Published for SISSA by Springer

Received: *August 23, 2021*
Revised: *December 3, 2021*
Accepted: *January 16, 2022*
Published: *January 28, 2022*

Swampland conditions for higher derivative couplings from CFT

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Abstract: There are effective field theories that cannot be embedded in any UV complete theory. We consider scalar effective field theories, with and without dynamical gravity, in D-dimensional anti-de Sitter (AdS) spacetime with large radius and derive precise bounds (analytically) on the coupling constants of higher derivative interactions $\phi^2\Box^k\phi^2$ by only requiring that the dual CFT obeys the standard conformal bootstrap axioms. In particular, we show that all such coupling constants, for even $k \geq 2$, must satisfy positivity, monotonicity, and log-convexity conditions in the absence of dynamical gravity. Inclusion of gravity only affects constraints involving the $\phi^2\Box^2\phi^2$ interaction which now can have a negative coupling constant. Our CFT setup is a Lorentzian four-point correlator in the Regge limit. We also utilize this setup to derive constraints on effective field theories of multiple scalars. We argue that similar analysis should impose nontrivial constraints on the graviton four-point scattering amplitude in AdS.

Keywords: Conformal Field Theory, Effective Field Theories, AdS-CFT Correspondence, Models of Quantum Gravity

ArXiv ePrint: 2104.11238

https://doi.org/10.1007/JHEP01(2022)176

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1 Introduction

By now it is well-known that not all effective field theories (EFTs) can be UV completed. One famous example is the EFT of a massless scalar with higher derivative interaction

$$S = \frac{1}{2}\int d^D x \left(-(\partial\phi)^2 + \mu\phi^2\Box^2\phi^2 + \cdots\right) , \tag{1.1}$$

which does not admit a UV completion for $\mu < 0$ [1]. Conceptually, this represents a substantial departure from our traditional understanding of Wilsonian EFTs. In fact, this remarkable result led to the proof of the 4D a-theorem establishing irreversibility of unitary renormalization group flows between conformal fixed points [2]. More generally this constraint is related to the idea of "swampland" of EFTs that cannot be obtained as a low energy approximation of a consistent theory of quantum gravity (oftentimes string theory) [3–8].

The constraint on the EFT (1.1) is not actually an accident, but part of a general feature of low energy EFTs with higher derivative interactions. There is a growing body of literature with similar precise bounds on IR couplings of an EFT from UV consistency [9–25]. All these bounds have one thing in common that they do not depend on the details of the UV completion. However, these types of bounds are generally derived under the assumption that the $2 \to 2$ scattering amplitude obeys (i) analyticity (in the usual regime), (ii) partial wave unitarity, (iii) crossing symmetry, and (iv) Regge boundedness conditions

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even in the UV. Such S-matrix based arguments can be unsatisfying since some of these assumptions (even though well-motivated) have not yet been rigorously established.[1]

Another technical challenge of these EFT arguments is to incorporate dynamical gravity mainly because of the graviton pole in the $2 \to 2$ scattering amplitude. Recently, an elegant framework has been introduced in [47] that bypasses this problem by studying scattering amplitudes at finite impact parameter (see [48] for related discussions). Under the same assumptions about the $2 \to 2$ scattering amplitude, this framework leads to non-trivial and rigorous two sided bounds on coupling constants of higher derivative interactions in $D > 4$ dimensions. The analysis necessarily requires that the $2 \to 2$ scattering amplitude $\mathcal{A}(s,t) < |s|^2$ for large s (at fixed $t < 0$). However, it is unclear whether this Regge boundedness condition is valid in the presence of dynamical gravity.[2] Nevertheless, these bounds provide compelling evidence in favor of the expectation that all higher derivative interactions must have order one coupling constants in the units of the UV cut-off scale. The main motivation of this paper is to derive similar bounds on EFTs in anti-de Sitter (AdS) spacetime where such loopholes can be avoided.

In this paper, we will address a closely related question: what scalar EFTs in AdS$_D$ cannot be embedded into a UV theory that is dual to a CFT$_{D-1}$ obeying the usual CFT axioms? We will provide a partial answer to this question by leveraging the huge advancement in constraining the space of consistent CFTs from well-established conformal bootstrap axioms (for a review see [56]). The main logic of our argument parallels recent developments in constraining EFTs in AdS (with or without dynamical gravity) from rigorous analysis in the dual CFT [26, 29, 32, 57–74]. For example, the sign constraint on the $\phi^2\Box^2\phi^2$ coupling in the EFT (1.1) can be derived in AdS from the conformal bootstrap [58].

The main advantage of our AdS argument is that the bounds follow directly from the conformal bootstrap axioms which are, both conceptually and technically, well-understood, at least at the level of four-point correlators. We will derive the $\phi^2\Box^2\phi^2$ constraint (with and without dynamical gravity) as a special case of an infinite set of similar constraints on higher derivative interactions of the form $\phi^2\Box^k\phi^2$ from a simple CFT setup.

We consider a scalar EFT in AdS$_D$ with an effective action[3]

$$S = S_{\rm EH} + \frac{1}{2}\int d^Dx\sqrt{-g}\left(-g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - m^2\phi^2 + \mu\sum_{k=2,3,4,\cdots}\left(\frac{\lambda_k}{n_k(\Delta)M^{2(k-2)}}\right)\phi^2\Box^k\phi^2\right) + \cdots\,, \tag{1.2}$$

[1] In recent years, significant progress has been made both in analytical and numerical approaches to the S-matrix bootstrap [26–46].

[2] Note that the Froissart bound [49–51] does not hold without a mass gap in the theory. Hence, the Regge boundedness condition $\mathcal{A}(s,t) < |s|^2$ is subtle whenever there are massless states in the theory, even in the absence of gravity. For example, the same issue persists even for the 4-point scattering amplitude of the dilaton that led to the proof of the 4D a-theorem in [2]. However, in that case, the Regge boundedness follows from conformal invariance of the UV fixed point [52, 53]. On the other hand, the same argument for the 4-point dilaton amplitude in 6D imposes a weaker condition $\mathcal{A}(s,t) < |s|^3$ [54, 55].

[3] We are ignoring ϕ^3, ϕ^4, $\phi^2\Box\phi^2$, and all other higher derivative interactions that cannot be written as $\phi^2\Box^k\phi^2$ since these interactions, as well as presence of other low spin ($J \le 1$) fields, will not affect the final bounds. However, these interactions can sometimes create obstruction to a flat space limit, especially at low spacetime dimensions. We will discuss this in section 4.3.

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where, S_{EH} is the Einstein-Hilbert action with a negative cosmological constant and M is the mass-scale of new physics. We allow for the possibility that the scalar field has a mass $0 \leq m^2 \ll M^2$. First, let us explain our convention. We have defined a positive coupling constant $\mu \geq 0$ which has the dimension $1/M^D$. The coupling constants λ_k are dimensionless and normalized by introducing (dimensionless and known) $\mathcal{O}(1)$ positive numerical factors $n_k(\Delta)$.[4] The choice of this particular normalization makes the final bounds rather simple. Moreover, without loss of generality, we will assume that λ_2 is order one, however, to begin with we do not assume that the other coupling constants are order one as well. Our goal is to derive necessary conditions (analytically) for the tree level EFT (1.2) to have a UV completion.[5] In particular, we will impose precise constraints on λ_k coupling constants, irrespective of the details of the UV physics, from the requirement that the dual CFT satisfies the bootstrap axioms.

There are non-trivial constraints on the EFT (1.2) even when gravity is non-dynamical ($G_N = 0$). So, first we focus on this simpler case. The EFT (1.2) in AdS with large radius $R_{\text{AdS}}M \gg 1$ enjoys a dual CFT description in $D-1$ spacetime dimensions. Specifically, it was shown by [75] and subsequent authors [76–87], that the scalar EFTs in AdS$_D$ of the form (1.2) are in one-to-one correspondence with perturbative solutions of crossing symmetry in CFT$_{D-1}$. This interacting dual CFT has a scalar primary operator $\mathcal{O}$ which is dual to the AdS field ϕ with dimension Δ given by $m^2R_{\text{AdS}}^2 = \Delta(\Delta - D + 1)$. Since there is no dynamical gravity, the stress tensor of the dual CFT must decouple from the low energy spectrum. This implies that we are in the limit of large central charge $c_T \to \infty$ with $R_{\text{AdS}}M \equiv \Delta_{\text{gap}}$ fixed (but large).[6] Of course, the dual CFT should be thought of as an "effective" CFT which is embedded in some bigger CFT satisfying the usual CFT axioms. We utilize this dual description to study CFT Regge correlators associated with the EFT (1.2). At the leading order in μ, these CFT Regge correlators grow in a very specific way within the regime of validity of the EFT (1.2). In fact, this type of Regge growth is known to be highly constrained by the argument of [88] (see section 6). In this paper, we revisit these bounds on the Regge growth of CFT correlators and show that they impose precise constraints on the EFT (1.2). In particular, in the limit of large $R_{\text{AdS}}M$ we conclude that the coupling constants λ_k, irrespective of the rest of the theory, must obey the following conditions for the EFT (1.2) (with $\mu \geq 0$) to be embedded into a UV theory that is dual to a CFT obeying the CFT axioms:[7]

- **Positivity** — For all even $k \geq 2$

$$\lambda_k > 0\,. \tag{1.3}$$

[4]The numerical factor $n_k(\Delta) > 0$ is defined in (4.17) as ratios of Γ-functions. Note that $n_2(\Delta) = n_4(\Delta) = 1$. Moreover, in the large AdS radius limit with finite and non-zero m, this factor $n_k(\Delta) = 1$ for all finite k. For large AdS radius, the numerical factor $n_k(\Delta)$ is non-trivial (i.e., $n_k(\Delta) \neq 1$) only in the massless limit (or for $k \gg mR_{\text{AdS}}$).

[5]We assume that the EFT (1.2) is weakly coupled such that G_N (if non-zero) and μ are small and of the same order in the units of the cut-off scale M.

[6]The central charge c_T is the overall coefficient of the CFT stress tensor two-point function.

[7]All bounds obtained in this paper are valid in spacetime dimensions $D \geq 4$. We also expect that our analysis is valid even for $D = 3$ as long as $0 \leq m^2 \ll M^2$ and the field ϕ has shift symmetry or $\mathbb{Z}_2$ symmetry.

- **Monotonicity** — λ_k as a function of even k decreases monotonically[8]

$$\lambda_{k+2} \leq \lambda_k \tag{1.4}$$

for all even $k \geq 2$.

- **Log-Convexity** — λ_k, for even k, satisfies a global log-convexity condition and hence for any even k_1, k_2, and k_3

$$\frac{1}{k_2-k_1}\ln\frac{\lambda_{k_1}}{\lambda_{k_2}} \geq \frac{1}{k_3-k_1}\ln\frac{\lambda_{k_1}}{\lambda_{k_3}}\,, \qquad k_3 > k_2 > k_1 \geq 2\,. \tag{1.5}$$

We emphasize that these constraints (for a pictorial depiction see figure 1) follow directly from analyticity, positivity, and crossing symmetry of CFT four-point correlators — properties that are contained in the conformal bootstrap axioms. The above conditions, among other things, imply that all higher derivative interactions $\phi^2\Box^k\phi^2$ with even k must have order one coupling constants in the units of the UV cut-off scale M. However, we believe that (1.3)–(1.5) are necessary conditions but they are far from being sufficient. For example, it is expected that similar bounds exist even for odd k. Whereas, our setup does not impose any restriction on the odd λ_k couplings other than all λ_k couplings in AdS, even or odd, with $k \geq 3$ must vanish when $\lambda_2 = 0$.[9]

Of course, the next key step is to include dynamical gravity ($G_N \neq 0$). In our setup, the inclusion of gravity is a rather trivial generalization. Now the central charge of the dual CFT is large $c_T \gg \Delta_{\text{gap}} \gg 1$ but finite. The bulk graviton contributes only to the leading growing term of the Regge correlator of the dual CFT. This immediately implies that the constraints (1.3)–(1.5) remain unchanged for all even $k \geq 4$. On the other hand, all conditions involving λ_2 now receive corrections from gravity. For example, gravity allows for the $\phi^2\Box^2\phi^2$ interaction to have a negative coupling constant

$$\mu\lambda_2 > -\pi N_D(\Delta) G_N R_{\text{AdS}}^2\,, \tag{1.6}$$

where $N_D(\Delta)$ is a positive order one numerical factor given in appendix F. It is however unclear how to extract a precise bound from (1.6) in the flat space limit. The AdS bound (1.6), as we will explain, suggests that in the flat space limit $\mu\lambda_2 M^D > -\varepsilon$, where ε is some small positive number. This is certainly consistent with the results of [47], however, we do not have a precise definition of ε. Nevertheless, this raises an interesting conceptual question of whether, and in what sense, the 4D a-theorem is valid in the presence of dynamical gravity.

Finally, we will generalize our analysis for EFTs of multiple scalar fields in AdS. The main motivation for this generalization is to demonstrate that there are other tools available when we go beyond a single scalar field. For example, the same CFT consistency

[8] It should be noted that this condition, unlike other two conditions, depends on our exact definition of the cut-off scale M. For an arbitrary definition of M, there must always exist a rescaling $M \to XM$ with order one X which makes the EFT consistent with the condition (1.4).

[9] This can be alternatively stated as $\lambda_2 > 0$ with $\mu \geq 0$. This condition is more subtle in the exact flat space limit, as we explain later. In flat space $\lambda_2 = 0$ does not necessarily requires $\lambda_k = 0$ for odd k. For an example see [55].

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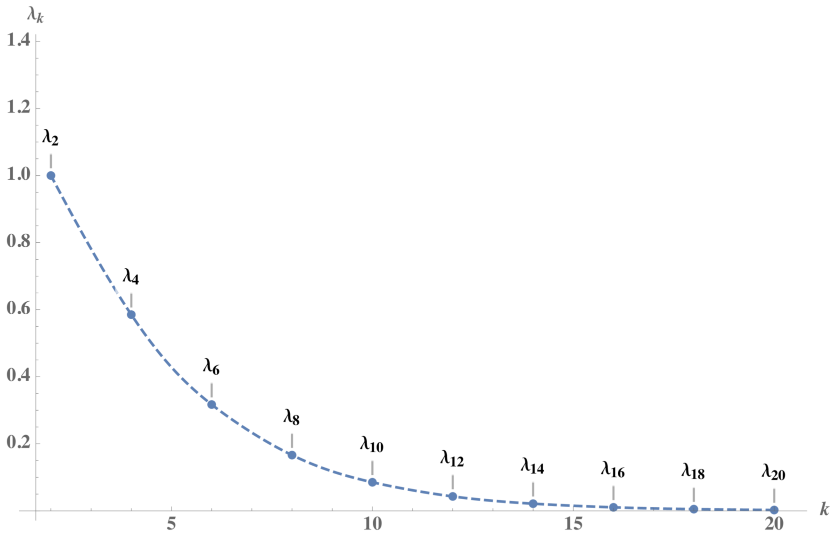

Figure 1. The coupling constants λ_k for even $k \geq 2$, without dynamical gravity, must obey the conditions (1.3)–(1.5). There is always some choice of the scale M (and μ) for which the coupling constants have this generic structure. When gravity is included, only bounds on λ_2 become weaker.

conditions of [88] now also impose two-sided bounds on odd $k \geq 3$ higher derivative interactions involving multiple fields (see [25] for similar bounds on flat space multi-field EFTs). Furthermore, for multiple fields there are interference effects that are also constrained by the CFT axioms leading to an infinite set of non-linear bounds among various higher derivative coupling constants.[10] These additional tools will certainly be useful for bounding the four-graviton scattering amplitude in AdS by using the dual CFT description.

Our CFT setup, from the dual gravity perspective, is probing local high energy scattering deep in the bulk. Since the local high energy scattering is insensitive to the spacetime curvature, on physical grounds we expect that AdS bounds obtained in this paper persist even in the flat space limit (other than the caveat mentioned after equation (1.6)). Indeed, we checked that weakly coupled string amplitudes satisfy all the conditions derived in this paper. However, there is one obvious but important issue that we must address. Any strict AdS inequality $A > 0$ must be regarded as $A \geq 0$ in the flat space limit since the $A = 0$ case can no longer be ruled out due to finite curvature effects.

An important feature of our AdS bounds is that they differ significantly for massive and massless scalars, especially when we take the flat space limit. In particular, when we take the large R_{AdS} limit (with fixed m), our bounds agree completely with the ones obtained from the flat space dispersive sum-rules under the same set of assumptions about the four-point amplitude as mentioned in the beginning.[11] On the other hand, when we take $m = 0$ first and then $R_{\text{AdS}} \to \infty$, two sets of bounds differ significantly. This perhaps indicates that the Regge boundedness condition of the flat space amplitude can break down in the presence of massless states (see section 5).

[10] Note that the same interference effects were utilized in [72] to derive non-linear bounds on the dilaton-axion effective action associated with 4D RG flows with global symmetry breaking.

[11] This is true even when we take $m \to 0$ after taking the large radius limit. See section 5.

At this stage, one may wish to compare our bounds with the flat space bounds of [47]. Indeed, there is some overlap between these two sets of bounds. Of course, from our CFT setup, it is not immediately clear how to obtain any constraints on odd k coupling constants for a single scalar field. Such bounds will perhaps require a more sophisticated CFT analysis. Nevertheless, we observe that our constraints, in the flat space limit, are consistent with the bounds of [47].

The rest of the paper is organized as follows. In section 2 we begin by explaining our general setup. In section 3 we review the bounds of [88] on certain CFT Regge correlators and explain how they follow from the conformal bootstrap axioms. We use this CFT constraints in section 4 to derive bounds (1.3)–(1.5) on the scalar EFT in AdS without dynamical gravity. Then in section 5 we discuss some implications of these constraints for massless external scalars in the flat space limit. Section 6 studies the consequences of the inclusion of gravity. In section 7 we extend our analysis to impose bounds on the EFT of two scalar fields. Section 8 contains our conclusions and final comments. Some additional aspects of our analysis are included in several appendices. In particular, in appendix A we demonstrate how Rindler positivity in CFT follows from unitarity and crossing symmetry.

2 Scalar EFT in AdS

We consider an EFT of a single massless or massive scalar field in AdS with higher derivative interactions. We start with the following low energy effective action with four-point interactions

$$\begin{aligned} S =& \frac{1}{16\pi G_N}\int d^Dx\sqrt{-g}\left(R+\frac{(D-1)(D-2)}{R_{\rm AdS}^2}\right)\\ &+\frac{1}{2}\int d^Dx\sqrt{-g}\left(-g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi-m^2\phi^2+\alpha_3\phi^3+\sum_{k=0}^{\infty}\mu_k\,\phi^2\Box^k\phi^2\right)+\cdots\,, \end{aligned} \tag{2.1}$$

where, α_3 and μ_k are coupling constants.[12] The AdS radius $R_{\rm AdS}$ is large but finite. Our goal is to derive constraints on the coefficients μ_k. In the process, the form of the effective action (1.2) will emerge automatically. Note that different higher derivative interactions, in general, can be suppressed by different scales. However, we will assume that all interactions are suppressed by some small coupling $0<\mu\ll 1$:

$$G_N,\alpha_3^2,\mu_k\sim\mu\,. \tag{2.2}$$

We intend to impose constraint on the weakly coupled effective theory and hence we work in the leading order in μ.[13] This will be implemented by keeping only tree level processes.

[12] Note that at the tree level the $k=1$ term can be removed by using the equation of motion. So, we will ignore the $k=1$ interaction completely.

[13] One can think of μ as the analog of the string coupling in string theory. Similarly, the cut-off scale M in the effective action (1.2) can be regarded as the string scale.

2.1 Other higher-derivative interactions

The observant reader may have noticed that the effective action (2.1) can have other higher derivative 4-ϕ interactions. For example, even in flat space there are multiple inequivalent 4-ϕ interactions with 12 or more derivatives. Now we argue that these other higher-derivative 4-ϕ interactions will not affect any of the bounds obtained in this paper (provided μ_2 is non-zero).

First, let us consider the flat space EFT (massive or massless) with 4-ϕ interactions. At the $2k$-derivative level, there are exactly two types of interactions:

$$a_k\phi^2\Box^k\phi^2 + \sum_{a,b,c>0}^{a+b+c=k} b_k^{\{a,b,c\}} \left(\partial^{\mu_1}\cdots\partial^{\mu_k}\phi\right)\left(\partial_{\mu_1}\cdots\partial_{\mu_a}\phi\right)\left(\partial_{\mu_1}\cdots\partial_{\mu_b}\phi\right)\left(\partial_{\mu_1}\cdots\partial_{\mu_c}\phi\right)\,. \quad (2.3)$$

All other possible interactions can be written in the above form by utilizing the free equation of motion and integration by parts. Note that the second term can only be non-zero and independent (when we use the equation of motion) for $k\geq 6$.[14]

Now we move on to the AdS case and replace $\partial_\mu \to \nabla_\mu$. In AdS, derivatives do not commute in general. So, one may construct several more terms from the second term of (2.3) by choosing different ordering of derivatives. However, different derivative orderings differ only by factors of $1/R_{\rm AdS}^2$

$$\left(\cdots\nabla_\mu\nabla_\nu\cdots\right)\phi - \left(\cdots\nabla_\nu\nabla_\mu\cdots\right)\phi \sim \frac{1}{R_{\rm AdS}^2}\left(\cdots\right)\phi \quad (2.4)$$

and hence derivative ordering is not important in the large $R_{\rm AdS}M$ limit, where M is the cut-off scale of the EFT (2.1). In this paper, we will impose constraints on the EFT (2.1) by studying the CFT Regge correlator $G(\eta,\sigma)$, as defined in section 3, of the dual CFT. In the CFT Regge limit $\sigma\to 0$, the first term of (2.3) contributes to $G(\eta,\sigma)$

$$a_k\phi^2\Box^k\phi^2 \qquad \Rightarrow \qquad \sim i\frac{a_k}{R_{\rm AdS}^{D+2k-4}}\frac{1}{\sigma^{k-1}}\,, \quad (2.5)$$

for even k. On the other hand, the Regge contribution from the second term of (2.3) grows as $\frac{1}{\sigma^{k-3}}$ or slower.[15] Similarly, contributions from $\phi^2\Box^k\phi^2$ with odd k are always subleading since they are suppressed by higher powers of $1/R_{\rm AdS}$. Hence, the leading contributions to $G(\eta,\sigma)$ in the Regge limit always come from $\phi^2\Box^k\phi^2$ terms with even k. This implies that the additional higher derivative 4-ϕ interactions of (2.3) will not affect the argument of this paper in any way. So, we can safely ignore these other higher derivative 4-ϕ interactions in the effective action (2.1). Moreover, we also see that odd k interactions of (2.1) are not bounded by our CFT argument.

From the flat space perspective the above discussion can be understood as follows. Roughly speaking, the CFT argument of this paper is only sensitive to the forward limit ($t=0$) of the tree level 4-point scattering amplitude (for large s) associated with the

[14] For example, for $k=3$ the second term can be equivalently written as $\phi^2\Box^3\phi^2$ plus terms with 4 or less derivatives.

[15] See section 4.4 for details.

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effective action (2.1). The second term of (2.3) does not contribute to the forward amplitude since $a, b, c > 0$. So, it is not surprising that these other higher derivative 4-ϕ interactions cannot affect any of the bounds obtained in this paper.

2.2 Dual CFT

We will impose constraints on this action from the consistency of the dual CFT$_d$, where $D = d+1$. The AdS theory (2.1) is dual to an interacting CFT in d-dimensions. The bulk field ϕ is dual to a scalar primary operator $\mathcal{O}$ with dimension $m^2R^2_{\text{AdS}} = \Delta(\Delta - d)$. The two-point function is completely fixed by conformal invariance[16]

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\rangle = \frac{(2\Delta - d)C_\Delta}{x_{12}^{2\Delta}}\,, \qquad C_\Delta = \frac{\Gamma[\Delta]}{\pi^{d/2}\Gamma[\Delta - d/2]}\,. \tag{2.6}$$

Of course, the graviton $h_{\mu\nu}$ is dual to the CFT stress tensor $T_{\mu\nu}$. The EFT (2.1) is a well behaved theory at energies below the cut-off scale M. Our goal is to impose constraints on the coupling constants by requiring that the EFT is the low energy description of a UV complete theory. Equivalently, in the CFT side we will assume that the dual CFT is well behaved. Next, we discuss exactly what we mean by a well behaved CFT.

2.3 CFT axioms

We make the assumption that the dual CFT obeys the Euclidean bootstrap axioms. In particular, we only make use of the following three properties:

(i) *OPE Unitarity* — All OPE coefficients of real operators are real.

(ii) *Crossing Symmetry* — CFT four-point correlators are crossing symmetric.

(iii) *Analyticity* — Lorentzian CFT four-point correlators are analytic in the usual domain (see figure 3).

These CFT properties are well-established and they imply rigorous non-perturbative constraints on certain Regge correlators as derived in [88]. We will use these constraints to derive precise bounds on the higher-derivative couplings of the EFT (2.1).

3 A review of the bounds on CFT Regge correlators

In this section we review the bounds of [88] on CFT Regge correlators for scalar external operators. Points $x \in \mathbb{R}^{1,d-1}$ in CFT$_d$ are denoted as follows:

$$x = (t, y, \vec{x}) \equiv (x^-, x^+, \vec{x})\,, \tag{3.1}$$

where, $x^\pm = t \pm y$ are lightcone coordinates. We study the Lorentzian CFT correlator[17]

$$G = \frac{\langle O_2(\mathbf{1})O_1(\boldsymbol{\rho})O_1^\dagger(-\boldsymbol{\rho})O_2^\dagger(-\mathbf{1})\rangle}{\langle O_2(\mathbf{1})O_2^\dagger(-\mathbf{1})\rangle\langle O_1(\boldsymbol{\rho})O_1^\dagger(-\boldsymbol{\rho})\rangle} \tag{3.2}$$

[16] For a review see appendix C.

[17] The Hermitian conjugatation in (3.2) acts only on operators, not on coordinates.

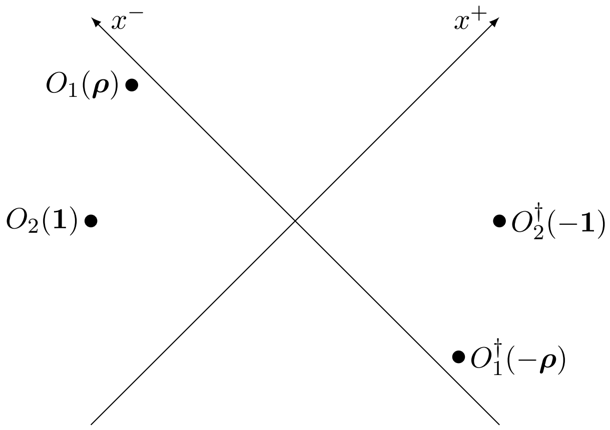

Figure 2. Lorentzian four-point correlator (3.2) where all operators are restricted to a 2d subspace.

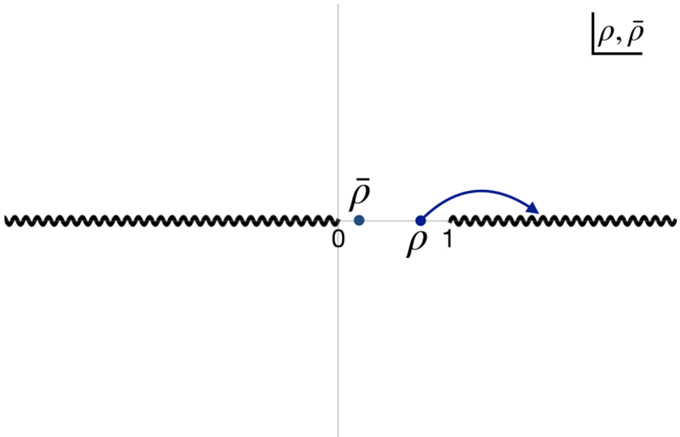

Figure 3. Analytic structure of the correlator (3.2) — branch cuts appear only when two operators become null separated. The Regge limit is obtained from the Euclidean correlator by analytically continuing ρ along the path shown.

of two arbitrary CFT scalar operators, where operators inside the correlator are ordered as written. All the points are restricted to be on a 2d subspace:

$$\begin{aligned} \mathbf{1} &\equiv (t=0, y=-1, \vec{0})\,, \qquad & \boldsymbol{\rho} &\equiv (x^- = \rho, x^+ = -\bar{\rho}, \vec{0})\,, \\ -\mathbf{1} &\equiv (t=0, y=1, \vec{0})\,, \qquad & -\boldsymbol{\rho} &\equiv (x^- = -\rho, x^+ = \bar{\rho}, \vec{0}) \end{aligned} \tag{3.3}$$

with $1 > \bar{\rho} > 0$ and $\rho > 1$, as shown in figure 2. The operator ordering in (3.2) is important since some of the operators, as shown in the figure 2, are timelike separated. This Lorentzian correlator can be obtained from the Euclidean correlator by analytically continuing ρ along the path shown in figure 3.

For later convenience, we parametrize

$$\rho = \frac{1}{\sigma}\,, \qquad \bar{\rho} = \sigma\eta \tag{3.4}$$

with $\eta > 0$ and hence $G \equiv G(\eta, \sigma)$. The CFT Regge limit can be reached by taking

$$\sigma \to 0\,, \qquad \text{with} \qquad \eta = \text{fixed} > 0 \tag{3.5}$$

of the correlator $G(\eta, \sigma)$. The Regge correlator $G(\eta, \sigma)$, as a function of complex σ, is analytic near $\sigma \sim 0$ (for $0 < \eta < 1$) on the lower-half σ-plane [88–90].

3.1 Boundedness of the Regge correlator

We can define another Lorentzian correlator

$$G_0(\eta, \sigma) = \frac{\langle O_2(\mathbf{1}) O_1(\boldsymbol{\rho}) O_2^\dagger(-\mathbf{1}) O_1^\dagger(-\boldsymbol{\rho})\rangle}{\langle O_2(\mathbf{1}) O_2^\dagger(-\mathbf{1})\rangle \langle O_1(\boldsymbol{\rho}) O_1^\dagger(-\boldsymbol{\rho})\rangle} \tag{3.6}$$

which is determined by Euclidean OPE and hence in the limit (3.5)

$$G_0(\eta, \sigma) = 1 + \cdots\,, \tag{3.7}$$

where dots represent terms that are suppressed by positive powers of σ. The Regge correlator $G(\eta, \sigma)$ is bounded by the "Euclidean" correlator $G_0(\eta, \sigma)$. In particular, for real σ with $|\sigma| < 1$ OPE unitarity and crossing symmetry imply

$$|G(\eta, \sigma)| \leq G_0(\eta, \sigma)\,, \tag{3.8}$$

where, $G_0(\eta, \sigma) > 0$. In the strict Regge limit (3.5), this simplifies to

$$|G(\eta, \sigma)| \leq 1 + \mathcal{O}(\sigma^a) \tag{3.9}$$

with $a > 0$.

More generally, the bound (3.8) follows from Rindler positivity as described in [88, 89]. For CFT correlators of scalar operators, Rindler positivity is a consequence of OPE unitarity and crossing symmetry. This is reviewed in appendix A (see also [90]).

3.2 CFT constraints

Next, we focus on Regge correlators with a very specific Regge behavior for some range of σ:

$$G(\eta, \sigma) = 1 + i \sum_{L=1,2,\cdots} \frac{c_L(\eta)}{\sigma^{L-1}}\,, \qquad \sigma_* \leq |\sigma| \ll \eta < 1 \tag{3.10}$$

up to terms that decay in the Regge limit. The cut-off σ_* dictates the regime of validity of the Regge expansion (3.10). Later we will relate σ_* to $1/\Delta_{\text{gap}}^2$.

The Regge correlator $G(\eta, \sigma)$, as a function of complex σ, is analytic near $\sigma \sim 0$ on the lower-half σ-plane [88, 89]. Using this analyticity property we can write a CFT dispersion relation for $c_L(\eta)$ [88]:

$$c_L(\eta) = \frac{1}{\pi} \int_{-R}^{R} d\sigma\, \sigma^{L-2} (1 - \text{Re}\, G(\eta, \sigma))\,, \qquad \sigma_* \leq R \ll \eta < 1\,, \tag{3.11}$$

where $L \geq 2$. The above relation leads to bounds on $c_L(\eta)$ for all $0 < \eta < 1$. Note that the left hand side does not depend on R. This implies Re $G(\eta, \sigma)$ deviates significantly from 1 only when $\sigma \lesssim \sigma_*$. This is closely related to the fact that the tree level 4-pt scattering amplitude for the EFT (2.1) has no imaginary part.

Positivity. The boundedness condition (3.9) immediately implies [88]

$$c_L(\eta) \geq 0\,, \qquad \text{for} \qquad \text{even } L \geq 2 \tag{3.12}$$

for $0 < \eta < 1$. One can worry whether the $\mathcal{O}(\sigma^a)$ correction terms in (3.9) can affect the above positivity condition for higher L. For the CFT dual to (2.1), we can always take a limit where R is small enough such that these corrections are suppressed.[18]

Parametric separation. The fact that Re $G(\eta, \sigma) \leq 1$ also implies [88]

$$\frac{c_{L+2}(\eta)}{c_L(\eta)} \leq \sigma_*^2\,, \qquad \frac{|c_{L+1}(\eta)|}{c_L(\eta)} \leq \sigma_* \tag{3.13}$$

for all even $L \geq 2$ and $0 < \eta < 1$. Therefore $|c_L(\eta)|$, as a function of L, must decrease monotonically as a power law. Furthermore, the above bound along with the condition (3.9) also require that the Regge correlator (3.10) is consistent only if

$$c_2 \lesssim \sigma_*\,. \tag{3.14}$$

Log-convexity for even *L*. The Cauchy-Schwarz inequality of integrable functions leads to the log-convexity condition for $c_L(\eta)$ with even L [88]:

$$\left(\frac{c_{L+2}(\eta)}{c_L(\eta)}\right)^2 \leq \frac{c_{L+4}(\eta)}{c_L(\eta)}\,, \qquad \text{for even} \quad L \geq 2 \tag{3.15}$$

and $0 < \eta < 1$.

Boundedness of odd *L*. There is no sign constraint on $c_L(\eta)$ with odd L. However, the absolute value of $c_L(\eta)$ for odd L is bounded [88]

$$|c_L(\eta)| \leq \sqrt{c_{L-1}(\eta) c_{L+1}(\eta)}\,, \qquad \text{for odd} \quad L \geq 3 \tag{3.16}$$

and $0 < \eta < 1$. This also follows from the positivity condition (3.9) and the Cauchy-Schwarz inequality.

Note that the chaos sign and the growth bounds of [91] are contained in the above consistency conditions. The condition (3.12) is a generalization of the chaos sign bound. Whereas, the condition (3.13) implies that the Regge correlator (3.10) must not grow faster than $1/\sigma$ within the regime of validity $\sigma_* \leq |\sigma| \ll \eta < 1$.

Finally, let us note that the above constraints hold for arbitrary external operators with or without spins (and not necessarily primary)[19] as long as the Regge correlator has the form (3.10). For such a general case, the positivity of the integrand in (3.11) follows from Rindler positivity.

[18] At the end of this section we will discuss more about these corrections.
[19] See [88] for details.

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3.3 Correction terms and validity of the CFT dispersion relation

All of the above constraints depend on the dispersion relation (3.11). So, it is only natural to ask whether there are corrections to this dispersion relation. In this section, we argue that any such correction terms do not affect the dispersion relation (3.11) since they are always suppressed for CFTs that are dual to some EFT in AdS. Casual readers may skip this subsection.

The first correction comes from the $\mathcal{O}(\sigma^a)$ terms of (3.9). Moreover, similar $\mathcal{O}(\sigma^a)$ correction terms can be present in the Regge expansion (3.10). So, the leading correction to the dispersion relation (3.11) comes from a term

$$\delta(1 - \text{Re } G(\eta, \sigma)) \sim (\delta c)\sigma^a \qquad \text{with} \qquad a > 0 \tag{3.17}$$

since all terms with negative a have integer a with imaginary coefficients.[20]

First, let us justify the dispersion relation (3.11) for the scenario where operators O_1 and O_2 are different. In this case, especially for CFTs that are dual to some EFT in AdS, it is easy to see that $a \geq d$ since CFT operators that are exchanged are either double trace operators or single trace operators with $a = \Delta \geq d$ from a bulk three-point interaction.[21] In particular, the contribution of a correction term (3.17) to the dispersion relation of c_2 is given by

$$\frac{1}{\pi}\int_{-R}^{R} d\sigma(1 - \text{Re } G(\eta, \sigma)) \sim c_2 + \text{Re } \delta c \int_{-R}^{R} d\sigma\sigma^a = c_2 + \mathcal{O}(1)|\delta c|R^{a+1} \tag{3.18}$$

where the line integrals are evaluated just below the real σ-axis. Now, note that the leading contribution to $c_2 \sim \frac{\mu_2}{R_{\text{AdS}}^D}$, whereas, the leading contribution to δc comes from scalar three-point couplings: $\delta c \sim \frac{\alpha_3^2}{R_{\text{AdS}}^{D-6}}$. Therefore, for any non-zero c_2, the correction term is suppressed for

$$R \ll \left(\frac{|\mu_2|}{\alpha_3^2}\right)^{\frac{1}{D}} \frac{1}{R_{\text{AdS}}^{6/D}}\,. \tag{3.19}$$

On the other hand, the cut-off σ_* scales as $1/R_{\text{AdS}}^2$.[22] Therefore, for large R_{AdS} we can always choose $1 \gg R \geq \sigma_*$ such that the correction term is parametrically suppressed for $D \geq 4$.[23] If three-point bulk interactions such as α_3 are absent, all other corrections (even from the bulk graviton exchange) to the sum-rule are more suppressed. Hence, the dispersive sum-rule for c_2 can always be trusted, at least for $D \geq 4$, for small $R \to 0$.

[20] It is important to note that any correction term with integer (positive or negative) power of σ and an imaginary coefficient cannot affect the sum-rule (3.11) [88].

[21] Let us recall that we are restricting to the case where all fields have $m^2 \geq 0$.

[22] This can be seen easily from the scaling of individual terms of the expansion (3.10) for the bulk theory (2.1). In particular, the expansion (3.10) for the Regge correlator of the dual CFT is an expansion in the quantity $1/\sigma R_{\text{AdS}}^2$, as can be seen from (4.2). The cut-off σ_* is controlled by the relative strength of consecutive terms in the expansion (3.10) and hence $\sigma_* \propto 1/R_{\text{AdS}}^2$.

[23] Note that $\frac{\alpha_3^2}{|\mu_2|} = m_0^6$, where m_0 is some mass scale. We are making the mild assumption that m_0 is not parametrically larger than the cut-off scale M associated with the bulk theory (2.1). More precisely, we are assuming that mass scales m_0 and M do not scale with R_{AdS}.

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Let us now analyze the relation (3.11) for higher L. Note that we can estimate:

$$1 - \text{Re } G(\eta, \sigma) \sim \frac{\pi c_2}{2\sigma_*} \tag{3.20}$$

which implies

$$\frac{1}{\pi} \int_{-\sigma_*}^{\sigma_*} d\sigma \, \sigma^{L-2} (1 - \text{Re } G(\eta, \sigma)) \sim c_L + \mathcal{O}(1) |\delta c| \sigma_*^{a+L-1} \tag{3.21}$$

where we see from (3.20) that $|c_L| \sim c_2 \sigma_*^{L-2}$ and hence the second term can be ignored just like before even for $L > 2$.[24]

When $O_1 = O_2$, there is a loophole in the above argument which we now fix. The disconnected Witten diagrams associated with the 4-pt correlator of the scalar operator of dimension Δ have the leading correction term $\sim \sigma^{2\Delta}$ with order 1 coefficient. However, one can subtract these contributions without affecting any of the bounds on c_L. For example, when Δ of the external scalar operator is an integer, we can replace $1 - \text{Re } G(\eta, \sigma)$ in the sum-rule (3.11) by $G_0^{\text{free}}(\eta, \sigma) - \text{Re } G(\eta, \sigma)$, where $G_0^{\text{free}}(\eta, \sigma)$ is the correlator (3.6) for the AdS theory (2.1) without any interactions. This new sum-rule holds because $G_0^{\text{free}}(\eta, \sigma)$ is analytic on the lower-half σ plane for integer Δ. Moreover, $G_0^{\text{free}}(\eta, \sigma) - \text{Re } G(\eta, \sigma)$ is positive on the real line up to correction terms that are exactly the same as the above discussion of nonidentical operators. So, we repeat the same argument again to conclude that the modified dispersive sum-rule for c_L is reliable for $D \geq 4$ and integer Δ. This is sufficient for us, since for any fixed m^2, we can always tune R_{AdS} such that Δ is an integer. In any case, for non-integer Δ one can still write a more general sum-rule for $G(\eta, \sigma)$ by subtracting contributions from the identity operator in all channels. The procedure is outlined in appendix B.

To summarize, we conclude that the CFT dual to the AdS theory (2.1) must obey the consistency conditions (3.12), (3.13), (3.15), and (3.16) for $D \geq 4$.

4 Constraining scalar EFT in AdS without gravity

The main goal of this section is to impose bounds on the EFT (2.1) from CFT consistency conditions. To that end, we compute contributions of each EFT interactions to the Lorentzian correlator

$$G(\eta, \sigma) = \frac{\langle \mathcal{O}(\mathbf{1}) \mathcal{O}(\boldsymbol{\rho}) \mathcal{O}(-\boldsymbol{\rho}) \mathcal{O}(-\mathbf{1}) \rangle}{\langle \mathcal{O}(\mathbf{1}) \mathcal{O}(-\mathbf{1}) \rangle \langle \mathcal{O}(\boldsymbol{\rho}) \mathcal{O}(-\boldsymbol{\rho}) \rangle} \tag{4.1}$$

in the Regge limit (3.5), where operator $\mathcal{O}$ is dual to the scalar field ϕ. First, we consider the purely non-gravitational case by setting $G_N = 0$. The leading contribution to the correlator $G(\eta, \sigma)$ comes from the disconnected Witten diagrams. The dominant subleading contribution comes from the tree level Witten diagrams that are shown in figure 4.

Before we proceed with the computation, let us review what is already known about the Regge limit. For example, from [75, 92] we know the scaling of the leading Regge

[24]For odd L, it is possible that $|c_L| \ll c_2 \sigma_*^{L-2}$ because of cancellations implying that the dispersion relation (3.11) is not reliable. However, in this case all of the CFT bounds for odd L are satisfied automatically.

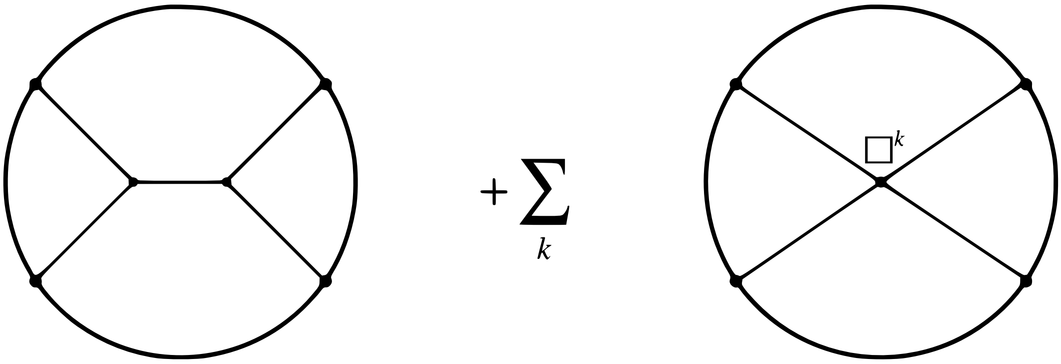

Figure 4. The tree-level Witten diagrams that are relevant in the Regge limit for $G_N = 0$. Of course, the exchange diagram should be summed over all channels.

contribution of each interaction in (2.1):

$$\begin{aligned} \alpha_3\phi^3 \qquad &\Rightarrow \qquad \sim i\alpha_3^2 R_{\mathrm{AdS}}^{6-D}\sigma\,, \\ \mu_{2n}\phi^2\Box^{2n}\phi^2 \qquad &\Rightarrow \qquad \sim i\frac{\mu_{2n}}{R_{\mathrm{AdS}}^{D+4n-4}}\frac{1}{\sigma^{2n-1}}\,, \\ \mu_{2n+1}\phi^2\Box^{2n+1}\phi^2 \qquad &\Rightarrow \qquad \sim i\frac{\mu_{2n+1}}{R_{\mathrm{AdS}}^{D+4n-2}}\frac{1}{\sigma^{2n-1}}\,, \end{aligned} \tag{4.2}$$

for integer n. From the scaling behavior (4.2) it is clear that the Regge correlator (4.1) has the desired expansion (3.10) up to terms that are suppressed by positive powers of σ. Notice that contributions of $\phi^2\Box^k\phi^2$ for odd k are always suppressed in the large R_{AdS} limit. Hence, the above scaling behavior implies that we can only impose constraints on interactions $\mu_{2n}\phi^2\Box^{2n}\phi^2$ for $n = 1, 2, \cdots$ from the CFT consistency conditions of the preceding section.

We observe that the leading contribution to c_L with even $L \geq 2$ comes entirely from the interaction $\phi^2\Box^L\phi^2$. Contributions from $k > L$ interactions to c_L are all suppressed in the large R_{AdS} limit. It is also clear from (4.2) that c_L with odd L are all zero. This simply follows from the fact that $\mathcal{O}$ is a real scalar operator.

So, it is sufficient for us to consider each interaction separately

$$S_L = \frac{\mu_L}{2}\int d^Dx\sqrt{-g}\phi^2\Box^L\phi^2\,, \tag{4.3}$$

with $L > 0$ being an even integer, where $d^Dx \equiv dzd^dx$. Note that

$$\frac{\mu_L}{2}\phi^2\Box^L\phi^2 \sim 2^{L-1}\mu_L\phi^2(\nabla_{\mu_1}\cdots\nabla_{\mu_L}\phi)^2 + \cdots\,, \tag{4.4}$$

where dots represent terms with lower number of derivatives after we impose the free equation of motion.

It is more convenient to first compute the Euclidean correlator and then analytically continue to obtain the Regge correlator. So, the on-shell Euclidean action associated with (4.3) is obtained from (C.25)

$$S_{\text{on-shell}}^{(L)} = -2^{L-1}\mu_L\int dzd^dx\sqrt{g}\,\phi^2(\nabla_{\mu_1}\cdots\nabla_{\mu_L}\phi)^2 + \cdots\,, \tag{4.5}$$

where again dots represent terms with lower number of derivatives that cannot contribute to c_L. This on-shell action can be further simplified by using the bulk-to-boundary propagator (C.5)

$$S^{(L)}_{\text{on-shell}} = -2^{L-1}\mu_L C_\Delta^4 \int_{\text{AdS}}\int_{\Phi^4} \tilde{K}_\Delta(z,x;x_1)\tilde{K}_\Delta(z,x;x_2) \\ \nabla_{\mu_1}\cdots\nabla_{\mu_L}\tilde{K}_\Delta(z,x;x_3)\nabla^{\mu_1}\cdots\nabla^{\mu_L}\tilde{K}_\Delta(z,x;x_4)\,, \tag{4.6}$$

where the derivatives are taken with respect to the bulk point $\{z,x\}$.[25] In the above expression, we have utilized the notations of [93]

$$\int_{\text{AdS}} \equiv \int dz d^d x\sqrt{g}\,, \qquad \int_{\Phi^4} \equiv \prod_{i=1}^{4}\int \Phi(x_i)d^d x_i\,, \qquad x_{ij}^2 = (x_i - x_j)^2\,. \tag{4.7}$$

Note that x represents points on the AdS boundary. Moreover, the reduced bulk-to-boundary propagator $\tilde{K}_\Delta(z,x;x')$ is defined in (D.3) to reduce clutter. The boundary value of the field $\phi(z,x)$ is given by $\Phi(x)$ which acts as the source for the CFT$_d$ primary operator $\mathcal{O}(x)$ in the usual way.

We can now use the identity (D.4) to write $S^{(L)}_{\text{on-shell}}$ in terms of the D-function which is defined in (D.1) in the standard way. We notice from equation (D.16) that all D-functions decay in the Regge limit $D_{\Delta_1\Delta_2\Delta_3\Delta_4}\sim\sigma$. On the other hand, x_{ij}^2 factors for the kinematics (D.2) can grow as $\sim 1/\sigma$. Therefore, terms in $S^{(L)}_{\text{on-shell}}$ that have the largest factors of x_{ij}^2 dominate in the Regge limit (3.5). This greatly simplifies the analysis since we only care about the growing part of the Regge correlator $G(\eta,\sigma)$. In particular, the leading Regge contribution from the on-shell action (4.6) comes from

$$S^{(L)}_{\text{on-shell}} = -2^{2L-1}\mu_L C_\Delta^4\left(\frac{\Gamma(\Delta+L)}{\Gamma(\Delta)}\right)^2\int_{\text{AdS}}\int_{\Phi^4} x_{34}^{2L}\tilde{K}_\Delta(z,x;x_1)\tilde{K}_\Delta(z,x;x_2) \\ \times\tilde{K}_{\Delta+L}(z,x;x_3)\tilde{K}_{\Delta+L}(z,x;x_4)+\cdots\,, \tag{4.8}$$

where, dots represent terms that will not contribute to c_L. It is now a straightforward exercise to show that the leading Regge contribution of the interaction (4.3) is

$$G(\eta,\sigma)\sim\mu_L\frac{(16\eta)^\Delta 2^{2L-1}C_\Delta^2}{(2\Delta-d)^2 R_{\text{AdS}}^{D+2L-4}}\left(\frac{\Gamma(\Delta+L)}{\Gamma(\Delta)}\right)^2\frac{16}{\sigma^L}D_{\Delta+L\ \Delta\ \Delta+L\ \Delta}(\eta,\sigma)\,, \tag{4.9}$$

where $D=d+1$. From the above result, we obtain an expression for c_L for even $L\geq 2$ in the limit of large R_{AdS} (with Δ fixed):

$$c_L(\eta) = \frac{\kappa_\Delta\mu_L}{R_{\text{AdS}}^{D+2L-4}}F_{2\Delta+L}(\eta) \tag{4.10}$$

where, κ_Δ is a positive coefficient independent of L

$$\kappa_\Delta = \frac{4}{\Gamma(\Delta)^2\Gamma\left(-\frac{D}{2}+\Delta+\frac{3}{2}\right)^2} \tag{4.11}$$

[25]The coefficient C_Δ is defined in (2.6).

and the F-function is given by using (D.17):

$$F_{2\Delta+L}(\eta) = \frac{1}{\eta^{\frac{L-1}{2}}} f_{\Delta+L\ \Delta\ \Delta+L\ \Delta}\left(-\frac{1}{2}\log(\eta)\right) . \tag{4.12}$$

which is positive for $0<\eta<1$. As we mentioned before, all $c_L(\eta)$ coefficients with odd L vanish exactly.

4.1 Bounds

We are now in a position to utilize the CFT constraints from section 3.2 to derive bounds on the EFT (2.1).

4.1.1 Positivity

First, we impose the condition (3.12). The fact that both κ_Δ and $F_{2\Delta+L}(\eta)$ for $0<\eta<1$ are positive immediately implies

$$\mu_k \geq 0\,, \qquad \text{for} \qquad \text{even } k\geq 2\,. \tag{4.13}$$

Moreover, saturation of any one of (4.13) necessarily requires that the all of them are saturated. These bounds are consistent with the flat space bound of [1] from analyticity and unitarity of $2\to 2$ scattering amplitudes. Note that there is no such positivity condition on μ_k with odd k from the CFT consistency conditions.

4.1.2 Scale suppression of higher derivative interactions

We now impose the condition (3.13). First, let us apply (3.13) to $L=2$:

$$\frac{\mu_4}{\mu_2} \leq \frac{R_{\text{AdS}}^4 F_{2\Delta+2}(\eta)}{F_{2\Delta+4}(\eta)}\sigma_*^2 \tag{4.14}$$

for all $0<\eta<1$, where we are assuming that the AdS theory is interacting ($\mu_2>0$). First thing we notice that a mass scale $M=\Delta_{\text{gap}}/R_{\text{AdS}}$ is emerging naturally where we have identified

$$\sigma_* \equiv \frac{1}{\Delta_{\text{gap}}^2}\sqrt{\frac{\Gamma(2\Delta+3)\Gamma\left(-\frac{D}{2}+2\Delta+\frac{9}{2}\right)}{\Gamma(2\Delta+1)\Gamma\left(-\frac{D}{2}+2\Delta+\frac{5}{2}\right)}}\,. \tag{4.15}$$

This definition of Δ_{gap} needs some explanation. It is expected that $\sigma_*\propto 1/\Delta_{\text{gap}}^{\#}$ since Re $G(\eta,\sigma)$ deviates significantly from 1 when $\sigma\lesssim\sigma_*$ implying a breakdown of (3.10). The exact power in (4.15) follows from the linear relationship between $M=R_{\text{AdS}}\Delta$. The order one numerical factor has been chosen such that in certain scenarios Δ_{gap} has the physical interpretation of the lightest heavy state exchanged.[26] Given a UV complete CFT dual and the low energy Regge behavior (3.10), one can compute σ_* from the sum-rule (3.11) with $R=\sigma_*$. Then (4.15) should be thought of as a precise definition of Δ_{gap}. The bulk cut-off scale is then given by the relation: $M=\Delta_{\text{gap}}/R_{\text{AdS}}$. Of course, this definition of

[26] We will make this more precise in section 5.

M is not unique. This definition is analogous to the definition of M in [47] and in certain cases these two definitions are exactly equivalent, as we show in section 5.

The strongest bound from (4.14) is obtained for the value of η that minimizes the right hand side. One can check that this is achieved in the limit $\eta \to 0$. Therefore, by using results from appendix D.3 we obtain a strict bound:

$$\frac{\mu_{k+2}}{\mu_k} \leq \frac{n_k(\Delta)}{n_{k+2}(\Delta)} \left(\frac{R_{\text{AdS}}}{\Delta_{\text{gap}}}\right)^4 \qquad \text{for} \qquad \text{even } k \geq 2 \tag{4.16}$$

where, $n_k(\Delta)$ is given by

$$n_k(\Delta) = \frac{\Gamma\left(2\Delta + k - \frac{D-1}{2}\right) \Gamma\left(2\Delta + k - 1\right)}{\Gamma\left(2\Delta - \frac{D-5}{2}\right) \Gamma\left(2\Delta + 1\right)} \left(\frac{\Gamma(2\Delta+3)\Gamma\left(2\Delta - \frac{D-9}{2}\right)}{\Gamma(2\Delta+1)\Gamma\left(2\Delta - \frac{D-5}{2}\right)}\right)^{1-\frac{k}{2}} \tag{4.17}$$

with $m^2 R_{\text{AdS}}^2 = \Delta(\Delta - D + 1)$. Of course, the bound (4.16) depends heavily on our definition of Δ_{gap} (4.15). Notice that $n_k(\Delta)$ is the same coefficient that appears in (1.2).

The bound (4.16) validates our expectation that higher derivative interactions $\phi^2 \Box^k \phi^2$ are suppressed by inverse powers of Δ_{gap} for even k. However, CFT consistency conditions of the preceding section do not impose similar constraints on $\phi^2 \Box^k \phi^2$ interactions with odd k. This perhaps suggests that our bounds are far from being optimal.

Let us make few comments about the coefficient $n_k(\Delta)$ which is a log-convex function of k. From (4.17) we find that $n_2(\Delta) = n_4(\Delta) = 1$ for any m^2 and D implying

$$\frac{\mu_4}{\mu_2} \leq \frac{1}{M^4}\,. \tag{4.18}$$

We will derive this relation in flat space by assuming the Regge boundedness condition: $\mathcal{A}(s, t = 0) < |s|^2$ for large s. In that case, M is the mass of the lightest massive state exchanged. This explains the choice (4.15).

Furthermore, note that $n_k(\Delta)$ is non-trivial only in the massless limit $m \to 0$. In particular, if we take $R_{\text{AdS}} \to \infty$ with fixed $0 < m \ll M$, we obtain

$$n_k(\Delta) \approx 1 \tag{4.19}$$

for all finite $k \ll m R_{\text{AdS}}$. Therefore, for non-zero m, all our bounds simplify greatly.

4.1.3 Log-convexity condition

The CFT condition (3.15) leads to a rather strong condition on couplings μ_k that does not depend on the exact definition of Δ_{gap} (or equivalently the scale M). The optimal bound again is achieved for $\eta \to 0$, yielding

$$\frac{\mu_{k+2}^2}{\mu_k \mu_{k+4}} \leq \frac{n_k(\Delta) n_{k+4}(\Delta)}{n_{k+2}(\Delta)^2} \qquad \text{for} \qquad \text{even } k \geq 2 \tag{4.20}$$

where, $n_k(\Delta)$ is defined in (4.17). The right hand side is exactly 1 for $k \ll m R_{\text{AdS}} \ll M R_{\text{AdS}}$, as discussed above.

Therefore, the EFT (2.1), in the absence of gravity, can be UV completed only when it satisfies (4.20), irrespective of how the cut-off scale M is defined. It would be nice to derive a similar bound for odd μ_k couplings.

4.1.4 Odd couplings

We still can say few things about the odd μ_k couplings. First of all, if $\mu_2 = 0$ the leading contribution to c_2 comes from μ_3. However, this contribution to c_2 changes sign as we tune η within the domain $0 < \eta < 1$ for $m^2 \geq 0$ (see appendix E). Hence μ_3 must vanish exactly. Then the condition (3.13) implies that

$$\mu_2 = 0 \qquad \Rightarrow \qquad \mu_k = 0 \tag{4.21}$$

for even or odd $k > 2$.[27] When $\mu_2 > 0$, the bound on odd μ_k couplings are rather weak. The above argument then implies that $\frac{|\mu_k|}{\mu_{k-1}} \lesssim R_{\rm AdS}^2$ for odd $k \geq 3$.

4.2 Final effective action

Let us now summarize the results of this section by writing the AdS scalar effective action (2.1) as follows

$$\begin{aligned} S[\phi] = &\frac{1}{2}\int d^D x\sqrt{-g}\left(-g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - m^2\phi^2 + \alpha_3\phi^3 + \mu_0\phi^4\right) \\ &+ \frac{\mu}{2}\int d^D x\sqrt{-g}\sum_{k=2,3,4,\cdots}\frac{\lambda_k}{n_k(\Delta)M^{2(k-2)}}\phi^2\Box^k\phi^2 + \cdots , \end{aligned} \tag{4.22}$$

where $M = \Delta_{\rm gap}/R_{\rm AdS}$ is the scale of new physics and the numerical factor $n_k(\Delta)$ is defined in (4.17). The scalar field can have mass but $0 \leq m^2 \ll M^2$. Note that $n_2(\Delta) = n_4(\Delta) = 1$ for any m^2 and D. For $k > 4$, $n_k(\Delta)$, in the large $R_{\rm AdS}$ limit, differs from 1 only for $m = 0$.[28]

So far gravity is non-dynamical $G_N = 0$. We have defined a positive coupling constant $\mu \geq 0$ which has the dimension $1/M^D$. The λ-coefficients are dimensionless, however, to begin with we do not assume that they are $\mathcal{O}(1)$. We do assume the theory is weakly coupled $\mu M^D \sim |\mu_0|M^{D-4} \sim \alpha_3^2 M^{D-6} \ll 1$ and hence analyze the theory at tree level.

The main goal of this paper is to address the question: when can this EFT be UV completed? Or equivalently what are the necessary conditions for this EFT to be embedded into a UV theory that is dual to a CFT with $\Delta_{\rm gap} \gg 1$ obeying the CFT axioms? In this section, we conclude that the EFT (4.22), with $\mu \geq 0$, must have the following properties ($D \geq 4$):

(i) $\lambda_k > 0$ for all even $k \geq 2$,

(ii) $\lambda_{k+2} \leq \lambda_k$ for all even $k \geq 2$,

(iii) $\frac{1}{k_2-k_1}\ln\frac{\lambda_{k_1}}{\lambda_{k_2}} \geq \frac{1}{k_3-k_1}\ln\frac{\lambda_{k_1}}{\lambda_{k_3}}$ for all even $k_3 > k_2 > k_1 \geq 2$.

The last condition follow directly from the local log-convexity condition (4.20). It should be noted again that the condition (ii) depends on the exact definition of $\Delta_{\rm gap}$ and hence the scale M. On the other hand, other two conditions do not depend on the exact definition

[27] We assume that $m^2 \geq 0$. For negative mass^2, see appendix E for comments.

[28] More generally, in the large $R_{\rm AdS}$ limit, $n_k(\Delta)$ with $k > 4$ differs from 1 only for $m \to 0$ and $R_{\rm AdS} \to \infty$ with $R_{\rm AdS}m$ fixed.

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of the scale M. For an arbitrary definition of M, the condition (ii) should be thought of in the following way. There must always exist a rescaling $M \to XM$, with order one X, which makes the EFT consistent with the condition (ii).

It should also be emphasized that (i)–(iii) are necessary conditions but we believe they are far from being sufficient. For example, it is expected that similar bounds exist even for odd k.[29] However, our argument does not impose any restriction on the odd λ_k couplings other than $|\lambda_k| \lesssim \Delta_{\rm gap}^2$.

4.3 Flat space limit

We end this section with some discussion on the flat space limit of the EFT (4.22). In this section we restrict to the massless case: $m = 0$. The flat space limit should be taken in the following way:

$$R_{\rm AdS} \to \infty\,, \qquad \Delta_{\rm gap} \to \infty \qquad \text{with} \qquad \frac{\Delta_{\rm gap}}{R_{\rm AdS}} = M = \text{fixed}\,. \tag{4.23}$$

In the massless case

$$n_k^{(0)} = \left(\frac{3}{2}\right)^{1-\frac{k}{2}} \frac{\Gamma\left(\frac{3(D-1)}{2}+k\right)\Gamma(2D+k-3)}{\Gamma\left(\frac{3D}{2}+\frac{1}{2}\right)\Gamma(2D-1)} (D(D+1)(2D-1)(3D+1))^{1-\frac{k}{2}} \tag{4.24}$$

increases fast for $k > 4$ as we increase k. In this limit, the constraints (i)–(iii) lead to bounds on the flat space EFT of a massless scalar. We can compare these flat space bounds with the results from [47] by relating various coupling constants:

$$g_2 = 4\mu\lambda_2\,, \quad g_3 = \frac{12\mu\lambda_3}{n_3^{(0)}M^2}\,, \quad g_4 = \frac{2\mu\lambda_4}{M^4}\,, \quad g_6 = \frac{\mu\lambda_6}{n_6^{(0)}M^8}\,, \quad \cdots\,. \tag{4.25}$$

In particular, in the absence of gravity we obtain

$$g_2,\ g_4,\ g_6, \cdots \geq 0\,. \tag{4.26}$$

Furthermore, with our definition of M, we find that the bound (4.18) agrees with the bound obtained in [47].

As we will explain in section 6, all bounds for g_k with $k > 2$ remains unaffected even when gravity is dynamical. Furthermore, we obtain a rather interesting inequality by applying (iii):

$$\frac{g_4^2}{g_2 g_6} \leq \frac{(2D+1)(3D+5)(3D+7)}{3D(2D-1)(3D+1)}\,. \tag{4.27}$$

It would be interesting to compare this bound with the analysis of [20]. It is possible to derive an infinite set of such constraints from (iii). Note that constraints involving g_2 will only be affected when gravity is turned on. Let us stress that there is a discreet difference between the massless case $m = 0$ and the massless limit $m \to 0$ when we take the flat space limit. We will discuss this in the next section.

[29] Note that the condition (4.21) has been implemented by assuming the EFT has the form (4.22) along with $\lambda_2 > 0$.

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Unlike [20, 47], our analysis is insensitive to ϕ^3 and ϕ^4 interactions of (4.22). However, we still need to pay attention to these interactions. In particular, the coupling constant α_3 for the ϕ^3 interaction has positive mass dimension for $D \leq 5$. So, this coupling can lead to large mixing effects in the dual CFT when we take the flat space limit $R_{\rm AdS} \to \infty$ [68]. In particular, when

$$|\alpha_3| R_{\rm AdS}^{3-D/2} \sim 1 \tag{4.28}$$

there is a large mixing between the naive generalized free field operator $\mathcal{O}$ and $[\mathcal{O}\mathcal{O}]_{n,0}$ because of the decay channel $\phi \to \phi\phi$. It is unclear whether the flat space bounds are reliable when the mixing effect is large. Nevertheless, we can avoid this issue for $D \leq 5$ by giving the bulk field ϕ some $\mathbb{Z}_2$ symmetry. Or we can take the flat space limit of the AdS theory (2.1) such that

$$R_{\rm AdS} M \gg 1 \qquad \text{with} \qquad |\alpha_3| R_{\rm AdS}^{3-D/2} \ll 1. \tag{4.29}$$

There is one more subtlety that we must address. When the $\phi^2\Box^2\phi^2$ interaction is absent in the AdS EFT (4.22), all the higher derivative interactions $\phi^2\Box^k\phi^2$ must also vanish. However, our analysis does not require this to be true in the exact flat space limit. For example, the coefficient of the $\phi^2\Box^2\phi^2$ interaction can be suppressed by $R_{\rm AdS}$ in such a way that the dual CFT is well behaved. Moreover, the EFT can have a $\phi^2\Box^3\phi^2$ interaction which is not suppressed by $R_{\rm AdS}$ but fine-tuned such that $c_2(\eta)$ is still positive. In this scenario, the $\phi^2\Box^2\phi^2$ interaction goes to zero in the flat space limit with a non-vanishing $\phi^2\Box^3\phi^2$ interaction. It has been recently conjectured that such EFTs emerge naturally in the IR from 6D supersymmetric RG flows on to the Higgs branch [55]. So, such RG flows in AdS_6 with finite radius are expected to generate a $\phi^2\Box^2\phi^2$ interaction for the dilaton which is suppressed by $1/R_{\rm AdS}^2$.

4.4 Other higher-derivative interactions

In this section, we figure out the Regge contribution of the second term of (2.3) in AdS. One can easily check that the leading Regge contribution of the second term comes from the on-shell action

$$\int_{\rm AdS}\int_{\Phi^4} x_{34}^{2c} x_{24}^{2b} x_{14}^{2a} \tilde{K}_{\Delta+c}(z,x;x_1)\tilde{K}_{\Delta+b}(z,x;x_2)\tilde{K}_{\Delta+a}(z,x;x_3)\tilde{K}_{\Delta+k}(z,x;x_4) \tag{4.30}$$

which grows slower than $\frac{1}{\sigma^{k-1}}$ since $a, b, c > 0$. Hence, for even k, the leading Regge contribution always comes from the first term of (2.3). This is sufficient to conclude that these other higher derivative 4-ϕ interactions do not affect any of the bounds obtained in this paper.

5 Flat space limit: massless & massive scalars

In this section, we compare bounds from the previous section with bounds obtained by studying flat space scattering amplitudes. We again start with the effective action (2.1) without dynamical gravity $G_N = 0$. For simplicity we take the three-point coupling $\alpha_3 = 0$,

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so that the tree level 4-point scattering amplitude does not have poles at m^2. The forward limit ($t = 0$) of the tree level 4-point scattering amplitude associated with the effective action (2.1) is given by

$$\mathcal{A}(s, t = 0) = 8 \sum_{I=0}^{\infty} \mu_{2I} s^{2I} \,. \tag{5.1}$$

At this point we make four assumptions: (1) the forward amplitude is bounded for large s:

$$\mathcal{A}(s, t = 0) < |s|^2 \,, \tag{5.2}$$

(2) the amplitude is analytic in the upper-half complex s-plane, (3) the amplitude obeys partial-wave unitarity implying Im $\mathcal{A}(s, t = 0) > 0$ for real s, (4) the amplitude is crossing symmetric.

These are the key assumptions which allow us to write a dispersive sum-rule for μ_{2I}. In particular, repeating the argument of [1], we can write

$$\mu_k = \frac{1}{4\pi} \int_{M_*^2}^{\infty} ds \frac{\text{Im } \mathcal{A}(s, t = 0)}{s^{k+1}} > 0 \,, \tag{5.3}$$

for all even $k \geq 2$, where M_* is the cut-off scale at which Im$\mathcal{A}(s = M_*^2, t = 0)$ becomes non-zero. The cut-off scale $M_* \propto M$, however, the two scales can be different in general by some order one proportionality constant.

From (5.3), we can also derive a monotonicity and a log-convexity conditions for even $k \geq 2$:

$$\frac{\mu_{k+2}}{\mu_k} \leq \frac{1}{M_*^4} \,, \qquad \frac{\mu_{k+2}^2}{\mu_k \mu_{k+4}} \leq 1 \,. \tag{5.4}$$

The second inequality can be used to derive a global log-convexity condition (1.5) for even μ_k.

Thus, under the above assumptions we showed that *the tree level amplitude, in the forward limit, has a polynomial expansion in* s^2 *with coefficients obeying (i) positivity, (ii) monotonicity, and (iii) log-convexity conditions.* At first sight, these conditions seem to be stronger than the flat space limit of the AdS conditions (1.3)–(1.5). For the remainder of this section we will address whether, and in what sense, the above bounds are related to the AdS bounds.

5.1 Massive scalars

An important feature of our AdS bounds is that they differ significantly for massive and massless scalars, especially when we take the flat space limit. First, we consider the massive case $0 < m \ll M$, where M is the cut-off scale defined in the previous section. We take the flat space limit by $MR_{\text{AdS}} \gg 1$, keeping m fixed. So, in this limit $\Delta \approx mR_{\text{AdS}} \gg 1$ and hence

$$n_k(\Delta) = 1 \tag{5.5}$$

for all $k \ll mR_{\text{AdS}}$. Therefore, in this case, for all $m > 0$ and $D \geq 4$ the AdS conditions (1.3)–(1.5) are identical to conditions (5.3) and (5.4) that were derived from the flat space sum-rule, provided we identify $M = M_*$. This provides compelling evidence in favor of the assumptions that were used to derive the sum-rule (5.3) for massive external scalars. Moreover, for massive scalars, as we will explain in the next section, conditions (5.3) and (5.4) remain valid for even $k \geq 4$ even when there is dynamical gravity. This is rather non-trivial since the validity of the Regge boundedness condition (5.2) is not obvious in the presence of the graviton.

5.2 Massless scalars

The situation is a lot more subtle for massless external scalars. We can start with $m = 0$ and then take the large R_{AdS} limit. One can also take a massless limit in which we first take the large R_{AdS} limit (with fixed m) and then $m \to 0$. Clearly, our bounds are different in these two limits. In the latter case, we again obtain (5.5). Hence, conditions (5.3) and (5.4) still hold. For example, in this case (4.27) becomes

$$\frac{g_4^2}{g_2 g_6} \leq 1\,. \tag{5.6}$$

On the other hand, if we take $m = 0$ first, we obtain bounds from section 4.3. In particular, now we have conditions (1.3)–(1.5) with $n_k(\Delta = D-1) \equiv n_k^{(0)}$ given by (4.24). These bounds are weaker than the conditions (5.3) and (5.4).

Furthermore, the massless limit can also be taken in a more general way: $R_{\text{AdS}} \to \infty$, $m \to 0$ with $\Delta =$ fixed. In this case, we again obtain the weaker set of bounds (1.3)–(1.5) with $n_k(\Delta)$ given by (4.17). Therefore, depending on how we take the massless limit (or equivalently the value of Δ), we obtain a different set of constraints. We recover the flat space conditions (5.3) and (5.4) only for $\Delta \gg 1$. This suggests that in general some of the assumptions that were used to derive the sum-rule (5.3) are not valid for massless scalars. This is perhaps not surprising since the Regge boundedness condition (5.2) can break down in the presence of massless states.

Nonetheless, we can still provide a general condition on the tree level amplitude of massless scalars which does not require any assumption other than the usual CFT-axioms. *The tree level amplitude of massless scalars, in the forward limit, has a polynomial expansion in s^2*

$$\mathcal{A}(s, t=0) = \sum_{I=0}^{\infty} \frac{c_{2I}}{n_{2I}(\Delta)} \left(\frac{s}{M^2}\right)^{2I} \tag{5.7}$$

with coefficients c_{2I} obeying (i) positivity, (ii) monotonicity, and (iii) log-convexity conditions (1.3)–(1.5) for $I \geq 1$. Of course, M and Δ are theory dependent but fixed for a specific four-point amplitude.[30] Whereas, the numerical coefficient $n_{2I}(\Delta)$ is theory independent and given by (4.17).

In the presence of gravity, $\mathcal{A}(s,t)$ has a pole at $t = 0$. However, $\mathcal{A}(s, t \to 0)$ still must satisfy the above condition for $I \geq 2$.

[30]The cut-off scale $M = XM_*$ is proportional to the mass M_* of the lightest particle exchanged. The proportionality factor $X \sim \mathcal{O}(1)$, however, it may differ from 1 in general. The parameter Δ should be regarded as a measure of the breakdown of the Regge boundedness condition (5.2).

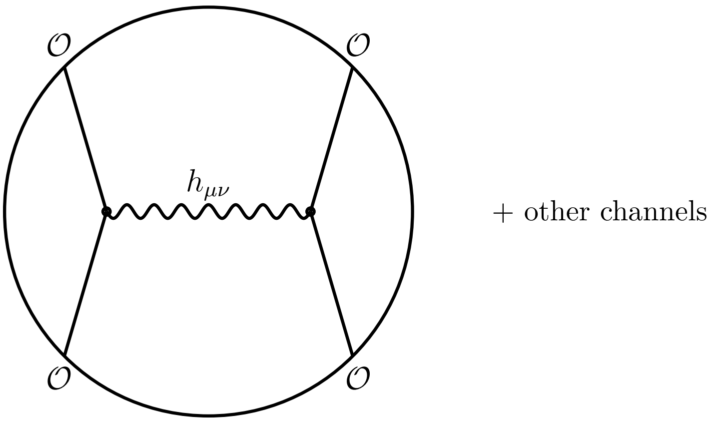

Figure 5. The leading gravitational contribution to the Lorentzian correlator (4.1) comes from the Witten diagram with a single graviton exchange.

6 Scalar EFT with gravity

We now discuss the effects of gravity on the bounds on the EFT (4.22) by turning on $G_N \neq 0$:

$$S[\phi, g] = \frac{1}{16\pi G_N}\int d^D x\sqrt{-g}\left(R + \frac{(D-1)(D-2)}{R_{\text{AdS}}^2}\right) + S[\phi]\,, \tag{6.1}$$

where, $S[\phi]$ is given by (4.22). We analyze the EFT at tree level, so we assume that the theory is weakly coupled as described by (2.2). Now the central charge of the dual CFT is large $c_T \gg \Delta_{\text{gap}} \gg 1$ but finite. We again compute the Lorentzian correlator (4.1) in the Regge limit (3.5), where operator $\mathcal{O}$ is dual to the scalar field ϕ. The leading contribution to the connected part of the correlator $G(\eta, \sigma)$ comes from Witten diagrams 4 plus the graviton exchange Witten diagram as shown in figure 5.

In the Regge limit (3.5), contribution from the channel $\mathcal{O}(\boldsymbol{\rho})\mathcal{O}(-\boldsymbol{\rho}) \to h_{\mu\nu} \to \mathcal{O}(\mathbf{1})\mathcal{O}(-\mathbf{1})$ grows as $1/\sigma$. The other channels do not contribute at all to the Regge growth. So, in the presence of gravity $c_L(\eta)$ for $L > 2$ remains unaffected. On the other hand, $c_2(\eta)$ receives a contribution from gravity. In particular, the gravitational contribution to $c_2(\eta)$ can be obtained from [61, 62, 94]

$$c_2(\eta)|_{\text{gravity}} = \frac{\pi G_N}{R_{\text{AdS}}^{D-2}}\kappa_\Delta \tilde{F}_g(\eta)\,, \tag{6.2}$$

where, the numerical factor κ_Δ is defined in (4.11). The function $\tilde{F}_g(\eta)$ is given by an integral of the harmonic functions $\Omega_{i\nu}$ in the hyperbolic space (see (D.18))

$$\tilde{F}_g(\eta) = \frac{1}{\sqrt{\eta}}\int_{-\infty}^{\infty} d\nu \frac{\Gamma\left(\frac{2\Delta+2-d/2+i\nu}{2}\right)^2\Gamma\left(\frac{2\Delta+2-d/2-i\nu}{2}\right)^2}{\nu^2+\left(\frac{d}{2}\right)^2}\Omega_{i\nu}\left(-\frac{1}{2}\log(\eta)\right) \tag{6.3}$$

where $D = d+1$. Therefore, the full $c_2(\eta)$ is given by

$$c_2(\eta) = \frac{\kappa_\Delta}{R_{\text{AdS}}^D}F_{2\Delta+2}(\eta)\left(\mu\lambda_2 + \pi G_N R_{\text{AdS}}^2\frac{\tilde{F}_g(\eta)}{F_{2\Delta+2}(\eta)}\right) > 0\,, \tag{6.4}$$

where the positivity follows from condition (3.12) for $0 < \eta < 1$. One can check that the optimal bound in this case is obtained in the limit $\eta \to 1$. In this limit, we find that λ_2 is now allowed to have negative values:

$$\lambda_2 > -\frac{\pi G_N R_{\text{AdS}}^2}{\mu} N_D(\Delta)\,, \tag{6.5}$$

where $N_D(\Delta)$ is an $\mathcal{O}(1)$ numerical factor given in appendix F. In particular, for the massless case we find

$$N_4 = 0.1775\,,\quad N_5 = 0.0882\,,\quad N_6 = 0.0525\,,\quad N_7 = 0.0348\,,\quad N_8 = 0.0247\,,\quad \cdots\,. \tag{6.6}$$

Note that the bound (6.5) cannot be saturated in a way which is consistent with the sum-rule (3.11).

So, we conclude that in the presence of gravity λ_2 is not required to be positive. This is consistent with the results of [47]. On the other hand, the bounds (i)–(iii) are still valid for all even $k \geq 4$. Before we proceed, we must note that λ_2, if negative, cannot be arbitrarily large even in the large R_{AdS} limit. To see that, we write (6.5) as:

$$\mu\lambda_2 M^D > -\frac{\Delta_{\text{gap}}^D}{c_T}\mathcal{O}(1)\,, \tag{6.7}$$

where c_T is the CFT central charge. Validity of our analysis requires that we take $c_T \to \infty$ first and then $\Delta_{\text{gap}} \to \infty$. This implies that we should use caution when we take the flat space limit. In particular, we must take R_{AdS} to be large such that $\frac{1}{\sqrt{G_N M^{D-2}}} \gg M R_{\text{AdS}} \gg 1$. Hence, the right hand side of the above expression remains small even in the flat space limit.

We now analyze the bound (4.16) in the presence of gravity. Since, $c_k(\eta)$ for $k \geq 4$ remains unchanged, we only need to analyze the $k = 2$ case. We assume that $\frac{G_N}{\mu M^2} \sim \mathcal{O}(1)$ so that the gravity effects are significant. The optimal bound, in the presence of gravity, is now obtained at the limit $\eta \to 1$ yielding

$$0 \leq \lambda_4 \leq \tilde{N}_D(\Delta)\left(\lambda_2 + \frac{\pi G_N R_{\text{AdS}}^2}{\mu} N_D(\Delta)\right)\,, \tag{6.8}$$

where $\tilde{N}_D(\Delta) > 1$ is given in appendix F. One may wish to recover the $G_N = 0$ result (ii) from the above inequality. The above bound is still valid when $G_N = 0$, however, it is not optimal. This is simply because of the order of limits. As we take $\frac{G_N}{\mu M^2} \to 0$, the optimal bound is obtained for a value of η which is close to zero and hence the upper bound of λ_4 approaches λ_2. The correction term from finite but small $\frac{G_N}{\mu M^2}$ now can be computed numerically, though we will have to leave this for the future.

Finally, we focus on the log-convexity condition $c_4(\eta)^2 \leq c_2(\eta)c_6(\eta)$ in the presence of gravity. We again assume that there is no parametric separation between G_N and μ in units of M: $\frac{G_N}{\mu M^2} \sim \mathcal{O}(1)$. Repeating the argument of the preceding section, however for $\eta \to 1$, we obtain

$$\frac{\lambda_4^2}{\lambda_6} \leq \frac{\tilde{N}_D(\Delta)}{\tilde{N}_D(\Delta+1)}\left(\lambda_2 + \frac{\pi G_N R_{\text{AdS}}^2}{\mu} N_D(\Delta)\right) \tag{6.9}$$

where, N-coefficients are given in appendix F. One can check that the pre-factor $\frac{\tilde{N}_D(\Delta)}{\tilde{N}_D(\Delta+1)} > 1$ and asymptotes to 1 for large Δ. Interestingly the ratio $\frac{\tilde{N}_D(\Delta)}{\tilde{N}_D(\Delta+1)}$ is independent of the spacetime dimension D.

6.1 Summary of bounds

Let us now summarize the results of this section. The EFT (6.1) has a well behaved CFT dual with $\Delta_{\text{gap}} \gg 1$, if and only the EFT, with $\mu \geq 0$, has the following properties ($D \geq 4$):

1. Conditions (i)–(iii) are satisfied for all even $k \geq 4$,
2. λ_2 is bounded from below by the relation (6.5),
3. λ_4 is bounded from above by the relation (6.8),
4. λ_2, λ_4, and λ_6 must satisfy the convexity condition (6.9).

Therefore, presence of gravity makes the EFT bounds weaker.

6.2 Flat space limit

Finally, let us make a few comments about the flat space limit of the above bounds. Clearly, the constraint (1) persists even in the flat space limit. These constraints, in flat space, are consistent with the bounds of [47].[31]

On the other hand, constraints (2)–(4) do not produce precise bounds for the flat space EFT. For example, consider the condition (2) in the flat space limit. As discussed before, the flat space limit should be taken such that $\frac{1}{\sqrt{G_N M^{D-2}}} \gg M R_{\text{AdS}} \gg 1$. Therefore, the condition (2), in the flat space limit, suggests that $\mu \lambda_2 M^D > -\varepsilon$, where ε is some small number. This is certainly consistent with the results of [47] (for $D \geq 5$), however, we do not have a precise definition of ε. This perhaps suggests that ε is theory dependent. Nevertheless, the important point is that ε is strictly positive.

7 Multiple scalar fields in AdS

In this section, we analyze EFTs of multiple scalars in AdS. The main motivation for this section is to demonstrate that there are additional constraints from the same CFT consistency conditions that must be satisfied when there are multiple fields. As we showed earlier, odd k interactions with a single scalar field are not constrained from our CFT analysis. However, in this section we will consider higher derivative interactions with multiple scalar fields to demonstrate that some odd k interactions are constrained from the CFT consistency conditions of section 3.2.

Furthermore, for multiple fields there are interference effects that are also constrained by the same CFT consistency conditions. These interference effects have been utilized in [72] to derive non-linear bounds on the dilaton-axion effective action associated with 4D

[31] It would be interesting to extend our analysis and compare with more recent results (such as [95]) on all order higher derivative couplings in different string theories.

RG flows with global symmetry breaking. In this section, we will derive such interference bounds in a systematic way.

For the purpose of demonstration of the general idea, we choose a simple theory: an EFT of two scalar fields (with the same mass) in AdS with $\mathbb{Z}_2$ symmetry and without gravity $G_N = 0$. We follow the convention of section 4.2 and start with the effective action[32]

$$\begin{aligned} S[\phi_1,\phi_2] &= \frac{1}{2}\int d^Dx\sqrt{-g}\left(-g^{\mu\nu}\nabla_\mu\phi_1\nabla_\nu\phi_1 - g^{\mu\nu}\nabla_\mu\phi_2\nabla_\nu\phi_2 - m^2(\phi_1^2+\phi_2^2)\right) \\ &+ \frac{\mu}{2}\sum_{k=2}^{\infty}\int \frac{d^Dx\sqrt{-g}}{n_k(\Delta)M^{2(k-2)}}\left(\lambda_k^{(1)}\phi_1^2\Box^k\phi_1^2 + \lambda_k^{(2)}\phi_2^2\Box^k\phi_2^2 + g_k\phi_1^2\Box^k\phi_2^2 + \tilde{g}_k\phi_1\phi_2\Box^k\phi_1\phi_2\right) \\ &+ \cdots , \end{aligned} \tag{7.1}$$

where $M = \Delta_{\text{gap}}/R_{\text{AdS}}$ is the scale of new physics, $\mu \geq 0$, and the numerical factor $n_k(\Delta)$ is defined in (4.17).[33] Note that λ and g coefficients are dimensionless. The argument of the previous sections still holds implying that both $\lambda_k^{(1)}$ and $\lambda_k^{(2)}$ must satisfy conditions (i)–(iii) independently. However, as we will show in this section, there are additional non-trivial constraints that involve g_k and $\tilde{g}_k$ couplings.

The AdS theory (7.1) is dual to an interacting CFT in $d = D-1$ dimensions. The bulk fields ϕ_1 and ϕ_2 are dual to two scalar operators $\mathcal{O}_1$ and $\mathcal{O}_2$ respectively, with dimensions $m^2R_{\text{AdS}}^2 = \Delta(\Delta-d)$. The two point functions are given by (2.6). Let us now consider a general four-point CFT correlator

$$G(\eta,\sigma) = \frac{\langle \mathcal{O}_B(\mathbf{1})\mathcal{O}_A(\boldsymbol{\rho})\mathcal{O}_A^\dagger(-\boldsymbol{\rho})\mathcal{O}_B^\dagger(-\mathbf{1})\rangle}{\langle \mathcal{O}_B(\mathbf{1})\mathcal{O}_B^\dagger(-\mathbf{1})\rangle\langle \mathcal{O}_A(\boldsymbol{\rho})\mathcal{O}_A^\dagger(-\boldsymbol{\rho})\rangle} \tag{7.2}$$

in the Regge limit (3.5). The operators are defined as

$$\mathcal{O}_A = \mathcal{O}_1 + a\mathcal{O}_2 , \qquad \mathcal{O}_B = \mathcal{O}_1 + b\mathcal{O}_2 , \tag{7.3}$$

where, a and b are arbitrary complex numbers. We repeat the calculation of section 4 and obtain an expression for $c_L(\eta)$ in the limit of large MR_{AdS} (with mR_{AdS} fixed):

$$\begin{aligned} c_L(\eta) &= \frac{\kappa_\Delta \mu F_{2\Delta+L}(\eta)}{n_L(\Delta)\Delta_{\text{gap}}^{D+2L-4}(1+|a|)^2(1+|b|)^2} \\ &\times\left(\lambda_L^{(1)} + \lambda_L^{(2)}|a|^2|b|^2 + \frac{1}{4}\tilde{g}_L\left(|a|^2+|b|^2\right) + \frac{1}{8}(2g_L+\tilde{g}_L)(a+a^*)(b+b^*)\right) \end{aligned} \tag{7.4}$$

for even $L \geq 2$, where κ_Δ is a positive coefficient independent of L, as defined in equation (4.11). On the other hand, $c_L(\eta)$ for odd L is non-zero. In particular for odd $L \geq 3$ we obtain

$$c_L(\eta) = \frac{\kappa_\Delta \mu F_{2\Delta+L}(\eta)}{n_L(\Delta)\Delta_{\text{gap}}^{D+2L-4}(1+|a|)^2(1+|b|)^2}\frac{1}{8}(\tilde{g}_L - 2g_L)(a-a^*)(b-b^*) . \tag{7.5}$$

[32] Note that we are ignoring $k = 0, 1$-interactions. These interactions as well as any other interaction that cannot be written in the form (7.1), if present, will not affect the bounds obtained in this section.

[33] Let us recall that in our convention $n_2(\Delta) = n_4(\Delta) = 1$ for all Δ and D. Moreover, for $\Delta_{\text{gap}} \gg mR_{\text{AdS}} \gg k$, we have $n_k = 1$.

7.1 Bounds

We are now in a position to derive bounds by utilizing the CFT consistency conditions of section 3.2. All CFT conditions of section 3.2 apply to $c_L(\eta)$ obtained in this section for $0 < \eta < 1$ and all choices of a and b. As we have discussed before, it is sufficient to derive constraints at the limit $\eta \to 0$. However, now the bounds will also depend on the particular choice of a and b.

Our CFT setup, as we discussed before, is probing local high energy scattering deep in the bulk. Since the local high energy scattering is insensitive to the spacetime curvature, the AdS bounds of this section remain valid even in the flat space limit.

7.1.1 Positivity for even k

The condition (3.12) now imposes

$$\lambda_k^{(1)} > 0\,, \qquad \lambda_k^{(2)} > 0\,, \qquad \tilde{g}_k > 0 \tag{7.6}$$

for even $k \geq 2$ generalizing the bound (i). Furthermore, now we can derive a non-linear interference bound by choosing a and b that minimize (7.4), yielding

$$|\tilde{g}_k + 2g_k| \leq 4\sqrt{\lambda_k^{(1)}\lambda_k^{(2)}} + \tilde{g}_k \tag{7.7}$$

for all even $k \geq 2$. Note that the above bounds are consistent with bounds obtained in [72] on the dilaton-axion effective action.

7.1.2 Monotonicity for even k

The condition (3.13) leads to the following monotonicity conditions:

$$\lambda_k^{(1)} \geq \lambda_{k+2}^{(1)}\,, \qquad \lambda_k^{(2)} \geq \lambda_{k+2}^{(2)}\,, \qquad \tilde{g}_k \geq \tilde{g}_{k+2} \tag{7.8}$$

for all even $k \geq 2$ generalizing the bound (ii). We can again derive a non-linear interference bound by optimizing with respect to a and b:

$$|(\tilde{g}_k - \tilde{g}_{k+2}) + 2(g_k - g_{k+2})| \leq 4\sqrt{(\lambda_k^{(1)} - \lambda_{k+2}^{(1)})(\lambda_k^{(2)} - \lambda_{k+2}^{(2)})} + (\tilde{g}_k - \tilde{g}_{k+2}) \tag{7.9}$$

for all even $k \geq 2$.

7.1.3 Boundedness for odd k

The condition (3.13) now imposes bounds also on odd k coupling constants. By optimizing with respect to a and b, we find that

$$|\tilde{g}_k - 2g_k| \leq 4\sqrt{\lambda_{k-1}^{(1)}\lambda_{k-1}^{(2)}} + \tilde{g}_{k-1} \tag{7.10}$$

for all odd $k \geq 3$. Note that there is a particular combination of interactions for any odd k which is not bounded from our argument.

7.1.4 Log-convexity for even k

To begin with, we can utilize (3.15) for different limits of a and b to obtain local log-convexity conditions: $\lambda^{(1)}_{k+2} \leq \sqrt{\lambda^{(1)}_k \lambda^{(1)}_{k+4}}$, $\lambda^{(2)}_{k+2} \leq \sqrt{\lambda^{(2)}_k \lambda^{(2)}_{k+4}}$, and $\tilde{g}_{k+2} \leq \sqrt{\tilde{g}_k \tilde{g}_{k+4}}$ for all even $k \geq 2$. These local conditions lead to the global log-convexity condition (iii) for $\lambda^{(1)}_k$, $\lambda^{(2)}_k$, and $\tilde{g}_k$ individually for even $k \geq 2$. Furthermore, there is again a more general local log-convexity condition:

$$C_{k+2}(a,b)^2 \leq C_k(a,b) C_{k+4}(a,b)\,, \qquad \text{even } k \geq 2 \tag{7.11}$$

for all real a and b, where

$$C_k(a,b) = 4\lambda^{(1)}_k + 4\lambda^{(2)}_k a^2 b^2 + \tilde{g}_k\,(a+b)^2 + 4 g_k ab > 0\,. \tag{7.12}$$

Of course, we can again write a global log-convexity condition for $C_k(a,b)$ as before.

Note that the strongest bound can be obtained by optimizing (7.11) with respect to a and b. The actual expression is not very illuminating and hence we will not transcribe it here.

7.1.5 Log-convexity for odd k

Odd k-interactions also obey a local (but not global) log-convexity condition. This can be obtained by using (3.16):

$$(\tilde{g}_k - 2g_k)^2 \leq \frac{1}{y^2}\left(2\lambda^{(1)}_{k-1} + 2\lambda^{(2)}_{k-1} y^2 + \tilde{g}_{k-1} y\right)\left(2\lambda^{(1)}_{k+1} + 2\lambda^{(2)}_{k+1} y^2 + \tilde{g}_{k+1} y\right) \tag{7.13}$$

for all odd $k \geq 3$ and $0 < y < \infty$. Of course, the optimal bound is obtained by minimizing the right hand side with respect to y.

Finally, we wish to note that now the fields can couple to a massive or a massless gauge field. However, that will not alter equations (7.4) or (7.5) and hence the bounds remain unchanged. On the other hand, when we couple the theory (7.1) to gravity, as we discussed in the previous section, it will contribute to $c_2(\eta)$. So, all bounds for $k \geq 4$ (even or odd) are valid even when $G_N \neq 0$.

7.2 Application: complex scalar field

Let us now consider a complex scalar field

$$S = \frac{1}{2}\int d^D x \sqrt{-g}\left(-\nabla_\mu \phi^\dagger \nabla^\mu \phi + m^2 \phi\phi^\dagger + \sum_{k=0}^{\infty} \frac{\mu}{n_k(\Delta) M^{2(k-2)}}\left(\alpha_k \phi^2 \Box^k {\phi^\dagger}^2 + \beta_k \phi\phi^\dagger \Box^k \phi\phi^\dagger\right)\right). \tag{7.14}$$

Results of this section apply to this EFT as well. In particular, bounds on this EFT can be obtained easily once we identify

$$\lambda^{(1)}_k = \lambda^{(2)}_k = \alpha_k + \beta_k\,, \qquad g_k = 2(\beta_k - \alpha_k)\,, \qquad \tilde{g}_k = 4\alpha_k\,. \tag{7.15}$$

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8 Conclusions & comments

In this paper we addressed the question of what EFTs in AdS_D cannot be embedded into a UV theory that is dual to a CFT_{D-1} obeying the usual CFT axioms. We considered EFTs of scalar fields in AdS spacetime of large radius and derived precise constraints (1.3)–(1.5) on the coupling constants of higher derivative interactions $\phi^2\Box^k\phi^2$ from the dual CFT. Our derivation of the bounds does not make any assumptions about the dual CFT beyond the well established conformal bootstrap axioms. Furthermore, we showed that inclusion of gravity only affects constraints involving the $\phi^2\Box^2\phi^2$ interaction which now can have a negative coupling constant even in $D = 4$. It is unclear whether this fact survives in the exact flat space limit. It will be interesting to explore this further since positivity of this interaction is essential in the proof of the 4D a-theorem.

Our CFT setup was a Lorentzian four-point correlator in the Regge limit which was designed to probe local high energy scattering deep in the AdS. We utilized the fact that the growth of this CFT Regge correlator is highly constrained from the argument of [88]. Conceptually, bounds obtained in this paper are closely related to the CFT Nachtmann theorem of [88, 96]. In fact, the CFT Nachtmann theorem was derived in [88] by starting from the same four-point correlator, however, in the Lorentzian lightcone limit ($\eta \to 0$, then $\sigma \to 0$). Moreover, the condition (1.3) can be derived from the CFT Nachtmann theorem (with some caveat, as we explain later) once we identify anomalous dimensions $\gamma_{n,\ell}$ of double-trace operators $[\mathcal{O}\mathcal{O}]_{n,\ell}$ are related to λ_ℓ (for even ℓ) as follows [75, 97]

$$\gamma_{n,\ell} \propto -\lambda_\ell \,. \tag{8.1}$$

On the other hand, constraints (1.4)–(1.5) are strictly stronger than what one obtains from the Nachtmann theorem.[34] Furthermore, one should exercise caution while applying the Nachtmann theorem to an “effective” CFT which is defined order by order in perturbation theory. Of course, even for such a CFT the Nachtmann theorem of [88, 96] does hold, however, identifying families of minimal twist operators can be subtle. It is particularly complicated when the family of minimal twist operators consists of different set of operators at different orders in perturbation theory. We emphasize that for “effective” CFTs constraints obtained from the CFT Regge limit are more reliable since they follow directly from the CFT sum-rule (3.11).

We have analyzed the EFT (1.2) at tree level. We note that the CFT consistency conditions of [88] that we have utilized in this paper apply even when we include corrections from EFT loops. In fact, the CFT consistency conditions of [88] (see section 6) hold even for arbitrary external CFT operators with or without spins (and not necessarily local or primary). So, it is a straightforward exercise to extend our analysis to derive bounds on the graviton four-point scattering amplitude in AdS by studying Regge correlators of the stress tensor operator in the dual CFT. It would be interesting to compare such bounds with similar classical bounds of [98] from “Classical Regge Growth” (CRG) conjecture and EFT bounds of [99, 100] from unitarity and crossing. We will have to leave this question for the future.

[34] This is a direct consequence of the fact that the order of limits $\eta, \sigma \to 0$ is non-trivial.

Finally, we end with some general comments about the swampland bounds on EFTs in flat space which are obtained by using various properties of 4-point scattering amplitudes. We mainly focus on two types of flat space arguments: (i) based on dispersive sum rules, (ii) based on positivity of the eikonal phase-shift. The first type of arguments, as explained in the introduction, lead to precise bounds, however, require some assumption about the Regge boundedness of the 4-point amplitude. Whereas, positivity of the eikonal phase-shift seems to be a more rigorous condition [101][35] which leads to non-trivial constraints [39, 67, 68, 101, 105–110], however, these constraints in some sense are parametric in nature. On the other hand, by now it is known that both types of bounds can be obtained in AdS from the same CFT sum-rule (3.11). So, roughly speaking our CFT Regge correlator (3.2) is the AdS analogous of the flat space finite impact parameter scattering amplitude $\mathcal{A}(s,\vec{b})$ of [20], since both capture two types of constraints described above. It would be interesting to derive the full set of constraints of [47] by viewing a flat space EFT as the flat space limit of the EFT in AdS.

More generally, it would be nice to unify the flat space bounds and the AdS bounds in a more systematic way. This can be achieved at the level of individual bounds, however a more useful goal would be to rigorously derive the Regge boundedness condition (and the closely related CRG condition of [92, 98]) of the flat space amplitude directly from the CFT axioms by taking the flat space limit.[36] It is tempting to translate the results of this paper in to a Regge boundedness condition for the flat space finite impact parameter scattering amplitude $\mathcal{A}(s,\vec{b})$ of [47] (or some variation of it) for arbitrary external states. In particular, CFT conditions of section 3.2 suggest

> *Any finite impact parameter scattering amplitude $\mathcal{A}(s,\vec{b})$ for large s cannot grow faster than s^2 within any range of s.*

In other words, there can be terms in $\mathcal{A}(s,\vec{b})$ that grow as $s^3, s^4, \cdots$ for large s but none of them can dominate within any range of s. Classical version of this statement is very similar to the CRG conjecture of [98], however, it is not equivalent since $\mathcal{A}(s,\vec{b})$ is in the impact parameter space. Note that this Regge boundedness condition, even if true, is weaker than what is required in [47]. Nevertheless, it is of importance to have a rigorous proof of the above Regge boundedness condition or some stronger version of it.

Acknowledgments

It is my pleasure to thank Simon Caron-Huot, Subham Dutta Chowdhury, Tom Hartman, Jonathan Heckman, Jared Kaplan, Suman Kundu, Shiraz Minwalla, and Hao Zhang for several helpful discussions over the years. I thank Dalimil Mazac, Joao Penedones, Leonardo Rastelli, and David Simmons-Duffin for helpful correspondence. I would also like to thank Madhuparna Pal for all the support during the Covid pandemic. I was supported in part by the Simons collaboration Grant on the Non-Perturbative Bootstrap.

[35] For some subtleties see [102–104].

[36] Note that the flat space limit is subtle when gravity is dynamical.

A Rindler positivity from conformal bootstrap

In this appendix we will show that Rindler positivity, for scalar external operators, follows from OPE unitarity and crossing symmetry. Consider the Euclidean correlator $(0 < \rho, \bar{\rho} < 1)$

$$\begin{aligned}\langle O_2(-\boldsymbol{\rho})O_1(\boldsymbol{\rho})O_1^\dagger(\mathbf{1})O_2^\dagger(-\mathbf{1})\rangle_E &= \frac{1}{(16\rho\bar{\rho})^{\frac{\Delta_1+\Delta_2}{2}}}\left(\frac{(1-\rho)(1-\bar{\rho})}{(1+\rho)(1+\bar{\rho})}\right)^{\Delta_{12}} \\ &\quad\times\sum_p c_{O_2O_1p}c_{O_2^\dagger O_1^\dagger p}(-1)^\ell g_{\Delta,\ell}^{\Delta_{21},\Delta_{12}}(z,\bar{z})\end{aligned} \tag{A.1}$$

where $\Delta_{12} = \Delta_1 - \Delta_2$ and cross-ratios are

$$z = \frac{4\rho}{(1+\rho)^2}\,, \qquad \bar{z} = \frac{4\bar{\rho}}{(1+\bar{\rho})^2}\,. \tag{A.2}$$

Unitarity ensures that $c_{O_2O_1p}c_{O_2^\dagger O_1^\dagger p} > 0$. Moreover, positivity of the conformal block expansion in $\rho, \bar{\rho}$ now implies

$$\langle O_2(-\boldsymbol{\rho})O_1(\boldsymbol{\rho})O_1^\dagger(\mathbf{1})O_2^\dagger(-\mathbf{1})\rangle_E = \frac{1}{(16\rho\bar{\rho})^{\frac{\Delta_1+\Delta_2}{2}}}\sum_{h,\bar{h}} b_{h,\bar{h}}\rho^h\bar{\rho}^{\bar{h}}\,, \qquad b_{h,\bar{h}} \geq 0 \tag{A.3}$$

where $h = \frac{1}{2}(\Delta \pm \ell)$ and $\bar{h} = \frac{1}{2}(\Delta \mp \ell)$. The sum is over all operators both primaries and their descendants. Note that $b_{h,\bar{h}} \geq 0$ also follows from reflection positivity, as shown in [58]. The above facts immediately implies that for $1 > z, \bar{z} > 0$

$$\begin{aligned}&\sum_p c_{O_2O_1p}c_{O_2^\dagger O_1^\dagger p}(-1)^\ell g_{\Delta,\ell}^{\Delta_{21},\Delta_{12}}(z,\bar{z}) \\ &\quad = ((1-z)(1-\bar{z}))^{\Delta_{21}/2}\sum_{h,\bar{h}} b_{h,\bar{h}}\left(\frac{1-\sqrt{1-z}}{1+\sqrt{1-z}}\right)^h\left(\frac{1-\sqrt{1-\bar{z}}}{1+\sqrt{1-\bar{z}}}\right)^{\bar{h}}\,.\end{aligned} \tag{A.4}$$

Rindler positivity. We now consider the correlator G of equation (3.2), however, in the Euclidean regime $(0 < \rho, \bar{\rho} < 1)$. In the direct channel expansion

$$G_E = \sum_p c_{O_1O_1^\dagger p}c_{O_2^\dagger O_2 p}g_{\Delta,\ell}^{0,0}(z,\bar{z}) \tag{A.5}$$

with

$$z = \frac{4\rho}{(1+\rho)^2}\,, \qquad \bar{z} = \frac{4\bar{\rho}}{(1+\bar{\rho})^2}\,. \tag{A.6}$$

The subscript E is there to remind ourselves that we are in the Euclidean regime. Positivity of this correlator is not obvious from the direct channel expansion. So, we expand in the crossed channel

$$G_E = \frac{(16\rho\bar{\rho})^{\Delta_2}}{((1-\rho)(1-\bar{\rho}))^{\Delta_1+\Delta_2}}\left(\frac{1}{(1+\rho)(1+\bar{\rho})}\right)^{\Delta_{21}}\sum_p c_{O_2O_1p}c_{O_2^\dagger O_1^\dagger p}(-1)^\ell g_{\Delta,\ell}^{\Delta_{21},\Delta_{12}}(z,\bar{z}) \tag{A.7}$$

where now cross-ratios are

$$z = \frac{(1-\rho)^2}{(1+\rho)^2}\,, \qquad \bar{z} = \frac{(1-\bar{\rho})^2}{(1+\bar{\rho})^2}\,. \tag{A.8}$$

Using the positive expansion (A.4), we can write

$$G_E = \frac{(16\rho\bar{\rho})^{\frac{\Delta_1+\Delta_2}{2}}}{((1-\rho)(1-\bar{\rho}))^{\Delta_1+\Delta_2}} \sum_{h,\bar{h}} b_{h,\bar{h}} \left(\frac{1-\sqrt{\rho}}{1+\sqrt{\rho}}\right)^{2h} \left(\frac{1-\sqrt{\bar{\rho}}}{1+\sqrt{\bar{\rho}}}\right)^{2\bar{h}} \tag{A.9}$$

which is positive for $0<\rho,\bar{\rho}<1$.

Lorentzian correlators. Rindler positivity is most useful in the Lorentzian regime $\rho>1$ and $0<\bar{\rho}<1$. So, we now consider this regime where some of the operators are time-like separated and hence operator ordering does matter. The positive ordered correlator G_0, as defined in (3.6), in the Lorentzian regime ($\rho>1$ and $0<\bar{\rho}<1$) is given directly by the Euclidean correlator and hence

$$G_0 = \frac{(16\rho\bar{\rho})^{\frac{\Delta_1+\Delta_2}{2}}}{(\rho(1-1/\rho)(1-\bar{\rho}))^{\Delta_1+\Delta_2}} \sum_{h,\bar{h}} b_{h,\bar{h}} \left(\frac{1-1/\sqrt{\rho}}{1+1/\sqrt{\rho}}\right)^{2h} \left(\frac{1-\sqrt{\bar{\rho}}}{1+\sqrt{\bar{\rho}}}\right)^{2\bar{h}} \geq 0\,. \tag{A.10}$$

This establishes Rindler positivity in the Lorentzian regime.

This leads to the other Lorentzian correlator G, as defined in (3.2) and another distinct Lorentzian correlator that we can define

$$G = \frac{\langle O_2(\mathbf{1})O_1(\boldsymbol{\rho})O_1^\dagger(-\boldsymbol{\rho})O_2^\dagger(-\mathbf{1})\rangle}{\langle O_2(\mathbf{1})O_2^\dagger(-\mathbf{1})\rangle\langle O_1(\boldsymbol{\rho})O_1^\dagger(-\boldsymbol{\rho})\rangle}\,, \qquad \tilde{G} = \frac{\langle O_1(\boldsymbol{\rho})O_2(\mathbf{1})O_2^\dagger(-\mathbf{1})O_1^\dagger(-\boldsymbol{\rho})\rangle}{\langle O_2(\mathbf{1})O_2^\dagger(-\mathbf{1})\rangle\langle O_1(\boldsymbol{\rho})O_1^\dagger(-\boldsymbol{\rho})\rangle}\,. \tag{A.11}$$

These Lorentzian correlators, in the regime $\rho>1$ and $0<\bar{\rho}<1$, are obtained from analytic continuations of the Euclidean correlator

$$\tilde{G} = \frac{(16\rho\bar{\rho})^{\frac{\Delta_1+\Delta_2}{2}}}{(\rho(1-1/\rho)(1-\bar{\rho}))^{\Delta_1+\Delta_2}} \sum_{h,\bar{h}} b_{h,\bar{h}} \left(\frac{1-1/\sqrt{\rho}}{1+1/\sqrt{\rho}}\right)^{2h} \left(\frac{1-\sqrt{\bar{\rho}}}{1+\sqrt{\bar{\rho}}}\right)^{2\bar{h}} e^{i\pi(2h-\Delta_\psi-\Delta_O)} \tag{A.12}$$

and similarly

$$G = \frac{(16\rho\bar{\rho})^{\frac{\Delta_1+\Delta_2}{2}}}{(\rho(1-1/\rho)(1-\bar{\rho}))^{\Delta_1+\Delta_2}} \sum_{h,\bar{h}} b_{h,\bar{h}} \left(\frac{1-1/\sqrt{\rho}}{1+1/\sqrt{\rho}}\right)^{2h} \left(\frac{1-\sqrt{\bar{\rho}}}{1+\sqrt{\bar{\rho}}}\right)^{2\bar{h}} e^{-i\pi(2h-\Delta_\psi-\Delta_O)}\,. \tag{A.13}$$

From the above expansions, we conclude that the Lorentzian correlators G_0, G, and $\tilde{G}$, in the regime $\rho>1$ and $0<\bar{\rho}<1$, obey the following properties:

$$G_0 \geq 0\,, \qquad G = \tilde{G}^*\,, \tag{A.14}$$

$$|G| \leq G_0\,, \qquad |\tilde{G}| \leq G_0\,. \tag{A.15}$$

B A sum-rule by subtracting the identity operator

In this section, we derive a sum-rule similar to (3.11) by subtracting the identity operator from all channels. This discussion is only important when $O_1 = O_2 = O$ in the correlator (3.6) with $\Delta_1 = \Delta_2 = \Delta$. We will restrict to real scalar operators, however, this discussion can be easily generalized for complex scalars.

In this case, we write (A.10) and (A.13) as

$$G_0 = \frac{(16\rho\bar{\rho})^{\Delta}}{\rho^{2\Delta}\left((1-1/\rho)(1-\bar{\rho})\right)^{2\Delta}} \left(1 + \sum_{h,\bar{h}\neq 0} b_{h,\bar{h}} \left(\frac{1-1/\sqrt{\rho}}{1+1/\sqrt{\rho}}\right)^{2h} \left(\frac{1-\sqrt{\bar{\rho}}}{1+\sqrt{\bar{\rho}}}\right)^{2\bar{h}}\right) \tag{B.1}$$

and

$$G = \frac{(16\rho\bar{\rho})^{\Delta} e^{2\pi i\Delta}}{\rho^{2\Delta}\left((1-1/\rho)(1-\bar{\rho})\right)^{2\Delta}} \left(1 + \sum_{h,\bar{h}\neq 0} b_{h,\bar{h}} \left(\frac{1-1/\sqrt{\rho}}{1+1/\sqrt{\rho}}\right)^{2h} \left(\frac{1-\sqrt{\bar{\rho}}}{1+\sqrt{\bar{\rho}}}\right)^{2\bar{h}} e^{-2\pi i h}\right) \tag{B.2}$$

by isolating the contribution from the identity operator. We can compare these correlators with correlators for the CFT which is dual to a free scalar theory in AdS. In this generalized free CFT, the corresponding correlators are

$$G_0^{\text{free}} = 1 + \frac{(16\rho\bar{\rho})^{\Delta}}{\rho^{2\Delta}\left((1-1/\rho)(1-\bar{\rho})\right)^{2\Delta}} + \frac{(16\rho\bar{\rho})^{\Delta}}{\rho^{2\Delta}\left((1+1/\rho)(1+\bar{\rho})\right)^{2\Delta}} \tag{B.3}$$

and

$$G^{\text{free}} = 1 + \frac{(16\rho\bar{\rho})^{\Delta} e^{2\pi i\Delta}}{\rho^{2\Delta}\left((1-1/\rho)(1-\bar{\rho})\right)^{2\Delta}} + \frac{(16\rho\bar{\rho})^{\Delta}}{\rho^{2\Delta}\left((1+1/\rho)(1+\bar{\rho})\right)^{2\Delta}} \,. \tag{B.4}$$

These two correlators of the generalized free theory are different only when Δ is not an integer. We now define subtracted correlators:

$$\delta G_0(\eta,\sigma) = G_0 - G_0^{\text{free}}\,, \qquad \delta G(\eta,\sigma) = G - G^{\text{free}}\,, \tag{B.5}$$

where η and σ are defined in (3.4). Moreover, note that for positive $|\sigma| < 1$ and $0 < \eta < 1$

$$\mathrm{Re}\left(\delta G_0(\eta,\sigma) - \delta G(\eta,\sigma)\right) = \sum_{h,\bar{h}\neq 0} b_{h,\bar{h}} \left(\frac{1-\sqrt{\sigma}}{1+\sqrt{\sigma}}\right)^{2h} \left(\frac{1-\sqrt{\eta\sigma}}{1+\sqrt{\eta\sigma}}\right)^{2\bar{h}} (1-\cos(2\pi h)) \geq 0 \tag{B.6}$$

which follows from $b_{h,\bar{h}} \geq 0$. This positivity is true for all unitary CFTs. For negative $|\sigma| < 1$, the same positivity condition can be derived by starting from $\tilde{G}$ correlator at positive σ.

Now we can perform a contour integral on the complex lower-half σ-plane, as described in [88]. This now yields a modified sum-rule for the expansion (3.10)

$$c_L(\eta) = \frac{1}{\pi}\int_{-R}^{R} d\sigma\; \sigma^{L-2}\mathrm{Re}\left(\delta G_0(\eta,\sigma) - \delta G(\eta,\sigma)\right)\,, \qquad \sigma_* \leq R \ll \eta < 1\,, \tag{B.7}$$

which is a more formal (and precise) version of the sum-rule (3.11).

The above sum-rule has one key advantage. In order to illustrate that we focus on CFTs that are dual to some EFT in AdS. Clearly these subtracted correlators come entirely from the interacting part of the AdS EFT. The possible corrections to the above sum-rule comes from terms

$$(\delta G_0(\eta,\sigma) - \delta G(\eta,\sigma)) \sim (\delta c)\sigma^a \qquad \text{with} \qquad a \geq d \tag{B.8}$$

where δc is obtained entirely from the interacting part of the EFT. Hence, the entire argument of section 3.3 about the correction terms now can be repeated implying that the consistency conditions (3.12), (3.13), (3.15), and (3.16) are valid even when Δ is non-integer.

The observant reader may have noticed that the correlator $\delta G_0(\eta,\sigma)$, in general, is not a well-defined object on the complex lower-half σ-plane. However, we can always define a function $\delta G_0^{(-)}(\eta,\sigma)$ which is analytic on the lower half σ-plane (minus the real line) and has the property $\mathrm{Re}\delta G_0^{(-)}(\eta,\sigma) = \delta G_0(\eta,\sigma)$ on the real line ($\mathrm{Im}\ \sigma \to 0_-$). For example, $\delta G_0(\eta,\sigma)$, in the limit $\sigma \to 0$, has terms like

$$\delta G_0(\eta,\sigma) \sim c_a|\sigma|^a \tag{B.9}$$

with positive a. We can define $\delta G_0^{(-)}(\eta,\sigma)$ as a function on the lower-half σ plane with terms

$$\delta G_0^{(-)}(\eta,\sigma) \sim c_a\left(1 + i\tan\left(\frac{\pi a}{2}\right)\right)\sigma^a \tag{B.10}$$

and derive the sum-rule (B.7) using $\delta G_0^{(-)}(\eta,\sigma)$. Clearly, any additional correction that can occur because of $\delta G_0^{(-)}(\eta,\sigma)$ will also obey (B.8) and hence the sum-rule (B.7) is valid for CFTs dual to any AdS EFT for $D \geq 4$. The sum-rule is valid even for $D = 3$ as long as $0 \leq m^2 \ll M^2$ and the ϕ^3 interaction is absent.

C Correlators of CFTs dual to EFTs in AdS

Derivation of our bounds depends heavily on determining the exact numerical factors. So, we review the computation of correlators in the AdS/CFT correspondence. The tree level Witten diagrams can be obtained from the Euclidean on-shell action:

$$e^{-S_{\text{on-shell}}[\Phi]} = \langle e^{\int \Phi\mathcal{O}}\rangle\,, \tag{C.1}$$

where Φ is the boundary value of the bulk field ϕ with CFT dual $\mathcal{O}$. For simplicity we will work in the Euclidean signature with the metric

$$ds^2 = \frac{dz^2 + \delta_{\mu\nu}dx^\mu dx^\nu}{z^2}\,. \tag{C.2}$$

We start with a single scalar field in AdS:

$$S = \frac{1}{2}\int d^{d+1}x\sqrt{g}\left[g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + m^2\phi^2\right] \tag{C.3}$$

which leads to the equation of motion

$$(\Box - m^2)\phi = 0\,. \tag{C.4}$$

Bulk-to-boundary propagator. This has the solution

$$\phi(z,x) = C_\Delta \int d^d x' \frac{z^\Delta}{(z^2+|x-x'|^2)^\Delta}\Phi(x') \equiv \int d^d x' K_\Delta(z,x;x')\Phi(x') \tag{C.5}$$

with $m^2 = \Delta(\Delta-d)$ and

$$C_\Delta = \frac{\Gamma[\Delta]}{\pi^{d/2}\Gamma[\Delta-d/2]}\,. \tag{C.6}$$

Note that the bulk to boundary propagator satisfies

$$(\Box(z,x)-m^2)K_\Delta(z,x;x') = 0\,. \tag{C.7}$$

Furthermore, note that

$$K_\Delta(z\to 0,x;x') = z^{d-\Delta}\left(\delta^d(x-x')+\mathcal{O}(z^2)\right) + z^\Delta\left(\frac{C_\Delta}{|x-x'|^{2\Delta}}+\mathcal{O}(z^2)\right)\,. \tag{C.8}$$

Bulk-to-bulk propagator. The bulk-to-bulk propagator is defined as the solution of the differential equation

$$(\Box(z,x)-m^2)G_\Delta(z,x;z',x') = \frac{1}{\sqrt{g(z,x)}}\delta(z-z')\delta^d(x-x')\,. \tag{C.9}$$

The propagator can be explicitly written as

$$G_\Delta(z,x;z',x') = -\frac{2^\Delta \xi^\Delta \Gamma(\Delta)\Gamma\left(-\frac{d}{2}+\Delta+\frac{1}{2}\right)}{(4\pi)^{\frac{d+1}{2}}\Gamma(-d+2\Delta+1)}\,{}_2F_1\left(\frac{\Delta}{2},\frac{\Delta}{2}+\frac{1}{2};-\frac{d}{2}+\Delta+1;\xi^2\right) \tag{C.10}$$

where,

$$\xi = \frac{2zz'}{z^2+z'^2+(x-x')^2}\,. \tag{C.11}$$

Let us also note the asymptotic behavior of the propagator

$$G_\Delta(z\to\epsilon,x;z',x') = -\frac{\epsilon^\Delta}{2\Delta-d}K_\Delta(z',x';x)\,. \tag{C.12}$$

C.1 CFT 2-pt functions

The on-shell action is given by

$$\begin{aligned} S_{\text{on-shell}} &= -\frac{1}{2}\int_{z=\epsilon} d^d x \frac{1}{z^{d-1}}\phi(z,x)\partial_z\phi(z,x) \\ &= -\frac{1}{4\epsilon^{d-1}}\int_{z=\epsilon} d^d x \partial_z(\phi(z,x))^2\,. \end{aligned} \tag{C.13}$$

This on-shell action can be evaluated by using the asymptotic expression for the bulk-to-boundary propagator yielding[37]

$$S_{\text{on-shell}} = -\frac{C_\Delta d}{2}\int d^d x d^d x' \frac{\Phi(x')\Phi(x)}{|x-x'|^{2\Delta}} - \frac{d-\Delta}{2\epsilon^{2\Delta-d}}\int d^d x \Phi(x')^2\,. \tag{C.14}$$

[37] The following identity can be useful:

$$\int d^d x \left(\frac{z}{z^2+x^2}\right)^{\Delta_1}\left(\frac{z}{z^2+|x-x'|^2}\right)^{\Delta_2} = \frac{\pi^{d/2}\Gamma(\Delta_1+\Delta_2-d/2)}{\Gamma(\Delta_1)\Gamma(\Delta_2)}\int_0^1 ds\, \frac{z^{\Delta_1+\Delta_2}s^{\Delta_2-1}(1-s)^{\Delta_1-1}}{\left(s(1-s)x'^2+z^2\right)^{\Delta_1+\Delta_2-d/2}}\,.$$

The divergent term can be removed by adding a counter-term at $z = \epsilon$:

$$S_{\text{ct}} = \frac{d-\Delta}{2\epsilon^d} \int_{z=\epsilon} d^d x \; \phi(z,x)^2 \,. \tag{C.15}$$

The on-shell counter-term also contributes a finite part

$$S_{\text{ct}} = \frac{d-\Delta}{2\epsilon^{2\Delta-d}} \int d^d x \Phi(x)^2 + C_\Delta (d-\Delta) \int d^d x d^d x' \frac{\Phi(x)\Phi(x')}{|x-x'|^{2\Delta}} \tag{C.16}$$

and hence the total on-shell action becomes

$$S_{\text{on-shell}} = -\frac{(2\Delta-d)C_\Delta}{2} \int d^d x d^d x' \frac{\Phi(x')\Phi(x)}{|x-x'|^{2\Delta}} \,. \tag{C.17}$$

Finally the two-point function is

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2) \rangle = \frac{(2\Delta-d)C_\Delta}{|x_1-x_2|^{2\Delta}} \,. \tag{C.18}$$

C.2 Perturbative expansion of the Euclidean on-shell action

We now study the following Euclidean bulk action

$$S = \int d^{d+1} x \sqrt{g} \left(\frac{1}{2}(\partial\phi)^2 + \frac{1}{2} m^2 \phi^2 + L_{\text{int}} \right) \,. \tag{C.19}$$

The bulk equation of motion is now given by

$$(\Box - m^2)\phi = \frac{\delta L_{\text{int}}}{\delta \phi} \,. \tag{C.20}$$

We can again write down a formal solution of the equation of motion

$$\phi(x,z) = \int d^d x' K_\Delta(z,x;x')\Phi(x') + \int d^d x' dz' \sqrt{g'} G_\Delta(z,x;z',x') \frac{\delta L_{\text{int}}}{\delta\phi}(z',x') \tag{C.21}$$

and the on-shell action is given by

$$\begin{aligned} S_{\text{on-shell}} = & -\frac{1}{4\epsilon^{d-1}} \int_{z=\epsilon} d^d x \partial_z (\phi(z,x))^2 + \frac{d-\Delta}{2\epsilon^d} \int_{z=\epsilon} d^d x \; \phi(z,x)^2 \\ & + \int d^d x dz \sqrt{g} \left(L_{\text{int}} - \frac{1}{2} \phi \frac{\delta L_{\text{int}}}{\delta \phi} \right) & \text{(C.22)} \\ \equiv & \; S_0 + S_{\text{ct}} + S_{\text{int}} \,. & \text{(C.23)} \end{aligned}$$

First, we find that

$$\begin{aligned} S_0 + S_{\text{ct}} = & -\frac{c_\Delta(2\Delta-d)}{2} \int_{z=\epsilon} d^d x_1 d^d x_2 \frac{\Phi(x_1)\Phi(x_2)}{|x_1-x_2|^{2\Delta}} \\ & + \frac{1}{2} \int_{z=\epsilon} d^d x_1 \int d^d x' dz' \sqrt{g} K_\Delta(x_1; z', x') \Phi(\vec{x}_1) \frac{\delta L_{\text{int}}}{\delta\phi}(z',x') \,. \end{aligned} \tag{C.24}$$

So the total Euclidean on-shell action can be written in a nice form

$$S_{\text{on-shell}} = -\frac{c_\Delta(2\Delta-d)}{2}\int_{z=\epsilon} d^dx_1 d^dx_2 \frac{\Phi(x_1)\Phi(x_2)}{|x_1-x_2|^{2\Delta}} + \int d^dxdz\sqrt{g}L_{\text{int}}(z,x)$$
$$-\frac{1}{2}\int d^dxdz\sqrt{g}\int d^dx'dz'\sqrt{g'}G_\Delta(z,x;z',x')\frac{\delta L_{\text{int}}}{\delta\phi}(z',x')\frac{\delta L_{\text{int}}}{\delta\phi}(z,x)\,, \quad \text{(C.25)}$$

where, the bulk field ϕ should be understood as

$$\phi(x,z) = \int d^dx' K_\Delta(z,x;x')\Phi(x') + \int d^dx'dz'\sqrt{g'}G_\Delta(z,x;z',x')\frac{\delta L_{\text{int}}}{\delta\phi}(z',x')\,. \quad \text{(C.26)}$$

We can use equation (C.26) to perform a perturbative expansion of (C.25). Note that contact diagrams receive contributions only from the second term in (C.25). On the other hand, both the second and the third term can contribute to an exchange diagram.

C.3 Example

Let us now consider the example

$$S = \int d^{d+1}x\sqrt{g}\left(\frac{1}{2}(\partial\phi)^2 + \frac{1}{2}m^2\phi^2 + \lambda_3\phi^3 + \lambda_4\phi^4\right)\,. \quad \text{(C.27)}$$

The bulk equation of motion is now given by

$$(\Box - m^2)\phi = 3\lambda_3\phi^2 + 4\lambda_4\phi^3 \equiv \frac{\delta L_{\text{int}}}{\delta\phi}\,. \quad \text{(C.28)}$$

Three-point function. We can now write down the cubic action by using (C.25):

$$S_{(3)} = \lambda_3\int d^dxdz\sqrt{g}\int d^dx_1d^dx_2d^dx_3 K_\Delta(z,x;x_1)K_\Delta(z,x;x_2)K_\Delta(z,x;x_3)\Phi(x_1)\Phi(x_2)\Phi(x_3) \quad \text{(C.29)}$$

and hence the tree-level three-point function is given by

$$\langle\mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\rangle = -6\lambda_3\int d^dxdz\sqrt{g}K_\Delta(z,x;x_1)K_\Delta(z,x;x_2)K_\Delta(z,x;x_3)\,. \quad \text{(C.30)}$$

For the sake of completeness let us note that [111]

$$\int d^dxdz\sqrt{g}K_{\Delta_1}(z,x;x_1)K_{\Delta_2}(z,x;x_2)K_{\Delta_3}(z,x;x_3) = \frac{a_{ijk}}{|x_1-x_2|^{\Delta_{12}}|x_1-x_3|^{\Delta_{13}}|x_3-x_2|^{\Delta_{32}}} \quad \text{(C.31)}$$

with

$$a_{ijk} = \frac{\Gamma\left(\frac{\Delta_{12}}{2}\right)\Gamma\left(\frac{\Delta_{32}}{2}\right)\Gamma\left(\frac{\Delta_{13}}{2}\right)\Gamma\left(\frac{\sum_i\Delta_i-d}{2}\right)}{2\pi^d\Gamma\left(\Delta_1-\frac{d}{2}\right)\Gamma\left(\Delta_2-\frac{d}{2}\right)\Gamma\left(\Delta_3-\frac{d}{2}\right)} \quad \text{(C.32)}$$

and $\Delta_{ij} = \Delta_i + \Delta_j - \Delta_k$.

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Four-point function. The four-point function receives contributions from both contact diagrams and exchanged diagrams. In the leading order the quartic on-shell action is given by

$$S_{(4)} = \lambda_4 \int d^dxdz\sqrt{g}\phi^4 + \frac{9}{2}\lambda_3^2 \int d^dxdz\sqrt{g} \int d^dx'dz'\sqrt{g'}G_\Delta(z,x;z',x')\phi^2(z,x)\phi^2(z',x') \,. \tag{C.33}$$

So the full four-point function is given by

$$\begin{aligned}\langle\mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle = &-(4!)\lambda_4(\text{contact Witten diagram})\\ &-(3!)^2\lambda_3^2(\text{three exchanged Witten diagrams})\,.\end{aligned} \tag{C.34}$$

D Properties of D-functions

The $D(\eta,\sigma)$-function in AdS$_{d+1}$ is defined as

$$D_{\Delta_1\Delta_2\Delta_3\Delta_4}(\eta,\sigma) = \int d^{d+1}x\sqrt{g}\prod_{i=1}^{4}\tilde{K}_{\Delta_i}(z,x;x_i) \tag{D.1}$$

where boundary x_i-points are given by (3.3):

$$x_1 = -x_2 = \boldsymbol{\rho}\,, \qquad x_4 = -x_3 = \mathbf{1}\,. \tag{D.2}$$

Note that $\tilde{K}$ is the reduced bulk to boundary propagator

$$\tilde{K}_\Delta(x') \equiv \tilde{K}_\Delta(z,x;x') = \frac{z^\Delta}{(z^2+|x-x'|^2)^\Delta}\,. \tag{D.3}$$

D.1 Some useful identities

The following identities will be very useful for us.

First identity. From [112], we write

$$\begin{aligned}g^{\mu\nu}\partial_\mu\tilde{K}_{\Delta_1}(z,x;x_1)\partial_\nu\tilde{K}_{\Delta_2}(z,x;x_2) = \Delta_1\Delta_2\Big(&\tilde{K}_{\Delta_1}(z,x;x_1)\tilde{K}_{\Delta_2}(z,x;x_2)\\ &-2x_{12}^2\tilde{K}_{\Delta_1+1}(z,x;x_1)\tilde{K}_{\Delta_2+1}(z,x;x_2)\Big)\,,\end{aligned} \tag{D.4}$$

where, derivatives are taken with respect to bulk coordinates.

Second identity. From [112], we can also write

$$D_{\Delta+1\ \Delta\ \Delta\ \Delta+1}(\eta,\sigma) = D_{\Delta\ \Delta+1\ \Delta+1\ \Delta}(\eta,\sigma)\,, \tag{D.5}$$
$$D_{\Delta+1\ \Delta\ \Delta+1\ \Delta}(\eta,\sigma) = D_{\Delta\ \Delta+1\ \Delta\ \Delta+1}(\eta,\sigma)\,. \tag{D.6}$$

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Third identity. Let us now write our D-functions in terms of $\mathcal{D}$-functions of [113][38]

$$\mathcal{D}_{\Delta\Delta+1\Delta+1\Delta}(u,v) = \frac{2\Gamma[\Delta]^2\Gamma[\Delta+1]^2}{\Gamma\left(2\Delta+1-h\right)}\frac{(16\rho\bar{\rho})^{\Delta+1}}{(1-\rho)(1-\bar{\rho})}D_{\Delta\Delta+1\Delta+1\Delta}(\eta,\sigma)\,, \tag{D.7}$$

$$\mathcal{D}_{\Delta+1\Delta\Delta+1\Delta}(u,v) = \frac{2\Gamma[\Delta]^2\Gamma[\Delta+1]^2}{\Gamma\left(2\Delta+1-h\right)}\frac{(16\rho\bar{\rho})^{\Delta+1}}{(1+\rho)(1+\bar{\rho})}D_{\Delta+1\Delta\Delta+1\Delta}(\eta,\sigma)\,, \tag{D.8}$$

$$\mathcal{D}_{\Delta\Delta\Delta\Delta}(u,v) = \frac{2\Gamma[\Delta]^4}{\Gamma\left(2\Delta-h\right)}(16\rho\bar{\rho})^{\Delta}D_{\Delta\Delta\Delta\Delta}(\eta,\sigma)\,, \tag{D.9}$$

where,

$$u = \frac{(1+\rho)^2(1+\bar{\rho})^2}{16\rho\bar{\rho}} = \frac{(1+\sigma)^2(1+\eta\sigma)^2}{16\eta\sigma^2}\,, \tag{D.10}$$

$$v = \frac{(1-\rho)^2(1-\bar{\rho})^2}{16\rho\bar{\rho}} = \frac{(1-\sigma)^2(1-\eta\sigma)^2}{16\eta\sigma^2}\,. \tag{D.11}$$

From [113], we can relate

$$\mathcal{D}_{\Delta\Delta+1\Delta+1\Delta}(u,v) = -\partial_v\mathcal{D}_{\Delta\Delta\Delta\Delta}(u,v)\,, \quad \mathcal{D}_{\Delta+1\Delta\Delta+1\Delta}(u,v) = -\partial_u\mathcal{D}_{\Delta\Delta\Delta\Delta}(u,v)\,. \tag{D.12}$$

Therefore, we can derive the following expression

$$\begin{aligned}
&(1-\rho)^3(1-\bar{\rho})^3D_{\Delta\Delta+1\Delta+1\Delta}(\eta,\sigma) + (1+\rho)^3(1+\bar{\rho})^3D_{\Delta+1\Delta\Delta+1\Delta}(\eta,\sigma)\\
&= -\frac{\Gamma\left(2\Delta+1-h\right)}{2\Gamma[\Delta]^2\Gamma[\Delta+1]^2(16\eta)^{\Delta-1}}\left(v^2\partial_v + u^2\partial_u\right)\mathcal{D}_{\Delta\Delta\Delta\Delta}(u,v)\\
&= -\frac{16\left(2\Delta-h\right)}{\Delta^2\eta^{\Delta-1}}\left(v^2\partial_v + u^2\partial_u\right)\eta^{\Delta}D_{\Delta\Delta\Delta\Delta}(u,v)\\
&= -\frac{16\left(2\Delta-h\right)}{\Delta^2}\left(\frac{f_1(\eta,\sigma)}{\eta^{\Delta-1}}\partial_\eta\eta^{\Delta} + f_2(\eta,\sigma)\partial_\sigma\right)D_{\Delta\Delta\Delta\Delta}(\eta,\sigma)\,,
\end{aligned} \tag{D.13}$$

where, $h = d/2$ and

$$f_1(\eta,\sigma) = \frac{(\eta+1)\left(\sigma^2\left(\eta\left(3\eta\sigma^2+\eta+8\right)+1\right)+3\right)}{16(\eta-1)\sigma^2}\,, \tag{D.14}$$

$$f_2(\eta,\sigma) = \frac{(1-\sigma)(\sigma+1)\left(\eta(\eta+2)\sigma^2+1\right)\left(\eta\left(\eta\sigma^2+2\right)+1\right)}{16(\eta-1)\sigma\left(\eta\sigma^2-1\right)}\,. \tag{D.15}$$

D.2 Regge limit of the D-functions

Following [93], in the Regge limit we obtain

$$D_{\Delta_1\Delta_2\Delta_3\Delta_4}(\eta,\sigma) = i\frac{\pi^d 2^{1-\sum_i\Delta_i}\sigma}{\eta^{\frac{\Delta_1+\Delta_2-1}{2}}\prod_i\Gamma(\Delta_i)}f_{\Delta_1\Delta_2\Delta_3\Delta_4}\left(-\frac{1}{2}\log\left(\eta\right)\right) \tag{D.16}$$

[38] Our $\mathcal{D}$-functions are $\bar{D}$ functions of [113].

where,

$$f_{\Delta_1\Delta_2\Delta_3\Delta_4}(s) = \int_{-\infty}^{\infty} d\nu \Omega_{i\nu}(s)\Gamma\left(\frac{\Delta_3+\Delta_4-d/2+i\nu}{2}\right)\Gamma\left(\frac{\Delta_3+\Delta_4-d/2-i\nu}{2}\right)$$
$$\times\Gamma\left(\frac{\Delta_1+\Delta_2-d/2+i\nu}{2}\right)\Gamma\left(\frac{\Delta_1+\Delta_2-d/2-i\nu}{2}\right). \tag{D.17}$$

Harmonic functions $\Omega_{i\nu}$ on H_{d-1} are known in any dimension [62]

$$\Omega_E(s) = -\frac{E\sin(\pi E)\Gamma\left(\frac{d-2}{2}+E\right)\Gamma\left(\frac{d-2}{2}-E\right)}{2^{d-1}\pi^{\frac{d+1}{2}}\Gamma\left(\frac{d-1}{2}\right)}$$
$$\times {}_2F_1\left(\frac{d-2}{2}+E, \frac{d-2}{2}-E, \frac{d-1}{2}, \frac{1-\cosh(s)}{2}\right). \tag{D.18}$$

D.3 F-function

The F-function is defined as

$$F_{2\Delta+L}(\eta) = \frac{1}{\eta^{\frac{L-1}{2}}} f_{\Delta+L\ \Delta\ \Delta+L\ \Delta}\left(-\frac{1}{2}\log(\eta)\right). \tag{D.19}$$

In the limit $\eta \to 0$, we obtain from (D.17) (see appendix D of [93]):

$$F_{2\Delta+L}(\eta\to 0) = 2\pi^{1-\frac{d}{2}}\Gamma\left(2\Delta+L-\frac{d}{2}\right)\Gamma(2\Delta+L-1)\,\eta^{\Delta}\ln\left(\frac{1}{\eta}\right) \tag{D.20}$$

where $D = d+1$.

D.4 Another identity

We can also derive an exact identity

$$\partial_\eta f_{\Delta\Delta\Delta\Delta}\left(-\frac{1}{2}\log\eta\right) = \frac{1-\eta}{(4\Delta-d)\eta^{3/2}} f_{\Delta+1\Delta\Delta+1\Delta}\left(-\frac{1}{2}\log\eta\right). \tag{D.21}$$

This will be useful later.

E Regge contributions of odd couplings

In this appendix, our goal is to establish (4.21). To this end, we first prove (4.21) for $k=3$. This will necessarily imply (4.21) for all odd $k \geq 3$, as we explain at the end.

Using the explicit form of the dilaton effective action (2.1), we obtain the leading on-shell Euclidean effective action for $k=3$:

$$S^{(k=3)}_{\text{on-shell}} = -\frac{\mu_3}{2}\int d^D x\,\sqrt{g}\phi^2\Box^3\phi^2\,. \tag{E.1}$$

The above on-shell action can be rewritten at the leading order in perturbation theory by using the bulk-to-boundary propagator. We notice from [93] that all D-functions decay in the Regge limit $D_{\Delta_1\Delta_2\Delta_3\Delta_4}(\rho,\bar\rho) \sim \frac{1}{\rho}$. On the other hand, x_{ij}^2 factors can grow as $\sim \rho$. Therefore, terms in (E.1) that have at least two factors of x_{ij}^2 can grow in the Regge limit (3.5). This greatly simplifies the analysis since we only care about the growing part of the Regge correlator. In particular, the on-shell four-point interaction (E.1) can be approximated as

$$\begin{aligned} S^{(k=3)}_{\text{on-shell}} \propto -\mu_3 \int_{\Phi^4}\int_{\text{AdS}} \Big(& -x_{12}^4 x_{34}^2 \tilde{K}_{\Delta+2}(z,x;x_1)\tilde{K}_{\Delta+2}(z,x;x_2)\tilde{K}_{\Delta+1}(z,x;x_3)\tilde{K}_{\Delta+1}(z,x;x_3) \\ & + \frac{2-d+4\Delta-2d\Delta+4\Delta^2}{2(\Delta+1)^2} x_{12}^2 x_{34}^2 \tilde{K}_{\Delta+1}(z,x;x_1)\tilde{K}_{\Delta+1}(z,x;x_2)\tilde{K}_{\Delta+1}(z,x;x_3)\tilde{K}_{\Delta+1}(z,x;x_3) \\ & + \frac{2\Delta-d}{2\Delta} x_{12}^4 \tilde{K}_{\Delta+2}(z,x;x_1)\tilde{K}_{\Delta+2}(z,x;x_2)\tilde{K}_{\Delta}(z,x;x_3)\tilde{K}_{\Delta}(z,x;x_3)\Big) + \cdots , \end{aligned} \tag{E.2}$$

where dots represent terms that do not contribute to the Regge growth. Note that we are not keeping track of the overall (positive) numerical factor, since our conclusion will not depend on it. It is now a straightforward exercise to compute the Regge contribution of the $k=3$ term:

$$\begin{aligned} G(\eta,\sigma) \sim \mu_3 \Big(& \frac{2-d+4\Delta-2d\Delta+4\Delta^2}{2(\Delta+1)^2}\rho^2 D_{\Delta+1\ \Delta+1\ \Delta+1\ \Delta+1}(\eta,\sigma) + \frac{2\Delta-d}{2\Delta}\rho^2 D_{\Delta+2\ \Delta\ \Delta+2\ \Delta}(\eta,\sigma) \\ & -\frac{1}{4}(1-\rho)^3(1-\bar\rho)^3 D_{\Delta+2\ \Delta+1\ \Delta+1\ \Delta+2}(\eta,\sigma) - \frac{1}{4}(1+\rho)^3(1+\bar\rho)^3 D_{\Delta+2\ \Delta+1\ \Delta+2\ \Delta+1}(\eta,\sigma) \\ & -\frac{1}{4}(1-\rho)^3(1-\bar\rho)^3 D_{\Delta+1\ \Delta+2\ \Delta+2\ \Delta+1}(\eta,\sigma) - \frac{1}{4}(1+\rho)^3(1+\bar\rho)^3 D_{\Delta+1\ \Delta+2\ \Delta+1\ \Delta+2}(\eta,\sigma)\Big) \\ & + \mathcal{O}\left(\sigma^0\right). \end{aligned} \tag{E.3}$$

In the above expression, we have also exploited the fact that all $D_{\Delta_1\Delta_2\Delta_3\Delta_4}(\rho,\bar\rho)$ functions with fixed $\Delta_1+\Delta_2=\Delta_3+\Delta_4$ have the same leading Regge behavior [93]. Moreover, in the Regge limit, one can also relate [93]

$$D_{\Delta+1\ \Delta+1\ \Delta+1\ \Delta+1}(\eta,\sigma) = \frac{(\Delta+1)^2}{\Delta^2} D_{\Delta+2\ \Delta\ \Delta+2\ \Delta}(\eta,\sigma) + \mathcal{O}\left(\sigma^2\right). \tag{E.4}$$

We can now use various identities discussed in appendix D.1 to obtain

$$\begin{aligned} G(\eta,\sigma) \sim i\frac{\mu_3}{\eta^{\Delta+\frac{1}{2}}\sigma}\Bigg(& \frac{-12(d-1)\Delta-3d+24\Delta^2+4}{2} f_{\Delta+1\ \Delta+1\ \Delta+1\ \Delta+1}\left(-\frac{1}{2}\log\eta\right) \\ & -\frac{3(\eta+1)}{\sqrt{\eta}} f_{\Delta+2\ \Delta+1\ \Delta+2\ \Delta+1}\left(-\frac{1}{2}\log\eta\right)\Bigg) + \mathcal{O}\left(\sigma^0\right), \end{aligned} \tag{E.5}$$

where, f-functions are given by (D.17). One now can check that the quantity inside the parentheses, for $m^2 \geq 0$, changes sign as we increase η. For example, for $\eta \to 0$ it is negative. Whereas, for $\eta \to 1$ it becomes positive for $m^2 \geq 0$. Hence, if $\mu_2 = 0$, then the condition (3.12) necessarily requires

$$\mu_3 = 0\,, \qquad m^2 \geq 0\,. \tag{E.6}$$

Furthermore, the condition (3.13) now also requires that

$$\mu_k = 0 \tag{E.7}$$

for all $k \geq 4$.

Interestingly, for negative m^2 there is always a range of Δ for which the quantity inside the parentheses does not change sign. In such a case, we can only derive a sign constraint on μ_3. The condition (3.13) now rules out all even μ_k with $k \geq 4$, however, odd μ_k with $k \geq 5$ are not ruled out. It is possible that CFT conditions (3.12) and (3.13) for $L > 2$ might rule out such a scenario. Nonetheless, we will restrict to $m^2 \geq 0$ to avoid this possible loophole.

F N-coefficients

F.1 $N_D(\Delta)$

$N_D(\Delta)$ is a numerical coefficient that appears in the bound of $\phi^2\Box^2\phi^2$ interaction in the presence of gravity. First, let us note that the Harmonic function (D.18) function has the following behavior in the limit $\eta \to 1$:

$$\tilde{\Omega}_{i\nu} = \Omega_{i\nu}\left(-\frac{1}{2}\log(\eta)\right)_{\eta\to 1} = \frac{2^{1-d}\pi^{-\frac{d}{2}-\frac{1}{2}}\nu\sinh(\pi\nu)\Gamma\left(\frac{d}{2}-i\nu-1\right)\Gamma\left(\frac{d}{2}+i\nu-1\right)}{\Gamma\left(\frac{d-1}{2}\right)}. \tag{F.1}$$

The $N_D(\Delta)$ coefficient is now given by the ratio:

$$N_D(\Delta) = \frac{\int_{-\infty}^{\infty} d\nu \frac{\Gamma\left(\frac{2\Delta+2-d/2+i\nu}{2}\right)^2\Gamma\left(\frac{2\Delta+2-d/2-i\nu}{2}\right)^2}{\nu^2+\left(\frac{d}{2}\right)^2}\tilde{\Omega}_{i\nu}}{\int_{-\infty}^{\infty} d\nu\Gamma\left(\frac{2\Delta+2-d/2+i\nu}{2}\right)^2\Gamma\left(\frac{2\Delta+2-d/2-i\nu}{2}\right)^2\tilde{\Omega}_{i\nu}} \tag{F.2}$$

where $D = d+1$. This factor can be easily computed in Mathematica. In particular, we find that for large Δ and $D > 4$:

$$N_D(\Delta \gg 1) \approx \frac{1}{(D-4)\Delta}. \tag{F.3}$$

F.2 $\tilde{N}_D(\Delta)$

The $\tilde{N}_D(\Delta)$ coefficient is now given by the ratio:

$$\tilde{N}_D(\Delta) = \frac{\Gamma\left(2\Delta - \frac{D-9}{2}\right)\Gamma(2\Delta+3)\int_{-\infty}^{\infty} d\nu\Gamma\left(\frac{2\Delta+2-d/2+i\nu}{2}\right)^2\Gamma\left(\frac{2\Delta+2-d/2-i\nu}{2}\right)^2\tilde{\Omega}_{i\nu}}{\Gamma\left(2\Delta - \frac{D-5}{2}\right)\Gamma(2\Delta+1)\int_{-\infty}^{\infty} d\nu\Gamma\left(\frac{2\Delta+4-d/2+i\nu}{2}\right)^2\Gamma\left(\frac{2\Delta+4-d/2-i\nu}{2}\right)^2\tilde{\Omega}_{i\nu}} \tag{F.4}$$

where $D = d+1$. Note that $\tilde{N}_D(\Delta) > 1$.

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