\rho^2\alpha'^2 + L^4\cos^4\alpha\right)}{L^2\rho^2\cos^2\alpha}, \\
\tilde{\hat{B}} &= 0, \qquad e^{-2\tilde{\hat{\Phi}}} = \frac{L^4\rho^2\cos^4\alpha\sin^2\chi}{\alpha'^2}.
\end{aligned} \tag{5.1}
$$

As the dilaton indicates, there is a singularity at $\cos\alpha = 0$. This is, indeed, a curvature singularity as can be seen from the 10d string frame Ricci scalar curvature:

$$
\tilde{\hat{\mathcal{R}}} = -\frac{4L^2\cos^2\alpha\sin^2\chi}{\rho^2\alpha'^2} - \frac{4\sin^2\chi\left(1 + \sin^2\chi\cos^2\alpha\right)}{L^2\cos^2\alpha}. \tag{5.2}
$$

The RR Fluxes are

$$
\begin{aligned}
\tilde{\hat{F}}_3 &= -\frac{8L^4\sin\alpha\cos^3\alpha d\alpha\wedge d\theta\wedge d\xi}{\alpha'}, \\
\tilde{\hat{F}}_7 &= -\frac{2\rho\alpha'(d\xi\wedge d\rho + \rho\sin\chi d\rho\wedge d\chi)\wedge d\mathrm{Vol}(AdS_5)}{L},
\end{aligned} \tag{5.3}
$$

where $\tilde{\hat{F}}_7 = -\star\tilde{\hat{F}}_3$. It is interesting to point out that only an F_3 flux survives, no F_5 is present. This situation is rather unexpected in the general framework of IIB solutions with an AdS_5 factor in light of the results of [6]. Since this background breaks SUSY, we are not subject to those restrictions.

We proceed to compute the Page charge supergravity and central charge for the field theory. Since $B_2 = 0$,

$$
Q_{P,D5} = \frac{1}{2\kappa_{10}^2 T_{D5}} \int_{\Sigma_3} F_3 = N_{D5}. \tag{5.4}
$$

With the normalization we have used, the equations of motion imply $L^4 = \frac{1}{2} N_{D5} \alpha'^2$, and therefore, the field theory central charge is:

$$\tilde{\hat{c}} = \frac{8\pi^5 L^8}{3\alpha'^4} = \frac{2\pi^5}{3} N_{D5}^2 \tag{5.5}$$

Here we can compare to the central charge after NATD and see that an Abelian T-duality on ξ does not change the central charge. We can then follow our previous comments and conclude that after the final T-duality, the dual QFT has not changed.

Let us present another possible T-duality, generating a different background.

5.2 T-dual on θ for the NATD of $AdS_5 \times S^5$

In this section, we perform a T-duality on the θ angle for the background of section 2.1.1 instead.

$$\begin{aligned}
\tilde{\hat{ds}}^2 &= ds^2(AdS_5) + 4L^2 d\alpha^2 + \frac{\alpha'^2}{L^2}\left(\frac{d\theta^2}{4\sin^2\alpha} + \frac{d\rho^2}{\cos^2\alpha}\right) + \frac{L^2\rho^2\alpha'^2\cos^2\alpha(d\xi^2\sin^2\chi + d\chi^2)}{\rho^2\alpha'^2 + L^4\cos^4\alpha} \\
\tilde{\hat{B}} &= \frac{\alpha'^3\rho^3 \sin\chi \, d\xi \wedge d\chi}{\alpha'^2\rho^2 + L^4\cos^4\alpha}, \qquad e^{-2\tilde{\hat{\Phi}}} = \frac{4L^4\cos^2\alpha\sin^2\alpha(\alpha'^2\rho^2 + L^4\cos^4\alpha)}{\alpha'^4}.
\end{aligned} \tag{5.6}$$

The RR fluxes are given by

$$\begin{aligned}
\tilde{\hat{F}}_1 &= \frac{8L^4\cos^3\alpha\sin\alpha \, d\alpha}{\alpha'^2}, \\
\tilde{\hat{F}}_3 &= \frac{8L^4\alpha'\rho^3\cos^3\sin\alpha\sin\chi \, d\alpha\wedge d\xi\wedge d\chi}{\alpha'^2\rho^2 + L^4\cos^4\alpha}, \\
\tilde{\hat{F}}_7 &= \frac{2\alpha'\rho}{L} d\rho \wedge d\mathrm{Vol}AdS_5, \\
\tilde{\hat{F}}_9 &= \frac{2\alpha'^2\rho^2\cos^4\alpha\sin\chi d\mathrm{Vol}AdS_5\wedge d\theta\wedge d\xi\wedge d\rho\wedge d\chi}{\alpha'^2\rho^2 + L^4\cos^4\alpha}
\end{aligned} \tag{5.7}$$

We again compute the Page charges, this time we see only the Page charge associated with $D7$-branes survives,

$$\begin{aligned}
Q_{P,D7} &= \frac{1}{2\kappa_{10}^2 T_{D7}} \int_{\Sigma_1} F_1 = N_{D7} \\
Q_{P,D5} &= \frac{1}{2\kappa_{10}^2 T_{D5}} \int_{\Sigma_3} F_3 - B_2 \wedge F_1 = N_{D5} = 0 \\
Q_{P,D3} &= \frac{1}{2\kappa_{10}^2 T_{D3}} \int_{\Sigma_5} F_5 - B_2 \wedge F_3 = N_{D3} = 0.
\end{aligned} \tag{5.8}$$

The $Q_{P,D7}$ imposes a relation $L^4 = \frac{1}{2} N_{D7} \alpha'^2$. Comparing these results to eq. (2.6), we see that B_2 is unchanged after the Abelian T-duality. We can, thus perform a large gauge transformation in a similar way as in section 4. Here we see that,

$$\Delta Q_{P,D3} = n N_{D7}, \quad \Delta Q_{P,D5} = 0 \tag{5.9}$$

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Finally, we apply the $L^4 \sim N$ relation above and find that the central charge is

$$\tilde{\hat{c}} = \frac{8\pi^5 L^8}{3\alpha'^4} = \frac{2\pi^5}{3} N_{D7}^2. \tag{5.10}$$

Again, we see that the central charge is invariant under the Abelian T-duality. Let us apply a similar logic for the case of $AdS_5 \times T^{1,1}$.

5.3 T-dual on ξ for the NATD of $AdS_5 \times T^{1,1}$

In this section we perform an Abelian T-duality along the ξ direction of the background presented in section 2.2.1.

The resulting NS sector is given by

$$\begin{aligned}
\tilde{\hat{ds}}^2 &= ds^2(AdS_5) + L^2\lambda_1^2(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2) + \frac{1}{P}\lambda_2^2 L^2\alpha'^2(\rho d\chi \sin\chi - d\rho\cos\chi)^2 \\
&\quad + \frac{\alpha'^2}{L^2 QP}\left(d\rho \sin\chi\left(\rho^2\alpha'^2 + \lambda^2\lambda_2^2 L^4\right) + \lambda^2\lambda_2^2 L^4 \rho d\chi\cos\chi\right)^2 \\
&\quad + \frac{1}{4\lambda^2\lambda_2^2 L^2\rho^2 Q\sin^2\chi}\left(\rho^2\alpha'^2\sin\chi\left(\left(\lambda^2-\lambda_2^2\right)d\rho\sin 2\chi + 2\rho d\chi N\right) - 2Q\,d\xi\right)^2, \\
\tilde{\hat{B}} &= \alpha'\cos\theta_1 d\phi_1\wedge(d\xi + \cos\chi d\rho - \rho\sin\chi d\chi), \quad e^{-2\tilde{\hat{\Phi}}} = \frac{L^4\lambda^2\lambda_2^2\rho^2\sin^2\chi}{\alpha'^2},
\end{aligned} \tag{5.11}$$

where, as in eq. (2.15), we have $N = \lambda^2\cos^2\chi + \lambda_2^2\sin^2\chi$, $Q = (\alpha'^2\rho^2 N + L^4\lambda^2\lambda_2^2)$ and $P = (\rho^2\alpha'^2\sin^2\chi + \lambda^2\lambda_2^2 L^4)$.

The non-trivial RR fields resulting are,

$$\begin{aligned}
\tilde{\hat{F}}_3 &= \frac{4L^4\lambda\lambda_1^2\lambda_2^2\sin\theta_1}{\alpha'} d\theta_1\wedge d\xi\wedge d\phi_1, \\
\tilde{\hat{F}}_7 &= \frac{\alpha'\rho}{8L}(d\mathrm{Vol}AdS_5\wedge(d\xi\wedge d\rho + \rho\sin\chi d\rho\wedge d\chi)).
\end{aligned} \tag{5.12}$$

The only non vanishing Page charge is given by

$$Q_{P,D5} = \frac{1}{2\kappa_{10}^2 T_{D5}}\int_{\Sigma_3} F_3 = N_{D5}. \tag{5.13}$$

This implies a condition, $L^4 = \frac{27}{2}N_{D5}\alpha'^2$. We can then compute the central charge,

$$\tilde{\hat{c}} = \frac{\pi^5 L^8}{162\alpha'^4} = \frac{9\pi^5}{8}N_{D5}^2 \tag{5.14}$$

Similar to section 5.1 above, we see that a T-duality on ξ does not change the central charge of the dual QFT.

Despite its relative simplicity, the results of section 6.1 show that this solution breaks SUSY. This is not surprising as it has been previously argued that the ξ isometry plays the role of the R-symmetry in the NATD of $AdS_5 \times T^{1,1}$ [25, 32].

5.4 T-dual on ϕ_1 of NATD of $AdS_5 \times T^{1,1}$

The background corresponding to applying T-duality on the ϕ_1 direction for the NATD of $AdS_5 \times T^{1,1}$ takes the form:

$$
\begin{aligned}
d\tilde{\hat{s}}^2 &= ds^2(AdS_5) + L^2\lambda_1^2 d\theta_1^2 \\
&+\frac{\alpha'^2}{L^2}\Bigg[\frac{\lambda_2^2 L^4}{P}(d\rho\cos\chi - \rho d\chi\sin\chi)^2 \\
&+\frac{1}{QW}\left(-\lambda^2 d\rho\cos\theta_1\cos\chi\left(\rho^2\alpha'^2+\lambda_2^4L^4\right)+\lambda^2\lambda_2^4L^4\rho d\chi\cos\theta_1\sin\chi + Qd\phi_1\right)^2 \\
&+\frac{1}{PQ}\left(d\rho\sin\chi\left(\rho^2\alpha'^2+\lambda^2\lambda_2^2L^4\right)+\lambda^2\lambda_2^2L^4\rho d\chi\cos\chi\right)^2 \\
&+\frac{1}{W^2}\lambda^2\lambda_1^2\lambda_2^2L^4\rho^2 d\xi^2\sin^4\theta_1\sin^2\chi\left(\lambda^2\lambda_2^2\rho^2\alpha'^2\cot^2\theta_1\sin^2\chi+\lambda_1^2 Q\right)\Bigg], \\
W\tilde{\hat{B}} &= \alpha'^3\lambda_1^2\rho^2\sin\chi d\xi\wedge\Big[\lambda^2\sin\chi d\phi_1 \\
&+\sin^2\theta_1\big((\lambda^2-\lambda_1^2)\cos\chi\sin\chi d\rho+N\rho d\chi\big)+\lambda^2\cos^2\theta_1\sin\chi\big(\rho\sin\chi d\chi-\cos\chi d\rho\big)\Big], \\
e^{-2\tilde{\hat{\Phi}}} &= \frac{L^4}{\alpha'^4}W,
\end{aligned}
\tag{5.15}
$$

where

$$
W = \alpha'^2\lambda^2\lambda_2^2\rho^2\cos^2\theta_1\sin^2\chi + \lambda_1^2\sin^2\theta_1(\alpha'^2\rho^2 N + L^4\lambda^2\lambda_2^4), \tag{5.16}
$$

where $P = (\rho^2\alpha'^2\sin^2\chi + \lambda^2\lambda_2^2L^4)$. The RR sector has the following non-vanishing field strengths;

$$
\begin{aligned}
\tilde{\hat{F}}_1 &= \frac{4L^4\lambda\lambda_1^4\sin\theta_1}{\alpha'^2}d\theta_1, \\
W\tilde{\hat{F}}_3 &= 4L^4\alpha'\lambda\lambda_1^6\rho^2\sin\chi\sin\theta_1 d\theta_1\wedge d\xi\wedge\Big[\lambda^2\cos\theta_1\sin\chi d\phi_1 \\
&+\sin^2\theta_1\big((\lambda^2-\lambda_1^2)\cos\chi\sin\chi d\rho+N\rho d\chi\big)+\lambda^2\cos^2\theta_1\sin\chi\big(\rho\sin\chi d\chi-\cos\chi d\rho\big)\Big].
\end{aligned}
\tag{5.17}
$$

For the sake of brevity, we only present the F_1 (if nontrivial) and F_3, and omit their corresponding Hodge duals for the rest of the results in this section. The corresponding Page charges and central charge are

$$
\begin{aligned}
Q_{P,D7} &= \frac{1}{2\kappa_{10}^2 T_{D7}}\int_{\Sigma_1} F_1 = N_{D7} \\
Q_{P,D5} &= \frac{1}{2\kappa_{10}^2 T_{D5}}\int_{\Sigma_3} F_3 - B_2\wedge F_1 = N_{D5} = 0 \\
Q_{P,D3} &= \frac{1}{2\kappa_{10}^2 T_{D3}}\int_{\Sigma_5} F_5 - B_2\wedge F_3 = N_{D3} = 0,
\end{aligned}
\tag{5.18}
$$

and

$$
\tilde{\hat{c}} = \frac{\pi^5 L^8}{162\alpha'^4} = \frac{9\pi^5}{8}N_{D5}^2, \tag{5.19}
$$

where, again we used $L^4 = \frac{27}{2}N_{D5}\alpha'^2$.

Section 6.1 shows that this solution preserves all supercharges of $AdS_5 \times T^{1,1}$ in the form of an SU(2)-structure defined on the new 6d internal space.

5.5 Shift $\phi_1 \to \phi_1 + \gamma\xi$, T-dual on ξ of NATD of $AdS_5 \times T^{1,1}$

In this section we consider mixing the two U(1)'s symmetries via a shift. The resulting one-parameter solutions have a much smaller singularity loci, however as we once more dualise on the R-symmetry we break SUSY.[6] Nonetheless AdS_5 solutions with parameters are uncommon so we believe this solution deserves some further study.

The NS sector is given by

$$\begin{aligned}
d\tilde{\tilde{s}}^2 &= ds^2(AdS_5) + L^2\lambda_1^2 d\theta_1^2 \\
&+\frac{1}{P}\left(\lambda_2 L\alpha' d\rho\cos\chi - \lambda_2 L\rho\alpha' d\chi\sin\chi\right)^2 + \frac{1}{Y}\lambda^2\lambda_1^2\lambda_2^2 L^2\rho^2\alpha'^2 d\phi_1{}^2\sin^2\theta_1\sin^2\chi \\
&+\frac{\alpha'^2}{L^2QY}\left(\rho d\chi\sin\chi\left(\rho^2\alpha'^2 N - \gamma\lambda^2\lambda_2^4L^4\cos\theta_1\right)\right. \\
&+\frac{\alpha'^2}{L^2QY}\Big(d\rho\cos\chi\big(\lambda^2\rho^2\alpha'^2\left(\gamma\cos\theta_1 + \sin^2\chi\right) \\
&-\lambda_2^2\rho^2\alpha'^2\sin^2\chi + \gamma\lambda^2\lambda_2^4L^4\cos\theta_1\big) - Q\,d\xi\Big)^2 \\
&+\frac{\alpha'^2\left(d\rho\sin\chi\left(\rho^2\alpha'^2 + \lambda^2\lambda_2^2L^4\right) + \lambda^2\lambda_2^2L^4\rho d\chi\cos\chi\right)^2}{L^2QP}, \\
\frac{2Y}{\alpha'}\tilde{\tilde{B}} &= \gamma\lambda_1^2\sin^2\theta_1\Big(\rho\sin\chi d\phi_1\wedge d\chi\left(-2\rho^2\alpha'^2N + \gamma\lambda^2\lambda_2^4L^4\cos\theta_1\right) \\
&+d\rho\wedge d\phi_1\left(\lambda^2\rho^2\alpha'^2\left(\gamma\cos\theta_1\cos\chi + \sin\chi\sin 2\chi\right)\right. \\
&\left.-2\,\lambda_2^2\rho^2\alpha'^2\sin^2\chi\cos\chi + \gamma\lambda^2\lambda_2^4L^4\cos\theta_1\cos\chi - 2Q\,d\xi\wedge d\phi_1\right)\Big) \\
&-\frac{1}{Q}\lambda^2\lambda_2^2\rho^2\alpha'^2\cos\theta_1\sin^2\chi\left(\gamma\cos\theta_1 + 1\right) \\
&\times\Big[\rho\sin\chi d\phi_1\wedge d\chi\left(2\rho^2\alpha'^2N + \lambda^2\lambda_2^4L^4\left(1-\gamma\cos\theta_1\right)\right) \\
&+d\rho\wedge d\phi_1\Big(\lambda^2\rho^2\alpha'^2\cos\chi\left(\cos 2\chi - \gamma\cos\theta_1\right) + \lambda_2^2\rho^2\alpha'^2\sin\chi\sin 2\chi \\
&+\lambda^2\lambda_2^4L^4\cos\chi\left(1-\gamma\cos\theta_1\right)\Big)2Q\,d\xi\wedge d\phi_1\Big], \\
e^{-2\tilde{\tilde{\Phi}}} &= \frac{L^4}{\alpha'^4}Y,
\end{aligned} \tag{5.20}$$

where $Y = \alpha'^2\lambda^2\lambda_2^2\rho^2\sin^2\chi(\gamma\cos\theta_1 + 1)^2 + \gamma^2\lambda_1^2\sin^2\theta_1 Q$. The above expression makes clear that a shift γ has substantially reduced the singular locus. For example, taking into consideration that Q is non-vanishing, now to get a singular dilaton we need $\sin\chi = 0$ simultaneously with $\sin\theta_1 = 0$; also $\rho = 0$ and $\sin\theta_1 = 0$.

[6] The same is also true if we perform $\xi \to \xi + \gamma\phi_1$ and T-dualise on ∂_{ϕ_1}. This time because we introduce a ϕ_1 dependence on the Killing spinor, which means the Kosmann derivative cannot vanish (see section 6.1).

The RR sector contains:

$$\tilde{\hat{F}}_1 = \frac{4\gamma\lambda\lambda_1^2\lambda_2^2 L^4 \sin\theta_1}{\alpha'^2} d\theta_1$$

$$\tilde{\hat{F}}_3 = 2\lambda\lambda_1^2\lambda_2^2 L^4\rho^2\alpha' \sin\theta_1 \sin\chi \Bigg[-\frac{1}{Q} d\rho\wedge d\theta_1\wedge d\phi_1 \Bigg(\frac{\lambda^2 \sin 2\chi \left(\rho^2\alpha'^2 + \lambda^2\lambda_2^2 L^4\right)}{P}$$
$$-\frac{1}{Y}\lambda_2^2\lambda^2 \sin\chi \left(\gamma\cos\theta_1 + 1\right)\left(\lambda^2\rho^2\alpha'^2 \left(2\gamma\cos\theta_1\cos\chi + \sin\chi\sin 2\chi\right) + 2\gamma\lambda^2\lambda_2^4 L^4 \cos\theta_1\cos\chi\right.$$
$$\left.-2\ \lambda_2^2\rho^2\alpha'^2 \sin^2\chi\cos\chi\right) - \frac{2}{P}\sin\chi\cos\chi\Bigg)$$
$$-\frac{2}{Y}\gamma\rho d\chi\wedge d\theta_1\wedge d\phi_1 \left(\lambda^2\lambda_2^2\cos\theta_1\sin^2\chi\left(\gamma\cos\theta_1+1\right) + \gamma\lambda_1^2 N\sin^2\theta_1\right)$$
$$-2\lambda^2\lambda_2^2 \sin\chi\left(\gamma\cos\theta_1+1\right) d\xi\wedge d\theta_1\wedge d\phi_1\Bigg]. \tag{5.21}$$

5.6 T-Dual on ξ of NATD of $AdS_5 \times Y^{p,q}$

In this section we perform an Abelian T-duality along the ξ direction on the background specified in section 2.3.1.

$$\tilde{\hat{ds}}^2 = ds^2(AdS_5) + L^2k^2 d\alpha^2 + \frac{L^2}{vw} dy^2$$
$$+\frac{1}{6L^2\rho^2 gm}\Big(g\left(72\alpha'^2\rho^2 d\xi\cot\chi(d\rho\sin\chi + \rho d\chi\cos\chi) + 36\alpha'^2\rho^2(d\rho\sin\chi+\rho d\chi\cos\chi)^2\right.$$
$$\left.+d\xi^2\left(36\alpha'^2\rho^2\cot^2\chi + L^4m^2\csc^2\chi\right)\right) + 6\alpha'^2\rho^2 m(d\xi + \rho d\chi\sin\chi - d\rho\cos\chi)^2\Big)$$
$$\tilde{\hat{B}} = -\frac{\alpha' h}{\sqrt{g}} d\alpha\wedge(d\xi - \cos\chi d\rho + \rho\sin\chi d\chi)$$
$$e^{-2\tilde{\hat{\Phi}}} = \frac{L^4 gm\rho^2\sin^2\chi}{6\alpha'^2}. \tag{5.22}$$

Just as in the cases of dualizing along ξ in the NATD of $AdS_5 \times S^5$ and $AdS_5 \times T^{1,1}$, we recover only an F_3 (and its Hodge dual),

$$\tilde{\hat{F}}_3 = \frac{2L^4 m}{9\alpha'} d\alpha\wedge dy\wedge d\xi. \tag{5.23}$$

We compute the Page charge associated with the N_{D5},

$$Q_{P,D5} = \frac{1}{2\kappa_{10}^2 T_{D5}} \int_{\Sigma_3} F_3 = N_{D5}. \tag{5.24}$$

This implies a condition, $L^4 = \frac{9}{l(y_2^2 - y_1^2 + 2(y_1 - y_2))} N_{D5}\alpha'^2$. We can then compute the central charge,

$$\tilde{\hat{c}} = \frac{3\pi^5}{4l(y_2^2 - y_1^2 + 2(y_1 - y_2))} N_{D5}^2. \tag{5.25}$$

Notice that this is the same result we found after the NATD of Y^{pq} with N_{D6} being replaced by N_{D5}.

Like the equivalent NATD-T solution of $AdS_5 \times T^{1,1}$, this solution breaks SUSY. We propose that this is for the same reason as above. Namely, the ξ isometry once more plays the role of the R-symmetry in the SU(2) transformed $AdS_5 \times Y^{p,q}$ solution.

5.7 T-dual on α of NATD of $AdS_5 \times Y^{p,q}$

As a final solution, we consider performing a further Abelian T-duality along ∂_α of the NATD of $AdS_5 \times Y^{p,q}$. As shown in section 6.2, such a solution preserves $\mathcal{N}=1$ SUSY in 4d via an SU(2)-structure defined on the 6-d internal space.

Performing the T-duality gives a NS sector of the form

$$\begin{aligned}
d\tilde{\hat{s}}^2 &= ds^2(AdS_5) + \frac{L^2}{vw}dy^2 + \frac{\alpha'^2}{36\alpha'^2\rho^2\cos^2\chi + L^4m^2}\Big(6L^2m\big(\rho\cos\chi d\chi + \sin\chi d\rho\big)^2 \\
&\quad + \frac{1}{L^2\Upsilon}\big((36\alpha'^2\rho^2 + L^4m^2)\cos\chi d\rho - L^4\rho m^2\sin\chi d\chi\big)^2\Big) + \frac{L^2\alpha'^2\rho^2}{\Theta}k^2gm\sin^2\chi d\xi^2 \\
&\quad + \frac{\alpha'^2}{6L^2\Upsilon\Theta}\Big(\Upsilon d\alpha + L^4\rho\sqrt{g}hm^2\sin\chi d\chi - \sqrt{g}h(36\alpha'^2\rho^2 + L^4m^2)\cos\chi d\rho\Big)^2, \\
\tilde{\hat{B}} &= \frac{\alpha'^3\rho^2}{\Theta}\sin\chi\Big(6gk^2\cos\chi(\sin\chi d\xi\wedge d\rho + \rho\cos\chi d\xi\wedge d\chi) \\
&\quad + mw\sin\chi(-\cos\chi d\xi\wedge d\rho + \rho\sin\chi d\xi\wedge d\chi) - \sqrt{g}mh\sin\chi d\alpha\wedge d\xi\Big), \\
e^{-2\tilde{\hat{\Phi}}} &= \frac{L^4}{6\alpha'^4}\Theta,
\end{aligned} \tag{5.26}$$

where $\Theta = \frac{1}{6}gk^2(36\alpha'^2\rho^2\cos^2\chi + L^4m^2) + \alpha'^2\rho^2wm\sin^2\chi$, and
$\Upsilon = g(36\alpha'^2\rho^2\cos^2\chi + L^4m^2) + 6\alpha'^2\rho^2m\sin^2\chi$.

While the RR Sector has no trivial fluxes

$$\begin{aligned}
F_1 &= \frac{2L^4}{9\alpha'^2}mdy \\
F_3 &= \frac{2\rho^2L^4\alpha'm}{9\Theta}\sin\chi\Big(6gk^2\cos\chi(\sin\chi d\xi\wedge d\rho + \rho\cos\chi d\xi\wedge d\chi) \\
&\quad + mw\sin\chi(-\cos\chi d\xi\wedge d\rho + \rho\sin\chi d\xi\wedge d\chi) - \sqrt{g}mh\sin\chi d\alpha\wedge d\xi\Big)\wedge dy.
\end{aligned} \tag{5.27}$$

We compute the Page charge associated with the N_{D7},

$$Q_{P,D7} = \frac{1}{2\kappa_{10}^2T_{D7}}\int_{\Sigma_3}F_1 = N_{D7}. \tag{5.28}$$

This implies a condition, $L^4 = \frac{9}{l(y_2^2-y_1^2+2(y_1-y_2))}N_{D5}\alpha'^2$. We can then compute the central charge,

$$\tilde{\hat{c}} = \frac{3\pi^5}{4l(y_2^2-y_1^2+2(y_1-y_2))}N_{D5}^2. \tag{5.29}$$

Notice that this is the same result we found after the NATD-T on ξ of Y^{pq} with N_{D5} being replaced by N_{D7}.

In summary, we have presented a set of new solutions. Some of them preserve minimal SUSY as will be proven in the next section. These backgrounds escape the classification of [6], in the sense that they present an AdS_5 factor in IIB, but without F_5 RR-field. More details will follow below.

6 G-structures of the NATD-T solutions

In this section we perform a SUSY analysis of some of the new solutions generated in this paper. We focus our attention on the NATD-T transformations of $AdS_5 \times T^{1,1}$ and $AdS_5 \times Y^{p,q}$ as these have relatively simple descriptions in terms of G-structures. As shown in appendix C further U(1) T-dualities performed on the NATD of $AdS_5 \times S^5$ break all the remaining SUSY.

In [67] necessary and sufficient conditions for the preservation of $\mathcal{N} = 1$ SUSY where established in terms of geometric quantities. For a solution with metric of the form $\mathbb{R}^{1,3} \times \mathcal{M}^6$ and non trivial RR sector these are,

1. The existence of either an SU(3) or SU(2) structure defined on the internal manifold $\mathcal{M}^6$.
2. A NS 3-form H_3 that is closed.
3. A RR polyform F which obey certain differential relations in terms of the previously mentioned quantities.

Let us briefly review what is required, we use the notation of [68] — see also the introduction of [69] for a nice review. We assume a metric of the form,

$$ds^2_{str} = e^{2A} dx^2_{1,3} + ds^2(\mathcal{M}^6), \tag{6.1}$$

where A is an arbitrary function on the internal space. The starting point is to introduce two Majorana-Weyl Killing spinors such that

$$\epsilon = \begin{pmatrix} \epsilon_1 \\ \epsilon_2 \end{pmatrix} \tag{6.2}$$

These are then further split into 4-d (ζ) and 6-d (η) components

$$\epsilon_1 = e^{\frac{A}{2}} e^{i\frac{\theta_+ + \theta_-}{2}} (\zeta_+ \otimes \eta^1_+ + \zeta_- \otimes \eta^1_-), \quad \epsilon_1 = e^{-i\frac{\theta_+ - \theta_-}{2}} e^{\frac{A}{2}} (\zeta_+ \otimes \eta^2_\mp + \zeta_- \otimes \eta^2_\pm), \tag{6.3}$$

where $\pm$ labels chirality and so the upper/lower signs are taken in the above in type IIA/IIB respectively. The internal spinors also obey the relation $(\eta^{1,2}_+)* = \eta^{1,2}_-$. It is possible to define two bi-spinors on the internal space

$$\Psi_\pm = e^A e^{i\theta_\pm} \eta^1_+ \otimes \eta^2_\pm \tag{6.4}$$

where $\theta_\pm$ are arbitrary phases with support on $\mathcal{M}^6$. These bi-spinors may then be mapped to polyforms under the Clifford map at which point the conditions for $\mathcal{N} = 1$ SUSY may be expressed as

$$\begin{aligned}
&(d - H\wedge)(e^{2A-\Phi})\Psi_\pm = 0, \qquad (6.5)\\
&(d - H\wedge)(e^{2A-\Phi})\Psi_\mp = e^{2A-\Phi} dA \wedge \bar{\Psi}_\mp + \frac{ie^{3A}}{8}\tilde{F}
\end{aligned}$$

where upper/lower signs are taken in type IIA/IIB, $\tilde{F} = \iota_{(e^{tx^1x^2x^3})}(F)$ and F is a sum over all the RR fields in the democratic formalism. The specific form of $\Psi_\pm$ is determined by the type of structure. The two cases we will deal with in this section are either SU(3)-structures which are characterised by parallel internal spinors or orthogonal SU(2) structures, for which $\eta_+^{1\dagger}\eta_+^2 = 0$.

In the case of SU(3)-structures, the pure spinors are,

$$\Psi_+ = -e^{i\theta_+}\frac{e^A}{8}e^{-iJ}, \quad \Psi_- = -ie^{i\theta_-}\frac{e^A}{8}\Omega_{\rm hol} \tag{6.6}$$

where J is a (1,1)-form and $\Omega_{\rm hol}$ is a holomorphic 3-form and they must satisfy,

$$J\wedge\Omega_{\rm hol} = 0, \quad J\wedge J\wedge J = \frac{4i}{3}\Omega_{\rm hol}\wedge\bar{\Omega}_{\rm hol}. \tag{6.7}$$

The components of these forms may be calculated in terms of the 6-d gamma matrices γ_a and the internal spinors via

$$J_{ab} = -i\eta_+^{1\dagger}\gamma_{ab}\eta_+^1, \qquad \left(\Omega_{\rm hol}\right)_{abc} = -i\eta_+^{1\dagger}\gamma_{abc}\eta_+^1. \tag{6.8}$$

For orthogonal SU(2)-structures, the pure spinors read,

$$\Psi_+ = -ie^{i\theta_+}\frac{e^A}{8}e^{-v\wedge w}\wedge\omega, \quad \Psi_- = ie^{i\theta_-}\frac{e^A}{8}(v+iw)\wedge e^{-ij}, \tag{6.9}$$

where j is real 2-form and ω is a holomorphic 2-form and $z = v + iw$ is a holomorphic 1-form. These must satisfy the SU(2) structure conditions

$$j\wedge\omega = \omega\wedge\omega = \iota_{\bar{z}}(\omega) = \iota_{\bar{z}}(j) = 0, \quad j\wedge j = \frac{1}{2}\omega\wedge\bar{\omega}. \tag{6.10}$$

The components of these forms may be calculated via,

$$\bar{z}_a = \eta_-^{1\dagger}\gamma_a\eta_+^2, \quad j_{ab} = -i\eta^{1\dagger}\gamma_{ab}\eta_+^1 + i\eta_+^{2\dagger}\gamma_{ab}\eta_+^2, \quad \omega_{ab} = \eta_-^{1\dagger}\gamma_{ab}\eta_-^2. \tag{6.11}$$

Let us now just state some previously derived results we shall be using. As reviewed at length in [25], a Non-Abelian T-duality on an SU(2)-isometry, has a preferred basis of vielbeins with respect to which the transformation of the RR sector is given by a simple bispinor transformation,

$$e^{\Phi_{IIB}}F_{IIB}\Omega^{-1} = e^{\Phi_{IIA}}F_{IIA}, \tag{6.12}$$

where

$$\Omega_{\rm SU(2)} = \Gamma^{(10)}\frac{1}{\sqrt{1+\zeta_a\zeta^a}}\left(-\Gamma_{123} + \zeta_1\Gamma_1 + \zeta_2\Gamma_2 + \zeta_3\Gamma_3\right), \tag{6.13}$$

and $F_{IIA/B}$ is the sum of all the democratic formalism RR fields.[7] The $\zeta's$ are defined in terms of the original vielbeins describing the SU(2) isometry of the background $e^a = e^{B_a}(\sigma_a + A_a)$ as

$$\zeta^a = v_a e^{-\sum_{b\neq a}B^b}. \tag{6.14}$$

[7]Strictly speaking one needs to map the RR-sector to bispinors under the Clifford map.

The U(1) Omega matrix in the preferred frame of Abelian T-duality, $e^\theta = e^B(d\theta + C)$ is simply

$$\Omega_{\mathrm{U}(1)} = \Gamma^{(10)}\Gamma_\theta \tag{6.15}$$

The action of T-duality on the MW Killing spinors is given by

$$\hat{\epsilon}_1 = \epsilon_1, \quad \hat{\epsilon}_2 = \Omega\epsilon_2, \tag{6.16}$$

which was proven for U(1) isometries in [70] and SU(2) isometries in [71]. The condition that SUSY is preserved is that the Kosmann derivative vanishes along the given isometry. The Kosmann derivative of the Killing spinor ϵ along a Killing vector K is given by

$$\mathcal{L}_K\epsilon = K^a\nabla_a\epsilon + \frac{1}{8}(dK)_{ab}\Gamma^{ab}\epsilon. \tag{6.17}$$

The vanishing of this object is equivalent to the independence of $\epsilon_{1,2}$ on the isometry directions in the appropriate preferred frame [71].

Finally, it was established in [26, 32, 37, 72] that the bi-spinors defined on a d-dimensional internal space transform as

$$\hat{\Psi}_\pm = \Psi_\mp\Omega, \tag{6.18}$$

at least up to conventionally dependent phases.[8]

6.1 G-structure for NATD-T of $AdS_5 \times T^{1,1}$

We start our analysis with the Klebanov-Witten solution [39] which possesses a metric that may be succinctly expressed in terms of the following vielbein basis (notice that in this section we rename $\theta_1 \to \theta$ and $\phi_1 \to \varphi$ and set $\alpha' = 1$),

$$\begin{aligned} &e^{x^\mu} = \frac{r}{L}x^\mu, \qquad e^r = \frac{L}{r}dr, \qquad e^\theta = L\lambda_1 d\theta, \qquad e^\varphi = L\lambda_1\sin\theta d\varphi, \\ &e^{1,2} = L\lambda_1\sigma_{1,2}, \qquad e^3 = L\lambda(\sigma + \cos\theta d\varphi). \end{aligned} \tag{6.19}$$

With respect to eq. (6.19) the projection conditions on the 10-d Majorana Killing spinor are

$$\Gamma_{r123}\epsilon = \epsilon, \quad \Gamma_{\theta\phi}\epsilon = \Gamma_{12}\epsilon. \tag{6.20}$$

These define a canonical SU(3)-structure with

$$J = e^{r3} + e^{\varphi\theta} + e^{21}, \quad \Omega_{\mathrm{hol}} = (e^r + ie^3)\wedge(e^\varphi + ie^\theta)\wedge(e^2 + ie^1). \tag{6.21}$$

From these and the warp factor on the Minkowski directions, $e^{2A} = \frac{r^2}{L^2}$, it is possible to construct two bi-spinors of the form in eq. (6.7), where $\theta_+ = \frac{\pi}{2}$ and $\theta_- = 0$.

We now turn our focus to the SU(2) T-dualised Klebanov-Witten solution. As eq. (6.19) is in the preferred frame and the associated killing spinor depends only on r [73],

[8] Though it is yet to be formally proven, it is likely that this will hold when the Kosmann derivative along the SU(2) directions vanishes.

the result of [71] implies that all SUSY is preserved under the SU(2) transformation. The transformed vielbeins are given by,

$$\begin{aligned}
\hat{e}^1 &= -\lambda_1 \frac{x_1(L^2\lambda_1\cos\xi+x_2\sin\xi)(dx_2+L^2\lambda\hat{\sigma}_3)+(-L^2x_2\lambda^2\cos\xi+(x_1^2+L^4\lambda^2\lambda_1^2)\sin\xi)dx_1}{L\Delta},\\
\hat{e}^2 &= \lambda_1 \frac{x_1(L^2\lambda_1\sin\xi-x_2\cos\xi)(dx_2+L^2\lambda\hat{\sigma}_3)-(L^2x_2\lambda^2\sin\xi+(x_1^2+L^4\lambda^2\lambda_1^2)\cos\xi)dx_1}{L\Delta},\\
\hat{e}^3 &= -\lambda\frac{x_1x_2dx_1+(x_2^2+L^4\lambda_1^4)dx_2-L^2x_1^2\lambda_1^2\hat{\sigma}_3}{L\Delta},
\end{aligned} \tag{6.22}$$

where

$$\Delta = L^4\lambda_1^4\lambda^2+\lambda_1x_1^2+\lambda^2x_2^2, \quad \hat{\sigma}_3 = d\xi+\cos\theta d\varphi. \tag{6.23}$$

and we have introduced coordinates

$$x_1 = \rho\sin\chi, \quad x_2 = \rho\cos\chi. \tag{6.24}$$

The matrix $\Omega_{\mathrm{SU}(2)}$ is defined in terms of

$$\zeta_1 = \frac{x_1}{L^2\lambda\lambda_1}\sin\xi, \quad \zeta_2 = \frac{x_1}{L^2\lambda\lambda_1}\cos\xi, \quad \zeta_3 = \frac{x_2}{L^2\lambda_1}, \tag{6.25}$$

which implies that that the dual killing spinor depends on ξ. There are two isometries on which on can perform a further U(1) T-duality, ∂_ϕ and ∂_ξ. A quick computation (for mathematica) gives

$$\mathcal{L}_{\partial_\xi}\hat{\epsilon} = \partial_\xi\hat{\epsilon}, \qquad \mathcal{L}_{\partial_{\phi_1}}\hat{\epsilon} = \partial_\phi\hat{\epsilon} \tag{6.26}$$

which shows that SUSY will only be preserved when one performs a further T-duality on ∂_ϕ. Let us now calculate the G-structure.

We first use the projections to simplify Ω and then rotate the preferred vielbein basis as $\tilde{e} = \mathcal{R}_1\hat{e}$ where

$$\mathcal{R}_1 = \frac{1}{\sqrt{1+\zeta_a\zeta^a}}\begin{pmatrix} 1 & 0 & 0 & \zeta_1 & \zeta_2 & \zeta_3\\ 0 & \sqrt{1+\zeta_a\zeta^a} & 0 & 0 & 0 & 0\\ 0 & 0 & \sqrt{1+\zeta_a\zeta^a} & 0 & 0 & 0\\ -\zeta_1 & 0 & 0 & 1 & \zeta_3 & -\zeta_2\\ -\zeta_2 & 0 & 0 & -\zeta_3 & 1 & \zeta_1\\ -\zeta_3 & 0 & 0 & \zeta_2 & -\zeta_1 & 1 \end{pmatrix} \tag{6.27}$$

and the ordering is $[r\theta\varphi123]$. Then, the Ω acting on ϵ_2 is drastically simplified to

$$\Omega_{\mathrm{SU}(2)}\epsilon_2 = \tilde{\Gamma}^r\epsilon_2. \tag{6.28}$$

More remarkable is the fact the the projectors in this basis are also unchanged so that

$$\tilde{\Gamma}_{r123}\epsilon = \epsilon, \quad \tilde{\Gamma}_{\theta\varphi}\epsilon = \tilde{\Gamma}_{12}\epsilon. \tag{6.29}$$

Specifically the vielbeins $\tilde{e}$ are given by

$$\begin{aligned}
\tilde{e}^r &= \frac{L^4\lambda\lambda_1^2 dr - r(x_1 dx_1 + x_2 dx_2)}{Lr\sqrt{\Delta}}, \\
\tilde{e}^\theta &= L\lambda_1 d\theta_1, \quad \tilde{e}^\varphi = L\lambda_1 \sin\theta_1 d\varphi, \\
\tilde{e}^1 &= -\frac{L\lambda_1\big(\sin\xi(x_1 dr + \lambda r dx_1) + \cos\xi r x_1 \lambda \hat{\sigma_3}\big)}{r\sqrt{\Delta}}, \\
\tilde{e}^2 &= -\frac{L\lambda_1\big(\cos\xi(x_1 dr + \lambda r dx_1) - \sin\xi r x_1 \lambda \hat{\sigma_3}\big)}{r\sqrt{\Delta}}, \\
\tilde{e}^3 &= -\frac{L}{r\sqrt{\Delta}}\big(\lambda x_2 dr + \lambda_1^2 r dx_2\big).
\end{aligned} \tag{6.30}$$

As shown at length in [37] it is now a rather simple matter to derive the G-structure of the dual of Klebanov-Witten, which turns out to be an orthogonal SU(2)-structure. The relevant forms and phases are,

$$\begin{aligned}
z &= v + iw = \tilde{e}^r + i\tilde{e}^3, \\
j &= \tilde{e}^{\varphi\theta} + \tilde{e}^{21}, \\
\omega &= (\tilde{e}^2 + i\tilde{e}^1) \wedge (\tilde{e}^\varphi + i\tilde{e}^\theta), \\
\tilde{\theta}_m &= \frac{\pi}{2}, \quad \tilde{\theta}_m = 0.
\end{aligned} \tag{6.31}$$

The next step is to perform an Abelian T-duality on the φ direction. In order to do this it is convenient to rotate to a frame in which $d\varphi$ appears in only one of the vielbein. This can be achieved by rotating the basis $\tilde{e}$ by

$$\begin{pmatrix}
-\frac{\lambda\lambda_1 x_1\cos\theta_1}{\sqrt{\Xi}} & -\frac{\lambda\lambda_1\sin\theta_1\left(\lambda_1^2L^2\cos\xi+x_2\sin\xi\right)}{\sqrt{\Xi}} & \frac{\lambda\lambda_1\sin\theta_1\left(\lambda_1^2L^2\sin\xi-x_2\cos\xi\right)}{\sqrt{\Xi}} & \frac{\lambda_1^2 x_1\sin(\theta1)}{\sqrt{\Xi}} \\
0 & \frac{x_2\cos\xi-\lambda_1^2L^2\sin\xi}{\sqrt{\lambda_1^4L^4+x_2^2}} & -\frac{\lambda_1^2L^2\cos\xi+x_2\sin\xi}{\sqrt{\lambda_1^4L^4+x_2^2}} & 0 \\
0 & -\frac{\lambda_1x_1\left(\lambda_1^2L^2\cos\xi+x_2\sin\xi\right)}{\sqrt{\Delta}\sqrt{\lambda_1^4L^4+x_2^2}} & \frac{\lambda_1x_1\left(\lambda_1^2L^2\sin\xi-x_2\cos\xi\right)}{\sqrt{\Delta}\sqrt{\lambda_1^4L^4+x_2^2}} & -\frac{\lambda\sqrt{\lambda_1^4L^4+x_2^2}}{\sqrt{\Delta}} \\
\frac{\sqrt{\Delta}\lambda_1\sin\theta_1}{\sqrt{\Xi}} & -\frac{\lambda^2\lambda_1x_1\cos\theta_1\left(\lambda_1^2L^2\cos\xi+x_2\sin\xi\right)}{\sqrt{\Delta}\sqrt{\Xi}} & \frac{\lambda^2\lambda_1x_1\cos\theta_1\left(\lambda_1^2L^2\sin\xi-x_2\cos\xi\right)}{\sqrt{\Delta}\sqrt{\Xi}} & \frac{\lambda\lambda_1^2x_1^2\cos\theta_1}{\sqrt{\Delta}\sqrt{\Xi}}
\end{pmatrix}, \tag{6.32}$$

where this matrix acts on the flat directions [φ123]. This gives a new vielbein basis

$$\begin{aligned}
\tilde{e}^{a\prime} &= \tilde{e}^a = e^a, & a &= x^\mu, r, \theta, \\
\tilde{e}^{\varphi\prime} &= \frac{L\lambda\lambda_1^2 x_1\sin\theta}{\sqrt{\Xi}} d\xi, & \tilde{e}^{1\prime} &= \frac{L\lambda_1}{\sqrt{x_2^2 + L^4\lambda_1^4}} dx_1, \\
\tilde{e}^{2\prime} &= \frac{x_1x_2 dx_1 + (x_2^2 + L^4\lambda_1^4)dx_2}{L\sqrt{\Delta}\sqrt{x_2^2 + L^4\lambda_1^4}} \\
\tilde{e}^{3\prime} &= \frac{L\lambda_1^2\big(\Xi d\varphi + \lambda^2\lambda_1^2x_1^2\cos\theta d\xi\big)}{\sqrt{\Delta}\sqrt{\Xi}},
\end{aligned} \tag{6.33}$$

where

$$\Xi = \Delta\lambda_1^2 \sin\theta^2 + \lambda^2\lambda_1^2 x_1^2 \cos\theta^2. \tag{6.34}$$

In this basis it is possible to follow the standard Abelian T-duality rules in the presence of RR fields [70]. In the frame eq. (6.30), the U(1) Ω matrix is given by,

$$\Omega_{\mathrm{U}(1)} = \Gamma^{(10)} \frac{\lambda_1\sqrt{\Delta}\sin\theta\tilde{\Gamma}^{\varphi} + \lambda x_1\cos\theta(-\cos\xi\tilde{\Gamma}^1 + \sin\xi\tilde{\Gamma}^2)}{\sqrt{\Xi}}. \tag{6.35}$$

As one expects the MW Killing spinors transform as,

$$\hat{\hat{\epsilon}}_1 = \epsilon_1, \quad \hat{\hat{\epsilon}}_2 = \Omega_{\mathrm{U}(1)}\Omega_{\mathrm{SU}(2)}\epsilon_2 \tag{6.36}$$

following the logic of [37] it would be advantageous to find a frame in which both $\Omega_{\mathrm{U}(1)}$ and $\Omega_{\mathrm{SU}(2)}$ are both simple. Such a frame is provided by the Lorentz transformation $\tilde{\tilde{e}} = \mathcal{R}_2\tilde{e}$ where

$$\mathcal{R}_2 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{\Delta}\lambda_1\sin\theta}{\sqrt{\Xi}} & 0 & \frac{\lambda\lambda_1 x_1\cos\theta\sin\xi}{\sqrt{\Xi}} & \frac{\lambda\lambda_1 x_1\cos\theta\cos\xi}{\sqrt{\Xi}} & 0 \\ 0 & 0 & \frac{\sqrt{\Delta}\lambda_1\sin\theta}{\sqrt{\Xi}} & -\frac{\lambda\lambda_1 x_1\cos\theta\cos\xi}{\sqrt{\Xi}} & \frac{\lambda\lambda_1 x_1\cos\theta\sin\xi}{\sqrt{\Xi}} & 0 \\ 0 & -\frac{\lambda\lambda_1 x_1\cos\theta\sin\xi}{\sqrt{\Xi}} & \frac{\lambda\lambda_1 x_1\cos\theta\cos\xi}{\sqrt{\Xi}} & \frac{\sqrt{\Delta}\lambda_1\sin\theta}{\sqrt{\Xi}} & 0 & 0 \\ 0 & -\frac{\lambda\lambda_1 x_1\cos\theta\cos\xi}{\sqrt{\Xi}} & -\frac{\lambda\lambda_1 x_1\cos\theta\sin\xi}{\sqrt{\Xi}} & 0 & \frac{\sqrt{\Delta}\lambda_1\sin\theta}{\sqrt{\Xi}} & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix} \tag{6.37}$$

One can check that this does indeed satisfy $\mathcal{R}_2\mathcal{R}_2^T = I$ and $\det\mathcal{R} = -1$. With respect to this basis we have

$$\Omega_{\mathrm{SU}(2)} = -\Gamma^{(10)}\tilde{\tilde{\Gamma}}^r, \quad \Omega_{\mathrm{U}(1)} = \Gamma^{(10)}\tilde{\tilde{\Gamma}}^{\varphi}, \tag{6.38}$$

and so

$$\hat{\hat{\epsilon}}_2 = \tilde{\tilde{\Gamma}}^{r\varphi}\epsilon_2. \tag{6.39}$$

The projections are only slightly modified to

$$\tilde{\tilde{\Gamma}}_{r123}\epsilon = -\epsilon, \quad \tilde{\tilde{\Gamma}}_{\theta\varphi}\epsilon = \tilde{\tilde{\Gamma}}_{12}\epsilon. \tag{6.40}$$

In this frame the Vielbein are,

$$\begin{aligned}
\tilde{\tilde{e}}^r &= \frac{\lambda\lambda_1^2 L^4 dr - r(x_1dx_1 + x_2dx_2)}{\sqrt{\Delta}Lr}, \\
\tilde{\tilde{e}}^\theta &= \frac{\lambda_1^2 L(\Delta r d\theta\sin\theta - \lambda x_1\cos\theta(\lambda r dx_1 + x_1 dr))}{\sqrt{\Delta}\sqrt{\Xi}r}, \\
\tilde{\tilde{e}}^\varphi &= \frac{\lambda^2\cos\theta\left(dx_2\left(\lambda_1^4L^4 + x_2^2\right) + x_1x_2dx_1\right) - \Delta d\varphi}{\sqrt{\Delta}L\sqrt{\Xi}}, \\
\tilde{\tilde{e}}^1 &= -\frac{\lambda_1^2 L}{\sqrt{\Xi}r}\Big(\cos\xi\lambda r x_1\cos\theta d\xi + \sin\xi\big(\lambda r(x_1\cos\theta d\theta + \sin\theta dx_1) + x_1\sin\theta dr\big)\Big), \\
\tilde{\tilde{e}}^2 &= -\frac{\lambda_1^2 L}{\sqrt{\Xi}r}\Big(-\sin\xi\lambda r x_1\sin\theta d\xi + \cos\xi\big(\lambda r(x_1\cos\theta d\theta + \sin\theta dx_1) + x_1\sin\theta dr\big)\Big), \\
\tilde{\tilde{e}}^3 &= \frac{L\left(\lambda_1^2 r dx_2 + \lambda x_2 dr\right)}{\sqrt{\Delta}r}.
\end{aligned} \tag{6.41}$$

It is possible to show that these solutions support an orthogonal SU(2) structure where

$$\begin{aligned}
&z = v + iw = -\tilde{\tilde{e}}^1 + i\tilde{\tilde{e}}^2,\\
&j = \tilde{\tilde{e}}^{3r} + \tilde{\tilde{e}}^{\varphi\theta},\\
&\omega = (\tilde{\tilde{e}}^{\varphi} + i\tilde{\tilde{e}}^{\theta}) \wedge (\tilde{\tilde{e}}^3 + i\tilde{\tilde{e}}^r).\\
&\tilde{\tilde{\theta}}_p = \frac{\pi}{2}, \quad \tilde{\tilde{\theta}}_m = 0.
\end{aligned} \tag{6.42}$$

Let us move to a similar study for the case of the NATD of $AdS_5 \times Y^{p,q}$.

6.2 G-structures of $Y^{p,q}$ NATD-T

The vielbeins of $Y^{p,q}$ in the frame favoured by NATD are,

$$\begin{aligned}
&e^{x^\mu} = \frac{r}{L}dx^\mu, \qquad e^r = \frac{L}{r}dr, \qquad e^y = \frac{L}{\sqrt{vw}}dy\,, \qquad e^\alpha = Lkd\alpha,\\
&e^{1,2} = \frac{L\sqrt{m}}{\sqrt{6}}\sigma_{1,2}, \qquad e^3 = L(\sqrt{g}\sigma_3 + hd\alpha).
\end{aligned} \tag{6.43}$$

With respect to this basis the projection conditions that the Majorana-Killing spinor ϵ obeys [74], can be succinctly expressed in terms the functions

$$\cos\kappa(y) = \frac{m}{3\sqrt{g}}, \qquad \sin\kappa(y) = -\frac{\sqrt{vw}}{6\sqrt{g}}, \tag{6.44}$$

as

$$\Gamma_{r\alpha}\epsilon = \Gamma_{y3}\epsilon, \quad \Gamma_{r123}\epsilon = (\cos\kappa + \sin\kappa\Gamma_{3\alpha})\epsilon. \tag{6.45}$$

From these it is possible to define an SU(3)-structure, however unlike in the case of the Klebanov-Witten background, this will not be canonical in the NATD frame eq. (6.43). Instead it takes the form

$$\begin{aligned}
J &= e^r \wedge (\cos\kappa e^3 + \sin\kappa e^\alpha) + (-\sin\kappa e^3 + \cos\kappa e^\alpha) \wedge e^y + e^{21},\\
\Omega_{\text{hol}} &= (e^r + i(\cos\kappa e^3 + \sin\kappa e^\alpha)) \wedge ((-\sin\kappa e^3 + \cos\kappa e^\alpha) + ie^y) \wedge (e^2 + ie^1).
\end{aligned} \tag{6.46}$$

Of course, this can be put into canonical form by performing a rotation in e^α, e^3 which is rather more reminiscent of the wrapped D5 solution (see for example [75]) rather than the Klebanov-Witten case. The difference between NATD and canonical structure frames makes the G-structure analysis of $Y^{p,q}$ more complicated than the previous example. Indeed it was shown in [32] that a similar rotation in the wrapped D5 solution leads to a dynamical SU(2)-structure in the NATD. However we will learn shortly that this is not the case for the NATD of $Y^{p,q}$. Indeed, as observed in [38] the structure is orthogonal.

The next step is to perform the NATD on the σ_i's. To keep things compact we once more express the dual coordinates as

$$x_1 = \rho\sin\chi, \qquad x_2 = \rho\cos\chi. \tag{6.47}$$

The dual vielbeins are given by

$$\begin{aligned}
\hat{e}^a &= e^a, \qquad a = x^\mu, r, y, \alpha,\\
\hat{e}^1 &= -\sqrt{m}\tfrac{x_1\left(dx_2+L^2(gd\xi+\sqrt{g}hd\alpha)\right)\left(L^2m\cos\xi+gx_2\sin\xi)\right)+\left(gx_1^2\sin\xi+L^2g(-6x_2\cos\xi+L^2m\sin\xi)\right)dx_1}{6\sqrt{6}\Pi}\\
\hat{e}^2 &= -\sqrt{m}\tfrac{x_1\left(dx_2+L^2(gd\xi+\sqrt{g}hd\alpha)\right)\left(L^2m\sin\xi-gx_2\cos\xi)\right)-\left(gx_1^2\cos\xi+L^2g(6x_2\sin\xi+L^2m\cos\xi)\right)dx_1}{6\sqrt{6}\Pi}\\
\hat{e}^3 &= -\tfrac{36x_2(x_1dx_1+x_2dx_2)\sqrt{g}-6L^2x_1^2(\sqrt{g}d\xi+hd\alpha)m+L^4\sqrt{g}m^2dx_2}{36L\Pi}.
\end{aligned} \tag{6.48}$$

The action on the spinor MW killing spinors is once more

$$\hat{\epsilon}_1 = \epsilon_1, \quad \hat{\epsilon}_2 = \Omega_{\mathrm{SU}(2)}\epsilon_2, \tag{6.49}$$

where in the frame of eq. (6.48) $\Omega_{\mathrm{SU}(2)}$ is given by eq. (6.13) with ζ^a

$$\zeta_1 = \frac{\sqrt{6}}{L^2\sqrt{gm}}x_1\sin\xi, \quad \zeta_2 = \frac{\sqrt{6}}{L^2\sqrt{gm}}x_1\cos\xi, \quad \zeta_3 = \frac{6}{L^2m}x_2. \tag{6.50}$$

One can once more calculate the Kosmann derivative at this stage and find that the only isometry preserving SUSY under a further Abelian T-duality is ∂_α. Using the projections in eq. (6.45) one arrives at,

$$\hat{\epsilon}_2 = (\cos\kappa\hat{\Gamma}^r + \sin\kappa\hat{\Gamma}^y + \zeta_a\hat{\Gamma}^a)\epsilon_2, \tag{6.51}$$

which suggests performing a rotation such that,

$$\begin{aligned}
\hat{e}^{r\prime} &= \cos\kappa\hat{e}^r + \sin\kappa\hat{e}^y,\\
\hat{e}^{y\prime} &= \cos\kappa\hat{e}^y - \sin\kappa\hat{e}^r,\\
\hat{e}^{a\prime} &= \hat{e}^a, \quad a \neq r, y
\end{aligned} \tag{6.52}$$

so that $\hat{\epsilon}_2$ takes the same form as it did for the Klebanov-Witten NATD. Indeed if we then rotate $\hat{e}^{r\prime}$ with the matrix $\mathcal{R}_1$ in eq. (6.27) but with ζ_a now defined by eq. (6.50) we find the orthogonal SU(2) structure,

$$\begin{aligned}
z &= v + iw = \tilde{e}^r + i\tilde{e}^3,\\
j &= \tilde{e}^{\alpha y} + \tilde{e}^{21},\\
\omega &= (\tilde{e}^\alpha + i\tilde{e}^y)\wedge(\tilde{e}^2 + i\tilde{e}^1)
\end{aligned} \tag{6.53}$$

$$\tilde{\theta}_p = 0, \qquad \tilde{\theta}_m = \frac{\pi}{2}. \tag{6.54}$$

The new vielbeins are given by

$$
\begin{aligned}
\tilde{e}^r =& \frac{L^4\sqrt{g}m\left(\cos\kappa d\log r+\sin\kappa\frac{dy}{\sqrt{vw}}\right)-6(x_1dx_1+x_2dx_2)}{6L\sqrt{\Pi}}\\
\tilde{e}^y =& L\cos\kappa d\log r-L\sin\kappa\frac{dy}{\sqrt{vw}},\qquad \tilde{e}^\alpha=Lkd\alpha,\\
\tilde{e}^1 =& -\frac{L\sqrt{m}}{\sqrt{6}\sqrt{\Pi}}\left[x_1\left(\sin\xi\left(\cos\kappa d\log r+\sin\kappa\frac{dy}{\sqrt{vw}}\right)+\cos\xi hd\alpha\right)+\sqrt{g}\big(\sin\xi dx_1+x_1\cos\xi d\xi\big)\right]\\
\tilde{e}^2 =& -\frac{L\sqrt{m}}{\sqrt{6}\sqrt{\Pi}}\left[x_1\left(-\cos\xi\big(\cos\kappa d\log r+\sin\kappa\frac{dy}{\sqrt{vw}}\right)+\sin\xi hd\alpha\right)\\
&+\sqrt{g}\big(-\cos\xi dx_1+x_1\sin\xi d\xi\big)\Big]\\
\tilde{e}^3 =& -\frac{L}{6\sqrt{\Pi}}\left(6x_2\sqrt{g}\left(\cos\kappa d\log r+\sin\kappa\frac{dy}{\sqrt{vw}}\right)+mdx_2\right),
\end{aligned}
\tag{6.55}
$$

where

$$
\Pi=\frac{1}{36}\big(6x_1^2+g(36x_2^2+L^4m^2)\big).
\tag{6.56}
$$

Finally we turn consider the structure of the $Y^{p,q}$ NATD-T on α solution, we omit the details of the the derivation and just present the result. The geometry also presents an orthogonal SU(2) structure with forms given by

$$
\begin{aligned}
z&=v+iw=\tilde{\tilde{e}}^\alpha-i\tilde{\tilde{e}}^r,\\
j&=\tilde{\tilde{e}}^{y3}+\tilde{\tilde{e}}^{21},\\
\omega&=(\tilde{\tilde{e}}^y+i\tilde{\tilde{e}}^3)\wedge(\tilde{\tilde{e}}^2+i\tilde{\tilde{e}}^1),
\end{aligned}
\tag{6.57}
$$

$$
\tilde{\tilde{\theta}}_p=\frac{\pi}{2}+\arctan\frac{6x_2}{L^2m},\qquad \tilde{\tilde{\theta}}_m=\xi.
\tag{6.58}
$$

where here the vielbein basis is

$$
\begin{aligned}
\tilde{\tilde{e}}^r &= \frac{L\sqrt{m}}{\sqrt{\Theta}}\left[x_1\sqrt{h^2+k^2}\left(\sin\beta d\log r+\cos\beta\frac{dy}{\sqrt{vw}}\right)+\sqrt{g}kdx_1\right],\\
\tilde{\tilde{e}}^y &= L\left(-\cos\beta d\log r+\sin\beta\frac{dy}{\sqrt{vw}}\right),\qquad \tilde{\tilde{e}}^\alpha=\frac{Ldx_1k\sqrt{gm}}{\sqrt{\Theta}}d\alpha,\\
\tilde{\tilde{e}}^1 &= \frac{L}{\sqrt{6}\sqrt{\Theta}\sqrt{36x_2^2+L^4m^2}}\left[\sqrt{g}k\big(36x_2^2+L^4m^2\big)\left(\sin\beta d\log r+\cos\beta\frac{dy}{\sqrt{vw}}\right)\right.\\
&\quad\left.-6m\sqrt{h^2+k^2}x_1dx_1\right],\\
\tilde{\tilde{e}}^2 &= \frac{\big(h^2+k^2\big)\big(36x_1x_2dx_1+(36x_2^2+L^4m^2)dx_2\big)-\sqrt{g}h\big(36x_2^2+L^4m^2\big)d\alpha}{\sqrt{6}L\sqrt{\Theta}\sqrt{h^2+k^2}\sqrt{36x_2^2+L^4m^2}},\\
\tilde{\tilde{e}}^3 &= \frac{d\alpha}{L\sqrt{h^2+k^2}},
\end{aligned}
\tag{6.59}
$$

where we have defined the following functions for concision

$$\cos\beta(y) = \frac{k\cos\kappa - h\sin\kappa}{\sqrt{h^2+k^2}}, \quad \sin\beta(y) = \frac{h\cos\kappa + k\sin\kappa}{\sqrt{h^2+k^2}}, \tag{6.60}$$

and

$$\Theta = 6k^2\Pi + h^2 m x_1^2. \tag{6.61}$$

In summary, we have shown in this section that two of the new backgrounds we obtained by a successive application of a NATD and a T-duality on α are SUSY preserving; both of them support an orthogonal SU(2)-structure.

6.3 Comments on supersymmetry and relation to other works

It has been established that NATD-T of the $T^{1,1}$ and $Y^{p,q}$ backgrounds are supersymmetric solutions when the final U(1) transformation is performed on ϕ or α respectively. Similar arguments run into trouble when the final T-duality is applied along the ξ direction. Indeed, this plays the role of the U(1) R-symmetry in the dual solutions which is inherited from the original backgrounds. SUSY preservation is fairly intuitive in the frame preferred by T-duality on either U(1) or SU(2) isometries. It merely requires the Killing spinor to be independent of the isometry directions [70, 71]. The frame dependence of this statement is akin to saying that a metric is stationary because it does not depend explicitly on time. The general statement in the case of a stationary metric is the existence of a time-like Killing vector. Here too, a general frame independent statement can be made in terms of the Kosmann derivative [71]. When the Kosmann derivative vanishes along the isometry then SUSY is preserved under a T-duality transformation.

It is worth commenting on related work which sought supersymmetric backgrounds with AdS_5 factors. The most relevant work in this context is the classification of [6] where a large class of supersymmetric solutions of IIB supergravity with AdS_5 were classified. A natural question we need to address is the place of the solutions generated in this manuscript in the above classification. There are two properties of the solutions classified in [6] that our solutions do not satisfy. Namely, in order to make further progress, [6] considered solutions with non-vanishing F_5 form and with trivial axion. As can be seen clearly in the explicit expressions for our backgrounds we have *(i)* $F_5 = 0$, *(ii) Nontrivial axion, that is,* $F_1 \neq 0$. *This class of solutions was not considered in* [6].[9]

There is also work along the lines of a systematic classification of solutions with AdS_5 which is relevant but somehow more restrictive since they demand the existence of an S^2 factor inside the M_5 submanifold [7]. In particular, [7] demands that the S^2 not be fibered over the M_3 manifold that complements it inside M_5. This direct product requirement is motivated by the goal of having $\mathcal{N}=2$ SCFT on the field theory side whereby this SU(2) would be dual to $SU(2)_R$; without this direct product it is not possible to get doublets under SU(2) which are required by SUSY. Clearly, our solutions evade this classifications as the S^2 part is always fibered over the remaining M_3 base.

[9]We thank D. Martelli for insightful comments and clarifications on this point.

Let us finally comment on a general property of the classification approach at large. In the papers under comparison, [6] and [7], but in the generic situation in IIB, it is assumed that supersymmetry imposes a holomorphic condition on the axion-dilaton [76]. Then, using the statement that in a compact manifold the only regular harmonic functions are constant functions one arrives at a constant axion-dilaton. By looking at the form of the dilaton in some of the various solutions presented, it does not look anything like a holomorphic function. In fact, in some cases it seems to depend on three coordinates, for example, (χ, ρ, θ_1). Therefore, the supersymmetry mechanism underlying our solutions seems to be of a quite different nature and deserves to be scrutinized further.

7 Comments on the dual field theory: a generalization of toric duality?

In this section, we will try to put together different comments on the field theories dual to our different backgrounds.

An important source of information comes from the role of Page charges. These are particularly useful in backgrounds with regular as well as fractional D-branes. The prototypical example was worked out in this language explicitly in [58]. It was shown there, that the transformation of the Page charges matches precisely the transformation of the rank of gauge groups under Seiberg duality. Namely, under large gauge transformation in the Klebanov-Strassler background, the Page charges transform exactly as the ranks of the gauge groups of the dual field theory. Given the transformation of the Page charges in the background we discussed in section 4, we believe it is plausible that there is a version of Seiberg duality at work. This duality does not involved the energy scale as is the case in the Klebanov-Strassler background [57]. Hence, we propose that this Seiberg-like duality, represented by large gauge transformations of the B_2 NS field, is a transformation between conformal theories. This interpretation was first made in [36] where the NATD of $AdS_4 \times \mathbb{CP}^3$ was studied, the solutions we consider here also exhibit such behaviour.

As mentioned above, we can always 'counter' a motion in the ρ-coordinate with a large gauge transformation of the B-field. this mixing of geometry and NS-fields points to some relation with non-geometric backgrounds.

The transformations in the ρ-coordinate and the large gauge transformations of the B-field that we discussed in section 4, implied a change in the Page charges of D6 and D4 branes given by

$$\Delta Q_{P,D6} = 0, \quad \Delta Q_{P,D4} = -nN_{D6}; \tag{7.1}$$

that reads exactly like the changes in Page charges for D5 and D3 branes in the cascade of dualities for the Klebanov-Strassler system.

Moreover, in the KS-case, it was shown that motions in the radial direction (an 'Energy' direction labelled τ in the KS background) implied changes in the quantity b_0 [58]. In order to keep the quantity b_0 bounded, a Seiberg duality was applied when flowing down or up in the radial coordinate τ. In the cases considered in this paper, and as first observed in [36], motion in the ρ-direction (that is *not* a motion in energies in the dual field theory) also implies the need for a change in description to keep b_0 bounded; identifying $\rho \sim \rho + \pi$ and changing description of the QFT every time we cross $\rho = n\pi$ with $(n = 1, 2, 3, 4 \ldots)$. This change in description can be seen, by analogy, as a Seiberg duality.

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Notice that the coordinate ρ is *not* periodic — in the same way that the radial τ-coordinate is certainly not periodic in the KS-system. However, as suggested in [36], there is a 'minimal cell' of length π in the ρ-coordinate. This is somewhat analogous to what happens for an H_2 manifold (or any other negatively curved Riemann manifold), that can be locally written in terms of coordinates (x, y) with metric $ds^2 \sim y^{-2}(dx^2 + dy^2)$. In principle (x, y) are unbounded, but the H_2 is described in terms of a minimal cell, with finite volume.

Notice also that while the Klebanov-Witten field theory is self-dual under Seiberg duality, the quiver dual to the NATD or our new backgrounds need not be. Indeed, examples of this sort have been studied in the context of toric duality. A toric duality on the gravity (geometry) side and a Seiberg duality on the field theory side have been shown to be equivalent [77–79]. The origin of the toric duality is the inherent ambiguity in the definition of a toric diagram that arises from unimodular transformations on the lattice defining the diagram. For example, in the case of Calabi-Yau toric three folds defined on $\mathbb{Z}_3$, the set of $\mathrm{SL}(3, \mathbb{C})$ transformation leaving the endpoints of the vector defining the toric diagram invariant gives the same toric variety. In those cases, toric duality has nothing to do with the energy scale of the dual field theory, as the CFT is conformal and all the gauge groups of the quiver have the same rank.

When we apply these ideas to our context, we argued by analogy that there is certain ambiguity in the range of the coordinate $\rho \in [n\pi, \pi(n + 1)]$; which whenever a 'boundary' is crossed, changes the field theory description, which leads to Seiberg dual versions (different toric manifolds) of the same theory. Either that or that we 'undo' the crossing with a large gauge transformation of the B-field, that would also change the vacuum where the QFT is defined.

An obvious comment we need to make regarding this analogy: the cone over the resulting compact 5-manifold, $\mathcal{C}(W_5)$, is not necessarily toric; it is certainly not Calabi-Yau. Therefore, the situation we are describing must be related to a non-toric and not Calabi-Yau extension of the standard argument for D3 branes place at a toric singularity.

Another quantity that provided important information is the central charge. We have found, along our different dualities that it is always possible to write the central charge in terms of the Page charge N_c characterizing the background in the form $c \sim N_c^2$. In the examples before the NATD $N_c = N_{D3}$ is the Page charge of D3 branes. After the NATD we found $N_c = N_{D6}$ is the number of D6 branes. In all cases, this is a characteristically 'gauge theoretic' behavior. It is also of interest the comparison with the backgrounds of [45, 46].

In the same line of central charges, we observe in all of our examples given in section 4, that the quotient of the central charges, before and after the NATD — is given by

$$\frac{c_{\text{before}}}{c_{\text{after}}} = 3\frac{N_{D3}^2}{N_{D6}^2}. \tag{7.2}$$

This gives an interesting hint about the complete description of these new field theories, for which we are finding a dual description.

Let us move to comment something about gauge couplings. Let us focus on the Klebanov-Witten system and its NATD. In the KW case, it was shown that the gauge couplings of the two groups are given by (we take, like above, $g_s=1$),

$$\frac{4\pi^2}{g_1^2}+\frac{4\pi^2}{g_2^2}=\pi e^{-\Phi}, \quad \frac{4\pi^2}{g_1^2}-\frac{4\pi^2}{g_2^2}=\pi e^{-\Phi}\left(\frac{1}{2\pi^2\alpha'}\int_{\Sigma_2}B_2-1\right). \tag{7.3}$$

Using these definitions, we obtain the expressions for two individual couplings $\frac{8\pi^2}{g_{1,2}^2}$

$$\frac{8\pi^2}{g_1^2}=2\pi e^{-\Phi}b_0, \quad \frac{8\pi^2}{g_2^2}=2\pi e^{-\Phi}(1-b_0), \tag{7.4}$$

where as above $4\pi^2\alpha' b_0=\int_{\Sigma_2}B_2$. It is from expressions like these in eq. (7.4), that the condition that b_0 is bounded in the interval $(0,1)$ is imposed. The coupling g_1^2 can be calculated using a D5 brane that wraps a two cycle inside the conifold with an electric field in its worldvolume. Conversely, one can equate the inverse coupling with the BI part of the Action of an Euclidean D1 wrapping the same cycle.

In our NATD geometries written in section 2.2.1, we can define a configuration representing an instanton in two ways. First, by wrapping an Euclidean D2 brane on the cycle parameterized by $\Sigma_3=[\theta_1,\phi_1,\xi]$, with $\chi=\frac{\pi}{2}$ and at constant ρ. We can also consider an Euclidean D0 brane, that extends along the direction ϕ_1, with $\chi=\frac{\pi}{2}$ and constant ρ.[10] Let us discuss in detail the calculation with an Euclidean D0 brane. The induced metric and gauge field on the cycle $\Sigma_1=[\phi_1]$ (restricted such that $2\chi=\pi$ and $\theta_1=0$) are,

$$\begin{aligned} ds^2_{D0} &= \frac{\alpha'^2L^2}{Q}\lambda^2\lambda_2^2\rho^2 d\phi_1^2, \\ C_1 &= 4\frac{\lambda\lambda_1^2\lambda_2^2L^4}{\alpha'^{3/2}}d\phi_1. \end{aligned} \tag{7.5}$$

We calculate the Born-Infeld and the Wess-Zumino parts of the Action for this D0,

$$\begin{aligned} S_{\rm BI} &= -T_{D0}\int d\phi_1 e^{-\Phi}\sqrt{\det[g_{\rm ind}]}=\frac{L^2}{\alpha'}\lambda\lambda_2 2\pi\rho=\sqrt{N_{D6}}\pi\rho\sim b_0. \\ S_{\rm WZ} &= T_{D0}\int C_1 = N_{D6}\pi. \end{aligned} \tag{7.6}$$

We have used the explicit values of $\lambda,\lambda_1,\lambda_2$ and the relation $2L^4=27\alpha'^2N_{D6}$, discussed above. We then equate this to the Action of an instanton $S_{\rm inst}=\frac{8\pi^2}{g^2}+i\Theta$, obtaining expressions

$$\frac{8\pi^2}{g^2}\sim b_0, \quad \Theta=\pi N_{D6}. \tag{7.7}$$

In the case of the calculation with the Euclidean D2 brane, things are quite analogous. There, we see that the induced metric and B-field on the three manifold are,

$$ds^2_{\rm ind}=L^2\lambda_1^2(d\theta_1^2+\sin^2\theta_1 d\phi_1^2)+\frac{\alpha'^2}{Q}L^2\lambda^2\lambda_2^2\rho^2(d\xi+\cos\theta_1 d\phi_1)^2; \quad B_2=0. \tag{7.8}$$

[10] Calculating the gauge coupling and theta angle, with a D6 brane extended in $R^{1,3}\times\Sigma_3$ and switching on an electric field on the Minkowski directions would give a similar result.

We then calculate the BI-Action and get

$$T_{D2}\int d\theta_1 d\phi_1 d\xi e^{-\Phi}\sqrt{\det[g_{\rm ind}]}=\frac{2\pi L^4\lambda_1^2\lambda_2\lambda}{\alpha'^2}\frac{\rho}{\pi}. \tag{7.9}$$

Equating the BI action with the Action for an instanton $S_{\rm BI}\sim\frac{1}{g^2}$, we obtain that the coupling defined in this way is

$$\frac{1}{g^2}\sim b_0. \tag{7.10}$$

These results make contact with the one summarised in eq. (7.4) for the Klebanov-Witten background and reinforces the point that the quantity b_0 should be bounded, as we imposed above. It would be interesting to attempt a similar calculation for the cases of $Y^{p,q}$ and S^5.

Let us finally comment on interesting future problems and close this section with a general comment about our procedure. It would be interesting to use the variables we introduced and apply our methodology to the full duality cascade represented by the Baryonic Branch of the Klebanov-Strassler theory (choosing a mesonic branch is also possible, but more complicated technically). Working out this problem will produce as anticipated in [32] a configuration in Massive IIA. It would be interesting to study the interplay (if any) between the usual Seiberg duality in the 'Energy' direction r and the one described in this section. Finally, we should point out that our smooth backgrounds and procedure can be thought of, as a way of defining a field theory. Indeed, any of our non-singular backgrounds defines the large N_c strong coupled regime of a (presumably new!) QFT. Obtaining properties of these field theories using the supergravity backgrounds is a way of learning about new QFTs. The set of variables and techniques developed in this work make this task clearer.

8 Conclusions

In this paper we have presented several genuinely new supergravity backgrounds. Whilst these new solutions are singular, it is worth pointing out that in some cases the singularity structure is very mild and one can hope for a smoothing mechanism.

Our solutions "evade" previous classification efforts [6] due to the fact that they, generically do not contain an F_5 and have non-trivial and non-holomorphic axion-dilaton. More recent classification efforts have focused on solutions with a round S^2 which is not fibered over the rest of the manifold [7]; our solutions certainly do not contain such S^2 factor.

We tried to clarify the extent to which field theory data (central charge, entanglement entropy) were invariant under NATD. Our analysis was limited to the concrete cases we tackled. We found an intriguing relation for the quotient between the central charges before and after the NATD, that seems to be universal. It would be interesting to prove this universality (if correct), in full generality, that is, including situations with generic B-field. We host the hope that such approach could shed some light on the extension to which the Ryu-Takayanagi formula goes beyond simple supergravity backgrounds, embodying deeper string-theoretic principles. We also presented a proposal for a Seiberg-like duality acting on the field theories dual to our backgrounds. This proposal was based on the study of

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supergravity quantities. By the same study, a relation with non-geometric backgrounds is suggested by the interplay between the motions in the ρ-coordinate and large gauge transformations of the B_2-field.

There are a few interesting venues that we believe are worth exploring. One interesting generalization, would be to attempt to generate more general solutions exploiting 'spinor rotations' like in [80–82]. Indeed, given that we know the SU(2)-structure of some of our new solutions, one could speculate with 'rotating' the structure to obtain new solutions. This approach should lead to interesting solutions with exciting gravity duals. It would also be interesting to have a better understanding of the central charge, when calculated using the M-theory backgrounds. Using our supergravity backgrounds, we can calculate different observables in the initial and final CFTs to compare them. This is probably a fruitful line of work, that will give information on the structure of the new CFTs. The relation with non-geometric backgrounds is of obvious interest. It would also be interesting to place the backgrounds studied here within the formalism recently developed in [83, 84].

Another interesting direction would be to pursue some of the guidelines of the analysis of Lunin and Maldacena in [10] where it was clarified that the gravity transformation corresponding to T-s-T applies to any field theory with a global $\mathrm{U}(1) \times \mathrm{U}(1)$ symmetry. We can similarly argue that our transformation applies to any field theory with a gravity dual and a $\mathrm{SU}(2) \times \mathrm{U}(1)$ global symmetry; it is important to remark that this global symmetry should be different from the R-symmetry so as to preserve supersymmetry. Ultimately, similar to how [10] presented the transformation as a symmetry of string theory compactified on a torus where there is a natural action on the torus complex parameter by $\mathrm{SL}(2, \mathbb{R})$, it should be possible to compactify on the appropriate manifold and formulate our NATD-T as a symmetry of the lower dimensional theory. This approach might shed some light on the structure of NATD as well. Finally, perhaps the most interesting question is related to the elusive h-deformation of $\mathcal{N} = 4$ SYM. Given the reduced number of symmetries of the supergravity background dual to the h-deformed $\mathcal{N} = 4$ SYM, the best chances for finding the background should rely on solution generating techniques. Naïvely, however, NATD in its present form is not an invertible transformation and thus prevents an approach mimicking that of Lunin and Maldacena in [10]. But probably an approach along the lines discussed in this paper and [71] may help to 'invert' the NATD. We hope to return to some of these questions in the future.

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A Review of non-Abelian and Abelian T-duality rules

A.1 Non-Abelian T-duality

We follow [25] in the generalized 3-step Büscher procedure and consider only backgrounds with an SU(2) isometry such that the metric can be written in the form

$$ds^2 = G_{\mu\nu}(x)dx^\mu dx^\nu + 2G_{\mu i}(x)dx^\mu L^i + g_{ij}(x)L^i L^j \tag{A.1}$$

where $\mu,\nu = 1,\dots 7$ and $i,j = 1,2,3$. The L^i's are the SU(2) Maurer-Cartan forms. ($L^i_\pm = -i\mathrm{Tr}(t^i g^{-1}\partial_\pm g)$). We also consider a similar decomposition of the antisymmetric 2-form,

$$B = \frac{1}{2}B_{\mu\nu}(x)dx^\mu \wedge dx^\nu + B_{\mu i}(x)dx^\mu \wedge L^i + \frac{1}{2}b_{ij}L^i \wedge L^j \tag{A.2}$$

The Lagrangian density for the NS sector fields is given below, where we omit the dilaton contribution. (The transformation of the dilaton is given below in A.10.)

$$\mathcal{L}_0 = Q_{AB}\partial_+ X^A \partial_- X^B \tag{A.3}$$

where $A,B = 1,\dots,10$ and

$$Q_{AB} = \left(\begin{array}{c|c} Q_{\mu\nu} & Q_{\mu i} \\ \hline Q_{i\mu} & E_{ij} \end{array}\right), \quad \text{and} \quad \partial_\pm X^A = \left(\partial_\pm X^\mu,\ L^i_\pm\right) \tag{A.4}$$

with

$$Q_{\mu\nu} = G_{\mu\nu} + B_{\mu\nu}, \quad Q_{\mu i} = G_{\mu i} + B_{\mu i}, \quad Q_{i\mu} = G_{i\mu} + B_{i\mu}, \quad E_{ij} = g_{ij} + b_{ij} \tag{A.5}$$

We then gauge the SU(2) isometry by changing derivatives to covariant derivatives according to, $\partial_\pm g \to D_\pm g = \partial_\pm g - A_\pm g$. The next step is to add a Lagrange multiplier term to A.3 to ensure the gauge fields, $A_\pm$ are non-dynamical.

$$-i\mathrm{Tr}(\alpha' v F_\pm), \quad F_\pm = \partial_+ A_- - \partial_- A_+ - [A_+, A_-] \tag{A.6}$$

We must now eliminate three of the variables by making a gauge fixing choice, described in detail in B.2 below. A natural choice is $g = \mathbb{I}$, so that all 3 of the Lagrange multipliers, v_i, become dual coordinates. The last step is to integrate out the gauge fields to obtain the dual Lagrangian density,

$$\hat{\mathcal{L}} = \hat{Q}_{AB}\partial_+ \hat{X}^A \partial_- \hat{X}^B \tag{A.7}$$

where we can read off the dual components of $\hat{Q}_{AB}$ from,

$$\hat{Q}_{AB} = \left(\begin{array}{c|c} Q_{\mu\nu} - Q_{\mu i}M^{-1}_{ij}Q_{j\nu} & Q_{\mu j}M^{-1}_{ji} \\ \hline -M^{-1}_{ij}Q_{j\mu} & M^{-1}_{ij} \end{array}\right), \quad \text{and} \quad \partial_\pm \hat{X}^A = \left(\partial_\pm X^\mu,\ \partial_\pm v^i\right) \tag{A.8}$$

where we have defined,

$$M_{ij} = E_{ij} + f_{ij}, \quad \text{with} \quad f_{ij} = \alpha' \epsilon_{ij}^{\ \ k} v_k \tag{A.9}$$

(Note that If we wish to carry through the correct factors of α', we must include one factor of α' in front of the v_i's.) The v_i's originating in the Lagrange multiplier term may now take on the role of dual coordinates, depending on the gauge fixing choice. We can identify the dual metric and $\hat{B}_2$ field as the symmetric and antisymmetric components of $\hat{Q_{AB}}$, respectively. The transformation of the dilation is given by

$$\hat{\Phi} = \Phi - \frac{1}{2} \ln \left(\frac{\det M}{\alpha'^3} \right) \tag{A.10}$$

A.1.1 RR flux transformation

In order to transform the RR Fluxes, one must construct a bispinor out of the RR forms and their Hodge duals, (in Type IIB)

$$P = \frac{e^{\Phi}}{2} \sum_{n=0}^{5} \not{F}_{2n} \tag{A.11}$$

where $\not{F}_p = \frac{1}{p!} \Gamma_{\mu_1 \ldots \mu_p} F_p^{\ \mu_1 \ldots \mu_p}$. Then, the dual fluxes arise from inverting Ω

$$\hat{P} = P \cdot \Omega^{-1} \tag{A.12}$$

where $\Omega = (A_0 \Gamma^1 \Gamma^2 \Gamma^3 + A_a \Gamma^a) \Gamma_{11} / \sqrt{\alpha'^3}$ and

$$A_0 = \frac{1}{\sqrt{1+\zeta^2}}, \qquad A^a = \frac{\zeta^a}{\sqrt{1+\zeta^2}} \tag{A.13}$$

where $\zeta^a = \kappa^a_{\ i} z^i$ with $\kappa^a_{\ i} \kappa^a_{\ j} = g_{ij}$ and $z^i = \frac{y^i}{\det \kappa}$, $y_i = b_i + v_i$.

A.2 Abelian T-duality

Our conventions for the Büscher rules of Abelian T-duality [70] for the NS sector, including the appropriate factors of α', are

$$\begin{aligned} \tilde{G}_{99} &= \frac{\alpha'}{G_{99}}, \qquad \tilde{G}_{9i} = -\frac{\alpha' B_{9i}}{G_{99}}, \quad \tilde{G}_{ij} = G_{ij} - \frac{\alpha'}{G_{99}} (G_{9i} G_{9j} - B_{9i} B_{9j}) \\ \tilde{B}_{9i} &= -\frac{G_{9i}}{G_{99}}, \qquad \tilde{B}_{ij} = B_{ij} - \frac{1}{G_{99}} (G_{9i} B_{9j} - B_{9i} G_{9j}) \end{aligned} \tag{A.14}$$

where x_9 is the direction with the U(1) isometry we wish to dualize along. One can construct the form $\tilde{B}_2$ using,

$$\tilde{B}_2 = \tilde{B}_{ij} dx_i \wedge dx_j + \alpha' \tilde{B}_{9i} dx_9 \wedge dx_i. \tag{A.15}$$

The dilaton transformation is

$$\tilde{\Phi} = \Phi - \frac{1}{2} \ln \frac{G_{99}}{\alpha'}. \tag{A.16}$$

A.2.1 RR flux transformation

Similar to the NATD RR Flux transformation, one must construct a bispinor out of the RR forms and their Hodge duals, (in Type IIA)

$$P = \frac{e^{\Phi}}{2}\sum_{n=0}^{4} \not{F}_{2n+1} \tag{A.17}$$

The dual fluxes arise from inverting Ω

$$\hat{P} = P \cdot \Omega^{-1} \tag{A.18}$$

where

$$\Omega = \frac{\alpha'}{\sqrt{G_{99}}}\Gamma_{11}\Gamma_9 \tag{A.19}$$

B Multiparametric families of solutions in Type IIA and Type IIB

B.1 NATD-s-T on ξ of $AdS_5 \times S^5$

Motivated by the work of Lunin-Maldacena [10] we also consider constructing a one-parameter family of solutions. Namely, inspired by the TsT transformation that lead [10] to the construction of a large class of gravity solutions with interesting field theory duals, we perform a shift with parameter γ such that

$$\theta \to \theta + \gamma\xi, \tag{B.1}$$

where ξ is a U(1) angle in the S^2 leftover from NATD and θ is the U(1) angle originating in the S^2 (see the first line in eq. (5.1) for our non-standard notation of the angles) that was unaffected by the NATD. If we then T-dualize along ξ, we obtain,

$$\begin{aligned}
d\tilde{\hat{s}}^2 &= 4ds^2(AdS_5) + 4L^2 d\alpha^2 + \frac{\alpha'^2 d\rho^2}{L^2\cos^2\alpha} + \frac{1}{W}\Big(L^2\alpha'^2\rho^2\sin^2 2\alpha\sin^2\chi d\theta^2 \\
&\quad + \frac{\alpha'^2}{L^2}(\rho^2(L^4\gamma^2\sin^2 2\alpha + \alpha'^2\rho^2\sin^2\chi)d\chi^2 - \alpha'^2 2\rho^3\sin\chi d\chi d\xi + (\alpha'^2\rho^2 + L^4\cos^4\alpha)d\xi^2)\Big), \\
\tilde{\hat{B}} &= \frac{4\alpha'\gamma\sin^2\alpha\, d\theta\wedge((\alpha'^2\rho^2 + L^4\cos^4\alpha)d\xi - \alpha'^2\rho^3\sin\chi d\chi)}{W} \\
e^{-2\tilde{\hat{\Phi}}} &= \frac{L^4}{\alpha'^4}\cos^2\alpha W
\end{aligned} \tag{B.2}$$

where

$$W = 4\gamma^2(\alpha'^2\rho^2 + L^4\cos^4\alpha)\sin^2\alpha + \alpha'^2\rho^2\cos^2\alpha\sin^2\chi. \tag{B.3}$$

As in the previous cases $\alpha = \pi/2$ is a singularity of the dilaton. We verified that this singularity is indeed a curvature singularity by direct computation of the Ricci scalar. The dual RR fluxes are,

$$\begin{aligned}
\tilde{\hat{F}}_1 &= \frac{8L^4}{\alpha'^2}\gamma\cos^3\alpha\sin\alpha d\alpha, \\
W\tilde{\hat{F}}_3 &= -8L^4\alpha'\rho^2\cos^3\alpha\sin\alpha\sin\chi d\alpha\wedge d\theta\wedge\left(4\rho\gamma^2\sin^2\alpha d\chi + \cos^2\alpha\sin\chi d\xi\right).
\end{aligned} \tag{B.4}$$

All of the Type IIB equations have been satisfied for this case.

B.2 Generic gauge fixing in NATD

In this section we discuss gauge ambiguities and consider new solutions that can be generated by exploiting these intrinsic ambiguities in the NATD procedure. As explained in detail in [25] and [23], the NATD procedure requires gauge fixing leading to potentially dim(G) degrees of freedom. For our case of SU(2), we have up to three parameters that can be exploited. Given a specific SU(2) matrix D^{ij}, we can perform an orthogonal transformation on the Lagrange multipliers via

$$\hat{v}_i = D_{ij} v^j \tag{B.5}$$

where $D^{ij} = Tr(t^i g t^j g^{-1})$, and which explicitly can be written as[11]

$$D_{ij} = \begin{pmatrix} C_{\beta_0}C_{\psi_0}C_{\phi_0} - S_{\psi_0}S_{\phi_0} & C_{\beta_0}C_{\phi_0}S_{\psi_0} + C_{\psi_0}S_{\phi_0} & -C_{\phi_0}S_{\beta_0} \\ -C_{\beta_0}C_{\psi_0}S_{\phi_0} - C_{\phi_0}S_{\psi_0} & C_{\psi_0}C_{\phi_0} - C_{\beta_0}S_{\psi_0}S_{\phi_0} & S_{\beta_0}S_{\phi_0} \\ C_{\psi_0}S_{\beta_0} & S_{\beta_0}S_{\psi_0} & C_{\beta_0} \end{pmatrix}. \tag{B.6}$$

We will define $v = \alpha'(x_1, x_2, x_3)$ and henceforth, set $\alpha' = 1$ and $L = 1$. Note that numerical coefficients in front of the x's may be used for convenience.

From eq. (B.5) one can see that $\hat{v}$ is covariant with respect to an SO(3) transformation. We would like to investigate how ambiguities might arise through the process of identifying the dual metric coordinates. In order to see how the parameters $(\beta_0, \psi_0, \phi_0)$ and (x_1, x_2, x_3) responds to a generic SO(3) transformation, say $O(\beta, \psi, \phi)$, we write $v' = Ov$ and $D' = ODO^T$. Here (β, ψ, ϕ) are generic angles for the SO(3)(SU(2)) transformation matrix. One may collect the coordinates and the parameters of D^{ij} in the six-tuple $\Phi = (x_1, x_2, x_3, \beta_0, \psi_0, \phi_0)$. Then, under an infinitesimal transformation, where $(\beta \approx \epsilon_1, \psi \approx \epsilon_2, \phi \approx \epsilon_3)$, we find $\delta\Phi = (\delta x_1, \delta x_2, \delta x_3, \delta\beta_0, \delta\psi_0, \delta\phi_0)$, where

$$\begin{aligned} \delta x_1 &= x_2\Lambda_1 - x_3\Lambda_2 \\ \delta x_2 &= -x_1\Lambda_1 \\ \delta x_3 &= x_1\Lambda_2 \\ \delta\beta_0 &= (-\Lambda_1 + \Lambda_2 \csc(\beta_0)\sin(\phi_0) + \Lambda_2\cot(\beta_0)\sin(\psi_0), \psi_0, \phi_0) \\ \delta\psi_0 &= \Lambda_2(\cos(\phi_0) - \cos(\psi_0)) \\ \delta\phi_0 &= \Lambda_1 - \Lambda_2\cot(\beta_0)\sin(\phi_0) - \Lambda_2\csc(\beta_0)\sin(\psi_0). \end{aligned} \tag{B.7}$$

where $\Lambda_1 \equiv \epsilon_1 + \epsilon_2$, and $\Lambda_2 \equiv \epsilon 3$. Λ_1 displays an isotropy which reduces the access of gauge fixing conditions to only SU(2)/U(1). This suggests that in this case there will always be gauge redundancy in the choice of dual coordinates [14].

We define the dual coordinates by pulling back these six coordinates onto the three manifold that will serve as the dual volume. Let $\xi = (\xi_1, \xi_2, \xi_3)$ denote the dual coordinates

[11] Shorthand notation has been used such that $C_x = \cos x$ and $S_x = \sin x$.

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and denote $\mathcal{X}$ as the pullback function that maps Φ into ξ. Then the pullback functions becomes,

$$\mathcal{X} : \Phi \rightarrow (x_1(\xi), x_2(\xi), x_3(\xi), \beta_0(\xi), \psi_0(\xi), \phi_0(\xi)) \equiv \Phi(\xi).$$

The dual metric is then constructed from frame fields given by,

$$e^a_j = \frac{\partial \hat{v}^a}{\partial \xi^j} = \frac{\partial \hat{v}^a}{\partial \Phi^L}\frac{\partial \Phi(\xi)^L}{\partial \xi^j},$$

where $L = 1 \cdots 6$. We can then determine the differential volume density of the internal dual metric via, $g_{ij} = e^a_i e^b_j \delta_{ab}$. If the volume, $\det(e)$, is zero, the pullback is tantamount to a "poor gauge fixing" choice as the coordinates have dependence on each other. Three degrees of freedom have been used to specify the dual frame and three remaining degrees of freedom serve as parameters that can span a family of dual volumes. One may ask how the dual space metric transforms under eq. (B.8) to see if there is residual symmetry. One strategy would be to compute the matrix,

$$c_{ab} = \frac{\partial^2 \det(e')}{\partial \Lambda_a \partial \Lambda_b}|_{\Lambda=0}, \quad a, b = 1, 2,$$

where here e'^a_j is the response of the dual frame fields to the infinitesimal transformations in eq. (B.8). When $\det(c_{ab})$ vanishes, this suggests that the two remaining parameters are not independent and residual symmetry exists.

As an example, consider the pullback $\mathcal{X}$ such that $\mathcal{X} : \Phi -> (\xi_1, \xi_2, \xi_3, \frac{1}{2}\pi, 0, 0)$. This gives the frame fields

$$e^a_j = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \tag{B.8}$$

and the $\det(e) = 1$. Had we chosen our pullback to be $\mathcal{X} : \Phi -> (\xi_1, \xi_2, \xi_3, \omega_1, \omega_2, \omega_2)$, where ω_1, ω_2, and ω_2 are constants. The frame fields become

$$e^a_j(\omega) = \begin{pmatrix} C_{\omega_1} C_{\omega_2} C_{\omega_3} - S_{\omega_2} S_{\omega_3} & C_{\omega_1} C_{\omega_3} S_{\omega_2} + C_{\omega_2} S_{\omega_3} & -C_{\omega_3} S_{\omega_1} \\ -C_{\omega_1} C_{\omega_2} S_{\omega_3} - C_{\omega_3} S_{\omega_2} & C_{\omega_2} C_{\omega_3} - C_{\omega_1} S_{\omega_2} S_{\omega_3} & S_{\omega_1} S_{\omega_3} \\ C_{\omega_2} S_{\omega_1} & S_{\omega_1} S_{\omega_2} & C_{\omega_1} \end{pmatrix}, \tag{B.9}$$

which is precisely an SO(3) matrix and therefore $\det(e) = 1$.

Below are the most obvious pullback choices for the dual volume. Extra parameters are labeled with a $_0$ subscript.

$$
\left(\begin{array}{cc}
\text{Dual Coordinates, } \xi & \det\,(e) \\
\hline
(x_1,x_2,x_3) & 1 \\
(x_1,x_2,\beta) & (x_2\sin\psi_0 + x_1\cos\psi_0) \\
(x_1,x_2,\psi) & 0 \\
(x_1,x_2,\phi) & \sin\beta_0\,(x_2\cos\psi_0 - x_1\sin\psi_0) \\
(x_1,x_3,\beta) & x_3\sin\psi_0 \\
(x_1,x_3,\psi) & x_1 \\
(x_1,x_3,\phi) & (x_1\cos\beta_0 - x_3\sin\beta_0\cos\psi_0) \\
(x_1,\beta,\psi) & x_1\,(x_{20}\sin\psi + x_1\cos\psi) \\
(x_1,\beta,\phi) & x_1\,(x_{30}\sin\beta - \cos\beta\,(x_{20}\sin\psi_0 + x_1\cos\psi_0)) \\
(x_1,\psi,\phi) & x_1\sin\beta_0\,(x_{20}\cos\psi - x_1\sin\psi) \\
(x_2,x_3,\beta) & x_3\cos\psi_0 \\
(x_2,x_3,\psi) & x_2 \\
(x_2,x_3,\phi) & (x_2\cos\beta_0 - x_3\sin\beta_0\sin\psi_0) \\
(x_2,\beta,\psi) & x_2\,(x_2\sin\psi + x_{10}\cos\psi) \\
(x_2,\beta,\phi) & x_2\,(x_{30}\sin\beta - \cos\beta\,(x_2\sin\psi_0 + x_{10}\cos\psi_0)) \\
(x_2,\psi,\phi) & x_2\sin\beta_0\,(x_2\cos\psi - x_{10}\sin\psi) \\
(x_3,\beta,\psi) & x_3\,(x_{20}\sin\psi + x_{10}\cos\psi) \\
(x_3,\beta,\phi) & x_3\,(x_3\sin\beta - \cos\beta\,(x_{20}\sin\psi_0 + x_{10}\cos\psi_0)) \\
(x_3,\psi,\phi) & x_3\sin\beta_0\,(x_{20}\cos\psi - x_{10}\sin\psi) \\
(\beta,\psi,\phi) & 0
\end{array}\right) \tag{B.10}
$$

Note that in some of the cases the parameters support the internal manifold's volume. For example, when $(x_1,\ x_2,\ \beta)$ are coordinates, the parameter ψ_0 cannot be set to zero, or else the metric will have zero volume. Therefore, certain "poor gauge fixing" choices can be remedied by introducing these parameters.

B.3 Multiparametric solutions of NATD of $AdS_5 \times T^{1,1}$

In the remaining sections we present a few examples of new Type IIA solutions with extra parameters, generated by exploiting the gauge fixing ambiguities of NATD discussed above. In all of the following examples, we have checked explicitly that all of the Type IIA equations are satisfied. Here we present an example using $AdS_5 \times T^{1,1}$.

1. $x_1,\ x_3,\ \psi$ are coordinates and x_{20} is an extra parameter ($\beta_0 = 0$, $\phi_0 = 0$)

$$
\begin{aligned}
\hat{ds}^2 &= ds^2(AdS_5) + \lambda_1^2\left(d\phi_1{}^2\sin^2\theta_1 + d\theta_1{}^2\right) \\
&\quad + \frac{1}{\Delta_0}\Big(\big(x_1^2+\lambda^2\lambda_2^2\big)dx_1^2 + \big(x_3^2+\lambda_2^4\big)dx_3^2 + \big(x_1^2+x_{20}^2\big)\lambda^2\lambda_2^2(\cos\theta_1\,d\phi_1 + d\psi)^2 \\
&\quad + 2\,dx_1\big(x_1x_3\,dx_3 - x_{20}\lambda^2\lambda_2^2(\cos\theta_1\,d\phi_1 + d\psi)\big)\Big) \\
\hat{B} &= \frac{1}{\Delta_0}\Big(\lambda_2^2x_{20}\,dx_1\wedge dx_3 + \big(\lambda^2x_1x_3\,dx_1 + \lambda_2^2(x_1^2+x_{20}^2)\,dx_3\big)\wedge d\psi \\
&\quad - \lambda^2\cos\theta_1\big((x_3^2+\lambda_2^4)\,dx_3 + x_1x_3\,dx_1\big)\wedge d\phi_1\Big) \\
e^{-2\hat{\Phi}} &= \Delta_0, \quad \Delta_0 = \lambda^2\left(\lambda_2^4 + x_3^2\right) + \lambda_2^2\left(x_1^2 + x_{20}^2\right)
\end{aligned} \tag{B.11}
$$

$$\hat{F}_2 = 4\lambda\lambda_1^2\lambda_2^2 \sin\theta_1 d\theta_1 \wedge d\phi_1 \tag{B.12}$$
$$\hat{F}_4 = \frac{1}{\Delta_0}(4\lambda\lambda_1^2\lambda_2^2 \sin\theta_1(\lambda_2^2 x_{20} dx_1 \wedge dx_3 \wedge d\theta_1 \wedge d\phi_1$$
$$+(\lambda_2^2(x_1^2+x_{20}^2)dx_3 - \lambda^2 x_1 x_3 dx_1) \wedge d\theta_1 \wedge d\phi_1 \wedge d\psi))$$

B.4 Multiparametric solutions of NATD of $AdS_5 \times S^5$

Here we present additional examples using $AdS_5 \times S^5$.

1. x_1, x_2, x_3 are coordinates, β_0, ϕ_0 ψ_0 are parameters

$$\begin{aligned}
\hat{ds}^2 &= 4ds^2(AdS_5) + 4d\alpha^2 + 4\sin^2\alpha d\theta^2 \\
&\quad + \frac{1}{\cos^2\alpha\Delta_1}((x_1^2+\cos^4\alpha)dx_1^2 + (x_2^2+\cos^4\alpha)dx_2^2 + (x_3^2+\cos^4\alpha)dx_3^2 \\
&\quad + 2x_1dx_1(x_2dx_2+x_3dx_3) + 2x_2x_3dx_2dx_3) \\
\hat{B} &= -\frac{1}{\Delta_1}(x_3dx_1\wedge dx_2 - x_2dx_1\wedge dx_3 + x_1dx_2\wedge dx_3) \\
e^{-2\hat{\Phi}} &= \cos^2\alpha\Delta_1
\end{aligned} \tag{B.13}$$

where $\Delta_1 = ((x_1^2+x_2^2+x_3^2)+\cos^4\alpha)$

$$\begin{aligned}
\hat{F}_2 &= 8L^4\cos^3\alpha\sin\alpha d\alpha\wedge d\theta, \\
\hat{F}_4 &= -\frac{8\cos^3\alpha\sin\alpha}{\Delta_1}(x_3dx_1\wedge dx_2 - x_2dx_1\wedge dx_3 + x_1dx_2\wedge dx_3)\wedge d\alpha\wedge d\theta
\end{aligned}$$

This is precisely the answer we would have obtained if we had chosen a general gauge fixing (i.e. (x_1, x_2, x_3, $\beta=0$, $\phi=0$ and $\psi=0$) with no extra parameters.

2. x_1, x_3, ψ are coordinates, x_{20}, β_0, ϕ_0 are parameters (though only x_{20} appears)

$$\begin{aligned}
\hat{ds}^2 &= 4ds^2(AdS_5) + 4d\alpha^2 + 4\sin^2\alpha d\theta^2 \\
&\quad + \frac{\sec^2\alpha}{\Delta_2}((x_1^2+\cos^4\alpha)dx_1^2 + (x_3^2+\cos^4\alpha)dx_3^2 + (x_1^2+x_{20}^2)\cos^4\alpha d\psi^2 \\
&\quad + 2dx_1(x_1x_3dx_3 + x_{20}\cos^4\alpha d\psi)) \\
\hat{B} &= -\frac{1}{\Delta_2}(x_{20}dx_1\wedge dx_3 + x_1x_3dx_1\wedge d\psi - (x_1^2+x_{20}^2)dx_3\wedge d\psi) \\
e^{-2\hat{\Phi}} &= \cos^2\alpha\Delta_2, \quad \Delta_2 = (x_1^2+x_{20}^2+x_3^2+\cos^4\alpha)
\end{aligned} \tag{B.14}$$

$$\begin{aligned}
\hat{F}_2 &= -4\cos^3\alpha\sin\alpha d\alpha\wedge d\theta, \\
\hat{F}_4 &= -\frac{4\cos^3\alpha}{\Delta_2}\,(x_{20}\sin\alpha dx_1\wedge dx_3\wedge d\alpha\wedge d\theta + x_1x_3\sin\alpha dx_1\wedge d\alpha\wedge d\theta\wedge d\psi \\
&\quad -(x_1^2+x_{20}^2)\sin\alpha dx_3\wedge d\alpha\wedge d\theta\wedge d\psi)
\end{aligned} \tag{B.15}$$

3. x_3, ψ, β are coordinates, x_{10}, x_{20}, ϕ_0 are parameters (though only x_{10} and x_{20} appear)

$$\begin{aligned}
\hat{ds}^2 &= 4ds^2(AdS_5) + 4d\alpha^2 + 4\sin^2\alpha d\theta^2 \\
&\quad + \frac{\cos^2\alpha}{\Delta_3}\Big((1 + x_3^2\sec^4\alpha)dx_3^2 + (x_{10}^2 + x_{20}^2)d\psi^2 + 2(x_{10}\cos\psi + x_{20}\sin\psi)dx_3 d\beta \\
&\quad -2x_3(x_{20}\cos\psi - x_{10}\sin\psi)d\psi d\beta + \\
&\quad \left(\frac{1}{2}(x_{10}^2 + x_{20}^2 + 2x_3^2 + (x_{10}^2 - x_{20}^2)\cos 2\psi) + 2x_{10}x_{20}\sin 2\psi\right)d\beta^2\Big) \\
\hat{B} &= \frac{1}{\Delta_3}((x_{10}^2 + x_{20}^2)dx_3\wedge d\psi + x_3(-x_{20}\cos\psi + x_{10}\sin\psi)dx_3\wedge d\beta \\
&\quad +(x_{10}^2 + x_{20}^2 + x_3^2)(x_{10}\cos\psi + x_{20}\sin\psi)d\beta\wedge d\psi) \\
e^{-2\hat{\Phi}} &= 4\cos^2\alpha\Delta_3, \quad \Delta_3 = (x_{10}^2 + x_{20}^2 + x_3^2 + \cos^4\alpha)
\end{aligned} \tag{B.16}$$

$$\begin{aligned}
\hat{F}_2 &= -4\cos^3\alpha\sin\alpha d\alpha\wedge d\theta, \\
\hat{F}_4 &= -\frac{4\cos^3\alpha\sin\alpha}{\Delta_3}(-(x_{10}^2 + x_{20}^2)dx_3\wedge d\psi\wedge d\alpha\wedge d\theta \\
&\quad +x_3(x_{20}\cos\psi - x_{10}\sin\psi)dx_3\wedge d\beta\wedge d\alpha\wedge d\theta \\
&\quad +(x_{10}^2 + x_{20}^2 + x_3^2)(x_{10}\cos\psi + x_{20}\sin\psi)d\psi\wedge d\beta\wedge d\alpha\wedge d\theta))
\end{aligned} \tag{B.17}$$

C Killing spinor on $AdS_5 \times S^5$

In this appendix we derive a Killing spinor for $AdS_5 \times S^5$ that is independent of the SU(2) directions on which the NATD is performed.

To start we choose the vielbein basis

$$e^{x^\mu} = \frac{2r}{L}dx^\mu, \qquad e^r = \frac{2L}{r}dr, \qquad e^i = L\cos\alpha\sigma_i, \tag{C.1}$$

$$e^4 = 2Ld\alpha, \qquad e^5 = 2L\sin\alpha d\theta, \tag{C.2}$$

where $i = 1, 2, 3$. With respect to this basis the non zero components of the spin connection are

$$\omega^{x^\mu r} = \frac{1}{2L}e^{x^\mu}, \quad \omega^{45} = -\frac{1}{2L}\cot\alpha e^5, \quad \omega^{i5} = -\frac{1}{2L}\tan\alpha e^i, \quad \omega^{ij} = \frac{1}{2L}\sec\alpha\epsilon_{ijk}e^k. \tag{C.3}$$

which clearly indicates a 5+5 split, so the gravitino variation along the AdS_5 and S^5 directions can be treated independently.[12] As F_5 is given by

$$F_5 = \frac{2}{L}\left(e^{tx^1x^2x^3r} - e^{12345}\right) \tag{C.4}$$

and we choose

$$\Gamma^{tx^1x^2x^3r12345}\epsilon = \Gamma_{AdS_5}\Gamma_{S^5}\epsilon = -\epsilon, \tag{C.5}$$

[12] Since the dilaton is constant and only the 5-form flux is non trivial the dilatino variation is automatically satisfied.

the AdS_5 part leads to[13]

$$\left(\nabla_\mu + \frac{i}{2L}\Gamma_{AdS_5}\Gamma_\mu\right)\epsilon = 0 \tag{C.6}$$

where $\mu = t, x^1, x^2, x^3, r$, which is a standard Killing spinor equation on AdS_5 and so for our purposes it is sufficient to solve the gravitino variation on the S^5 directions. This is given by

$$\left(\nabla_a - \frac{i}{2L}\Gamma_{S^5}\Gamma_a\right)\epsilon = 0 \tag{C.7}$$

where $a = 1, 2, 3, 4, 5$. If we make the assumption that ϵ is independent of the SU(2) directions this gives the following set of coupled differential and algebraic equations

$$\begin{aligned}
(2\partial_\alpha + i\Gamma_{1235})\epsilon &= 0, \\
(2\partial_\theta - \cos\alpha\Gamma_{45} - i\sin\alpha\Gamma_{1234})\epsilon &= 0, \\
(\Gamma_{23} - \sin\alpha\Gamma_{14} - i\cos\alpha\Gamma_{2345})\epsilon &= 0, \\
(\Gamma_{13} + \sin\alpha\Gamma_{24} - i\cos\alpha\Gamma_{1345})\epsilon &= 0, \\
(\Gamma_{12} - \sin\alpha\Gamma_{34} - i\cos\alpha\Gamma_{1245})\epsilon &= 0.
\end{aligned} \tag{C.8}$$

These reduce to a projection

$$\Gamma_{45}\epsilon = (\cos\alpha + i\sin\alpha\Gamma_{1235})\epsilon \tag{C.9}$$

and two differential equations

$$\begin{aligned}
(2\partial_\alpha + i\Gamma_{1235})\epsilon &= 0, \\
(2\partial_\theta + i)\epsilon &= 0.
\end{aligned} \tag{C.10}$$

The whole Killing spinor then takes the form

$$\epsilon = \mathcal{M}(AdS_5)e^{-\frac{i}{2}\theta}e^{-\frac{i\alpha}{2}\Gamma_{1235}}\eta, \tag{C.11}$$

where η is a constant spinor obeying

$$\Gamma_{45}\eta = \eta, \tag{C.12}$$

and $\mathcal{M}(AdS_5)$ is a matrix which commutes with the projection and depends on the AdS_5 directions. There for a total of 16 real supercharges are preserved.

As we have found a Killing spinor preserving $\mathcal{N} = 2$ SUSY in 4-d which is independent of the SU(2) direction [71] tells us that this is the SUSY preserve by the NATD solution, confirming the result of [23].

The NATD solution of $AdS_5 \times S^5$ contains two U(1) isometries, ∂_θ and ∂_ξ. We have checked that in the preferred frame of NATD the the Kosmann derivative in each case reduces to

$$\mathcal{L}_{\partial_\theta}\hat{\epsilon} = \partial_\theta\hat{\epsilon}, \quad \mathcal{L}_{\partial_\xi}\hat{\epsilon} = \partial_\xi\hat{\epsilon}. \tag{C.13}$$

[13] Here $\epsilon = \epsilon_1 + i\epsilon_2$, where ϵ_i are the MW Killing spinors in 10-d.

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The dual MW Killing spinors are given in this frame by

$$\hat{\epsilon}_1 = \epsilon_1, \quad \hat{\epsilon}_2 = \Omega \epsilon_2. \tag{C.14}$$

where

$$\Omega = \frac{-L^2 \Gamma_{123} + \rho \big(\sin\chi \cos\xi \Gamma_1 + \sin\chi \sin\xi \Gamma_2 + \cos\chi \Gamma_3 \big)}{\sqrt{\rho^2 + L^4 \cos^4 \alpha}}. \tag{C.15}$$

Thus it is easy to see that a further Abelian T-duality along θ or ξ will break SUSY completely because the angular dependence in eq. (C.11) ensures that neither Kosmann derivative can vanish [71].

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