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What is a chiral 2d CFT? And what does it have to do with extremal black holes?

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Abstract: The near horizon limit of the extremal BTZ black hole is a "self-dual orbifold" of AdS_3. This geometry has a null circle on its boundary, and thus the dual field theory is a Discrete Light Cone Quantized (DLCQ) two dimensional CFT. The same geometry can be compactified to two dimensions giving AdS_2 with a constant electric field. The kinematics of the DLCQ show that in a consistent quantum theory of gravity in these backgrounds there can be no dynamics in AdS_2, which is consistent with older ideas about instabilities in this space. We show how the necessary boundary conditions eliminating AdS_2 fluctuations can be implemented, leaving one copy of a Virasoro algebra as the asymptotic symmetry group. Our considerations clarify some aspects of the chiral CFTs appearing in proposed dual descriptions of the near-horizon degrees of freedom of extremal black holes.

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Contents

1 Introduction

In the vicinity of their horizons, extremal black holes in many dimensions, both in flat and anti-de Sitter spaces, contain an AdS_2 component with a constant electric field.[1] Proposed dualities between AdS_2 space and a conformal quantum mechanics [2–8] or a chiral 1+1 dimensional conformal field theory (CFT) [7, 9] have been used to explain the statistical degeneracy of extremal black holes. In [9, 10] it was shown the AdS_2 geometry with a constant electric field can be understood as the compactification of an orbifold of AdS_3 with a null boundary. Systematically applying the rules of the AdS/CFT correspondence then suggests that the dual theory on the 1+1 dimensional boundary is a Discrete Light Cone Quantized CFT [8–11]. Because of the highly boosted kinematics of a DLCQ theory, only one chiral sector of the 2d CFT survives. Such chiral theories thus seem to appear universally in the dual descriptions of extremal black holes.

In this paper, we develop aspects of this DLCQ - extremal black hole correspondence. The essential features can be understood by considering the extremal BTZ geometry, which itself appears in the near-horizon geometry of many asymptotically flat or AdS black holes. It is well known that the BTZ black holes are dual to thermal ensembles in a 1+1 dimensional CFT. Thermal ensembles in a single chiral sector of this CFT are dual to the extremal black holes and explain their statistical degeneracy. Taking a limit which focuses on the vicinity of the BTZ horizon gives a locally AdS_3 geometry that is a circle fibration over an AdS_2 base. From the three dimensional perspective this is precisely the self-dual orbifold of [10, 12]. Dimensionally reducing over the circle fibre gives an AdS_2 geometry with an electric flux — precisely the spacetime appearing in [9]. As we will show in sections 2

[1] This statement is an actual theorem in four and five dimensions, under certain isometry assumptions and for extremal black holes with finite area horizons [1].

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and 3, the same focusing limit applied to the CFT dual to BTZ effectively applies a DLCQ procedure that isolates the chiral sector carrying the extremal black hole entropy. Thus, one chiral set of Virasoro generators of the CFT is frozen in this limit, in the sense that there are no physical states charged under them. It turns out that the same chiral sector also contains the $SL(2,\mathbb{R})$ isometries of the AdS_2 geometry, while the surviving $SL(2,\mathbb{R})$ in the limiting chiral CFT appears as an enhancement of the U(1) symmetry of the circle fibration. Specifically, we show that there exists a consistent set of boundary conditions on the fluctuations of the near horizon extremal BTZ metric, as in the Brown-Henneaux analysis [13], that enhances the U(1) isometry to an asymptotic chiral Virasoro algebra. This is consistent with recent proposals that the description of extremal black holes in terms of an AdS_2 throat requires asymptotic boundary conditions eliminating AdS_2 excitations and enhancing a U(1) appearing in the geometry to a Virasoro symmetry [14, 15].

Usually in the AdS/CFT duality, the isometries of spacetime are realized in the dual as global symmetries which then organize the representations of physical states. The surprise here is that the $SL(2,\mathbb{R})$ symmetry inherited in the CFT from the spacetime isometries acts trivially on the space of physical states. This has two implications. First, the chiral duals to the near-horizon geometry of extremal black holes are incapable of describing non-extremal excitations. Second, even after the addition of an electric field to AdS_2, $2d$ quantum gravity with this asymptotics has no dynamics. This is consistent with the idea that finite energy excitations in AdS_2 destroy its asymptotic structure [16]. These two points are related to the fact that non-extremal black holes do not have AdS_2 throats. Similarly, in the classic setting of the D1-D5-string, extremal black holes arise from chiral excitations, and non-extremality requires excitations of both left and right movers.

The self-dual orbifold and AdS_2 with a flux also appear in the near horizon limit of the extremal Kerr black hole in four dimensions [14] suggesting the appearance of a chiral CFT dual. However, in this setting (as in [17–19]) the near-horizon AdS geometries appear in a "warped" way, with their metric multiplied by a function of another angular direction in the overall spacetime. We suggest that reduction over this additional direction can give rise to an effective three dimensional gravity with a negative cosmological constant with the self-dual orbifold as a solution. The dual description of this space as a chiral $2d$ CFT then explains the statistical degeneracy of Kerr.

Note added: In the last stage of preparation of this article two papers appeared on the arXiv [20, 21] arguing that there is no dynamics in the chiral $2d$ CFT proposed to be dual to the near horizon extremal Kerr geometry, in agreement with our results.

2 Near horizon extremal BTZ is dual to DLCQ of a $2d$ CFT

BTZ black holes are three dimensional, asymptotically AdS_3 spacetimes with metric [22, 23]

$$ds^2 = -\frac{(r^2-r_+^2)(r^2-r_-^2)}{r^2\ell^2}dt^2 + \frac{\ell^2 r^2}{(r^2-r_+^2)(r^2-r_-^2)}dr^2 + r^2\left(d\phi - \frac{r_+r_-}{\ell r^2}dt\right)^2. \tag{2.1}$$

They have ADM angular momentum and mass

$$\frac{J}{2} = \frac{r_+r_-}{\ell^2}, \qquad M = \frac{r_+^2+r_-^2}{\ell^2} \tag{2.2}$$

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given in terms of two parameters: the inner and outer horizons $r_\pm$. These are locally AdS_3 spacetimes, differing from global AdS_3 by a quotient under a discrete identification. This is the origin of the periodicity in ϕ in (2.1), i.e. $\phi \sim \phi + 2\pi$. Regularity of the metric requires $|J| \leq M$. The BTZ black holes also appear as components in the near-horizon geometry of black holes in many dimensions with both vanishing and negative cosmological constants (e.g. see [24, 25]). The *extremal* BTZ black holes ($M = J$) have coincident inner and outer horizons

$$M = J \quad \Longrightarrow \quad r_+ = r_- \equiv r_h \,. \tag{2.3}$$

Globally, the generator of the discrete quotient of AdS_3 giving rise to the extremal black hole lies in a different conjugacy class from the generator giving rise to the non-extremal black hole [26].

According to the AdS/CFT correspondence, quantum gravity in AdS_3 is dual to a $2d$ conformal field theory (CFT) with equal left and right central charges c [13]

$$c = \frac{3\ell}{2G_3} \,, \tag{2.4}$$

where G_3 is Newton's constant in three dimensions. The BTZ black holes are thermal states in this CFT having left and right-moving temperatures

$$T_R = \frac{1}{4\pi}\frac{r_+ - r_-}{\ell} \,, \qquad T_L = \frac{1}{4\pi}\frac{r_+ + r_-}{\ell} \,, \tag{2.5}$$

with energy and angular momentum:

$$L_0 - \frac{c}{24} = M - J, \qquad \bar{L}_0 - \frac{c}{24} = M + J. \tag{2.6}$$

(In our conventions M and J are both dimensionless; their natural units are given by the AdS_3 radius ℓ.) In the extremal ($M = J$) black hole the right-movers are in the ground state[2]

$$L_0 = \frac{c}{24}; \qquad T_R = 0 \tag{2.7}$$

while the left moving temperature $T_L = \frac{1}{2\pi}\frac{r_h}{\ell}$ and $\bar{L}_0$ are arbitrary. The extremal BTZ entropy (and that of higher dimensional black holes of which it is the near horizon limit) is accounted for by the statistical degeneracy of such a chiral CFT sector with $\bar{L}_0 - c/24 = 2M$, at least when $\bar{L}_0 \gg c/24$.

We will see that the chiral sector that is responsible for the extremal entropy can be isolated by taking a near-horizon limit of the extremal BTZ black hole [9]. It is convenient to do this in another set of coordinates [27]

$$\hat{u} = t/\ell - \phi \,, \qquad \hat{v} = t/\ell + \phi, \qquad r^2 - r_+^2 = \ell^2 e^{2\rho} \,, \tag{2.8}$$

[2]In theories with supersymmetry these are indeed the obvious ground states in the RR sector. This condition can even correspond to ground states in the NS sector, because in many examples the quantum numbers L_0 and $\bar{L}_0$ are not exactly identical to the standard CFT quantum numbers but can e.g. receive contributions from gauge fields which make them spectral flow invariant, in which case this condition really implies that the states have to be chiral primary. Although we have no proof that $L_0 = c/24$ always implies that the states have to be ground states of some sort, we will continue to refer to these states as ground states and hope that this will not cause any confusion.

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in which the metric takes the form[3]

$$ds^2 = r_+^2\, d\hat{u}^2 + \ell^2\, d\rho^2 - \ell^2 e^{2\rho}\, d\hat{u}\, d\hat{v}\ . \tag{2.10}$$

The variables $\hat{u},\ \hat{v}$ have a periodicity

$$\{\hat{u}, \hat{v}\} \sim \{\hat{u} - 2\pi, \hat{v} + 2\pi\}\,. \tag{2.11}$$

On the cylindrical boundary of AdS_3 ($\rho \to \infty$), $d\hat{u}$ and $d\hat{v}$ become null directions. Thus the two chiral Virasoro algebras of the dual conformal field theory are associated to asymptotic reparameterizations $\hat{u} \to f(\hat{u})$ and $\hat{v} \to g(\hat{v})$.

Since the horizon is located at $\rho \to -\infty$, we take the near horizon limit

$$\rho = \rho_0 + r, \quad u = \hat{u}\,\frac{r_+}{\ell}, \quad v = \frac{e^{2\rho_0}\ell}{r_+}\,\hat{v}, \quad \{u, v\} \sim \left\{u - 2\pi\frac{r_+}{\ell}, v + 2\pi\frac{\ell}{r_+}e^{2\rho_0}\right\} \quad (\rho_0 \to -\infty) \tag{2.12}$$

while keeping $r,\ u,\ v$ and r_+ fixed.[4] (See [9] for the first discussion of this limit.) The resulting metric, which describes the geometry in the vicinity of the extremal horizon,

$$ds^2 = \ell^2(du^2 + dr^2 - e^{2r}\, du\, dv) \tag{2.13}$$

is identical in form to (2.10) but there is a crucial difference. In the $\rho_0 \to -\infty$ limit, the identification (2.11) becomes

$$\{u, v\} \sim \left\{u - 2\pi\frac{r_+}{\ell}, v\right\}. \tag{2.14}$$

Thus, the boundary of (2.13) ($r \to \infty$) is a "null cylinder" — it has a metric conformal to $du\, dv$, the standard lightcone metric on a cylinder, but has a compact *null* direction (u). The periodicity of u encodes the temperature of the left-moving thermal state that gave the original extremal BTZ black hole its statistical degeneracy.

Rewriting the radial coordinate as $y = e^{2r}$ gives

$$ds^2 = \frac{\ell^2}{4}\left(-y^2\, dv^2 + \frac{dy^2}{y^2}\right) + \ell^2\left(du - \frac{1}{2}\, y\, dv\right)^2. \tag{2.15}$$

This is an S^1 fibration over AdS_2 which arises as a discrete identification of AdS_3. The generator of this discrete group sits inside the $SL(2,\mathbb{R})_L$ subgroup of the initial $SL(2,\mathbb{R})_L \times SL(2,\mathbb{R})_R$ isometry group of AdS_3 [9, 10]. To be precise, the parametrization of $SL(2,\mathbb{R})$ (i.e. AdS_3) that is relevant for the metric (2.15) is

$$\mathcal{G} = \begin{pmatrix} 1 & 0 \\ \frac{v}{2} & 1 \end{pmatrix} \begin{pmatrix} \sqrt{y} & \sqrt{y} \\ \frac{-1}{2\sqrt{y}} & \frac{1}{2\sqrt{y}} \end{pmatrix} \begin{pmatrix} e^u & 0 \\ 0 & e^{-u} \end{pmatrix}, \tag{2.16}$$

[3] For later use note that a generic BTZ metric in the $\hat{u}, \hat{v}, \rho$ coordinate system takes the form [27]

$$ds^2 = \ell^2\left[L^+ d\hat{u}^2 + L^- d\hat{v}^2 + d\rho^2 - (e^{2\rho} + L^+L^- e^{-2\rho}) d\hat{u}d\hat{v}\right]\ . \tag{2.9}$$

where $L^\pm = \frac{1}{4\ell^2}(r_+ \pm r_-)^2$. Recalling (2.5), $L^+ = (2\pi T_L)^2,\ L^- = (2\pi T_R)^2$.

[4] Despite the resemblance of the limit (2.12) and the coordinate changes one makes in taking the Penrose limit, (2.12) is not a Penrose limit, as the geometry we obtain after the limit is not a plane-wave.

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in terms of which the metric (2.15) is

$$ds^2 = \frac{\ell^2}{2}\mathrm{tr}(\mathcal{G}^{-1}\,d\mathcal{G})^2 \tag{2.17}$$

Under $u \to u - 2\pi r_+/\ell$, $\mathcal{G}$ is identified by the right action of

$$\begin{pmatrix} e^{-2\pi r_+/\ell} & 0 \\ 0 & e^{2\pi r_+/\ell} \end{pmatrix}. \tag{2.18}$$

The isometry group is $\mathrm{SL}(2,\mathbb{R})_R \times \mathrm{U}(1)_L$, the first factor corresponding to the isometries of the AdS_2 base.[5] On the boundary of the spacetime these isometries act to reparameterize the non-compact coordinate v. In fact, this geometry is precisely the *self-dual orbifold* of Coussaert and Henneaux [12]. The present coordinate system covers only part of the global spacetime described in [10, 12].

Since (2.13) is asymptotically locally AdS_3, we expect the dual field theory to still be a two dimensional conformal field theory, but defined on a boundary null cylinder. To understand what that means, we can follow [10] and regulate the CFT by cutting off the self-dual orbifold at a fixed, large radius. Following the usual AdS/CFT reasoning, this implements a UV cutoff in the field theory. We will remove the cutoff by sending $r \to \infty$. At any fixed r, the metric (2.13) is conformal to

$$ds^2 = du^2 - e^{2r}\,du\,dv \tag{2.19}$$

Now consider a standard cylinder with its usual Cartesian metric $ds^2 = -dt_0^2 + d\phi_0^2$ and $\{\phi_0, t_0\} \sim \{\phi_0 - \beta, t_0\}$. We will use coordinates

$$u_1 = t_0 - \phi_0 \;\;;\;\; t_1 = 2t_0 \quad\Longrightarrow\quad ds^2 = du_1^2 - du_1\,dt_1 \;\;;\;\; \{u_1, t_1\} \sim \{u_1 + \beta, t_1\}\,. \tag{2.20}$$

We now boost the cylinder with a rapidity 2γ ($\tilde{u}_1 = e^{2\gamma}u_1$) and then reparameterize the boosted cylinder so that the identification is still occurring at fixed t_1. The metric then becomes

$$ds^2 = e^{-4\gamma}\left(d\tilde{u}_1^2 - e^{2\gamma}d\tilde{u}_1\,dt_1\right) \;\;;\;\; \{\tilde{u}_1, t_1\} \sim \{\tilde{u}_1 + \beta e^{2\gamma}, t_1\} \tag{2.21}$$

Rescaling the coordinates as $\tilde{u}_1 \to e^{-2\gamma}\tilde{u}_1$ and $t_1 \to e^{-2\gamma}t_1$ gives the metric

$$ds^2 = d\tilde{u}_1^2 - e^{2\gamma}d\tilde{u}_1\,dt_1 \;\;;\;\; \{\tilde{u}_1, t_1\} \sim \{\tilde{u}_1 + \beta, t_1\}\,. \tag{2.22}$$

Thus, metrics on fixed r surfaces of the near-horizon BTZ metric (2.19) are conformal to a boosted cylinder. As $r \to \infty$ the boost becomes infinite, precisely realizing the procedure defined by Seiberg [28] for realizing the Discrete Light Cone Quantization (DLCQ) of a field theory. In section 3 we will show that following the usual kinematics of DLCQ, only one chiral sector of the CFT dual to AdS_3 will survive at finite energies.

We can also see the latter by directly examining the near-horizon limit (2.12). Acting in the CFT dual to AdS_3, the near horizon limit of the extremal BTZ black hole focuses

[5] Strictly speaking, these $\mathrm{SL}(2,\mathbb{R})$ transformations include U(1) gauge transformations compensating the transformation of the gauge field on AdS_2.

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in on energies so low that they lie below the black hole mass gap, thus eliminating all non-extremal dynamics [9] (also see [8]). This will isolate one chiral sector (the left-movers), since non-extremal, finite energy excitations necessarily involve excitations of the right-movers also. Explicitly, the infinite rescaling in the coordinate $\hat{v}$ relates translations as

$$\partial_v \sim e^{-2\rho_0}\, \partial_{\hat{v}}\,. \tag{2.23}$$

Thus, recalling the $\partial_{\hat{v}}$ is the right-moving Hamiltonian in the CFT dual to AdS_3, any finite-energy right-moving excitation, i.e. any excitation $|s\rangle$ with $\partial_{\hat{v}}|s\rangle = (L_0 - c/24)|s\rangle \neq 0$, will be infinitely blue shifted in the Hamiltonian ∂_v that is well defined in the $\rho_0 \to -\infty$ limit. In other words, we should only be keeping the states satisfying

$$\partial_{\hat{v}}|s\rangle = (L_0 - c/24)|s\rangle = 0 \tag{2.24}$$

which are the ground states in the right-moving sector.

We can also directly follow how the near-horizon limit (2.12) acts on the left and right moving Virasoro generators of the CFT dual to AdS_3. These generators are

$$L_n - \frac{c}{24}\delta_{n,0} = e^{in\hat{v}}\frac{\partial}{\partial\hat{v}}, \qquad \bar{L}_n - \frac{\bar{c}}{24}\delta_{n,0} = e^{in\hat{u}}\frac{\partial}{\partial\hat{u}}\,. \tag{2.25}$$

As $\rho_0 \to -\infty$ in the near-horizon limit (2.12), it is evident that $\bar{L}_n$ are essentially unchanged while the L_n annihilate all the *finite energy* states because of the condition (2.24).

3 DLCQ of a 2*d* CFT is a chiral CFT

In the previous section we reviewed how the near-horizon geometry of extremal BTZ is dual to the DLCQ of a $2d$ CFT. We now examine how such theories are quantized. Consider a $2d$ CFT on a cylinder

$$ds^2 = -dt^2 + d\phi^2 = -du'\, dv' \quad ; \quad u' = t - \phi,\ v' = t + \phi \tag{3.1}$$

where ϕ is a circle with radius R. Here

$$\{\phi, t\} \sim \{\phi + 2\pi R, t\} \quad ; \quad \{u', v'\} \sim \{u' - 2\pi R, v' + 2\pi R\} \tag{3.2}$$

Let $P^{u'}$ and $P^{v'}$ denote momentum operators in the v' and u' directions respectively. Their eigenvalues

$$P^{v'} = \left(h + n - \frac{c}{24}\right)\frac{1}{R}, \qquad P^{u'} = \left(h - \frac{c}{24}\right)\frac{1}{R}, \qquad n \in \mathbb{Z} \tag{3.3}$$

are given in terms of the quantized momentum n along the S^1, the $2d$ central charge c and an arbitrary value of h with $h \geq 0$ and $h + n \geq 0$. These are related to the eigenvalues of the standard operators $L_0, \bar{L}_0$ used in radial quantization on the plane by $\bar{L}_0 = h + n$ and $L_0 = h$. We will assume that the $2d$ CFT is non-singular, and therefore that the spectrum is discrete.

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Following Seiberg [28], consider a boost with rapidity γ

$$u' \to e^{\gamma} u', \qquad v' \to e^{-\gamma} v' \,. \tag{3.4}$$

The boost leaves metric (3.1) invariant.[6] However the identifications are now

$$\{u', v'\} \sim \{u' - 2\pi R e^{\gamma}, v' + 2\pi R e^{-\gamma}\} \,. \tag{3.5}$$

We want to match the boundary structure appearing in the boundary of the near horizon geometry with the DLCQ of the starting boundary cylinder. To do so, consider the limit $\gamma \to \infty$ with $R\, e^{\gamma}$ fixed. This describes a null cylinder geometry with metric $ds^2 = -du'\, dv'$ and u' a compact null direction. The same infinite boost was presented in different coordinates in (2.20)–(2.22). However, notice that since $v' \to e^{-\gamma} v' = e^{-\gamma}(t + \phi)$ and $0 \leq \phi \leq 2\pi\, R$, as $\gamma \to \infty$ any finite changes in v' come from changes in t. Thus, in the limit, $dv' \propto dt$ and $ds^2 = -du'\, dv' \approx -e^{-\gamma} du'\, dt$ which is conformal to the dominant piece of the metric in (2.22).

More explicitly, the periodicities of the boundary coordinates under the limit $\gamma \to \infty$ with $R_- \equiv R\, e^{\gamma}$ fixed are

$$\begin{pmatrix} \phi \\ t \end{pmatrix} \sim \begin{pmatrix} \phi \\ t \end{pmatrix} + \begin{pmatrix} 2\pi R \\ 0 \end{pmatrix} \; - \text{infinite boost} \to \; \begin{pmatrix} u' \\ v' \end{pmatrix} \sim \begin{pmatrix} u' \\ v' \end{pmatrix} + \begin{pmatrix} 2\pi R_- \\ 2\pi R_- e^{-2\gamma} \end{pmatrix} \tag{3.6}$$

We can now identify $\{u', v'\}$ with the lightcone boundary coordinates of AdS_3 in (2.12) via $u' = u(\ell/r_+)R_-$ and $v' = v(r_+/\ell)R_-$. Then, comparing (2.12) and (3.6), it is evident that the action of the near horizon limit on u, v precisely reproduces the identifications induced by the infinite boost in DLCQ. Thus, from this perspective also, the dual to the near-horizon geometry of the extremal BTZ black hole should be the DLCQ of the 1+1 dimensional CFT dual to AdS_3.

Because of the kinematics of the DLCQ boosts,

$$P^{v'} = \left(h + n - \frac{c}{24}\right) \frac{e^{-\gamma}}{R}, \qquad P^{u'} = \left(h - \frac{c}{24}\right) \frac{e^{\gamma}}{R} \,. \tag{3.7}$$

Keeping $P^{u'}$ (momentum along v') finite in the $\gamma \to \infty$ limit requires $h = c/24$. This leads to

$$P^{v'} = n \cdot \frac{e^{-\gamma}}{R} = \frac{n}{R_-} \,. \tag{3.8}$$

Thus the DLCQ limit (3.7) freezes the right moving sector. Equivalently, it generates an infinite energy gap in this sector, while the gap in the left-moving sector (whose energy is measured by $P^{v'}$) is kept finite. All physical finite energy states in this limit only carry momentum along the compact null direction u'. Therefore, the DLCQ $\gamma \to \infty$ limit defines a Hilbert space $\mathcal{H}$

$$\mathcal{H} = \{|\text{anything}\rangle_L \otimes |c/24\rangle_R\} \,. \tag{3.9}$$

[6] In our previous analysis the cylinder in coordinates (2.20) was boosted but also reparameterized — this is why the metric transformed to (2.21).

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It is worth noting that the extremal D1-D5-p black hole (whose near horizon limit is the BTZ black hole) is precisely dual to states of this form with the right movers in the RR ground state, and the left movers in a highly excited state the statistical degeneracy of which explains the black hole entropy [29].

Since the spectrum of the DLCQ theory is chiral we might wonder what remains in this limit of the Virasoro algebra of the CFT we started with. Denoting the right moving Virasoro generators by L_m, all states $L_m \,|\, c/24\rangle$ $(m < 0)$ have infinite energy in the DLCQ limit, since their action always changes the right-moving energy. Explicitly, consider the generators

$$L_q \sim e^{iqv'} \frac{\partial}{\partial v'}\,, \qquad \bar{L}_p \sim e^{ipu'} \frac{\partial}{\partial u'}\,, \tag{3.10}$$

with $L_0 - c/24$, $\bar{L}_0 - c/24$ being generators of translations along v' and u' respectively. After the boost (3.4) the quantization conditions for p, q become:

$$q = \frac{k}{Re^{-\gamma}} = \frac{k}{R_-} e^{2\gamma}, \qquad p = m \cdot \frac{1}{Re^{\gamma}} = \frac{m}{R_-}, \qquad k, m \in \mathbb{Z}. \tag{3.11}$$

Thus, there is a single copy of the Virasoro algebra, generated by $\bar{L}_p$, which survives the limit. This is acting on the left movers, as expected from the spectrum defining the Hilbert space of the theory. Notice the generators of this algebra are acting on the compact direction of the DLCQ null cylinder.

Summary. The DLCQ of a non-singular $2d$ CFT freezes the right moving sector to its ground states $|\, c/24\rangle$ while keeping the full left moving sector. Hence, the DLCQ limit gives a chiral $2d$ CFT with the same central charge as the original one. Applied to the BTZ black hole (section 2), we learn that the near-horizon geometry of extremal BTZ is dual to one chiral sector of the $2d$ CFT with central charge $c = 3\ell/2G_3$ that is dual to AdS_3 gravity. The surviving chiral sector is in the state in which it was placed to realize the dual to an extremal black hole, namely a thermal state at a temperature $T_L = T_{DLCQ} = R_-/(2\pi)$, corresponding to the left-moving thermal state $|\, c/24\rangle \otimes |\, T = R_-/2\pi\rangle$ in the Hilbert space of the CFT dual to AdS_3.

4 Asymptotic symmetries and the chiral Virasoro algebra

In the AdS/CFT correspondence, the isometries of spacetime manifest themselves as global symmetries of the dual field theory, and physical states are organized in representations of the isometry group. For this reason, various authors [10, 30, 31] have considered how the physical states of fields in the near-horizon BTZ geometry (2.13) or (2.15) transform under the $\mathrm{SL}(2,\mathbb{R}) \times \mathrm{U}(1)$ isometry group. Now recall that the DLCQ analysis of the dual field theory in the previous section showed that the physical states of this theory must live in a chiral CFT. It would have been natural to expect that the $\mathrm{SL}(2,\mathbb{R})$ isometries provide the global part of the associated Virasoro algebra. The surprise is that this is not the case. Specifically, the $\mathrm{SL}(2,\mathbb{R})$ isometries are associated to reparameterizations of the non-compact coordinate v on the boundary, while the physical states only carry momentum along the compact null direction u on which only the $\mathrm{U}(1)$ part of the isometry group acts.

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Thus, AdS/CFT is telling us that physical states cannot be charged under the $\mathrm{SL}(2,\mathbb{R})$ isometry group associated to the AdS_2 base in (2.15).

Why would a consistent quantum theory of gravity around the near-horizon BTZ background (2.15) require the absence of excitations in the AdS_2 base of this geometry? Perhaps because any such fluctuations would cause the space to "fragment" leading to the appearance of multiple boundaries to the spacetime [16]. In the next section we will compactify (2.15) and examine its stability to excitations in the AdS_2 base. Below we will simply accept the lesson from the analysis of the dual DLCQ field theory and implement boundary conditions for the spacetime that preserve only the predicted spectrum.

Boundary conditions. To this end, we will follow the asymptotic symmetry group analysis of Brown and Henneaux [13] by identifying the boundary conditions for "allowed" metric fluctuations close to the spacetime boundary. First recall the Brown-Henneaux boundary conditions for AdS_3. In the $\hat{u}, \hat{v}, r$ coordinates [27], where the background AdS_3 metric takes the form $ds^2 = \ell^2(\frac{dr^2}{r^2} - 2r^2 d\hat{u}d\hat{v})$ these boundary conditions at large r are [13]

$$\delta g_{\hat{u}\hat{u}} \sim \delta g_{\hat{v}\hat{v}} \sim \delta g_{\hat{u}\hat{v}} \sim \mathcal{O}(1), \qquad \delta g_{rr} \sim \mathcal{O}\left(\frac{1}{r^4}\right), \quad \delta g_{r\hat{u}} \sim \delta g_{r\hat{v}} \sim \mathcal{O}\left(\frac{1}{r^3}\right). \tag{4.1}$$

Order one fluctuations in $\delta g_{\hat{u}\hat{u}}$, $\delta g_{\hat{v}\hat{v}}$ correspond to normalizable modes in the dual $2d$ CFT and these may be chosen arbitrarily. For example, writing a generic BTZ black hole in the $\hat{u}, \hat{v}$ coordinates, the constant parts of $g_{\hat{u}\hat{u}}$ and $g_{\hat{v}\hat{v}}$ determine the ADM mass and angular momentum of the black hole (2.9). Thus, order $\mathcal{O}(1)$ fluctuations in $\delta g_{\hat{u}\hat{u}}$, $\delta g_{\hat{v}\hat{v}}$ correspond to changing the mass and angular momentum in the dual $2d$ CFT. A general deformation of $\delta g_{\hat{u}\hat{u}}$, $\delta g_{\hat{v}\hat{v}}$ would be non-extremal and would thus excite both chiral sectors of the dual CFT. By contrast, we want to restrict to extremal excitations. Recalling the form of BTZ metric (2.9), one may easily observe that imposing the extremality condition $L_0 = c/24$ requires a more stringent boundary condition on the variations in $g_{\hat{v}\hat{v}}$. The arguments of section 2 and 3 for taking the DLCQ limit and in particular (3.7) then suggest that we should replace the boundary condition on $g_{\hat{v}\hat{v}}$ by

$$\delta g_{\hat{v}\hat{v}} \sim \mathcal{O}\left(\frac{1}{r^2}\right). \tag{4.2}$$

The remainder of the Brown-Henneaux boundary conditions in (4.1) can be kept intact. Further analysis shows that these are forming a set of consistent boundary conditions. In fact this set is equivalent to choosing a subset of (4.1) that preserve the null nature of the non-compact coordinate v (up to transformations which are trivial at large r).

Asymptotic Symmetry Group. The asymptotic symmetry group (ASG) of a spacetime is the set of symmetry transformations (diffeomorphisms) which preserve the boundary conditions modulo the set of diffeomorphisms the generators of which vanish (reduce to a boundary integral) after implementation of the boundary conditions. Equipped with the above boundary conditions we can compute the ASG for the case of the near horizon extremal BTZ or the self-dual orbifold of AdS_3. We seek diffeomorphisms (vector fields ζ) whose action on the metric (Lie derivative $\mathcal{L}_\zeta g$) generates metric fluctuations compatible

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with the above boundary conditions. More mathematically, if $g_{\alpha\beta} = g^0_{\alpha\beta} + \delta g_{\alpha\beta}$, where $g^0_{\alpha\beta}$ stands for the asymptotic metric, then one is looking for vector fields ζ satisfying

$$(\pounds_\zeta g)_{\alpha\beta} \sim \delta g_{\alpha\beta}\,, \tag{4.3}$$

where the symbol $\sim$ stands for same order of magnitude in the large r expansion sense.

Since our boundary conditions are closely related but more restrictive than those of Brown-Henneaux [13], we can use their explicit analysis of the generators of the asymptotic symmetry group and simply impose the additional constraint on δg_{vv} (4.2) on them. The allowed diffeomorphisms are

$$\zeta^u = 2f(u) + \frac{1}{2r^2} g''(v) + \mathcal{O}(r^{-4}) \tag{4.4a}$$

$$\zeta^v = 2g(v) + \frac{1}{2r^2} f''(u) + \mathcal{O}(r^{-4}), \tag{4.4b}$$

$$\zeta^r = -r\left(f'(u) + g'(v)\right) + \mathcal{O}(r^{-1}) \tag{4.4c}$$

$$g'''(v) = 0 \quad \Longrightarrow \quad g = A + B\,v + C\,v^2\,. \tag{4.5}$$

Here, the connection to the Brown-Henneaux diffeomorphisms is made explicit: the diffeomorphisms generated by $\zeta = \zeta^\alpha \partial_\alpha$ of (4.4) are exactly those of Brown-Henneaux [13] and the constraint $\delta g_{vv} = \mathcal{O}(\frac{1}{r^2})$ is implemented by (4.5). One set of allowed diffeomorphisms is specified by a periodic function $f(u) = f(u + 2\pi)$. The analysis of generators of these diffeomorphisms follows directly from those of Brown and Henneaux and they lead to a *chiral Virasoro algebra* at central charge $c = 3\ell/2G_3$ (2.4).[7] The remaining three parameter family of diffeomorphisms in (4.5) describes the $\mathrm{SL}(2,\mathbb{R})$ isometries of the self-dual orbifold.

The isometries of the original extremal black holes were just a $\mathrm{U}(1)\times\mathrm{U}(1)$. In that case a Brown-Henneaux analysis with the extremal constraint would have also yielded (4.4) with the constraint $g''' = 0$. However, in the original geometry g has to be a periodic function which restricts the solutions to the constraint to $g = A$ only. The process of taking the near horizon limit led to an identification in u alone, and thus, g need not be periodic, allowing the three parameter solution above. The isometry generators that appear in this way, are not simply related to the $SL(2,\mathbb{R})$ generated by $L_0, L_{\pm 1}$ (2.25).

5 AdS_2 quantum gravity and dual chiral CFTs

Consider the two-dimensional Einstein-Maxwell-Dilaton theory with a negative cosmological constant:

$$S = \frac{\ell}{8G_3}\int d^2x\sqrt{-g}\left[e^\psi\left(R + \frac{2}{\ell^2}\right) - \frac{\ell^2}{4}e^{3\psi}F_{\mu\nu}F^{\mu\nu}\right] \tag{5.1}$$

where $F_{\mu\nu}$ is the $U(1)$ field strength. This action has an AdS_2 solution with curvature $R = -\frac{8}{\ell^2}$, constant ψ and constant electric flux:

$$ds^2 = -\frac{\ell^2}{r^2}(-dt^2 + dr^2), \qquad F_{tr} = \frac{2Q}{r^2}, \qquad e^{-\psi} = Q\,. \tag{5.2}$$

[7] Our analysis here suggests that the Left and Right CFT's introduced in [32] may be identical. This point deserves further investigation.

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This action may be obtained from the dimensional reduction of the $3d$ Einstein-Hilbert action with $3d$ Newton constant G_3 and cosmological constant $-1/\ell^2$ via restriction to the massless sector of the Kaluza-Klein tower.[8] Likewise the reduction of the near-horizon BTZ geometry (2.15) to two dimensions is precisely (5.2). The radius of the extremal BTZ horizon becomes ℓQ. The action (5.1) has another two parameter family of solutions in which ψ is not a constant [9] — these lift to generic BTZ black holes.

Because of this connection between two and three dimensions, we expect that quantum gravity around the background (5.2) is dual to a subsector of the DLCQ chiral CFT that is developed in section 2 and 3, and is only fully consistent when embedded in string theory with all the resulting additional degrees of freedom. The electric field strength Q is related to the DLCQ compactification scale R_- in (3.6) while the central charge is related to the $2d$ Newton constant: $c = 3\ell/(2G_3) = 3/(4\pi G_2)$.

Quantum gravity in the AdS_2 background (5.2) was explored in [30] from the perspective of the spacetime conformal field theory, and in [31] from the perspective of the boundary stress tensor. Both of the papers consider spectra including states charged under the $\mathrm{SL}(2,\mathbb{R})$ isometry group of AdS_2, and analyze a Virasoro algebra which includes this $\mathrm{SL}(2,\mathbb{R})$. However, as shown in previous sections, a consistent quantum theory of gravity in this background should not have any states charged under the isometry group. The reason for this is that excitations supported in AdS_2 back-react strongly and can modify the asymptotic structure of the spacetime [16].

To see this, let us write the two dimensional metric in a gauge in which the metric is conformally flat

$$ds^2 = e^{2\phi(\sigma^+,\sigma^-)}\, d\sigma^+ d\sigma^- \quad , \quad 0 \le \sigma^\pm \le \pi \quad , \tag{5.3}$$

and consider the variation of the action (5.1) with respect to the $2d$ metric. We find

$$\nabla_+\nabla_+ e^{\psi} = 8\pi G_2 T_{++} \tag{5.4}$$

and similarly for the $--$ component. If we regard (5.1) as arising from compactification of a three dimensional theory, besides the contributions from ψ and the gauge field, we can also include all contributions of massive Kaluza-Klein modes in T_{++}. We may now follow the discussion in section 2.2 of [16] (see eqs. (2.16) and (2.17) there). Integrating (5.4) against $e^{-2\phi}d\sigma^+$, we obtain

$$e^{-2\phi}\partial_+ e^{\psi}|_{\sigma^+=0} - e^{-2\phi}\partial_+ e^{\psi}|_{\sigma^+=\pi} = -8\pi G_2 \int d\sigma^+ e^{-2\phi} T_{++} \tag{5.5}$$

and similarly for T_{--}. Assuming a null energy condition ($T_{++} \ge 0$), any state with non-vanishing T_{++} requires at least one of the two terms on the left hand side of this equation to be non-zero. Since $e^{-2\phi}$ vanishes quadratically near the boundary of AdS_2, this implies that e^{ψ} must diverge at one of the AdS_2 boundaries. This is inconsistent with the constant

[8] An analysis of Schwinger pair creation of charged particles in AdS_2 in the presence of a constant electric field was performed in [33]. A bound between the mass of particle excitations and the background electric field was derived to ensure the stability of these backgrounds. This bound is satisfied in supersymmetric $AdS_2 \times S^2$ spacetimes and is also saturated for the two dimensional vacuum solution discussed here.

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value $e^{-\psi} = Q$ in (5.2), which is related from the three dimensional point of view to the compactification radius. This shows that preserving the boundary conditions requires $T_{++} = 0$ and a similar argument requires $T_{--} = 0$. Thus perturbations cannot have any dependence on σ^+, σ^-, as their back-reaction would destroy the boundary of the geometry. The background geometry (5.2) has an $SL(2,\mathbb{R})$ isometry, and if perturbations do not depend on σ^+, σ^-, then the perturbation cannot break the $SL(2,\mathbb{R})$ symmetry either. In other words, all degrees of freedom transform trivially under $SL(2,\mathbb{R})$, in agreement with the analysis in previous section.[9]

This argument used the fact that AdS_2 has two disconnected boundaries. In section 2 the analysis of the CFT dual was carried out in coordinates that only intersected a single boundary, but it was shown in [10] that, globally, the self-dual orbifold geometry has two boundaries, each of which is a null cylinder carrying a DLCQ of a CFT. To see this, transform the coordinates in (2.15) as

$$y = \cos\tau\, \cosh z + \sinh z \quad ; \qquad v = \frac{\sin\tau\, \cosh z}{\cos\tau\, \cosh z + \sinh z}\,, \tag{5.6}$$

so that the self-dual orbifold metric becomes

$$ds^2 = \frac{\ell^2}{4}\left(-\cosh^2 z\, d\tau^2 + dz^2\right) + \ell^2\left(du + A'\right)^2 \tag{5.7}$$

where A' is a gauge field with constant field strength in global AdS_2. This is the global self-dual orbifold of [10]. The entire range of v is covered by a finite range of global time τ. Thus each patch of the form (2.15) intersects one boundary of the global spacetime at either $z = \pm\infty$.

In view of this, both the near-horizon limit of extremal BTZ (2.13) and the $3d$ uplift of (5.2) can be regarded *globally* as dual to *two* DLCQ CFTs, each giving rise to one chiral theory (see [8, 10] for discussion). From this perspective we can presumably view the description of the self-dual orbifold as a thermal state in a single CFT as emerging from tracing over the Hilbert space living in one of the boundaries. This is in analogy with the usual treatment of the eternal BTZ black hole as either an entangled state in two CFTs defined on the two boundaries of the geodesically complete spacetime, or as a thermal state in a single CFT [36]. The statistical degeneracy of the thermal state in the chiral CFT dual to the spacetime (2.15) then measures the area of the familiar Poincare horizon of this coordinate patch (see [4] for a similar perspective). One difference between BTZ and the self-dual orbifold is that while the BTZ boundaries are causally disconnected, a light ray can travel between the two boundaries of the global self-dual orbifold [10]. The possible interactions that this seeds between the two boundaries have not been studied.

In this global context there is another piece of evidence that the AdS_2 base of the self-dual orbifold cannot be consistently excited. It was shown in [31] that the most general

[9] The fact that AdS_2 "fragments" in this way has led to the suggestion [16] that the dual of a one-dimensional conformal field theory should involve a sum over tree-like geometries with many different asymptotic AdS_2 boundaries. While some partial progress has been made in developing this picture [34, 35], it is still unclear whether this is the right way to think about AdS_2, or whether it eventually will lead to connection with the fuzzball proposal, and we will not further pursue this possibility in this paper.

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solution of the dimensional reduction of $3d$ gravity with a negative cosmological constant in a particular gauge can be put in the form

$$g_{\mu\nu}dx^\mu dx^\nu = d\eta^2 - \frac{1}{4}(h_0(t)e^{2\eta/L} + h_1(t)e^{-2\eta/L})^2 dt^2. \tag{5.8}$$

At the boundary $\eta \to \infty$, the boundary metric is determined by $h_0(t)$. One can choose a coordinate t such that $h_0 = 1$. The subleading behavior is determined by $h_1(t)$. The diffeomorphisms that preserve this gauge and leave h_0 unchanged were determined by [31]. However it turns out that while these are normalizable deformations of the boundary at $\eta \to +\infty$, they are not normalizable deformations at the other boundary $\eta \to -\infty$ — i.e. they change h_1. In fact, there are no deformations at all which both preserve the gauge and are normalizable at both boundaries, except the isometries. This again suggests that it is not possible to deform AdS_2 without disrupting the spacetime boundary.

6 Extremal Kerr black hole and its dual chiral CFT

The extremal $4d$ Kerr black hole is given by

$$ds^2 = -\frac{\Delta}{R^2}\left(d\hat{t} - a\sin^2\theta d\hat{\phi}\right)^2 + \frac{\sin^2\theta}{R^2}\left((\hat{r}^2+a^2)d\hat{\phi} - a d\hat{t}\right)^2 + \frac{R^2}{\Delta}d\hat{r}^2 + R^2 d\theta^2\,, \tag{6.1}$$

where

$$R^2 = \hat{r}^2 + a^2\cos^2\theta, \qquad \Delta = (\hat{r}-a)^2\ . \tag{6.2}$$

Its ADM mass and angular momentum are function of the horizon size a

$$M = a, \qquad J = \frac{a^2}{G_4}\ . \tag{6.3}$$

In the quantum theory, J is quantized (to half integers) in units of $\hbar$. This black hole has zero Hawking temperature and its Bekenstein-Hawking entropy is

$$S_{BH} = \frac{2\pi M^2}{\hbar G_4} = \frac{2\pi}{\hbar}\ J\ . \tag{6.4}$$

In the near horizon $\epsilon \to 0$ limit

$$\hat{r} = a + \epsilon\ r, \quad \hat{t} = \frac{2at}{\epsilon}, \quad \hat{\phi} = \phi + \frac{t}{\epsilon}, \tag{6.5}$$

while keeping the un-hatted parameters and coordinates fixed, we obtain the near horizon extremal Kerr (NHEK) geometry [14, 37]

$$ds^2 = 2G_4 J\,\Omega(\theta)^2\left[-r^2dt^2 + \frac{dr^2}{r^2} + d\theta^2 + \Lambda(\theta)^2\ (d\varphi + r dt)^2\right]\,, \tag{6.6}$$

where $\varphi \in [0, 2\pi]$, $0 \le \theta \le \pi$ and

$$\Omega(\theta)^2 = \frac{1+\cos^2\theta}{2}, \qquad \Lambda(\theta) = \frac{2\sin\theta}{1+\cos^2\theta}. \tag{6.7}$$

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This metric at a given θ has the form of a warped circle fibration over AdS_2 in which the fiber radius depends on the angle θ. If Λ and Ω were constants this would be precisely the self-dual orbifold of (2.15) times a circle. Indeed, as emphasized in [14], constant θ slices look like squashed self-dual orbifolds. The coordinates in (6.6) cover only part of the spacetime, with a boundary at $r \to \infty$ — globally, like the self-dual orbifold, there are two boundaries. One sees similar squashed geometries with AdS_2 and AdS_3 factors in decoupling limits of near-extremal black holes in anti-de Sitter space [17–19]. (Also see [38–47].)

The Kerr black hole is invariant under time and angular $\hat{\phi}$ translations. This isometry group is enhanced to $SL(2,\mathbb{R}) \times U(1)$ in the near horizon, just as in the self-dual orbifold. The $U(1)$ is generated by ∂_φ, whereas the $SL(2,\mathbb{R})$ acts both on the AdS_2 subspace and along the fiber to preserve the form of $d\varphi + rdt$ [14].

In [14], the asymptotic symmetry group preserving certain boundary conditions for the fluctuations of the NHEK was calculated. The corresponding diffeomorphisms they found were of the form

$$\zeta_\lambda = \lambda(\varphi)\partial_\varphi - r\lambda(\varphi)'\partial_r \,. \tag{6.8}$$

These generate a chiral Virasoro algebra. In [14] it was proposed that this Virasoro algebra should be understood as the symmetry group of a chiral $2d$ CFT dual to quantum gravity around the near horizon Kerr geometry. The central charge of this chiral CFT was computed to be

$$c_{Ext.\ Kerr} = 12J \ . \tag{6.9}$$

The NHEK is then associated with a thermal state of the chiral $2d$ CFT at temperature $T_{NHEK} = 1/2\pi$. Upon applying the Cardy formula for the entropy of $2d$ CFTs, the Bekenstein-Hawking entropy of the extremal Kerr black hole (6.4) is reproduced. The consistency of the boundary conditions proposed in [14] required the vanishing of the charge of the $U(1)_\tau \in SL(2,\mathbb{R})$, i.e.

$$E_R = 0 \tag{6.10}$$

in the notation used in [14], for all physical states. Thus, like for the self-dual orbifold, there are no physical excitations of the AdS_2 factor in the geometry. The $E_R = 0$ condition acts like the restriction to extremality in the BTZ black hole that we studied in section 2.

The analogies between the Kerr-CFT construction [14] and the analysis of the self-dual orbifold in previous sections suggests that chiral CFT of [14] is the DLCQ of an ordinary two dimensional conformal field theory. Ideally, we would like to find a consistent Kaluza-Klein reduction of gravity in the NHEK geometry to the three-dimensional self-dual orbifold. As a first step, we make a connection between the NHEK geometry and $3d$ gravity with a negative cosmological constant. For the NHEK geometry we consider then the four dimensional metric reduction ansatz:

$$ds^2 = L^2\,\Omega^2\left[-\partial_\sigma\beta(t,\,\sigma)\,\left(-dt^2 + d\sigma^2\right) + d\theta^2 + \Lambda^2\,\left(d\varphi + \beta(t,\,\sigma)dt\right)^2\right] , \tag{6.11}$$

where $\Omega^2 = (1+\cos^2\theta)/2$ and $\Lambda = 2\sin\theta/(1+\cos^2\theta)$. The equation of motion derived for β using this ansatz and the four dimensional Einstein equation *without* a cosmological

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constant is identical to the equation of motion obtained from the three-dimensional ansatz

$$ds^2 = \frac{\ell^2}{4}\left[-\partial_\sigma\beta(t,\,\sigma)\,\left(-dt^2+d\sigma^2\right)+\left(d\varphi+\beta(t,\,\sigma)dt\right)^2\right]\,. \tag{6.12}$$

and Einstein's equation *with* a cosmological constant

$$R_{3\mu\nu}+\frac{2}{\ell^2}g_{3\mu\nu}=0. \tag{6.13}$$

Here R_3 is the Ricci tensor computed for the $3d$ metric. Although this obviously does not show that there should exist a Kaluza-Klein reduction from four to three dimensions which reduces the NHEK geometry to the self-dual orbifold of AdS_3, it does show that the two theories share some dynamics.

We can also derive the central charge derived in [14] from the $4d$ NHEK geometry, by matching parameters with the three-dimensional reduction ansatz. To do this, note first that the above $3d$ equation of motion can be obtained from the Lagrangian

$$\mathcal{L}_3=\sqrt{-\det g_3}\left(R_3+\frac{2}{\ell^2}\right), \tag{6.14}$$

which describes $3d$ gravity in the presence of a negative cosmological constant. The $3d$ Newton constant is then computed by integrating over the compact direction θ in our reduction ansatz

$$\frac{1}{G_3}=\frac{2L^2\int_0^\pi d\theta\;\Omega^2\Lambda}{G_4\;\ell}=\frac{4L^2}{G_4\;\ell}\,. \tag{6.15}$$

Thus the $3d$ action is

$$S_3=\frac{1}{16\pi G_3}\int d^3x\;\mathcal{L}_3\,, \tag{6.16}$$

Note that its vacuum solution is an AdS_3 with radius $R_{AdS}=\ell$. Since $L^2=2G_4J$, using the Brown-Henneaux formula for the central charge, we have

$$c=\frac{3R_{AdS}}{2G_3}=12J\,. \tag{6.17}$$

This matches (6.9). We earlier showed that the AdS_3 central charge also matches the central charge of the chiral CFT that is dual to self-dual orbifold.

This suggests the proposed chiral $2d$ CFT dual to extremal Kerr [14] is the DLCQ of a $2d$ CFT with the following identifications: (a) The DLCQ compactification radius R_- is an arbitrary physical scale and has been set equal to one in the Kerr/CFT analysis [14], (b) The $E_R=0$ condition in [14] is mapped to $L_0=c/24$ DLCQ condition, (c) The extremal Kerr ADM angular momentum J is equal to the light-cone momentum P^+ of the DLCQ description.

One should note that identifying the chiral $2d$ CFT duals proposed for extremal black holes [14, 15] as the DLCQ of a $2d$ CFT also explains why we can use Cardy's formula to count the number of states. If we only knew that the states had to form representations of a single Virasoro algebra, we would not be able to use modular invariance, and unitarity alone does not determine the asymptotic growth of the number of states. Still, there are

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to our knowledge no general statements about the asymptotic growth of the number of states of the form $|c/24\rangle_R \otimes |\text{anything}\rangle$ in an arbitrary CFT. If the left-movers are Ramond ground states, and it is a theory with supersymmetry, one can estimate the number of states of this form using the elliptic genus and its modular properties [48, 49], and it would be interesting to establish similar results for more general CFT's.

While our results have provided some evidence that DLCQ of a CFT is dual to the near-horizon extremal Kerr, it would have been more satisfactory to have a consistent and complete reduction of $4d$ gravity with NHEK boundary conditions [14] to $3d$ gravity with a cosmological constant. In a similar setting where squashed AdS_3 factors appear in a decoupling limit of R-charged black holes in AdS_4 and AdS_5, progress towards such a reduction has been made [17–19].

7 Discussion

In this paper we have shown that the near-horizon limit of the extremal BTZ black hole, which leads to the so-called self-dual orbifold geometry, is dual to the DLCQ of a non-chiral 2d CFT, which is a *chiral 2d CFT* with the same central charge. We have also provided evidence that various "chiral CFTs" that have appeared in the literature as dual CFTs to extremal black holes should really be thought of as DLCQ of ordinary two-dimensional CFTs. This, among other things, justifies the use of Cardy formula to account for the extremal black hole entropy using this chiral CFT duals. It would be desirable to develop this picture in more detail. In particular, it would be interesting to study correlation functions in the DLCQ theory and the corresponding bulk-boundary dictionary. Another outstanding problem is to establish more rigorously that generic extremal black holes, upon taking a near-horizon limit, are indeed dual (once suitable boundary conditions are imposed) to the DLCQ of a conformal field theory. If this is indeed the case, one would expect that the parent $2d$ CFT of the DLCQ theory might also have a string theoretic realization, e.g. in the form of a warped AdS_3 solution of string theory. In other words, one might seek some sort of map from extremal black hole solutions to AdS_3 solutions. We have seen hints of such a map in [17–19], but whether it exists in the general case is unclear.

One curiosity about the self-dual orbifold geometry is that it is dual to thermal state in a DLCQ CFT. The *ground state* of the DLCQ theory does not appear to have a bona fide geometric dual.[10] This is unlike AdS_3 gravity with a standard cylindrical boundary where the ground state describes empty AdS and thermal states describe black holes. This seems to be a general feature of gauge-gravity duality for DLCQ field theories [50–52].

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[10] Specifically, it is *not* dual to a very near horizon limit of the $M = 0$ BTZ black hole as one can explicitly check.

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