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Measurement of two-photon decay width of $\chi_{c2}(1P)$ in $\gamma\gamma o \chi_{c2}(1P) o J/\psi\gamma$ at Belle



The Belle collaboration

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ABSTRACT: We report the measurement of the two-photon decay width of $\chi_{c2}(1P)$ in two-photon processes at the Belle experiment. We analyze the process $\gamma\gamma \to \chi_{c2}(1P) \to J/\psi\gamma$, $J/\psi \to \ell^+\ell^-$ ($\ell = e$ or μ) using a data sample of 971 fb⁻¹ collected with the Belle detector at the KEKB e^+e^- collider. In this analysis, the product of the two-photon decay width of $\chi_{c2}(1P)$ and the branching fraction is determined to be $\Gamma_{\gamma\gamma}(\chi_{c2}(1P))\mathcal{B}(\chi_{c2}(1P) \to J/\psi\gamma)\mathcal{B}(J/\psi \to \ell^+\ell^-) = 14.8 \pm 0.3(\text{stat.}) \pm 0.7(\text{syst.}) \,\text{eV}$, which corresponds to $\Gamma_{\gamma\gamma}(\chi_{c2}(1P)) = 653 \pm 13(\text{stat.}) \pm 31(\text{syst.}) \pm 17(\text{B.R.}) \,\text{eV}$, where the third uncertainty is from $\mathcal{B}(\chi_{c2}(1P) \to J/\psi\gamma)$ and $\mathcal{B}(J/\psi \to \ell^+\ell^-)$.

Keywords: e^+ - e^- Experiments, Quarkonium

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1 Introduction

The two-photon decay widths $(\Gamma_{\gamma\gamma})$ of mesonic states provide important information for testing models based on quantum chromodynamics (QCD), which describes quark-antiquark systems. In particular, it is important to measure the two-photon decay widths of a P-wave charmonium, whose description is at the intersection between perturbative and non-perturbative QCD. Predictions of the two-photon decay width of $\chi_{c2}(1P)$ ($\Gamma_{\gamma\gamma}(\chi_{c2}(1P))$) have a wide range of values between 280 eV and 930 eV in various theoretical calculations [1–11]. Therefore, precise measurements will help to improve our understanding of quarkonium states.

Several experiments [12–15] have reported the measurement of $\Gamma_{\gamma\gamma}(\chi_{c2}(1P))$. In general, two approaches have been used to determine $\Gamma_{\gamma\gamma}(\chi_{c2}(1P))$, either measuring the two-photon decay $\chi_{c2}(1P) \to \gamma\gamma$ or two-photon collisions $\gamma\gamma \to \chi_{c2}(1P)$. In the former approach, CLEO-c and BES III data give $\Gamma_{\gamma\gamma}(\chi_{c2}(1P)) = 555 \pm 58 \pm 32 \pm 28 \,\text{eV}$ [12] and $\Gamma_{\gamma\gamma}(\chi_{c2}(1P)) = 586 \pm 16 \pm 13 \pm 29 \,\text{eV}$ [13], respectively, using the production and decay processes $\psi(2S) \to \chi_{c2}(1P)\gamma$, $\chi_{c2}(1P) \to \gamma\gamma$. The uncertainties are statistical, systematic and from $\mathcal{B}(\psi(2S) \to \chi_{c2}(1P)\gamma)$ and the total width of $\chi_{c2}(1P)$ ($\Gamma_{\chi_{c2}(1P)}$), respectively. Hereafter, the first and second uncertainties are statistical and systematic. In the latter approach, Belle and CLEO III data give $\Gamma_{\gamma\gamma}(\chi_{c2}(1P)) = 596 \pm 58 \pm 48 \pm 16 \,\text{eV}$ [14] and

¹We recalculate the values in CLEO-c and BES III using $\mathcal{B}(\psi(2S) \to \chi_{c2}(1P)\gamma) = (9.52 \pm 0.20)\%$ and $\Gamma_{\chi_{c2}(1P)} = 1.97 \pm 0.09$ MeV from PDG [16].

 $\Gamma_{\gamma\gamma}(\chi_{c2}(1P)) = 582 \pm 59 \pm 50 \pm 15 \,\text{eV}$ [15], respectively, using $\gamma\gamma \to \chi_{c2}(1P) \to J/\psi\gamma$ in two-photon processes at e^+e^- colliders. The third uncertainty is from $\mathcal{B}(\chi_{c2}(1P) \to J/\psi\gamma)$ and $\mathcal{B}(J/\psi \to \ell^+\ell^-)$. At present, the precision of the experimental value of $\Gamma_{\gamma\gamma}(\chi_{c2}(1P))$ using two-photon production is much lower than the value measured in two-photon decay.

In this study, we report an updated measurement of $\Gamma_{\gamma\gamma}(\chi_{c2}(1P))$ in the analysis of $\gamma\gamma \to \chi_{c2}(1P) \to J/\psi\gamma$, $J/\psi \to \ell^+\ell^-$ ($\ell=e$ or μ) using a data sample of 971 fb⁻¹ at Belle. The previous measurement by Belle [14] was performed using a data sample of 32.6 fb⁻¹. The increased precision of this measurement will be crucial to check the consistency between the two approaches of measuring $\Gamma_{\gamma\gamma}(\chi_{c2}(1P))$ and to test the applicability of the existing theoretical models [1–11].

The measurement principle of $\Gamma_{\gamma\gamma}(\chi_{c2}(1P))$ in this analysis is as follows. The production cross section of a resonance R via two-photon processes at e^+e^- colliders is given by

$$\sigma(e^+e^- \to e^+e^-R) = \int \sigma(\gamma\gamma \to R; W) L_{\gamma\gamma}(W) dW, \tag{1.1}$$

where W is the energy of R in the center-of-mass (c.m.) frame of the two photons emitted from the e^+e^- beam, $\sigma(\gamma\gamma\to R;W)$ is the production cross section of R for two-photon collisions at the energy W and $L_{\gamma\gamma}(W)$ is the luminosity function [17], which is defined as the probability density of two-photon emission from the e^+e^- beam, with the energy W. If the total width of the resonance is sufficiently small compared with its mass, eq. (1.1) can be expressed as [17]

$$\sigma(e^+e^- \to e^+e^-R) = 4\pi^2(2J+1)\frac{L_{\gamma\gamma}(m_R)\Gamma^R_{\gamma\gamma}}{m_R^2},$$
 (1.2)

where J, $\Gamma_{\gamma\gamma}^R$ and m_R are the spin quantum numbers, the two-photon decay width and the mass of R, respectively. From the observed number of events, the two-photon decay width of the resonance R can be determined from eq. (1.2) as:

$$\Gamma_{\gamma\gamma}^R \mathcal{B}(R \to \text{final state}) = \frac{m_R^2 N_R}{4\pi^2 (2J+1) \left(\int \mathcal{L}dt\right) \eta L_{\gamma\gamma}(m_R)},$$
(1.3)

where N_R , $\int \mathcal{L}dt$, η and $\mathcal{B}(R \to \text{final state})$ are the observed number of R events in the two-photon process, the integrated luminosity at e^+e^- collisions, the detection efficiency and the branching ratio, respectively.

2 The Belle detector and data sample

We use a data sample, collected with the Belle detector [18, 19] at the KEKB e^+e^- asymmetric-energy collider [20, 21], corresponding to a total integrated luminosity of 971 fb⁻¹ collected at or near the $\Upsilon(1S)$, $\Upsilon(2S)$, $\Upsilon(3S)$, $\Upsilon(4S)$ and $\Upsilon(5S)$ resonances. The Belle detector is a large-solid-angle magnetic spectrometer that consists of a silicon vertex detector (SVD), a 50-layer central drift chamber (CDC), an array of aerogel threshold

²We recalculate the values in Belle and CLEO III using $\mathcal{B}(\chi_{c2}(1P) \to J/\psi \gamma) = (19.0 \pm 0.5)\%$ and $(J/\psi \to \ell^+\ell^-) = (11.93 \pm 0.05)\%$ from PDG [16].

Cherenkov counters (ACC), a barrel-like arrangement of time-of-flight scintillation counters (TOF), and an electromagnetic calorimeter comprised of CsI(Tl) crystals (ECL) located inside a super-conducting solenoid coil that provides a 1.5 T magnetic field. An iron flux-return located outside of the coil is instrumented to detect K_L^0 mesons and to identify muons (KLM). Events are selected with an OR of the two-track trigger and the "HiE" trigger, providing a 98.3% trigger efficiency. The condition for the two-track trigger is CDC hits from two charged tracks with either associated ECL clusters with an energy sum $> 0.5 \, \text{GeV}$, or associated KLM hits. The condition for the HiE trigger is an energy sum $> 1.0 \, \text{GeV}$ over all ECL clusters.

We use the TREPS generator [22] for Monte Carlo (MC) simulation of the two-photon process. For the signal, we generate the processes $e^+e^- \to e^+e^-\chi_{c2}(1P)$, $\chi_{c2}(1P) \to J/\psi\gamma$, $J/\psi \to \ell^+\ell^-$ ($\ell = e \text{ or } \mu$). The effect of J/ψ radiative decays to $\ell^+\ell^-\gamma$ is simulated using PHOTOS [23]. The generated events are processed by the full detector simulation based on GEANT3 [24]. TREPS is also used to calculate the luminosity function based on the equivalent photon approximation [17]. When we estimate the overall signal detection efficiency for the determination of $\Gamma_{\gamma\gamma}(\chi_{c2}(1P))$, the signal MC is generated under the following condition. We prepare the signal MC at the e^+e^- c.m. energy corresponding to the $\Upsilon(4S)$ resonance. This c.m. energy comprises the majority of the used data sample and differences of the beam energy within the data sample are estimated to have an effect of less than 0.1% on the cross-section and the detection efficiency of the signal processes. Similarly, we use a mass of $3.556 \,\mathrm{GeV}/c^2$ for $\chi_{c2}(1P)$ and we neglect the finite width, since its effect on the cross-section and the detection efficiency is also estimated to be small, about 0.4%, and these effects are added to the systematics. For the angular distribution of the decay in the signal MC of $\chi_{c2}(1P)$, we assume $a_2 = -0.11$, $a_3 = 0.00$ and a pure $\lambda = 2$ state, where a_2 , a_3 and λ are defined as the fractional multipole amplitudes of M2 and E3 transitions and the helicity of the $\chi_{c2}(1P)$ with respect to the $\gamma\gamma$ axis, respectively. (See appendix A for details.)

3 Event selection

We select an e^+e^- or $\mu^+\mu^-$ pair with a signal photon to reconstruct $\chi_{c2}(1P)$. To determine $\Gamma_{\gamma\gamma}(\chi_{c2}(1P))$, we select quasi-real two photon collisions, requiring that both recoiling beam particles are undetected ("zero-tag mode"). This event selection is similar to the previous Belle analysis [14].

For the e^+e^- or $\mu^+\mu^-$ selection, we require that only two oppositely charged tracks are present in the event. These tracks have to fulfill the following conditions in the laboratory frame: $-0.47 \le \cos\theta \le 0.82$, where θ is the polar angle; $p_T \ge 0.4 \,\text{GeV}/c$, where p_T is the transverse momentum; $|dz| \le 3$ cm and $dr \le 1$ cm, where dz and dr are the impact parameters relative to the beam interaction point (IP) along the z axis defined as the direction opposite that of the e^+ beam and the transverse plane, respectively; $|\Delta dz| \le 1$ cm, where Δdz is the difference between the dz's of the two tracks; $\cos\alpha > -0.997$, where α is the opening angle of the two tracks, to reject the cosmic-ray backgrounds. Events are identified as e^+e^- ($\mu^+\mu^-$) if both tracks have $E/pc \ge 0.8$ ($E/pc \le 0.4$), where E and E are the energy deposit on ECL and the measured momentum, respectively. Events rejected by

this criterion are mainly charged hadron backgrounds. In the case the tracks are identified as an e^+e^- pair, their momentum is corrected for the effect of bremsstrahlung as explained in the following. If there are one or more photons which have a total energy between 0.02 GeV and 0.2 GeV within a cone with an angle of 3° around the direction of electron momentum, the energy of these photons is added to that of the electron track. This procedure also reduces the width of the J/ψ radiative tail.

For the signal photon from $\chi_{c2}(1P)$ decay, we require just one cluster in the ECL with an energy $E_{\gamma} \geq 0.2 \,\text{GeV}$ under the condition that the cluster is isolated from the nearest charged track by an angle greater than 18.2° .

To reconstruct the $\chi_{c2}(1P)$ produced in the two-photon process in the zero-tag mode, the following conditions are required. The scalar sum of the momenta of the two tracks must be less than $6.0\,\mathrm{GeV}/c$ and the total energy deposited in the ECL must be less than 6.0 GeV. These requirements reject e^+e^- annihilation processes. To reject the initial-stateradiation (ISR) process, only events with $M_{\rm rec}^2 > 5.0\,{\rm GeV}^2/c^4$ are selected, where $M_{\rm rec}^2$ is the square of the recoil mass, which is defined as $M_{\text{rec}}^2 = (E_{\text{beam}}^* - E_{+-}^*)^2/c^4 - |p_{+-}^*|^2/c^2$. We define E_{beam}^* , E_{+-}^* and $|\boldsymbol{p}_{+-}^*|$ as the sum of e^+e^- beam energies, the sum of the energy of two tracks and the absolute value of the sum of the momentum vector of two tracks in the c.m. frame of the e^+e^- beam, respectively. The signal $\chi_{c2}(1P)$ produced in quasi-real two-photon collisions are selected with a p_T^* -balance requirement. This requirement is that the absolute value of the total transverse momentum vector in the c.m. frame of the e^+e^- beam, which is defined as $|\boldsymbol{p}_{\mathrm{T}}^{*\mathrm{tot}}|=|\boldsymbol{p}_{\mathrm{T}}^{*+}+\boldsymbol{p}_{\mathrm{T}}^{*-}+\boldsymbol{p}_{\mathrm{T}}^{*\gamma}|$, must be less than $0.15\,\mathrm{GeV}/c$, where $p_{\rm T}^{*+}$, $p_{\rm T}^{*-}$ and $p_{\rm T}^{*\gamma}$ are the transverse momentum vectors of ℓ^+ , ℓ^- and the signal photon, respectively. Furthermore, we require that $|p_T^{*+} + p_T^{*-}|$ be larger than $0.1 \,\text{GeV}/c$ to reject $\ell^+\ell^-$ pair produced in two-photon process with a fake photon. After the event selection except for the selection using $|p_{\rm T}^{*+} + p_{\rm T}^{*-}|$ and $|p_{\rm T}^{*\rm tot}|$, the scatter plots of $|p_{\rm T}^{*\rm tot}|$ versus $|p_T^{*+} + p_T^{*-}|$ for the data sample and the signal MC are shown in figure 1 (a) and (b), respectively. There are two clusters of events at $|\boldsymbol{p}_{\mathrm{T}}^{*+} + \boldsymbol{p}_{\mathrm{T}}^{*-}| \approx 0 \,\mathrm{GeV}/c$ and $|\boldsymbol{p}_{\mathrm{T}}^{*\mathrm{tot}}| \approx 0 \,\mathrm{GeV}/c$ in figure 1 (a). The former corresponds to the $\ell^+\ell^-$ pair production in two-photon process with a fake photon. The latter corresponds to the exclusive $\ell^+\ell^-\gamma$ final state in two-photon process, which is the dominant background component in this analysis; in figure 1 (a), the signal events are not clear due to the large background at this stage of the analysis. A clear cluster at $|p_T^{*tot}| \approx 0 \,\text{GeV}/c$ can be seen in the signal MC events in figure 1 (b). The region corresponding to the selections is drawn by the dotted red lines in figure 1 (a) and (b).

4 $\chi_{c2}(1P)$ signal extraction

Figure 2 shows the scatter plot of the invariant mass difference, $\Delta M = M_{+-\gamma} - M_{+-}$, versus the invariant mass of the two tracks (M_{+-}) for the data sample after the event selection, where $M_{+-\gamma}$ is the invariant mass of the selected two tracks and the signal photon candidate. As expected, a well separated cluster, corresponding to the signal of $\chi_{c2}(1P) \to J/\psi\gamma$ is evident in figure 2. To define the J/ψ sideband events and the J/ψ signal candidates, we set the J/ψ sideband regions $(2.65\,\text{GeV}/c^2 < M_{+-} < 3.00\,\text{GeV}/c^2$ and $3.15\,\text{GeV}/c^2 < M_{+-} < 3.50\,\text{GeV}/c^2)$ and the J/ψ signal region $(3.06\,\text{GeV}/c^2 \le M_{+-} \le 3.13\,\text{GeV}/c^2)$, respectively.

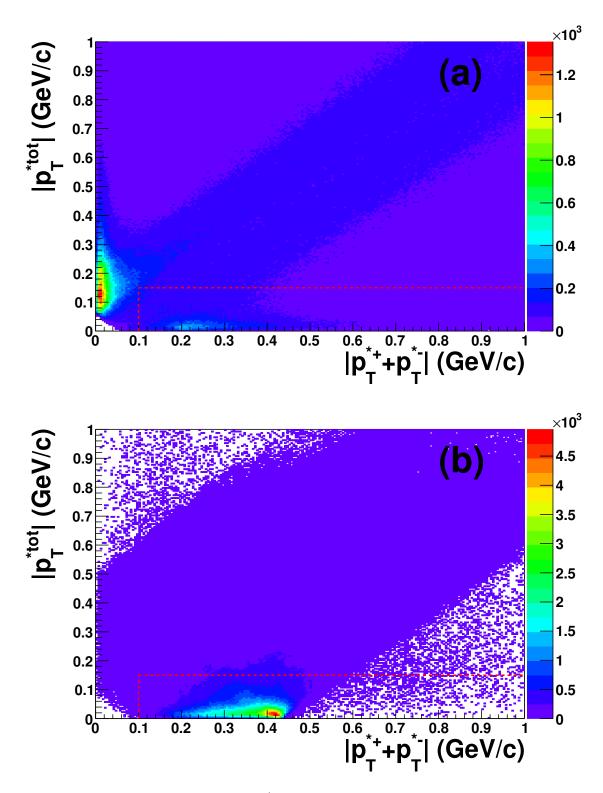


Figure 1. Scatter plots of $|\boldsymbol{p}_{\mathrm{T}}^{*\mathrm{tot}}|$ versus $|\boldsymbol{p}_{\mathrm{T}}^{*+} + \boldsymbol{p}_{\mathrm{T}}^{*-}|$ after the event selection except for the selections using $|\boldsymbol{p}_{\mathrm{T}}^{*+} + \boldsymbol{p}_{\mathrm{T}}^{*-}|$ and $|\boldsymbol{p}_{\mathrm{T}}^{*\mathrm{tot}}|$: (a) for the data sample and (b) for the signal MC. Dotted red lines show the selected region according to the selections using $|\boldsymbol{p}_{\mathrm{T}}^{*+} + \boldsymbol{p}_{\mathrm{T}}^{*-}|$ and $|\boldsymbol{p}_{\mathrm{T}}^{*\mathrm{tot}}|$.

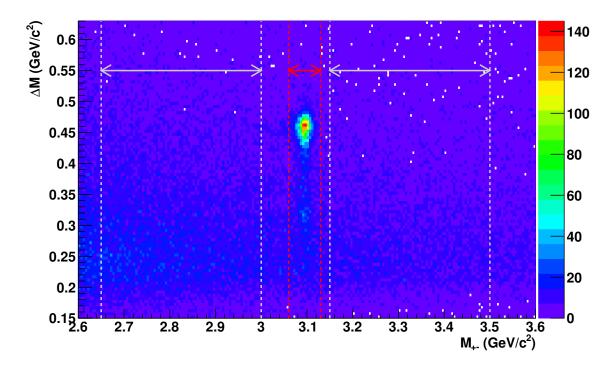


Figure 2. Scatter plot of the invariant mass difference (ΔM) versus the invariant mass of the two tracks (M_{+-}) for the data sample after the event selection combined with e^+e^- and $\mu^+\mu^-$ pairs. The J/ψ signal region and J/ψ sideband regions are drawn by the red and gray dotted line, respectively.

To take the radiative tail of the J/ψ signal into account, the J/ψ sideband regions are defined to be asymmetric with respect to the J/ψ signal region.

Non- J/ψ background is expected to be the dominant background component in this analysis, because spin-1 meson production is suppressed in quasi-real two-photon collisions. We use the J/ψ sideband events for the data sample to study the background component. Figure 3 (a) shows the mass difference distribution in J/ψ sideband events for the data sample fitted by the following empirical function from the previous Belle study [14] using a binned maximum-likelihood fit: $A(\Delta M - a)^{-b}/(1 + e^{-c(\Delta M - d)})$, where a, b, c and d are shape parameters, and A is the normalization parameter. From this fit we obtain the expected shape of the background component in the ΔM distribution. Furthermore, the number of background events expected in the signal region is found to be 9966 \pm 32 events by scaling the number of events in the sideband.

In the previous Belle measurement [14], the number of $\chi_{c2}(1P)$ signal events was estimated by subtracting the number of background events from the total in a narrow $\chi_{c2}(1P)$ signal region. However, this method suffered from a low detection efficiency and a large systematic uncertainty. In this analysis, we use a fit method to improve these points. To extract the $\chi_{c2}(1P)$ signal from the ΔM distribution in J/ψ signal candidates, we perform a binned extended maximum-likelihood fit with probability density functions (PDFs) corresponding to $\chi_{c0}(1P)$ and $\chi_{c2}(1P)$ signal and background components. The signal of $\chi_{c0}(1P)$ is expected to be non-negligible. On the other hand, the influence of

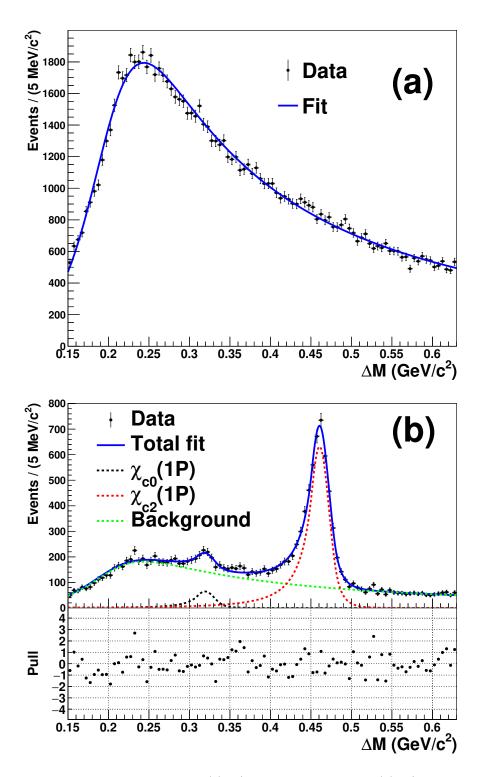


Figure 3. Mass difference distribution for (a) J/ψ sideband events and (b) J/ψ signal candidates combined with e^+e^- and $\mu^+\mu^-$ pairs. The solid blue curves and the black points with error bars show the overall fit result and the data, respectively. The dotted black, red and green curves show the $\chi_{c0}(1P)$, $\chi_{c2}(1P)$ and background components in (b), respectively. The pull plot in (b) shows the residual between the overall fit result and the data divided by the uncertainty in each bin.

 $\chi_{c1}(1P)$ is negligibly small because $\chi_{c1}(1P)$ is a spin-1 meson suppressed in quasi-real two photon collision. The ratio of the production and decay process for $\chi_{c1}(1P) \to J/\psi\gamma$ to $\chi_{c2}(1P) \to J/\psi \gamma$ is estimated to be only 4×10^{-4} in this analysis [8, 25]. A double-sided Crystal Ball function³ is empirically suitable for the $\chi_{c0}(1P)$ and $\chi_{c2}(1P)$ signal shape [26]. The tail parameters $(n_l, \alpha_l, n_r, \alpha_r)$ of the $\chi_{c2}(1P)$ signal PDF are fixed to the values obtained from the MC study of the signal, where the total width was taken into account. The parameters μ and σ in the $\chi_{c2}(1P)$ signal PDF are floated. The tail parameters and σ of the $\chi_{c0}(1P)$ signal PDF are fixed to those of the $\chi_{c2}(1P)$ signal PDF since there is only a small contribution from the $\chi_{c0}(1P)$ and the width of the $\chi_{c0}(1P)$ signal shape is expected to be close to that of the $\chi_{c2}(1P)$ signal shape obtained from the MC study. Furthermore, the mean of the $\chi_{c0}(1P)$ signal PDF is constrained to that of the $\chi_{c2}(1P)$ signal PDF with the mass difference between $\chi_{c0}(1P)$ and $\chi_{c2}(1P)$ taken from the PDG [16]. The background PDF is taken to be the same function as described in the study of J/ψ sideband events with the shape parameters (a, b, c, d) fixed to the values from the fit result shown in figure 3 (a). From a fitter test using toy MC simulations based on the shape of signal MCs and the background shape estimated from the study of J/ψ sideband events, we confirm that the fit method is stable and has no bias. Figure 3 (b) shows the mass difference distribution in J/ψ signal candidates for the data sample fitted by this method. The fitted $\chi_{c2}(1P)$ signal and background yields are 5131.2 \pm 97.4 and 10079 \pm 140 events, respectively. The number of background events estimated from this fit is consistent with that estimated from the study of the J/ψ sideband events.

Figure 4 shows the comparison of the signal MC with background-subtracted data for events with ΔM from $0.42\,\mathrm{GeV}/c^2$ to $0.49\,\mathrm{GeV}/c^2$ for $|\boldsymbol{p}_\mathrm{T}^{*\mathrm{tot}}|$, $|\cos\theta_\gamma^{+-\gamma}|$, $\cos\theta_-^{+-}$ and $\Delta\phi$, respectively; J/ψ sidebands are used to estimate the background. The definition of variables in figure 4 is as follows. $|\cos\theta_\gamma^{+-\gamma}|$ is defined as the polar angle of the signal photon in the $\ell^+\ell^-\gamma$ c.m. frame; $\cos\theta_-^{+-}$ is defined as the polar angle of the negatively charged lepton in the $\ell^+\ell^-$ c.m. frame; $\Delta\phi$ is defined as the difference in the azimuthal angle between the momentum vectors of the two leptons in the laboratory frame. The signal MC is normalized to the observed number of events in the data sample in figure 4. There is a clear peak due to the two-photon process at small $|\boldsymbol{p}_\mathrm{T}^{*\mathrm{tot}}|$ values in figure 4 (a). Good agreement is seen between the data and the signal MC for the $|\boldsymbol{p}_\mathrm{T}^{*\mathrm{tot}}|$ distribution. Furthermore, the signal MC shows good agreement with the data sample for the $|\cos\theta_\gamma^{+-\gamma}|$, $\cos\theta_-^{+-}$ and $\Delta\phi$ distributions. The simulation of angular distributions for the final state particles can be performed well. The overall signal detection efficiency in this analysis is estimated to be 7.36% using the signal MC.

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f(x; n_{l}, \alpha_{l}, n_{r}, \alpha_{r}, \mu, \sigma)
= N(n_{l}/|\alpha_{l}|)^{n_{l}} \exp(-|\alpha_{l}|^{2}/2) [(n_{l}/|\alpha_{l}|) - |\alpha_{l}| - (x - \mu)/\sigma)]^{-n_{l}}, \text{ if } (x - \mu)/\sigma \leq -\alpha_{l}
= N(n_{r}/|\alpha_{r}|)^{n_{r}} \exp(-|\alpha_{r}|^{2}/2) [(n_{r}/|\alpha_{r}|) - |\alpha_{r}| + (x - \mu)/\sigma)]^{-n_{r}}, \text{ if } (x - \mu)/\sigma \geq \alpha_{r}
= N \exp[(-(x - \mu)^{2})/(2\sigma^{2})], \text{ otherwise.}
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N is a normalization parameter. n_l and α_l are parameters of the left-hand side tail, n_r and α_r are parameters of the right-hand side tail, and μ and σ are the peak position and width of the gaussian term.

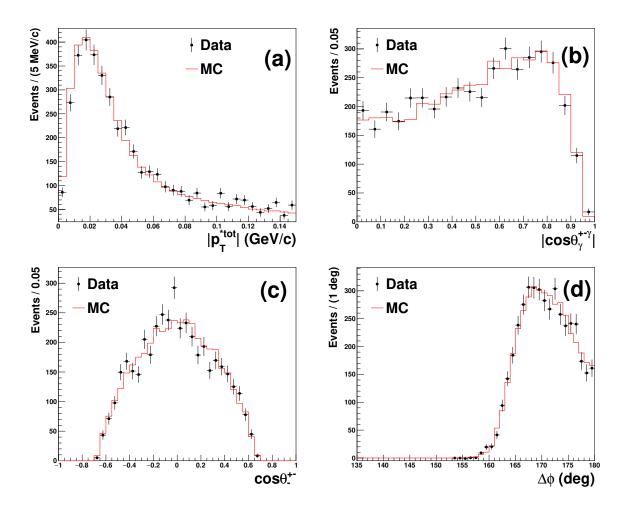


Figure 4. Comparisons of the signal MC (red histogram) with the background sample subtracted data sample estimated from J/ψ sideband events combined with e^+e^- and $\mu^+\mu^-$ pairs (black points with error bars) between $\Delta M = 0.42~{\rm GeV}/c^2$ and $0.49~{\rm GeV}/c^2$ for (a) $|\boldsymbol{p}_{\rm T}^{*\rm tot}|$; (b) $|\cos\theta_{\gamma}^{+-\gamma}|$; (c) $\cos\theta_{-}^{+-}$; (d) $\Delta\phi$. There are no entries in the $\Delta\phi<135^{\circ}$. The signal MC is normalized to the observed number of events in data sample.

5 Treatment of peaking background from ISR $\psi(2S)$ production

A peaking background is anticipated from the process chain $e^+e^- \to \gamma_{\rm ISR}\psi(2S)$, $\psi(2S) \to \chi_{c2}(1P)\gamma$, $\chi_{c2}(1P) \to J/\psi\gamma$, $J/\psi \to \ell^+\ell^-$. Events where the ISR photon and the photon from $\psi(2S)$ decay are undetected have the same final state as the $\gamma\gamma \to \chi_{c2}(1P)$ signal. Therefore, we need to estimate the expected number of peaking background events. The ISR $\psi(2S)$ production cross section has been precisely measured in Belle [27]. To estimate the peaking background detection efficiency, we prepare the peaking background MC by using the PHOKHARA generator [28]. The requirements for angles of charged tracks and square of recoil mass mostly remove the ISR events. However, some double-ISR events where an ISR photon is emitted from each beam, remain as their topology is similar to two-photon collision events. The peaking background detection efficiency at the $\Upsilon(4S)$ is evaluated to be 0.55% in this analysis. The expected number of peaking background

events and its uncertainty are estimated using the ISR $\psi(2S)$ production cross section from the Belle study, the product of the relevant branching fractions and the peaking background detection efficiency. Taking the data sample with the different beam energies into account, the total expected number of peaking background events is estimated to be 170.9 ± 9.5 events: the proportion of peaking background events in the $\chi_{c2}(1P)$ signal yield is only 3.3%. The influence of the peaking background is not visible in any of the signal candidate distributions. We finally evaluate the observed number of $\chi_{c2}(1P)$ signal events in two-photon processes by subtracting the total expected number of peaking background events from the $\chi_{c2}(1P)$ signal yield.

To check the accuracy of the total expected number of peaking background events, we compare the peaking background MC and the data sample for the p_z^{*tot} distribution with the "loose" event selection, where p_z^{*tot} is the sum of the z component of momentum for the final state particles $(\ell^+\ell^-\gamma)$ in the c.m. frame of the e^+e^- beams. In the loose event selection, only the requirement for the angles of the charged tracks $(-0.47 \le \cos \theta \le 0.82)$ does not apply. This requirement effectively rejects the peaking background and makes the characteristic peak due to the ISR process invisible on the p_z^{*tot} distribution. Figure 5 shows the p_z^{*tot} distribution in $\chi_{c2}(1P)$ signal candidates for the MC, which consists of the signal MC and the peaking background MC, and the background-subtracted data sample, where we have applied a sideband subtraction similar to the original signal analysis, changing the J/ψ sideband region to $2.965 \, {\rm GeV}/c^2 < M_{+-} < 3.000 \, {\rm GeV}/c^2$ and $3.150 \, {\rm GeV}/c^2 <$ $M_{+-} < 3.185 \,\mathrm{GeV}/c^2$, for the ΔM region between $0.42 \,\mathrm{GeV}/c^2$ and $0.49 \,\mathrm{GeV}/c^2$ after the standard (figure 5 (a)) and the loose event selection (figure 5 (b)). Unlike the original signal analysis, the distribution of the mass difference in the wide sideband region is different from that in narrow sideband region for the peaking background process. Therefore, we use the narrow sideband region. The requirement for the angles of the charged tracks removes 90.0% of the ISR $\psi(2S) \to \chi_{c2}(1P)\gamma$ background between figure 5 (a) and (b). There is a clear peak due to the peaking background at about $-4.5\,\mathrm{GeV}/c$ on the $p_z^{*\mathrm{tot}}$ distribution in figure 5 (b), and good agreement between the MC and the data sample in this region, validating the estimate of the peaking background in ΔM .

6 Systematic uncertainties

Table 1 shows the summary of systematic uncertainties for the measurement of the twophoton decay width of $\chi_{c2}(1P)$. We apply efficiency correction for lepton identification for the two tracks in the signal MC. The lepton identification correction and its uncertainty are estimated from a study of $e^+e^- \to e^+e^-\ell^+\ell^-$ events. The $\chi_{c2}(1P)$ signal MC assumes pure helicity-2 production; the associated systematic uncertainty is estimated by varying the relevant parameter by its measured uncertainty, which changes the $\ell^+\ell^-\gamma$ angular distribution (see appendix A), and noting the resulting change in the signal detection efficiency. The uncertainty from the track finding efficiency is evaluated to be 0.3% per track using $\tau \to \pi \pi^0 \nu$, $\pi^0 \to \gamma e^+e^-$ events. The uncertainty on the J/ψ detection efficiency due to the definition of the J/ψ signal region is estimated by evaluating the difference in the detector resolution on the M_{+-} distribution between data and MC simulation.

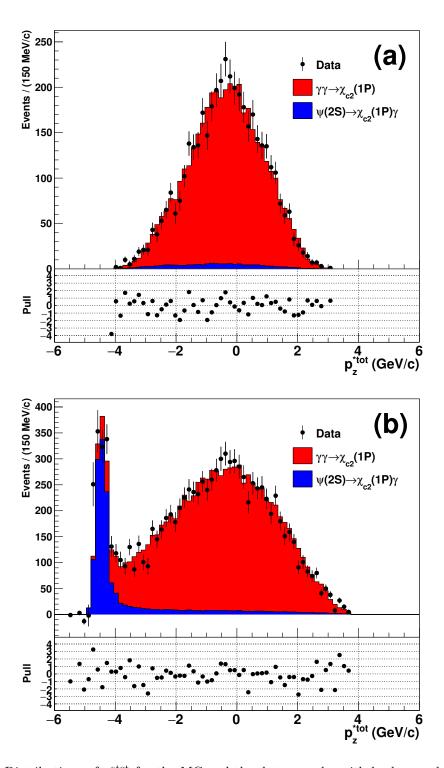


Figure 5. Distributions of $p_z^{*\text{tot}}$ for the MC and the data sample, with background (estimated from J/ψ sideband events) subtracted (black points with error bars) after (a) the standard event selection and (b) the loose event selection. The peaking background component (blue histogram) in the MC is normalized based on the total expected number of peaking background events. The signal component (red histogram) in the MC is normalized based on the number of events in the peaking-background-subtracted data sample.

The systematic uncertainty for the photon detection efficiency is estimated with radiative Bhabha events. Since we require just one candidate for the signal photon from $\chi_{c2}(1P)$ decay in the event selection, signal events are rejected when we detect extra photons. The uncertainty in the associated inefficiency is estimated using the difference in the probability of detected extra photons between data and MC simulation using the $e^+e^- \to e^+e^-\mu^+\mu^$ process with a p_T^* -balance requirement for $\mu^+\mu^-$. The trigger efficiency estimated from the signal MC is 98.3%. We estimate the associated uncertainty by comparing the ratios of the different sub-triggers between signal MC and experimental data. We use the signal MC with only 10.58 GeV corresponding to $\Upsilon(4S)$ as e^+e^- c.m. beam energy. The effect of the different e^+e^- beam energies is evaluated based on the product of overall signal detection efficiency and luminosity function, taking their luminosity-weighted average in the ratio to the $\Upsilon(4S)$ value. The uncertainty due to neglecting the $\chi_{c2}(1P)$ total width in the signal MC for the overall signal detection efficiency is estimated using dedicated signal MC that takes the total width into account. The validity and uncertainty of the ΔM fit method for $\chi_{c2}(1P)$ signal extraction are estimated from a fitter test using toy MCs based on the shape of the signal MCs and the experimental background component from the J/ψ sideband events. The estimated uncertainty includes the fit bias of $\chi_{c2}(1P)$ signal yield and the uncertainty of the PDFs estimated by fitter tests where the fixed shape parameters are varied. The uncertainty on the luminosity function calculated by TREPS includes the effect of uncertainties in the form factor, the radiative correction in the two-photon reaction, and the difference in the approximation model for two-photon processes estimated from the comparison between TREPS and a full-diagram calculation [29] in the $e^+e^- \rightarrow e^+e^-\mu^+\mu^$ process. The systematic uncertainty on the integrated luminosity is estimated to be 1.4%. In addition to these sources, we estimate the uncertainty on the photon energy resolution which is applied to the signal photon in the signal MC. The photon energy resolution correction and its uncertainty are estimated from a study of $D^{*0} \to D^0 \gamma$ events. The effect on the overall signal detection efficiency is evaluated to be negligibly small. The total systematic uncertainty is found to be 4.7%, adding the various contributions in quadrature.

7 Determination of two-photon decay width of $\chi_{c2}(1P)$

Subtracting the total expected number of peaking background events from the $\chi_{c2}(1P)$ signal yield, the observed number of signal $\chi_{c2}(1P)$ events in two-photon processes is estimated to be 4960.3 \pm 97.9 events. From eq. (1.3), the two-photon decay width of $\chi_{c2}(1P)$ is determined by

$$\Gamma_{\gamma\gamma}(\chi_{c2}(1P))\mathcal{B}(\chi_{c2}(1P) \to J/\psi \gamma)\mathcal{B}(J/\psi \to \ell^+\ell^-) = \frac{m_{\chi_{c2}(1P)}^2 N_{\text{sig}}}{20\pi^2 \left(\int \mathcal{L}dt\right) \eta L_{\gamma\gamma}(m_{\chi_{c2}(1P)})}. \quad (7.1)$$

We substitute $m_{\chi_{c2}(1P)} = 3.556 \,\text{GeV}/c^2$, $\int \mathcal{L}dt = 971 \,\text{fb}^{-1}$, $\eta = 7.36\%$ and $L_{\gamma\gamma}(m_{\chi_{c2}(1P)}) = 7.70 \times 10^{-4} \,\text{GeV}^{-1}$, respectively. From eq. (7.1), the measured value is

$$\Gamma_{\gamma\gamma}(\chi_{c2}(1P))\mathcal{B}(\chi_{c2}(1P) \to J/\psi\gamma)\mathcal{B}(J/\psi \to \ell^+\ell^-) = 14.8 \pm 0.3(\text{stat.}) \pm 0.7(\text{syst.}) \text{ eV}.$$
(7.2)

Source	Systematic uncertainty
Lepton ID efficiency correction	0.8%
Angular distribution	1.2%
Tracking efficiency	0.6%
J/ψ detection efficiency	2.4%
Photon detection efficiency	2.0%
Inefficiency due to extra photons	1.0%
Trigger efficiency	0.9%
Different e^+e^- beam energies	0.1%
Neglecting total width	0.4%
Fit method	0.6%
Luminosity function	2.3%
Integrated luminosity	1.4%
Total	4.7%

Table 1. Summary of the systematic uncertainties for $\Gamma_{\gamma\gamma}(\chi_{c2}(1P))$.

This result corresponds to

$$\Gamma_{\gamma\gamma}(\chi_{c2}(1P)) = 653 \pm 13(\text{stat.}) \pm 31(\text{syst.}) \pm 17(\text{B.R.}) \text{ eV},$$
 (7.3)

where the third uncertainty is from $\mathcal{B}(\chi_{c2}(1P) \to J/\psi \gamma) = (19.0 \pm 0.5)\%$ and $\mathcal{B}(J/\psi \to \ell^+\ell^-) = (11.93 \pm 0.05)\%$ [16]. Table 2 and figure 6 show the summary and comparison of experimental results for $\Gamma_{\gamma\gamma}(\chi_{c2}(1P))$, respectively. This measurement is the most precise measurement of $\Gamma_{\gamma\gamma}(\chi_{c2}(1P))$ in two-photon processes and consistent with the previous Belle result [14] and the other experimental results [12, 13, 15]. Precision of the present measurement is almost the same as that of the most precise measurement from the $\chi_{c2}(1P)$ decay of BES III [13].

8 Conclusion

We measure the two-photon decay width of $\chi_{c2}(1P)$ in the analysis of $\gamma\gamma \to \chi_{c2}(1P) \to J/\psi\gamma$, $J/\psi \to \ell^+\ell^-$ ($\ell = e$ or μ) using the 971 fb⁻¹ data sample at Belle. The measured value is $\Gamma_{\gamma\gamma}(\chi_{c2}(1P))\mathcal{B}(\chi_{c2}(1P) \to J/\psi\gamma)\mathcal{B}(J/\psi \to \ell^+\ell^-) = 14.8 \pm 0.3 \text{(stat.)} \pm 0.7 \text{(syst.)}$ eV. This result corresponds to $\Gamma_{\gamma\gamma}(\chi_{c2}(1P)) = 653 \pm 13 \text{(stat.)} \pm 31 \text{(syst.)} \pm 17 \text{(B.R.)}$ eV, where the third uncertainty is from $\mathcal{B}(\chi_{c2}(1P) \to J/\psi\gamma)$ and $\mathcal{B}(J/\psi \to \ell^+\ell^-)$. The result in this paper is the most precise measurement of $\Gamma_{\gamma\gamma}(\chi_{c2}(1P))$ in two-photon processes and has a compatible precision with that from the $\chi_{c2}(1P)$ decay of BES III [13]. This measurement supersedes the previous Belle result [14].

A Angular distribution

In the decay process $\chi_{c2}(1P) \to J/\psi\gamma$, E1, M2 and E3 transitions are allowed. Furthermore, the helicity of $\chi_{c2}(1P)$ with respect to the $\gamma\gamma$ axis can have $\lambda = 0$ or 2 in the production

Experiment [Ref.]	Measured value	$\Gamma_{\gamma\gamma}(\chi_{c2}(1P)) \text{ (eV)}$
This measurement	$\Gamma_{\gamma\gamma}(\chi_{c2}(1P)) \times \mathcal{B}_1 \times \mathcal{B}_2 = 14.8 \pm 0.3 \pm 0.7 \text{eV}$	$653 \pm 13 \pm 31 \pm 17^{\mathrm{a}}$
Previous Belle [14]	$\Gamma_{\gamma\gamma}(\chi_{c2}(1P)) \times \mathcal{B}_1 \times \mathcal{B}_2 = 13.5 \pm 1.3 \pm 1.1 \text{eV}$	$596 \pm 58 \pm 48 \pm 16^{\mathrm{a,b}}$
CLEO III $[15]$	$\Gamma_{\gamma\gamma}(\chi_{c2}(1P)) \times \mathcal{B}_1 \times \mathcal{B}_2 = 13.2 \pm 1.4 \pm 1.1 \text{eV}$	$582 \pm 59 \pm 50 \pm 15^{\mathrm{a,b}}$
CLEO-c [12]	$\mathcal{B}_3 \times \mathcal{B}_4 \times 10^5 = 2.68 \pm 0.28 \pm 0.15$	$555 \pm 58 \pm 32 \pm 28^{\mathrm{c,d}}$
BES III $[13]$	$\mathcal{B}_3 \times \mathcal{B}_4 \times 10^5 = 2.83 \pm 0.08 \pm 0.06$	$586 \pm 16 \pm 13 \pm 29^{\rm c,d}$

^a Third uncertainty is associated with the uncertainties of $\mathcal{B}(\chi_{c2}(1P) \to J/\psi \gamma)$ and $\mathcal{B}(J/\psi \to \ell^+\ell^-)$.

Table 2. Summary of experimental results for $\Gamma_{\gamma\gamma}(\chi_{c2}(1P))$, where $\mathcal{B}_1 \equiv \mathcal{B}(\chi_{c2}(1P) \to J/\psi \gamma)$, $\mathcal{B}_2 \equiv \mathcal{B}(J/\psi \to \ell^+\ell^-)$, $\mathcal{B}_3 \equiv \mathcal{B}(\psi(2S) \to \chi_{c2}(1P)\gamma)$, $\mathcal{B}_4 \equiv \mathcal{B}(\chi_{c2}(1P) \to \gamma\gamma)$.

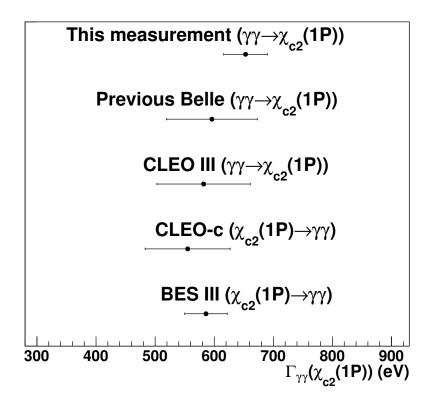


Figure 6. Comparison of experimental results for $\Gamma_{\gamma\gamma}(\chi_{c2}(1P))$, where the error bars show the combined uncertainties from all sources added in quadrature. This measurement supersedes the previous Belle result [14].

^b The results is recalculated by using $\mathcal{B}(\chi_{c2}(1P) \to J/\psi\gamma) = (19.0 \pm 0.5)\%$ and $\mathcal{B}(J/\psi \to \ell^+\ell^-) = (11.93 \pm 0.05)\%$ from PDG [16].

^c Third uncertainty is associated with the uncertainties of $\mathcal{B}(\psi(2S) \to \chi_{c2}(1P)\gamma)$ and the total width of $\chi_{c2}(1P)$.

^d The results is recalculated by using $\mathcal{B}(\psi(2S) \to \chi_{c2}(1P)\gamma) = (9.52 \pm 0.20)\%$ and $\Gamma_{\chi_{c2}(1P)} = 1.97 \pm 0.09$ MeV from PDG [16].

process. Taking these conditions in $\gamma\gamma \to \chi_{c2}(1P)$, $\chi_{c2}(1P) \to J/\psi\gamma$, $J/\psi \to \ell^+\ell^-$ into account, the normalized angular distribution of the final state, which is written as $\hat{W}(\theta, \theta^*, \phi^*)$, is given by eq. (A.1). In this equation, θ is the polar angle of the photon from $\chi_{c2}(1P)$ decay with respect to the $\gamma\gamma$ axis in the $\ell^+\ell^-\gamma$ c.m. frame; θ^* and ϕ^* are the polar and azimuthal angles of ℓ^- , respectively, in the x-z plane of the J/ψ and $\ell^+\ell^-$ c.m. frame, where the z-axis is along the direction of the J/ψ and the x-axis is in the $J/\psi\gamma$ scattering plane; A_k shows the helicity amplitudes of $\chi_{c2}(1P)$ with respect to the $\gamma J/\psi$ axis, where k is the absolute value of helicity; a_1 , a_2 and a_3 are the fractional multipole amplitudes of E1, M2 and E3 transitions, respectively. The relationship between A_k and a_i is also shown; A_k and a_i are normalized; the sign of a_1 is conventionally defined to be positive; and R is the ratio of $\lambda = 2$ to the whole ($\lambda = 0$ or 2).

In the signal MC, we assume $a_2 = -0.11 \pm 0.01$ and $a_3 = 0.00 \pm 0.01$ [16] for the $\chi_{c2}(1P) \to J/\psi\gamma$. For the state of helicity of $\chi_{c2}(1P)$ with respect to the $\gamma\gamma$ axis, we assume a pure $\lambda = 2$ state, based on the measurement $R = 1.000 \pm 0.018$ by BES III experiment⁴ [13].

$$\frac{64\pi^2}{15} \hat{W}(\theta, \theta^*, \phi^*) = K_1 + K_2 \cos^2 \theta + K_3 \cos^4 \theta + (K_4 + K_5 \cos^2 \theta + K_6 \cos^4 \theta) \cos^2 \theta^* \\ + (K_7 + K_8 \cos^2 \theta + K_9 \cos^4 \theta) \sin^2 \theta^* \cos 2\phi^* \\ + (K_{10} + K_{11} \cos^2 \theta) \sin 2\theta \sin 2\theta^* \cos \phi^*$$

$$(A.1)$$

$$\begin{cases} 16K_1 = 4A_0^2 + 6A_2^2 + R(2A_0^2 + 8A_1^2 - 5A_2^2) \\ 8K_2 = 3(-4A_0^2 + 8A_1^2 - 2A_2^2 + R(2A_0^2 - 8A_1^2 + 3A_2^2)) \\ 16K_3 = (6A_0^2 - 8A_1^2 + A_2^2)(6 - 5R) \\ 16K_4 = 4A_0^2 + 6A_2^2 + R(2A_0^2 - 8A_1^2 - 5A_2^2) \\ 8K_5 = 3(-4A_0^2 - 8A_1^2 - 2A_2^2 + R(2A_0^2 + 8A_1^2 + 3A_2^2)) \\ 16K_6 = (6A_0^2 + 8A_1^2 + A_2^2)(6 - 5R) \\ 8K_7 = \sqrt{6}(3R - 2)A_0A_2 \\ K_8 = \sqrt{6}(1 - R)A_0A_2 \\ 8K_9 = \sqrt{6}(5R - 6)A_0A_2 \\ 8K_{10} = \sqrt{3}\left(2A_0A_1 + \sqrt{6}A_1A_2 - R\left(A_0A_1 + 3\sqrt{3/2}A_1A_2\right)\right) \\ 8\sqrt{3}K_{11} = (5R - 6)\left(3A_0A_1 + \sqrt{3/2}A_1A_2\right) \end{cases}$$

$$\sum_{k=0}^2 A_k^2 = \sum_{i=1}^3 a_k^2 = 1$$

$$\begin{cases} A_0 = \sqrt{\frac{1}{10}}a_1 + \sqrt{\frac{1}{6}}a_2 - \sqrt{\frac{6}{15}}a_3 \\ A_1 = \sqrt{\frac{3}{10}}a_1 + \sqrt{\frac{1}{6}}a_2 - \sqrt{\frac{6}{15}}a_3 \\ A_2 = \sqrt{\frac{6}{10}}a_1 - \sqrt{\frac{1}{3}}a_2 + \sqrt{\frac{1}{15}}a_3 \end{cases}$$

 $^{^4\}mathrm{We}$ take the linear sum of the statistical and systematic uncertainties, conservatively, for our uncertainty evaluation.

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