

# Logarithmic correction to the entropy of extremal black holes in $\mathcal{N} = 1$ Einstein-Maxwell supergravity

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**ABSTRACT:** We study one-loop covariant effective action of “non-minimally coupled”  $\mathcal{N} = 1$ ,  $d = 4$  Einstein-Maxwell supergravity theory by heat kernel tool. By fluctuating the fields around the classical background, we study the functional determinant of Laplacian differential operator following Seeley-DeWitt technique of heat kernel expansion in proper time. We then compute the Seeley-DeWitt coefficients obtained through the expansion. A particular Seeley-DeWitt coefficient is used for determining the logarithmic correction to Bekenstein-Hawking entropy of extremal black holes using quantum entropy function formalism. We thus determine the logarithmic correction to the entropy of Kerr-Newman, Kerr and Reissner-Nordström black holes in “non-minimally coupled”  $\mathcal{N} = 1$ ,  $d = 4$  Einstein-Maxwell supergravity theory.

**KEYWORDS:** Black Holes, Black Holes in String Theory, Extended Supersymmetry, AdS-CFT Correspondence

ARXIV EPRINT: [2007.11497](https://arxiv.org/abs/2007.11497)

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## Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>2</b>	<b>Effective action analysis via heat kernel</b>	<b>4</b>
2.1	Loop expansion and effective action	4
2.2	Heat kernel calculational framework and methodology	6
<b>3</b>	<b>“Non-minimal” <math>\mathcal{N} = 1</math> Einstein-Maxwell supergravity theory</b>	<b>7</b>
3.1	Supergravity solution and classical equations of motion	8
<b>4</b>	<b>Heat kernels in “non-minimal” <math>\mathcal{N} = 1, d = 4</math> EMSGT</b>	<b>9</b>
4.1	Bosonic sector	9
4.2	Fermionic sector	9
4.3	Total Seeley-DeWitt coefficients	13
<b>5</b>	<b>Logarithmic correction of extremal black holes in “non-minimal” <math>\mathcal{N} = 1, d = 4</math> EMSGT</b>	<b>13</b>
5.1	General methodology for computing logarithmic correction using Seeley-DeWitt coefficient	13
5.2	Extremal black holes in “non-minimal” $\mathcal{N} = 1, d = 4$ EMSGT and their logarithmic corrections	15
<b>6</b>	<b>Concluding remarks</b>	<b>17</b>
<b>A</b>	<b>Trace calculations</b>	<b>18</b>

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## 1 Introduction

Studying the spectrum of effective action is one of the exciting topics in quantum field theory at all times as it contains the relevant information of any theory. It provides the knowledge of physical amplitudes and quantum effects of the fields present in the theory. The analysis of effective action by perturbative expansion of fields in various loops is a powerful way to study a particular theory. Heat kernel proves to be an efficient tool for analysis of effective action at one-loop level [1, 2]. The basic principle of this heat kernel method is to study an asymptotic expansion for the spectrum of the differential operator of effective action. It helps us in the investigation of one-loop divergence, quantum anomalies, vacuum polarization, Casimir effect, ultraviolet divergence, spectral geometry, etc. . Within heat kernel expansion method, there are various approaches [1–28] to analyze the effective action such as covariant perturbation expansion, higher derivative expansion, group theoretic approach, diagram technique, wave function expansion, Seeley-DeWitt expansion, etc.<sup>1</sup> However, among them, Seeley-DeWitt expansion [13] is better over other

<sup>1</sup>For a general review regarding several approaches of heat kernel expansion, one can refer [13].

methods for various reasons stated here. The other methods possess either some model dependency or background geometry dependency. Whereas the Seeley-DeWitt expansion is more general and standard. This neither depends upon background geometry nor on supersymmetry and is applicable for all background field configuration. These properties make the Seeley-DeWitt approach more appropriate on those manifolds where other approaches fail to work. In the regime of black hole physics, the heat kernel expansion coefficients are found to be very useful for computing the logarithmic corrections to the entropy of black holes [29–39] and the current paper will mainly focus on this direction.

This paper is motivated by a list of reported works [29–40]. In [29], Sen et al. determined the logarithmic corrections of extremal Kerr-Newman type black holes in non-supersymmetric 4D Einstein-Maxwell theory using the quantum entropy function formalism [41–43]. Here the essential coefficient,  $a_4$ , is calculated following the general Seeley-DeWitt approach [13]. In our previous work [30], we have also followed the same approach as [29] and computed the logarithmic correction to the entropy of extremal black holes in minimal  $\mathcal{N} = 2$  Einstein-Maxwell supergravity theory (EMSGT) in 4D. The computation of the heat kernel coefficients for extended supersymmetric Einstein-Maxwell theory for  $\mathcal{N} \geq 2$  is presented in [31, 32] by Larsen et al.. However, the authors have used the strategy of field redefinition approach within the Seeley-DeWitt technique to compute the required coefficients and obtained the logarithmic corrected entropy of various types of non-extremal black holes. In contrast, we are interested in the computation of logarithmic correction to the entropy of extremal black holes by using the general approach [13] for the computation of required coefficients. In [33–36], the heat kernel coefficients are computed via the wave-function expansion approach and then used for the determination of logarithmic corrections to the entropy of different extremal black holes in  $\mathcal{N} = 2$ ,  $\mathcal{N} = 4$ , and  $\mathcal{N} = 8$  EMSGTs. Again, in [37, 38], on-shell and off-shell approaches have been used to determine the required coefficients. In these wave function expansion, on-shell and off-shell methods, one should have explicit knowledge of symmetry properties of the background geometry as well as eigenvalues and eigenfunctions of Laplacian operator acting over the background. Whereas the Seeley-DeWitt expansion approach [13], followed in [29, 30, 40] as well as in the present work, is applicable for any arbitrary background geometry.

Logarithmic corrections are special quantum corrections to the Bekenstein-Hawking entropy of black holes that can be completely determined from the gravity side without being concerned about the UV completion of the theory [29–39]. In the context of quantum gravity, logarithmic corrections can be treated as the ‘infrared window’ into the microstates of the given black holes. Over the last few decades, supergravities are the most intensively used platform to investigate logarithmic corrections within string theory. Supergravity theories can be realized as the low energy gravity end of compactified string theories (mainly on Calabi-Yau manifold), and this makes logarithmic correction results in supergravities a more significant IR testing grounds for string theories.<sup>2</sup> Following the developments discussed in the previous paragraph, the analysis of the effective action in  $\mathcal{N} = 1, d = 4$  EMSGT is highly inquisitive for the determination of logarithmic correction to the entropy

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<sup>2</sup>An excellent comparative review of logarithmic corrections and their microscopic consistency in string theory can be found in [44, 45].

of extremal black holes. In the current paper, we prefer to do this study via the heat kernel tool following the Seeley-DeWitt expansion approach [13] and use the results into the framework of the quantum entropy function formalism [41–43]. The attractor mechanism for various black holes in  $\mathcal{N} = 1$  supergravity is presented in [46]. In [47], the authors have studied local supersymmetry of  $\mathcal{N} = 1$  EMSGT in four dimensions. Moreover, we aim to find the logarithmic correction to the black hole entropy of various extremal black hole solutions such as Reissner-Nordström, Kerr and Kerr-Newman black holes in a general class of  $\mathcal{N} = 1$  EMSGT in four dimensions. Our results will provide a strong constraint to an ultraviolet completion of the theory if an independent computation is done to determine the same entropy corrections from the microscopic description.

In the current paper, we particularly consider a “non-minimal”  $\mathcal{N} = 1$  EMSGT<sup>3</sup> in four dimensions on a compact Riemannian manifold without boundary. Here the gaugino field of the vector multiplet is non-minimally coupled to the gravitino field of the supergravity multiplet through the gauge field strength. We fluctuate the fields in the theory around an arbitrary background and construct the quadratic fluctuated (one-loop) action. We determine the functional determinant of one-loop action and then compute the first three Seeley-DeWitt coefficients for this theory. The coefficients are presented in (4.20). Then employing quantum entropy function formalism, the particular coefficient  $a_4$  is used to determine the logarithmic correction to Bekenstein-Hawking entropy for Kerr-Newman, Kerr and Reissner-Nordström black holes in their extremal limit. The results are expressed in (5.25) to (5.27).

Earlier, the logarithmic correction to the extremal Reissner-Nordström black hole entropy in two different classes of  $\mathcal{N} = 1, d = 4$  EMSGT (truncated from  $\mathcal{N} = 2$  and minimal) are predicted in [48]. In the “truncated”  $\mathcal{N} = 1$  theory, all the field couplings are considered in such a way that the  $\mathcal{N} = 2$  multiplets can be kinematically decomposed into  $\mathcal{N} = 1$  multiplets. For this truncation, one needs to assume a list of approximations (see section 4.1 of [48]). In the “minimal”  $\mathcal{N} = 1$  theory, the coupling term between gravitino (of supergravity multiplet) and gaugino (of vector multiplet) field is customized so that there exist minimal couplings between the supergravity multiplet and vector multiplets.<sup>4</sup> But in the present paper, we will proceed with a third and more general class of  $\mathcal{N} = 1$  theory — the “non-minimal”  $\mathcal{N} = 1$  EMSGT, where no such approximations are assumed for field couplings, and the general non-minimal coupling between the supergravity and vector multiplet is also turned on. All the three classes of  $\mathcal{N} = 1$  theory are distinct and unable to reproduce each other’s results because of the different choice of field couplings. Both the “truncated” and “minimal”  $\mathcal{N} = 1$  theories are approximated, while the “non-minimal”  $\mathcal{N} = 1$  EMSGT is a general one. In work [48], logarithmic corrections for only extremal Reissner-Nordström are achieved using  $\mathcal{N} = 2$  data and a particular local supersymmetrization process. On the other hand, our plan is to calculate logarithmic corrections to the entropy of extremal Reissner-Nordström, Kerr and Kerr-Newman black holes by directly

<sup>3</sup>For details of this theory, refer to section 3.

<sup>4</sup>In general, there exist a non-minimal coupling term  $\bar{\psi}F\lambda$  between gravitino  $\psi$  (of supergravity multiplet) and gaugino  $\lambda$  (of vector multiplets) via the vector field (of vector multiplets) strength  $F$  in an  $\mathcal{N} = 1$  theory [47]. The “minimally coupled”  $\mathcal{N} = 1$  class assumed a constant vector strength matrix  $F$  [48].

analyzing the quadratic fluctuated action of the general “non-minimal”  $\mathcal{N} = 1$  EMSGT. As discussed, our results are unique, novel and impossible to predict from [48] and vice versa.

The present paper is arranged as follows. In section 2, we have briefly revised the heat kernel technique for the analysis of effective action and introduced the methodology prescribed by Vassilevich et al. [13] for the computation of required Seeley-DeWitt coefficients. We described the four dimensional “non-minimal”  $\mathcal{N} = 1$  Einstein-Maxwell supergravity theory and the classical equations of motion for the action in section 3. In section 4, we carried out the heat kernel treatment to obtain the first three Seeley-DeWitt coefficients of the “non-minimal”  $\mathcal{N} = 1, d = 4$  EMSGT. The coefficients for bosonic fields are summarized from our earlier work [30], whereas the first three Seeley-DeWitt coefficients for fermionic fields are explicitly computed. In section 5, we first discussed a general approach to determine the logarithmic correction to entropy of extremal black holes using Seeley-DeWitt coefficients via quantum entropy function formalism. Using our results for the Seeley-DeWitt coefficient, we then calculate logarithmic corrections to the entropy Kerr-Newman, Kerr, Reissner-Nordström extremal black holes in “non-minimal”  $\mathcal{N} = 1, d = 4$  EMSGT. We summarize our results in section 6. Some details of our calculation are given in appendix A.

## 2 Effective action analysis via heat kernel

In this section, we are going to discuss the one-loop effective action in 4D Euclidean space-time using the Seeley-DeWitt expansion approach of heat kernel and review the basic calculation framework of this approach from [13]. As initiated by Fock, Schwinger and DeWitt [15, 49–51], a Green function is introduced and studied by the heat kernel technique for this analysis. The integral of this Green function over the proper time coordinate is a convenient way to study the one-loop effective action. The proper time method defines the Green function in the neighborhood of light cone, which made it suitable for studying UV divergence and spectral geometry of effective action in one-loop approximation [52].

### 2.1 Loop expansion and effective action

We begin with a review of the heat kernel approach demonstrated in [13] for studying the one-loop effective action by computing the functional determinant of quantum fields fluctuated around the background. We also analyze the relationship between effective action and heat kernels.<sup>5</sup>

We consider a set of arbitrary fields  $\varphi_m$ ,<sup>6</sup> on four dimensional compact, smooth Riemannian manifold without boundary. Then, the generating functional for Green function with these fields  $\varphi_m$  in a Euclidean path integral representation<sup>7</sup> is expressed as

$$\mathcal{Z} = \int [D\varphi_m] \exp(-\mathcal{S}[\varphi_m]), \tag{2.1}$$

$$\mathcal{S}[\varphi_m] = \int d^4x \sqrt{g} \mathcal{L}(\varphi_m, \partial_\mu \varphi_m), \tag{2.2}$$

<sup>5</sup>For an explicit elaboration, one can refer to [30].

<sup>6</sup>Note that this set also includes metric field.

<sup>7</sup>The Euclidean path integral is followed by Wick rotation. Note that we have set  $\hbar = c = 1$  and  $G_N = \frac{1}{16\pi}$  throughout the paper.

where  $[D\varphi_m]$  denotes the functional integration over all the possible fields  $\varphi_m$  present in the theory.  $\mathcal{S}$  is the Euclidean action with lagrangian density  $\mathcal{L}$  carrying all information of fields in the manifold defined by metric  $g_{\mu\nu}$ . We fluctuate the fields around any background solution to study the effect of perturbation in the action,

$$\varphi_m = \bar{\varphi}_m + \tilde{\varphi}_m, \tag{2.3}$$

where  $\bar{\varphi}_m$  and  $\tilde{\varphi}_m$  symbolize the background and fluctuated fields, respectively. We are actually interested in the quantum characteristics of the action around a classical background, which serves as a semi-classical system in an energy scale lower than the plank scale. For that, we consider the classical solutions  $\bar{\varphi}_m$  of the theory as a background and the Taylor expansion of the action (2.2) around the classical background fields up to quadratic fluctuation fields yields,

$$\mathcal{S} \approx \mathcal{S}_{\text{cl}} + \mathcal{S}_2 = \mathcal{S}_{\text{cl}} + \int d^4x \sqrt{g} \tilde{\varphi}_m \Lambda^{mn} \tilde{\varphi}_n, \tag{2.4}$$

where  $\mathcal{S}_{\text{cl}}$  is the classical action that only depends on the background.  $\mathcal{S}_2$  denotes the quadratic interaction part of the quantum fields in action via the differential operator  $\Lambda$ . After imposing the quadratic fluctuation (2.4), the Gaussian integral of generating functional (2.1) gives the form of one-loop effective action  $W$  as [1, 28]

$$W = \frac{\chi}{2} \ln \det(\Lambda) = \frac{\chi}{2} \text{tr} \ln \Lambda \quad \forall \begin{cases} \chi = +1 : \text{Bosons} \\ \chi = -1 : \text{Fermions} \end{cases}, \tag{2.5}$$

where the sign of  $\chi$  depicts the position of  $\det\Lambda$  in the denominator or numerator for bosonic or fermionic field taken under consideration.<sup>8</sup>  $W$  contains the detailed information of the fluctuated quantum fields at one-loop level. In order to analyze the spectrum of the differential operator  $\Lambda$  acted on the perturbed fields, we use the heat kernel technique by defining the heat kernel  $K$  as [1, 13]

$$K(x, y; s) = \langle x | \exp(-s\Lambda) | y \rangle, \tag{2.6}$$

where  $K(x, y; s)$  is the Green function between the points  $x$  and  $y$  that satisfies the heat equation,

$$(\partial_s + \Lambda_x)K(x, y; s) = 0, \tag{2.7}$$

with the boundary condition,

$$K(x, y; 0) = \delta(x, y). \tag{2.8}$$

Here the proper time  $s$  is known as the heat kernel parameter. The one-loop effective action  $W$  is related to the trace of heat kernel, i.e., heat trace  $D(s)$  via [13, 30]

$$W = -\frac{1}{2} \int_{\epsilon}^{\infty} \frac{1}{s} ds \chi D(s), \quad D(s) = \text{tr} (e^{-s\Lambda}) = \sum_i e^{-s\lambda_i}, \tag{2.9}$$

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<sup>8</sup> $\exp(-W) = \int [D\tilde{\varphi}_m] \exp(-\int d^4x \sqrt{g} \tilde{\varphi} \Lambda \tilde{\varphi}) = \det^{-\chi/2} \Lambda$  [32].

where  $\epsilon$  is the cut-off limit<sup>9</sup> of effective action (2.9) to counter the divergence in the lower limit regime and  $\lambda_i$ 's are non zero eigenvalues of  $\Lambda$  which are discrete, but they may also be continuous.  $D(s)$  is associated with a perturbative expansion in proper time  $s$  as [13, 15–27]

$$D(s) = \int d^4x \sqrt{\bar{g}} K(x, x; s) = \int d^4x \sqrt{\bar{g}} \sum_{n=0}^{\infty} s^{n-2} a_{2n}(x). \quad (2.10)$$

The proper time expansion of  $K(x, x; s)$  gives a series expansion where the coefficients  $a_{2n}$ , termed as Seeley-DeWitt coefficients [13, 15–20], are functions of local background invariants such as Ricci tensor, Riemann tensor, gauge field strengths as well as covariant derivative of these parameters. Our first task is to compute these coefficients  $a_{2n}$ ,<sup>10</sup> for an arbitrary background on a manifold without boundary. We then emphasize the applicability of a particular Seeley-DeWitt coefficient for determining the logarithmic correction to the entropy of extremal black holes.

## 2.2 Heat kernel calculational framework and methodology

In this section, we try to highlight the generalized approach for the computation of Seeley-DeWitt coefficients. For the four dimensional compact, smooth Riemannian manifold without boundary<sup>11</sup> under consideration, we will follow the procedure prescribed by Vassilevich in [13] to compute the Seeley-DeWitt coefficients. We aim to express the required coefficients in terms of local invariants.<sup>12</sup>

We start by considering an ansatz  $\tilde{\varphi}_m$ , a set of fields satisfying the classical equations of motion of a particular theory. We then fluctuate all the fields around the background as shown in (2.3) and the quadratic fluctuated action  $\mathcal{S}_2$  is defined as

$$\mathcal{S}_2 = \int d^4x \sqrt{\bar{g}} \tilde{\varphi}_m \Lambda^{mn} \tilde{\varphi}_n. \quad (2.11)$$

We study the spectrum of elliptic Laplacian type partial differential operator  $\Lambda^{mn}$  on the fluctuated quantum fields on the manifold. Again,  $\Lambda^{mn}$  should be a Hermitian operator for the heat kernel methodology to be applicable. Therefore, we have,

$$\tilde{\varphi}_m \Lambda^{mn} \tilde{\varphi}_n = \pm [\tilde{\varphi}_m (D^\mu D_\mu) G^{mn} \tilde{\varphi}_n + \tilde{\varphi}_m (N^\rho D_\rho)^{mn} \tilde{\varphi}_n + \tilde{\varphi}_m P^{mn} \tilde{\varphi}_n], \quad (2.12)$$

where an overall +ve or -ve sign needs to be extracted out in the quadratic fluctuated form of bosons and fermions, respectively.  $G$  is effective metric in field space, i.e., for vector field  $G^{\mu\nu} = \bar{g}^{\mu\nu}$ , for Rarita-Schwinger field  $G^{\mu\nu} = \bar{g}^{\mu\nu} \mathbb{I}_4$  and for gaugino field  $G = \mathbb{I}_4$ . Here  $\mathbb{I}_4$  is the identity matrix associated with spinor indices of fermionic fields.  $P$  and  $N$  are arbitrary matrices,  $D_\mu$  is an ordinary covariant derivative acting on the quantum fields. Moreover, while dealing with a fermionic field (especially Dirac type) whose operator, say  $O$  is linear, one has to first make it Laplacian of the form (2.12) using the following relation [28, 31, 36]

$$\ln \det(\Lambda^{mn}) = \frac{1}{2} \ln \det(O^{mp} O_p^{n\dagger}). \quad (2.13)$$

<sup>9</sup> $\epsilon$  limit is controlled by the Planck parameter,  $\epsilon \sim l_p^2 \sim G_N \sim \frac{1}{16\pi}$ .

<sup>10</sup> $a_{2n+1} = 0$  for a manifold without boundary.

<sup>11</sup>The same methodology is also applicable for manifold with a boundary defined by some boundary conditions (see section 5 of [13]).

<sup>12</sup>Kindly refer our earlier works [30, 40] for details on this discussion.



Equation (2.12) can be rewritten in the form below

$$\tilde{\varphi}_m \Lambda^{mn} \tilde{\varphi}_n = \pm (\tilde{\varphi}_m (\mathcal{D}_\mu \mathcal{D}^\mu) \mathbb{I}^{mn} \tilde{\varphi}_n + \tilde{\varphi}_m E^{mn} \tilde{\varphi}_n), \quad (2.14)$$

where  $E$  is endomorphism on the field bundle and  $\mathcal{D}_\mu$  is a new effective covariant derivative defined with field connection  $\omega_\mu$ ,

$$\mathcal{D}_\mu = D_\mu + \omega_\mu. \quad (2.15)$$

The new form of (2.14) of  $\Lambda$  is also minimal, Laplacian, second-order, pseudo-differential type as desired. One can obtain the form of  $\mathbb{I}$ ,  $\omega_\rho$  and  $E$  by comparing (2.12) and (2.14) with help of the effective covariant derivative (2.15),

$$\tilde{\varphi}_m (\mathbb{I})^{mn} \tilde{\varphi}_n = \tilde{\varphi}_m G^{mn} \tilde{\varphi}_n, \quad (2.16)$$

$$\tilde{\varphi}_m (\omega_\rho)^{mn} \tilde{\varphi}_n = \tilde{\varphi}_m \left( \frac{1}{2} N_\rho \right)^{mn} \tilde{\varphi}_n, \quad (2.17)$$

$$\tilde{\varphi}_m E^{mn} \tilde{\varphi}_n = \tilde{\varphi}_m F^{mn} \tilde{\varphi}_n - \tilde{\varphi}_m (\omega^\rho)^{mp} (\omega_\rho)_p{}^n \tilde{\varphi}_n - \tilde{\varphi}_m (D^\rho \omega_\rho)^{mn} \tilde{\varphi}_n. \quad (2.18)$$

And, the field strength  $\Omega_{\alpha\beta}$  associated with the curvature of  $\mathcal{D}_\mu$  is obtained via

$$\tilde{\varphi}_m (\Omega_{\alpha\beta})^{mn} \tilde{\varphi}_n \equiv \tilde{\varphi}_m [\mathcal{D}_\alpha, \mathcal{D}_\beta]^{mn} \tilde{\varphi}_n. \quad (2.19)$$

The Seeley-DeWitt coefficients for a manifold without boundary can be determined from the knowledge of  $\mathbb{I}$ ,  $E$  and  $\Omega_{\alpha\beta}$ . In the following, we give expressions for the first three Seeley-DeWitt coefficients  $a_0$ ,  $a_2$ ,  $a_4$  in terms of local background invariant parameters [13, 26, 27],

$$\begin{aligned} \chi(4\pi)^2 a_0(x) &= \text{tr} (\mathbb{I}), \\ \chi(4\pi)^2 a_2(x) &= \frac{1}{6} \text{tr} (6E + R\mathbb{I}), \\ \chi(4\pi)^2 a_4(x) &= \frac{1}{360} \text{tr} \left\{ 60RE + 180E^2 + 30\Omega^{\mu\nu} \Omega_{\mu\nu} + (5R^2 - 2R^{\mu\nu} R_{\mu\nu} + 2R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma}) \mathbb{I} \right\}, \end{aligned} \quad (2.20)$$

where  $R$ ,  $R_{\mu\nu}$  and  $R_{\mu\nu\rho\sigma}$  are the usual curvature tensors computed over the background metric.

There exist many studies about other higher-order coefficients, e.g.,  $a_6$  [27],  $a_8$  [5, 6],  $a_{10}$  [7], and higher heat kernels [8, 9]. However, we will limit ourselves up to  $a_4$ , which helps to determine the logarithmic divergence of the string theoretic black holes. Apart from this, the coefficient  $a_2$  describes the quadratic divergence and renormalization of Newton constant and  $a_0$  encodes the cosmological constant at one-loop approximation [31, 32].

### 3 “Non-minimal” $\mathcal{N} = 1$ Einstein-Maxwell supergravity theory

The gauge action of pure  $\mathcal{N} = 1$  supergravity, containing spin 2 graviton with superpartner massless spin 3/2 Rarita-Schwinger fields, was first constructed and the corresponding supersymmetric transformations were studied by Ferrara et al. [53]. Later, this work is extended in [47] by coupling an additional vector multiplet (matter coupling) to the pure  $\mathcal{N} = 1$  supergravity. Here the vector multiplet is non-minimally coupled to the supergauged



action of  $\mathcal{N} = 1$  theory. The resultant “non-minimal”  $\mathcal{N} = 1$   $d = 4$  EMSGT is the object of interest of the current paper. Two other special classes of  $\mathcal{N} = 1$  supergravity theories, namely “minimally coupled”  $\mathcal{N} = 1$  supergravity theory and  $\mathcal{N} = 1$  theory obtained from truncation of  $\mathcal{N} = 2$  theory, were also studied by Ferrara et al. in [48, 54]. This “non-minimal”  $\mathcal{N} = 1, d = 4$  EMSGT has the same on-shell bosonic and fermionic degrees of freedom. The bosonic sector is controlled by a 4D Einstein-Maxwell theory [29], while the fermionic sector casts a non-minimal exchange of graviphoton between the gravitino and gaugino species. We aim to calculate the Seeley-DeWitt coefficients in the mentioned theory for the fluctuation of its fields. We also summarize the classical equations of motion and identities for the theory.

### 3.1 Supergravity solution and classical equations of motion

The “non-minimal”  $\mathcal{N} = 1$  EMSGT in four dimensions is defined with the following field contents: a gravitational spin 2 field  $g_{\mu\nu}$  (graviton) and an abelian spin 1 gauge field  $A_\mu$  (photon) along with their superpartners, massless spin 3/2 Rarita-Schwinger  $\psi_\mu$  (gravitino) and spin 1/2  $\lambda$  (gaugino) fields. Note that the gaugino field ( $\lambda$ ) interacts non-minimally with the gravitino field ( $\psi_\mu$ ) via non-vanishing gauge field strength.

The theory is described by the following action [47]

$$\mathcal{S} = \int d^4x \sqrt{g} \mathcal{L}_{\text{EM}} + \int d^4x \sqrt{g} \mathcal{L}_f, \quad (3.1)$$

where,

$$\mathcal{L}_{\text{EM}} = \mathcal{R} - F^{\mu\nu} F_{\mu\nu}, \quad (3.2)$$

$$\mathcal{L}_f = -\frac{1}{2} \bar{\psi}_\mu \gamma^{[\mu\rho\nu]} D_\rho \psi_\nu - \frac{1}{2} \bar{\lambda} \gamma^\rho D_\rho \lambda + \frac{1}{2\sqrt{2}} \bar{\psi}_\mu F_{\alpha\beta} \gamma^\alpha \gamma^\beta \gamma^\mu \lambda. \quad (3.3)$$

$\mathcal{R}$  is the Ricci scalar curvature and  $F_{\mu\nu} = \partial_{[\mu} A_{\nu]}$  is the field strength of gauge field  $A_\mu$ .

Let us consider an arbitrary background solution denoted as  $(\bar{g}_{\mu\nu}, \bar{A}_\mu)$  satisfying the classical equations of motion of Einstein-Maxwell theory. The solution is then embedded in  $\mathcal{N} = 1$  supersymmetric theory in four dimensions. The geometry of action (3.1) is defined by the Einstein equation,

$$R_{\mu\nu} - \frac{1}{2} \bar{g}_{\mu\nu} R = 8\pi G_N T_{\mu\nu} = 2\bar{F}_{\mu\rho} \bar{F}_\nu{}^\rho - \frac{1}{2} \bar{g}_{\mu\nu} \bar{F}^{\rho\sigma} \bar{F}_{\rho\sigma}, \quad (3.4)$$

where  $\bar{F}_{\mu\nu} = \partial_{[\mu} \bar{A}_{\nu]}$  is the background gauge field strength. Equation (3.4) also defines background curvature invariants in terms of field strength tensor. For such a geometrical space-time, the trace of (3.4) yields  $R = 0$  as a classical solution. Hence one can ignore the terms proportional to  $R$  in further calculations. The reduced equation of motion for the  $\mathcal{N} = 1, d = 4$  EMSGT is thus given as

$$R_{\mu\nu} = 2\bar{F}_{\mu\rho} \bar{F}_\nu{}^\rho - \frac{1}{2} \bar{g}_{\mu\nu} \bar{F}^{\rho\sigma} \bar{F}_{\rho\sigma}. \quad (3.5)$$

The theory also satisfies the Maxwell equations and Bianchi identities of the form,

$$D^\mu \bar{F}_{\mu\nu} = 0, \quad D_{[\mu} \bar{F}_{\nu\rho]} = 0, \quad R_{\mu[\nu\theta\phi]} = 0. \quad (3.6)$$

The equations of motion (3.5), (3.6) and the condition  $R = 0$  play a crucial role in simplifying the Seeley-DeWitt coefficients for the  $\mathcal{N} = 1$ ,  $d = 4$  EMSGT theory and express them in terms of local background invariants.

## 4 Heat kernels in “non-minimal” $\mathcal{N} = 1$ , $d = 4$ EMSGT

In this section, we study the spectrum of one-loop action corresponding to the quantum fluctuations in the background fields for the “non-minimal”  $\mathcal{N} = 1$  EMSGT in a compact four dimensional Riemannian manifold without boundary. We will focus on the computation of the first three Seeley-DeWitt coefficients for the fluctuated bosonic and fermionic fields present in the concerned theory using the heat kernel tool, as described in section 2. In the later sections, we throw light on the applicability of these computed coefficients from the perspective of logarithmic correction to the entropy of extremal black holes.

### 4.1 Bosonic sector

Here we are interested in the analysis of quadratic order fluctuated action  $\mathcal{S}_2$  and computation of the Seeley-DeWitt coefficients for non-supersymmetric 4D Einstein-Maxwell theory with only bosonic fields, i.e.,  $g_{\mu\nu}$  and  $A_\mu$ , described by the action (3.2). We consider the following fluctuations of the fields present in the theory

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \tilde{g}_{\mu\nu}, \quad A_\mu = \bar{A}_\mu + \tilde{A}_\mu. \quad (4.1)$$

The resulting quadratic order fluctuated action  $\mathcal{S}_2$  is then studied and the required coefficients were calculated following the general Seeley-DeWitt technique [13] for heat kernel. The first three Seeley-DeWitt coefficients for bosonic fields read as<sup>13</sup>

$$\begin{aligned} (4\pi)^2 a_0^{\text{B}}(x) &= 4, \\ (4\pi)^2 a_2^{\text{B}}(x) &= 6\bar{F}^{\mu\nu}\bar{F}_{\mu\nu}, \\ (4\pi)^2 a_4^{\text{B}}(x) &= \frac{1}{180}(199R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + 26R_{\mu\nu}R^{\mu\nu}). \end{aligned} \quad (4.2)$$

### 4.2 Fermionic sector

In this subsection, we deal with the superpartner fermionic fields of  $\mathcal{N} = 1$ ,  $d = 4$  EMSGT, i.e.,  $\psi_\mu$  and  $\lambda$ . As discussed in section 3, these fermionic fields are non-minimally coupled. The quadratic fluctuated action for these fields is given in (3.1). Before proceeding further, it is necessary to gauge fix the action for gauge invariance. We achieve that by adding a gauge fixing term  $\mathcal{L}_{\text{gf}}$  to the lagrangian (3.3),

$$\mathcal{L}_{\text{gf}} = \frac{1}{4}\bar{\psi}_\mu\gamma^\mu\gamma^\rho D_\rho\gamma^\nu\psi_\nu. \quad (4.3)$$

This gauge fixing process cancels the gauge degrees of freedom and induces ghost fields to the theory described by the following lagrangian,

$$\mathcal{L}_{\text{ghost},f} = \bar{b}\gamma^\rho D_\rho\tilde{c} + \bar{e}\gamma^\rho D_\rho\tilde{e}. \quad (4.4)$$

---

<sup>13</sup>One may go through our earlier work [30] for explicit computation of Seeley-DeWitt coefficients for the bosonic gravity multiplet.

In (4.4)  $\tilde{b}$ ,  $\tilde{c}$  are Faddeev-Popov ghosts, and  $\tilde{e}$  is a ghost associated with the unusual nature of gauge fixing [33]. These ghost fields are bosonic ghost having spin half statistics in ghost lagrangian. The final quadratic fluctuated gauge fixed action for the fermionic sector, thus obtained along with unphysical ghost fields is expressed as

$$\mathcal{S}_{2,F} = \int d^4x \sqrt{\bar{g}} (\mathcal{L}_f + \mathcal{L}_{\text{gf}} + \mathcal{L}_{\text{ghost},f}), \quad (4.5)$$

where we have,

$$\begin{aligned} \mathcal{L}_f + \mathcal{L}_{\text{gf}} &= \frac{1}{4} \bar{\psi}_\mu \gamma^\nu \gamma^\rho \gamma^\mu D_\rho \psi_\nu - \frac{1}{2} \bar{\lambda} \gamma^\rho D_\rho \lambda \\ &+ \frac{1}{4\sqrt{2}} \bar{\psi}_\mu \bar{F}_{\alpha\beta} \gamma^\alpha \gamma^\beta \gamma^\mu \lambda - \frac{1}{4\sqrt{2}} \bar{\lambda} \gamma^\nu \gamma^\alpha \gamma^\beta \bar{F}_{\alpha\beta} \psi_\nu. \end{aligned} \quad (4.6)$$

Our primary task in this subsection is to compute the Seeley-DeWitt coefficients for action (4.5) and then add it to the coefficients obtained for the bosonic part (4.2).

Let us begin with the computation of required coefficients for fermionic fields without ghost fields. The corresponding quadratic fluctuated action can be written as

$$- \frac{1}{2} \int d^4x \sqrt{\bar{g}} \tilde{\varphi}_m O^{mn} \tilde{\varphi}_n, \quad (4.7)$$

where  $O$  is the differential operator which can be expressed in Dirac form as<sup>14</sup>

$$\begin{aligned} \tilde{\varphi}_m O^{mn} \tilde{\varphi}_n &= -\frac{i}{2} \bar{\psi}_\mu \gamma^\nu \gamma^\rho \gamma^\mu D_\rho \psi_\nu + i \bar{\lambda} \gamma^\rho D_\rho \lambda \\ &- \frac{i}{2\sqrt{2}} \bar{\psi}_\mu \bar{F}_{\alpha\beta} \gamma^\alpha \gamma^\beta \gamma^\mu \lambda + \frac{i}{2\sqrt{2}} \bar{\lambda} \gamma^\nu \gamma^\alpha \gamma^\beta \bar{F}_{\alpha\beta} \psi_\nu. \end{aligned} \quad (4.8)$$

Since we consider the Euclidean continuation of  $\mathcal{N} = 1$  supergravity in 4D Einstein-Maxwell theory, the gamma matrices are Hermitian. Hence, the differential operator  $O^{mn}$  is linear, Dirac type and Hermitian. In order to compute the Seeley-DeWitt coefficients following the general technique described in section 2.2, it is to be made Laplacian. Following (2.13) we have,

$$\begin{aligned} \tilde{\varphi}_m \Lambda^{mn} \tilde{\varphi}_n &= \tilde{\varphi}_m \left\{ (O)^m_p (O)^{pn} \right\} \tilde{\varphi}_n \\ &= \bar{\psi}_\mu \left\{ -\frac{1}{4} \gamma^\tau \gamma^\rho \gamma^\mu \gamma^\nu \gamma^\sigma \gamma_\tau D_\rho D_\sigma + \frac{1}{8} \gamma^\alpha \gamma^\beta \gamma^\mu \gamma^\nu \gamma^\theta \gamma^\phi \bar{F}_{\alpha\beta} \bar{F}_{\theta\phi} \right\} \psi_\nu \\ &+ \bar{\lambda} \left\{ \frac{1}{8} \gamma^\tau \gamma^\alpha \gamma^\beta \gamma^\theta \gamma^\phi \gamma_\tau \bar{F}_{\alpha\beta} \bar{F}_{\theta\phi} - \gamma^\rho \gamma^\sigma D_\rho D_\sigma \right\} \lambda \\ &+ \bar{\psi}_\mu \left\{ -\frac{1}{4\sqrt{2}} \gamma^\tau \gamma^\rho \gamma^\mu \gamma^\alpha \gamma^\beta \gamma_\tau (D_\rho \bar{F}_{\alpha\beta} + \bar{F}_{\alpha\beta} D_\rho) + \frac{1}{2\sqrt{2}} \gamma^\alpha \gamma^\beta \gamma^\mu \gamma^\rho \bar{F}_{\alpha\beta} D_\rho \right\} \lambda \\ &+ \bar{\lambda} \left\{ \frac{1}{4\sqrt{2}} \gamma^\tau \gamma^\alpha \gamma^\beta \gamma^\nu \gamma^\rho \gamma_\tau \bar{F}_{\alpha\beta} D_\rho - \frac{1}{2\sqrt{2}} \gamma^\rho \gamma^\nu \gamma^\alpha \gamma^\beta (D_\rho \bar{F}_{\alpha\beta} + \bar{F}_{\alpha\beta} D_\rho) \right\} \psi_\nu. \end{aligned} \quad (4.9)$$

<sup>14</sup>The spinors in (4.6) satisfy the Majorana condition. But in order to follow the heat kernel method, we first compute the Seeley-DeWitt coefficients for Dirac spinors and then obtain the same for the Majorana spinors.

The form of  $\Lambda^{mn}$  can further be simplified by using various identities and ignoring terms dependent on the Ricci scalar  $R$ ,

$$\begin{aligned}
 \tilde{\varphi}_m \Lambda^{mn} \tilde{\varphi}_n &= -\mathbb{I}_4 \bar{g}^{\mu\nu} \bar{\psi}_\mu D^\rho D_\rho \psi_\nu - \mathbb{I}_4 \bar{\lambda} D^\rho D_\rho \lambda + \bar{\psi}_\mu \left\{ R^{\mu\nu} - \frac{1}{2} \gamma^\nu \gamma^\theta R^\mu{}_\theta \right. \\
 &\quad \left. + \frac{1}{2} \gamma^\mu \gamma^\theta R^\nu{}_\theta - \frac{1}{2} \gamma^\rho \gamma^\sigma R^{\mu\nu}{}_{\rho\sigma} + \frac{1}{8} \gamma^\alpha \gamma^\beta \gamma^\mu \gamma^\nu \gamma^\theta \gamma^\phi \bar{F}_{\alpha\beta} \bar{F}_{\theta\phi} \right\} \psi_\nu \\
 &\quad + \frac{1}{4} \bar{\lambda} \left\{ \left( \gamma^\phi \gamma^\alpha \gamma^\beta \gamma^\theta + \gamma^\theta \gamma^\beta \gamma^\alpha \gamma^\phi \right) \bar{F}_{\alpha\beta} \bar{F}_{\theta\phi} \right\} \lambda \\
 &\quad - \frac{1}{2\sqrt{2}} \bar{\lambda} \left\{ \gamma^\rho \gamma^\nu \gamma^\alpha \gamma^\beta (D_\rho \bar{F}_{\alpha\beta}) \right\} \psi_\nu \\
 &\quad - \frac{1}{2\sqrt{2}} \bar{\psi}_\mu \left\{ \left( \gamma^\beta \gamma^\rho \gamma^\mu \gamma^\alpha + \gamma^\alpha \gamma^\mu \gamma^\rho \gamma^\beta \right) (D_\rho \bar{F}_{\alpha\beta}) \right\} \lambda \\
 &\quad - \frac{1}{2\sqrt{2}} \bar{\psi}_\mu \left\{ \left( \gamma^\beta \gamma^\rho \gamma^\mu \gamma^\alpha + \gamma^\alpha \gamma^\mu \gamma^\rho \gamma^\beta - \gamma^\alpha \gamma^\beta \gamma^\mu \gamma^\rho \right) \bar{F}_{\alpha\beta} \right\} D_\rho \lambda \\
 &\quad + \frac{1}{2\sqrt{2}} \bar{\lambda} \left\{ \left( \gamma^\rho \gamma^\alpha \gamma^\beta \gamma^\nu + \gamma^\nu \gamma^\beta \gamma^\alpha \gamma^\rho - \gamma^\rho \gamma^\nu \gamma^\alpha \gamma^\beta \right) \bar{F}_{\alpha\beta} \right\} D_\rho \psi_\nu.
 \end{aligned} \tag{4.10}$$

The Laplacian operator  $\Lambda$  obtained in (4.10) fits in the essential format (2.12) required for the heat kernel analysis, thus providing the expressions of  $\mathbb{I}$ ,  $N^\rho$  and  $P$  as

$$\tilde{\varphi}_m \mathbb{I}^{mn} \tilde{\varphi}_n = \bar{\psi}_\mu \mathbb{I}_4 g^{\mu\nu} \psi_\nu + \bar{\lambda} \mathbb{I}_4 \lambda, \tag{4.11}$$

$$\begin{aligned}
 \tilde{\varphi}_m P^{mn} \tilde{\varphi}_n &= \bar{\psi}_\mu \left\{ -R^{\mu\nu} \mathbb{I}_4 + \frac{1}{2} \gamma^\nu \gamma^\alpha R^\mu{}_\alpha \right. \\
 &\quad \left. - \frac{1}{2} \gamma^\mu \gamma^\alpha R^\nu{}_\alpha + \frac{1}{2} \gamma^\eta \gamma^\rho R^{\mu\nu}{}_{\eta\rho} - \frac{1}{8} \gamma^\alpha \gamma^\beta \gamma^\mu \gamma^\nu \gamma^\theta \gamma^\phi \bar{F}_{\alpha\beta} \bar{F}_{\theta\phi} \right\} \psi_\nu \\
 &\quad - \frac{1}{4} \bar{\lambda} \left\{ \gamma^\phi \gamma^\alpha \gamma^\beta \gamma^\theta + \gamma^\theta \gamma^\beta \gamma^\alpha \gamma^\phi \right\} \bar{F}_{\alpha\beta} \bar{F}_{\theta\phi} \lambda \\
 &\quad + \frac{1}{2\sqrt{2}} \bar{\psi}_\mu \left\{ \left( \gamma^\beta \gamma^\rho \gamma^\mu \gamma^\alpha + \gamma^\alpha \gamma^\mu \gamma^\rho \gamma^\beta \right) D_\rho \bar{F}_{\alpha\beta} \right\} \lambda \\
 &\quad + \frac{1}{2\sqrt{2}} \bar{\lambda} \left\{ \gamma^\rho \gamma^\nu \gamma^\alpha \gamma^\beta D_\rho \bar{F}_{\alpha\beta} \right\} \psi_\nu,
 \end{aligned} \tag{4.12}$$

$$\begin{aligned}
 \tilde{\varphi}_m (N^\rho)^{mn} \tilde{\varphi}_n &= \frac{1}{2\sqrt{2}} \bar{\psi}_\mu \left\{ \left( \gamma^\beta \gamma^\rho \gamma^\mu \gamma^\alpha + \gamma^\alpha \gamma^\mu \gamma^\rho \gamma^\beta - \gamma^\alpha \gamma^\beta \gamma^\mu \gamma^\rho \right) \bar{F}_{\alpha\beta} \right\} \lambda \\
 &\quad - \frac{1}{2\sqrt{2}} \bar{\lambda} \left\{ \left( \gamma^\rho \gamma^\alpha \gamma^\beta \gamma^\nu + \gamma^\nu \gamma^\beta \gamma^\alpha \gamma^\rho - \gamma^\rho \gamma^\nu \gamma^\alpha \gamma^\beta \right) \bar{F}_{\alpha\beta} \right\} \psi_\nu.
 \end{aligned} \tag{4.13}$$

One can extract  $\omega^\rho$  from  $N^\rho$  by using (2.17), which on further simplification using gamma matrices properties, reads as

$$\begin{aligned}
 \tilde{\varphi}_m (\omega^\rho)^{mn} \tilde{\varphi}_n &= \bar{\psi}_\mu \left\{ -\frac{1}{4\sqrt{2}} \gamma^\theta \gamma^\phi \gamma^\rho \gamma^\mu \bar{F}_{\theta\phi} + \frac{1}{\sqrt{2}} \gamma^\tau \gamma^\rho \bar{F}^\mu{}_\tau - \frac{1}{\sqrt{2}} \gamma^\tau \gamma^\mu \bar{F}^\rho{}_\tau \right\} \lambda \\
 &\quad + \bar{\lambda} \left\{ \frac{1}{4\sqrt{2}} \gamma^\nu \gamma^\rho \gamma^\theta \gamma^\phi \bar{F}_{\theta\phi} + \frac{1}{\sqrt{2}} \gamma^\rho \gamma^\tau \bar{F}^\nu{}_\tau - \frac{1}{\sqrt{2}} \gamma^\nu \gamma^\tau \bar{F}^\rho{}_\tau \right\} \psi_\nu.
 \end{aligned} \tag{4.14}$$

The strategy is to express the kinetic differential operator  $\Lambda$  in the form that fits in the prescription (2.14) and find the required quantities. Determination of  $E$  and  $\Omega_{\alpha\beta}$  from  $P$ ,  $N^\rho$  and  $\omega^\rho$  is now straightforward by using (2.18), (2.19), (4.12) and (4.14).<sup>15</sup> One can then

<sup>15</sup>The full expression of  $E$  and  $\Omega_{\alpha\beta}$  are given in (A.2) and (A.7).

calculate the required trace values from the expression of  $\mathbb{I}$ ,  $E$  and  $\Omega_{\alpha\beta}$ . The results are

$$\begin{aligned}
 \text{tr}(\mathbb{I}) &= 16 + 4 = 20, \\
 \text{tr}(E) &= -8\bar{F}^{\mu\nu}\bar{F}_{\mu\nu}, \\
 \text{tr}(E^2) &= 10(\bar{F}^{\mu\nu}\bar{F}_{\mu\nu})^2 + 3R^{\mu\nu}R_{\mu\nu} + 2R^{\mu\nu\theta\phi}R_{\mu\nu\theta\phi} - 2R_{\alpha\beta\theta\phi}\bar{F}^{\alpha\beta}\bar{F}^{\theta\phi}, \\
 \text{tr}(\Omega^{\alpha\beta}\Omega_{\alpha\beta}) &= -\frac{13}{2}R^{\mu\nu\theta\phi}R_{\mu\nu\theta\phi} + 12R_{\alpha\beta\theta\phi}\bar{F}^{\alpha\beta}\bar{F}^{\theta\phi} - 6R^{\mu\nu}R_{\mu\nu} \\
 &\quad - 60(\bar{F}^{\mu\nu}\bar{F}_{\mu\nu})^2.
 \end{aligned} \tag{4.15}$$

In the above trace result of  $\mathbb{I}$ , the values 16 and 4 correspond to the degrees of freedom associated with the gravitino and gaugino fields, respectively. So, the relevant Seeley-DeWitt coefficients for the gauged fermionic part, computed using (4.15) in (2.20) for the Majorana fermions, are

$$\begin{aligned}
 (4\pi)^2 a_0^f(x) &= -10, \\
 (4\pi)^2 a_2^f(x) &= 4\bar{F}^{\mu\nu}\bar{F}_{\mu\nu}, \\
 (4\pi)^2 a_4^f(x) &= -\frac{1}{144} \left( 41R^{\mu\nu\theta\phi}R_{\mu\nu\theta\phi} + 64R^{\mu\nu}R_{\mu\nu} \right).
 \end{aligned} \tag{4.16}$$

Here a -ve sign corresponding to the value of  $\chi$  from (2.20) for fermion spin-statistics and a factor of 1/2 for considering Majorana degree of freedom are imposed manually in determining the above coefficients.

Next, we aim to compute the Seeley-DeWitt coefficients for the ghost fields described by the lagrangian (4.4). The ghost fields  $\tilde{b}$ ,  $\tilde{c}$  and  $\tilde{e}$  are three minimally coupled Majorana spin 1/2 fermions. So, the contribution of Seeley-DeWitt coefficients from the ghost fields will be -3 times of the coefficients of a free Majorana spin 1/2 field,

$$a_{2n}^{\text{ghost},f}(x) = -3a_{2n}^{1/2,f}(x), \tag{4.17}$$

where  $a_{2n}^{1/2,f}(x)$  is the Seeley-DeWitt coefficients for a free Majorana spin 1/2 field calculated in [40]. The Seeley-DeWitt coefficients for ghost sector are

$$\begin{aligned}
 (4\pi)^2 a_0^{\text{ghost},f}(x) &= 6, \\
 (4\pi)^2 a_2^{\text{ghost},f}(x) &= 0, \\
 (4\pi)^2 a_4^{\text{ghost},f}(x) &= -\frac{1}{240} \left( 7R^{\mu\nu\theta\phi}R_{\mu\nu\theta\phi} + 8R_{\mu\nu}R^{\mu\nu} \right).
 \end{aligned} \tag{4.18}$$

The net Seeley-DeWitt coefficients for the fermionic sector can be computed by summing up the results (4.16) and (4.18),

$$\begin{aligned}
 (4\pi)^2 a_0^F(x) &= -4, \\
 (4\pi)^2 a_2^F(x) &= 4\bar{F}^{\mu\nu}\bar{F}_{\mu\nu}, \\
 (4\pi)^2 a_4^F(x) &= -\frac{1}{360} \left( 113R^{\mu\nu\theta\phi}R_{\mu\nu\theta\phi} + 172R^{\mu\nu}R_{\mu\nu} \right).
 \end{aligned} \tag{4.19}$$

### 4.3 Total Seeley-DeWitt coefficients

The total Seeley-DeWitt Coefficients obtained by adding the bosonic contribution (4.2) and the fermionic contribution (4.19) would be,

$$\begin{aligned}
 (4\pi)^2 a_0^{\text{B+F}}(x) &= 0, \\
 (4\pi)^2 a_2^{\text{B+F}}(x) &= 10 \bar{F}^{\mu\nu} \bar{F}_{\mu\nu}, \\
 (4\pi)^2 a_4^{\text{B+F}}(x) &= \frac{1}{24} \left( 19 R^{\mu\nu\theta\phi} R_{\mu\nu\theta\phi} - 8 R^{\mu\nu} R_{\mu\nu} \right).
 \end{aligned}
 \tag{4.20}$$

The coefficient  $a_4^{\text{B+F}}$  obtained in (4.20) is independent of the terms  $R_{\mu\nu\theta\phi} \bar{F}^{\mu\nu} \bar{F}^{\theta\phi}$  and  $(\bar{F}^{\mu\nu} \bar{F}_{\mu\nu})^2$ , and only depends upon the background metric which predicts the rotational invariance of the result under electric and magnetic duality.

## 5 Logarithmic correction of extremal black holes in “non-minimal” $\mathcal{N} = 1, d = 4$ EMSGT

In this section, we discuss the applicability of Seeley-DeWitt coefficients in the area of black hole physics. We use our results, particularly  $a_4$  coefficient for the computation of logarithmic correction part of extremal black hole entropy in “non-minimal”  $\mathcal{N} = 1, d = 4$  EMSGT.

### 5.1 General methodology for computing logarithmic correction using Seeley-DeWitt coefficient

We review the particular procedure adopted in [29, 30, 33–36] for determining the logarithmic correction of entropy of extremal black holes using quantum entropy function formalism [41–43]. An extremal black hole is defined with near horizon geometry  $\text{AdS}_2 \times \mathcal{K}$ , where  $\mathcal{K}$  is a compactified space fibered over  $\text{AdS}_2$  space. The quantum corrected entropy ( $S_{\text{BH}}$ ) of an extremal black hole is related to the near horizon partition function ( $\mathcal{Z}_{\text{AdS}_2}$ ) via  $\text{AdS}_2/CFT_1$  correspondence [41] as

$$e^{S_{\text{BH}} - E_0 \beta} = \mathcal{Z}_{\text{AdS}_2}, \tag{5.1}$$

where  $E_0$  is ground state energy of the extremal black hole carrying charges and  $\beta$  is the boundary length of regularized  $\text{AdS}_2$ . We denote coordinate of  $\text{AdS}_2$  by  $(\eta, \theta)$  and that of  $\mathcal{K}$  by  $(y)$ . If the volume of  $\text{AdS}_2$  is made finite by inserting an IR cut-off  $\eta \leq \eta_0$  for regularization, then the one-loop correction to the partition function  $\mathcal{Z}_{\text{AdS}_2}$  is presented as

$$\mathcal{Z}_{\text{AdS}_2}^{\text{1-loop}} = e^{-W}, \tag{5.2}$$

with the one-loop effective action,

$$\begin{aligned}
 W &= \int_0^{\eta_0} d\eta \int_0^{2\pi} d\theta \int_{\text{hor}} d^2 y \sqrt{\bar{g}} \Delta \mathcal{L}_{\text{eff}} \\
 &= 2\pi (\cosh \eta_0 - 1) \int_{\text{hor}} d^2 y G(y) \Delta \mathcal{L}_{\text{eff}},
 \end{aligned}
 \tag{5.3}$$

where  $G(y) = \sqrt{g}/\sinh \eta$  and  $\Delta\mathcal{L}_{\text{eff}}$  is the lagrangian density corresponding to the one-loop effective action. The integration limit over  $(y)$  depends upon the geometry associated with  $\mathcal{K}$  space. The above form of  $W$  can be equivalently interpreted as [35, 36]

$$W = -\beta\Delta E_0 + \mathcal{O}(\beta^{-1}) - 2\pi \int_{\text{hor}} d^2y G(y)\Delta\mathcal{L}_{\text{eff}}, \quad (5.4)$$

where  $\beta = 2\pi \sinh \eta_0$  and the shift in the ground state energy  $\Delta E_0 = -\int d^2y G(y)\Delta\mathcal{L}_{\text{eff}}$ . For the extremal black holes, the  $W$  form (5.4) extracts the one-loop corrected entropy ( $\Delta S_{\text{BH}}$ ) from the  $\beta$  independent part of the relation (5.1) as

$$\Delta S_{\text{BH}} = 2\pi \int_{\text{hor}} d^2y G(y)\Delta\mathcal{L}_{\text{eff}}. \quad (5.5)$$

These one-loop corrections serve as logarithmic corrections to the entropy of extremal black holes if one considers only massless fluctuations in the one-loop. In the standard heat kernel analysis of the one-loop effective lagrangian  $\Delta\mathcal{L}_{\text{eff}}$ , only the  $a_4(x)$  Seeley-DeWitt coefficient provides a term proportional to logarithmic of horizon area  $A_H$  [29, 30],

$$\Delta\mathcal{L}_{\text{eff}} \simeq -\frac{\chi}{2} \left( a_4(x) - (Y-1)K^{\text{zm}}(x, x; 0) \right) \ln A_H, \quad (5.6)$$

where  $K^{\text{zm}}(x, x; 0)$  is the heat kernel contribution for the modes of  $\Lambda$  having vanishing eigenvalues and  $Y$  is the scaling dimension associated with zero mode corrections of massless fields under considerations. For extremal black holes defined in large charge and angular momentum limit, the present set up computes the logarithmic correction into two parts — a local part ( $\Delta S_{\text{local}}$ ) controlled by the  $a_4(x)$  coefficient and a zero mode part ( $\Delta S_{\text{zm}}$ ) controlled by the zero modes of the massless fluctuations in the near-horizon,<sup>16</sup>

$$\Delta S_{\text{local}} = -\pi \int_{\text{hor}} d^2y G(y)\chi a_4(x) \ln A_H, \quad (5.7)$$

$$\Delta S_{\text{zm}} = -\frac{1}{2} \left( \sum_{r \in \text{B}} (Y_r - 1)M^r - \sum_{r \in \text{F}} (2Y_r - 1)M^r \right) \ln A_H. \quad (5.8)$$

$M_r$  and  $Y_r$  are respectively the number of zero modes and scaling dimension for different boson (B) and fermion (F) fluctuations. These quantities take different values depending on dimensionality and types of field present. In four dimensions we have [29, 31–37, 39],

$$Y_r = \begin{cases} 2 & : \text{metric,} \\ 1 & : \text{gauge fields,} \\ 1/2 & : \text{spin 1/2 fields,} \\ 3/2 & : \text{gravitino fields,} \end{cases} \quad (5.9)$$

$$M_r = \begin{cases} 3 + X & : \text{metric in extremal limit,} \\ 0 & : \text{gravitino field for non-BPS solutions,} \end{cases} \quad (5.10)$$

---

<sup>16</sup>Total logarithmic correction is obtained as  $\Delta S_{\text{BH}} = \Delta S_{\text{local}} + \Delta S_{\text{zm}}$ .



where  $X$  is the number of rotational isometries associated with the angular momentum  $J$  of black holes,

$$X = \begin{cases} 1 & \forall J_3 = \text{constant}, J^2 = \text{arbitrary}, \\ 3 & \forall J_3 = J^2 = 0. \end{cases} \quad (5.11)$$

One should not be concerned about the  $M_r$  values of the gauge and gaugino fields because their particular  $Y_r$  values suggest that they have no zero-mode correction contribution in the formula (5.8). The local mode contribution (5.7) for the extremal solutions of the  $\mathcal{N} = 1, d = 4$  EMSGT can be computed from the coefficient  $a_4(x)$  (4.20) in the near-horizon geometry.

## 5.2 Extremal black holes in “non-minimal” $\mathcal{N} = 1, d = 4$ EMSGT and their logarithmic corrections

An  $\mathcal{N} = 1, d = 4$  EMSGT can have Kerr-Newman, Kerr and Reissner-Nordström solutions. The metric  $\bar{g}$  of general Kerr-Newman black hole with mass  $M$ , charge  $Q$  and angular momentum  $J$  is given by

$$\begin{aligned} ds^2 = & -\frac{r^2 + b^2 \cos^2 \psi - 2Mr + Q^2}{r^2 + b^2 \cos^2 \psi} dt^2 \\ & + \frac{r^2 + b^2 \cos^2 \psi}{r^2 + b^2 - 2Mr + Q^2} dr^2 + (r^2 + b^2 \cos^2 \psi) d\psi^2 \\ & + \left( \frac{(r^2 + b^2 \cos^2 \psi)(r^2 + b^2) + (2Mr - Q^2)b^2 \sin^2 \psi}{r^2 + b^2 \cos^2 \psi} \right) \sin^2 \psi d\phi^2 \\ & + 2 \frac{(Q^2 - 2Mr)b}{r^2 + b^2 \cos^2 \psi} \sin^2 \psi dt d\phi. \end{aligned} \quad (5.12)$$

where  $b = J/M$ . The horizon radius is given as

$$r = M + \sqrt{M^2 - (Q^2 + b^2)}. \quad (5.13)$$

At the extremal limit  $M \rightarrow \sqrt{b^2 + Q^2}$ , the classical entropy of the black hole in near horizon becomes

$$S_{\text{cl}} = 16\pi^2(2b^2 + Q^2), \quad (5.14)$$

and the extremal near horizon Kerr-Newman metric is given by [29]

$$\begin{aligned} ds^2 = & (Q^2 + b^2 + b^2 \cos^2 \psi)(d\eta^2 + \sinh^2 \eta d\theta^2 + d\psi^2) \\ & + \frac{(Q^2 + 2b^2)^2}{(Q^2 + b^2 + b^2 \cos^2 \psi)} \sin^2 \psi \left( d\chi - i \frac{2Mb}{Q^2 + 2b^2} (\cosh \eta - 1) d\theta \right)^2. \end{aligned} \quad (5.15)$$

For the purpose of using the  $a_4(x)$  result (4.20) in the relation (5.7), we consider the background invariants [55, 56],

$$R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} = \frac{8}{(r^2 + b^2 \cos^2 \psi)^6} \left\{ 6M^2(r^6 - 15b^2r^4 \cos^2 \psi + 15b^4r^2 \cos^4 \psi - b^6 \cos^6 \psi) - 12MQ^2r(r^4 - 10r^2b^2 \cos^2 \psi + 5b^4 \cos^4 \psi) + Q^4(7r^4 - 34r^2b^2 \cos^2 \psi + 7b^4 \cos^4 \psi) \right\}, \quad (5.16)$$

$$R^{\mu\nu}R_{\mu\nu} = \frac{4Q^4}{(r^2 + b^2 \cos^2 \psi)^4},$$

$$G(y)_{\text{KN}} = G(\psi) = (Q^2 + b^2 + b^2 \cos^2 \psi)(Q^2 + 2b^2) \sin \psi.$$

In the extremal limit, one can find [29]

$$\int_{\text{hor}} d\psi d\phi G(\psi) R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} = \frac{8\pi}{e(e^2 + 1)^{5/2}(2e^2 + 1)} \left\{ 3(2e^2 + 1)^2 \tan^{-1} \left( \frac{e}{\sqrt{e^2 + 1}} \right) + e\sqrt{e^2 + 1}(-8e^6 - 20e^4 - 8e^2 + 1) \right\}, \quad (5.17)$$

$$\int_{\text{hor}} d\psi d\phi G(\psi) R^{\mu\nu} R_{\mu\nu} = \frac{2\pi}{e(e^2 + 1)^{5/2}(2e^2 + 1)} \left\{ 3(2e^2 + 1)^2 \tan^{-1} \left( \frac{e}{\sqrt{e^2 + 1}} \right) + e\sqrt{e^2 + 1}(8e^2 + 5) \right\}, \quad (5.18)$$

where  $e = b/Q$ . The local mode contribution in the logarithmic correction to the entropy of Kerr-Newman extremal black holes in  $\mathcal{N} = 1, d = 4$  EMSGT can be drawn by using the results (4.20), (5.17) and (5.18) in the relation (5.7),

$$\Delta S_{\text{local,KN}} = \frac{1}{48} \left\{ -\frac{51(2e^2 + 1) \tan^{-1} \left( \frac{e}{\sqrt{e^2 + 1}} \right)}{e(e^2 + 1)^{5/2}} + \frac{(152e^6 + 380e^4 + 168e^2 - 9)}{(e^2 + 1)^2(2e^2 + 1)} \right\} \ln A_H. \quad (5.19)$$

For Kerr black holes we set the limit  $e \rightarrow \infty$  in (5.19), which gives the local mode contribution,

$$\Delta S_{\text{local,Kerr}} = \frac{19}{12} \ln A_H. \quad (5.20)$$

And for Reissner-Nordström black holes, we set the limit  $e \rightarrow 0$  in (5.19), yielding the following result

$$\Delta S_{\text{local,RN}} = -\frac{5}{4} \ln A_H. \quad (5.21)$$

The extremal black holes in  $\mathcal{N} = 1, d = 4$  Einstein-Maxwell supergravity theory are non-BPS in nature [46, 48]. Hence, the zero mode contribution to the correction in entropy of non-BPS extremal Kerr-Newman, Kerr and Reissner-Nordström black holes are computed

by using (5.10) to (5.11) in the relation (5.8) as

$$\Delta S_{\text{zm,KN}} = -\frac{1}{2}(2-1)(3+1) \ln A_H = -2 \ln A_H, \quad (5.22)$$

$$\Delta S_{\text{zm,Kerr}} = -\frac{1}{2}(2-1)(3+1) \ln A_H = -2 \ln A_H, \quad (5.23)$$

$$\Delta S_{\text{zm,RN}} = -\frac{1}{2}(2-1)(3+3) \ln A_H = -3 \ln A_H. \quad (5.24)$$

Merging the above local and zero mode contributions, the total logarithmic correction  $\Delta S_{BH}$  ( $= \Delta S_{\text{local}} + \Delta S_{\text{zm}}$ ) to the entropy of extremal Kerr-Newman, Kerr and Reissner-Nordström black holes in “non-minimal”  $\mathcal{N} = 1, d = 4$  EMSGT are computed as

$$\Delta S_{\text{BH,KN}} = \frac{1}{48} \left\{ -\frac{51(2e^2 + 1) \tan^{-1} \left( \frac{e}{\sqrt{e^2 + 1}} \right)}{e(e^2 + 1)^{5/2}} + \frac{(152e^6 + 380e^4 + 168e^2 - 9)}{(e^2 + 1)^2(2e^2 + 1)} \right\} \ln A_H - 2 \ln A_H, \quad (5.25)$$

$$\Delta S_{\text{BH,Kerr}} = \frac{19}{12} \ln A_H - 2 \ln A_H = -\frac{5}{12} \ln A_H, \quad (5.26)$$

$$\Delta S_{\text{BH,RN}} = -\frac{5}{4} \ln A_H - 3 \ln A_H = -\frac{17}{4} \ln A_H. \quad (5.27)$$

## 6 Concluding remarks

In summary, we have analyzed the trace anomalies of quadratic fluctuated fields in the “non-minimal”  $\mathcal{N} = 1, d = 4$  EMSGT and calculated the first three Seeley-DeWitt coefficients using the approach described in section 2.2. An essential advantage of the Seeley-DeWitt coefficient results (4.20) is that these are not background-geometry specific and hence can be used for any arbitrary solution within the “non-minimal”  $\mathcal{N} = 1, d = 4$  EMSGT.

As an application, we have used the third coefficient  $a_4$  in the quantum entropy function formalism and computed the local contribution of logarithmic correction to the Bekenstein-Hawking entropy of extremal black holes in the “non-minimal”  $\mathcal{N} = 1, d = 4$  EMSGT. The zero mode contribution of the logarithmic correction to the entropy has been computed separately by analyzing the number of zero modes and the scaling dimensions of the fields in the theory. We, therefore, obtain the net logarithmic correction to the Bekenstein-Hawking entropy of extremal Kerr-Newman (5.25), Kerr (5.26) and Reissner-Nordström (5.27) non-BPS black holes in the “non-minimal”  $\mathcal{N} = 1, d = 4$  EMSGT. These results are new and significant. In contrast, the logarithmic correction to the entropy of extremal Reissner-Nordström black holes in two other classes of  $\mathcal{N} = 1, d = 4$  EMSGT are computed in [48]. However, our work deals with a more general  $\mathcal{N} = 1, d = 4$  EMSGT theory where the vector multiplet is non-minimally coupled the supergravity multiplet. Due to the presence of non-minimal coupling, it is impossible to reproduce the results of [48] from our work and vice-versa. It will also be interesting to reproduce these results by other methods of the heat kernel. The calculated results may also give some valuable inputs for developing insight regarding  $\mathcal{N} = 1$  Einstein-Maxwell supergravity theory in four dimensions and can also provide an infrared window to any microscopic theory for the same.

## Acknowledgments

The authors would like to thank Ashoke Sen and Rajesh Kumar Gupta for valuable discussions and useful comments on the work. We also acknowledge IISER-Bhopal for the hospitality during the course of this work.

## A Trace calculations

Here, we will explicitly show the expression of  $E$  and  $\Omega_{\alpha\beta}$ , and the computation of traces of  $E$ ,  $E^2$  and  $\Omega_{\alpha\beta}\Omega^{\alpha\beta}$  for the fermionic sector of  $\mathcal{N} = 1$ ,  $d = 4$  EMSGT. The trace results will be used to calculate  $a_4$  required in the logarithmic correction to the entropy of extremal black holes. In trace calculations, we have used the following identities [30]

$$\begin{aligned} (D_\rho \bar{F}_{\mu\nu})(D^\rho \bar{F}^{\mu\nu}) &= R_{\mu\nu\rho\sigma} \bar{F}^{\mu\nu} \bar{F}^{\rho\sigma} - R^{\mu\nu} R_{\mu\nu}, \\ (D_\mu \bar{F}_\rho{}^\nu)(D_\nu \bar{F}^{\rho\mu}) &= \frac{1}{2}(R_{\mu\nu\rho\sigma} \bar{F}^{\mu\nu} \bar{F}^{\rho\sigma} - R^{\mu\nu} R_{\mu\nu}). \end{aligned} \quad (\text{A.1})$$

The expression of  $E$  by using  $\omega_\rho$  (4.14) will be

$$\begin{aligned} \tilde{\varphi}_m E^{mn} \tilde{\varphi}_n &= \bar{\psi}_\mu E^{\psi\mu\psi\nu} \psi_\nu + \bar{\lambda} E^{\lambda\lambda} \lambda + \bar{\psi}_\mu E^{\psi\mu\lambda} \lambda + \bar{\lambda} E^{\lambda\psi\nu} \psi_\nu, \\ &= \bar{\psi}_\mu \left\{ -\frac{3}{8} \gamma^\alpha \gamma^\beta \gamma^\mu \gamma^\nu \gamma^\theta \gamma^\phi \bar{F}_{\alpha\beta} \bar{F}_{\theta\phi} - \frac{1}{4} \gamma^\theta \gamma^\phi \gamma^\alpha \gamma^\mu \bar{F}_{\theta\phi} \bar{F}^\nu{}_\alpha \right. \\ &\quad - \frac{1}{4} \gamma^\theta \gamma^\phi \gamma^\alpha \gamma^\nu \bar{F}_{\theta\phi} \bar{F}^\mu{}_\alpha + \frac{1}{8} \bar{g}^{\mu\nu} \gamma^\alpha \gamma^\beta \gamma^\theta \gamma^\phi \bar{F}_{\alpha\beta} \bar{F}_{\theta\phi} - \frac{3}{2} \gamma^\theta \gamma^\phi \bar{F}^{\mu\nu} \bar{F}_{\theta\phi} \\ &\quad - 3 \gamma^\alpha \gamma^\beta \bar{F}^\mu{}_\alpha \bar{F}^\nu{}_\beta + \frac{1}{2} \gamma^\mu \gamma^\alpha R^\nu{}_\alpha - \gamma^\nu \gamma^\alpha R^\mu{}_\alpha + \frac{1}{2} \gamma^\eta \gamma^\rho R^{\mu\nu}{}_{\eta\rho} \\ &\quad \left. + \frac{3}{4} \gamma^\mu \gamma^\nu \bar{F}^{\theta\phi} \bar{F}_{\theta\phi} - \frac{1}{2} \bar{g}^{\mu\nu} \mathbb{I}_4 \bar{F}^{\theta\phi} \bar{F}_{\theta\phi} + \mathbb{I}_4 R^{\mu\nu} \right\} \psi_\nu + \bar{\lambda} \left\{ \frac{1}{2} \gamma^\alpha \gamma^\beta \gamma^\theta \gamma^\phi \bar{F}_{\alpha\beta} \bar{F}_{\theta\phi} \right\} \lambda \\ &\quad + \bar{\psi}_\mu \left\{ -\frac{3}{4\sqrt{2}} \gamma^\theta \gamma^\phi \gamma^\rho \gamma^\mu D_\rho \bar{F}_{\theta\phi} + \frac{1}{\sqrt{2}} \gamma^\phi \gamma^\rho D_\rho \bar{F}^\mu{}_\phi + \frac{1}{\sqrt{2}} \gamma^\theta \gamma^\phi D^\mu \bar{F}_{\theta\phi} \right\} \lambda \\ &\quad + \bar{\lambda} \left\{ -\frac{3}{4\sqrt{2}} \gamma^\nu \gamma^\rho \gamma^\theta \gamma^\phi D_\rho \bar{F}_{\theta\phi} - \frac{1}{\sqrt{2}} \gamma^\rho \gamma^\theta D_\rho \bar{F}^\nu{}_\theta + \frac{1}{\sqrt{2}} \gamma^\theta \gamma^\phi D^\nu \bar{F}_{\theta\phi} \right\} \psi_\nu. \end{aligned} \quad (\text{A.2})$$

**Trace of  $E$ .**

$$\text{tr}(E) = \text{tr}(E^{\psi\mu}{}_{\psi\mu} + E^\lambda{}_\lambda + E^{\psi\mu}{}_\lambda + E^\lambda{}_{\psi\mu}). \quad (\text{A.3})$$

From (A.2), we have

$$\begin{aligned} \text{tr}(E^{\psi\mu}{}_{\psi\mu}) &= -4 \bar{F}^{\mu\nu} \bar{F}_{\mu\nu}, & \text{tr}(E^{\psi\mu}{}_\lambda) &= 0, \\ \text{tr}(E^\lambda{}_\lambda) &= -4 \bar{F}^{\mu\nu} \bar{F}_{\mu\nu}, & \text{tr}(E^\lambda{}_{\psi\mu}) &= 0. \end{aligned} \quad (\text{A.4})$$

Equations (A.3) and (A.4) gives the  $\text{tr}(E)$  (4.15).

**Trace of  $E^2$ .**

$$\text{tr}(E^2) = \text{tr}\left(E^{\psi_\mu}_{\psi_\nu} E^{\psi_\nu}_{\psi_\mu} + E^\lambda_\lambda E^\lambda_\lambda + E^{\psi_\mu}_\lambda E^\lambda_{\psi_\mu} + E^\lambda_{\psi_\mu} E^{\psi_\mu}_\lambda\right). \quad (\text{A.5})$$

Using eq. (A.2), we have

$$\begin{aligned} \text{tr}(E^{\psi_\mu}_{\psi_\nu} E^{\psi_\nu}_{\psi_\mu}) &= 5R^{\mu\nu} R_{\mu\nu} + 2(\bar{F}^{\mu\nu} \bar{F}_{\mu\nu})^2 + 2R^{\mu\nu\theta\phi} R_{\mu\nu\theta\phi}, \\ \text{tr}(E^\lambda_\lambda E^\lambda_\lambda) &= 8(\bar{F}^{\mu\nu} \bar{F}_{\mu\nu})^2 - 4R^{\mu\nu} R_{\mu\nu}, \\ \text{tr}(E^{\psi_\mu}_\lambda E^\lambda_{\psi_\mu}) &= R^{\mu\nu} R_{\mu\nu} - R_{\mu\nu\theta\phi} \bar{F}^{\mu\nu} \bar{F}^{\theta\phi}, \\ \text{tr}(E^\lambda_{\psi_\mu} E^{\psi_\mu}_\lambda) &= R^{\mu\nu} R_{\mu\nu} - R_{\mu\nu\theta\phi} \bar{F}^{\mu\nu} \bar{F}^{\theta\phi}. \end{aligned} \quad (\text{A.6})$$

Equations (A.5) and (A.6) yields the  $\text{tr}(E^2)$  (4.15).

**Trace of  $\Omega_{\alpha\beta} \Omega^{\alpha\beta}$ .** The expression for  $\Omega_{\alpha\beta}$  using  $\omega_\rho$  (4.14) is

$$\begin{aligned} \tilde{\varphi}_m(\Omega_{\alpha\beta})^{mn} \tilde{\varphi}_n &= \bar{\psi}_\mu(\Omega_{\alpha\beta})^{\psi_\mu\psi_\nu} \psi_\nu + \bar{\lambda}(\Omega_{\alpha\beta})^{\lambda\lambda} \lambda + \bar{\psi}_\mu(\Omega_{\alpha\beta})^{\psi_\mu\lambda} \lambda + \bar{\lambda}(\Omega_{\alpha\beta})^{\lambda\psi_\nu} \psi_\nu \\ &= \bar{\psi}_\mu \left\{ \mathbb{I}_4 R^{\mu\nu}_{\alpha\beta} + \frac{1}{4} \bar{g}^{\mu\nu} \gamma^\xi \gamma^\kappa R_{\alpha\beta\xi\kappa} + [\omega_\alpha, \omega_\beta]^{\psi_\mu\psi_\nu} \right\} \psi_\nu \\ &\quad + \bar{\lambda} \left\{ \frac{1}{4} \gamma^\theta \gamma^\phi R_{\alpha\beta\theta\phi} + [\omega_\alpha, \omega_\beta]^{\lambda\lambda} \right\} \lambda \\ &\quad + \bar{\psi}_\mu \left\{ D_{[\alpha} \omega_{\beta]}^{\psi_\mu\lambda} \right\} \lambda + \bar{\lambda} \left\{ D_{[\alpha} \omega_{\beta]}^{\lambda\psi_\nu} \right\} \psi_\nu. \end{aligned} \quad (\text{A.7})$$

One can calculate  $\text{tr}(\Omega_{\alpha\beta} \Omega^{\alpha\beta})$  as

$$\begin{aligned} \text{tr}(\Omega^{\alpha\beta} \Omega_{\alpha\beta}) &= \text{tr} \left\{ (\Omega_{\alpha\beta})_{\psi_\mu}^{\psi_\nu} (\Omega^{\alpha\beta})_{\psi_\nu}^{\psi_\mu} + (\Omega_{\alpha\beta})_\lambda^\lambda (\Omega^{\alpha\beta})_\lambda^\lambda \right. \\ &\quad \left. + (\Omega_{\alpha\beta})_{\psi_\mu}^\lambda (\Omega^{\alpha\beta})_\lambda^{\psi_\mu} + (\Omega_{\alpha\beta})_\lambda^{\psi_\nu} (\Omega^{\alpha\beta})_{\psi_\nu}^\lambda \right\}, \end{aligned} \quad (\text{A.8})$$

where

$$\begin{aligned} &(\Omega_{\alpha\beta})_{\psi_\mu}^{\psi_\nu} (\Omega^{\alpha\beta})_{\psi_\nu}^{\psi_\mu} \\ &= \left\{ \underbrace{\mathbb{I}_4 R_{\mu\nu}^{\alpha\beta}}_{R_1} + \frac{1}{4} \underbrace{\bar{g}_{\mu\nu} \gamma^\xi \gamma^\kappa R_{\alpha\beta\xi\kappa}}_{R_2} + \underbrace{\left( (\omega_\alpha)_{\psi_\mu}^\lambda (\omega_\beta)_{\psi_\nu}^{\psi_\mu} - (\omega_\beta)_{\psi_\mu}^\lambda (\omega_\alpha)_{\psi_\nu}^{\psi_\mu} \right)}_{R_3} \right\} \\ &\quad \times \left\{ \underbrace{\mathbb{I}_4 R_{\nu\mu}^{\alpha\beta}}_{S_1} + \frac{1}{4} \underbrace{\bar{g}_{\nu\mu} \gamma^\sigma \gamma^\kappa R^{\alpha\beta}_{\sigma\kappa}}_{S_2} + \underbrace{\left( (\omega^\alpha)_{\psi_\nu}^\lambda (\omega^\beta)_{\psi_\mu}^{\psi_\nu} - (\omega^\beta)_{\psi_\nu}^\lambda (\omega^\alpha)_{\psi_\mu}^{\psi_\nu} \right)}_{S_3} \right\}, \end{aligned} \quad (\text{A.9})$$

$$\begin{aligned} &(\Omega_{\alpha\beta})_\lambda^\lambda (\Omega^{\alpha\beta})_\lambda^\lambda = \left\{ \frac{1}{4} \gamma^\theta \gamma^\phi R_{\alpha\beta\theta\phi} + \underbrace{\left( (\omega_\alpha)_\lambda^{\psi_\mu} (\omega_\beta)_{\psi_\mu}^\lambda - (\omega_\beta)_\lambda^{\psi_\mu} (\omega_\alpha)_{\psi_\mu}^\lambda \right)}_{R_5} \right\} \\ &\quad \times \left\{ \frac{1}{4} \gamma^\rho \gamma^\sigma R^{\alpha\beta}_{\rho\sigma} + \underbrace{\left( (\omega^\alpha)_\lambda^{\psi_\nu} (\omega^\beta)_{\psi_\nu}^\lambda - (\omega^\beta)_\lambda^{\psi_\nu} (\omega^\alpha)_{\psi_\nu}^\lambda \right)}_{S_5} \right\}, \end{aligned} \quad (\text{A.10})$$

$$(\Omega_{\alpha\beta})_{\psi_\mu}^\lambda (\Omega^{\alpha\beta})_\lambda^{\psi_\mu} = \underbrace{\left( D_\alpha (\omega_\beta)_{\psi_\mu}^\lambda - D_\beta (\omega_\alpha)_{\psi_\mu}^\lambda \right)}_{R_6} \times \underbrace{\left( D^\alpha (\omega^\beta)_\lambda^{\psi_\mu} - D^\beta (\omega^\alpha)_\lambda^{\psi_\mu} \right)}_{S_6}, \quad (\text{A.11})$$

and

$$(\Omega_{\alpha\beta})_{\lambda}^{\psi\nu} (\Omega^{\alpha\beta})_{\psi\nu}^{\lambda} = \underbrace{(D_{\alpha}(\omega_{\beta})_{\lambda}^{\psi\nu} - D_{\beta}(\omega_{\alpha})_{\lambda}^{\psi\nu})}_{R_7} \times \underbrace{(D^{\alpha}(\omega^{\beta})_{\psi\nu}^{\lambda} - D^{\beta}(\omega^{\alpha})_{\psi\nu}^{\lambda})}_{S_7}. \quad (\text{A.12})$$

Then,

$$\begin{aligned} \text{tr}(R_1 S_1) &= -4R^{\mu\nu\theta\phi} R_{\mu\nu\theta\phi}, & \text{tr}(R_3 S_3) &= 6R^{\mu\nu} R_{\mu\nu} - 36(\bar{F}^{\mu\nu} \bar{F}_{\mu\nu})^2, \\ \text{tr}(R_2 S_2) &= -2R^{\mu\nu\theta\phi} R_{\mu\nu\theta\phi}, & \text{tr}(R_4 S_4) &= -\frac{1}{2}R^{\mu\nu\theta\phi} R_{\mu\nu\theta\phi}, \\ \text{tr}(R_1 S_2) &= 0, & \text{tr}(R_4 S_5) &= -2R^{\mu\nu} R_{\mu\nu}, \\ \text{tr}(R_2 S_1) &= 0, & \text{tr}(R_5 S_4) &= -2R^{\mu\nu} R_{\mu\nu}, \\ \text{tr}(R_1 S_3) &= -4R_{\mu\nu\theta\phi} \bar{F}^{\mu\nu} \bar{F}^{\theta\phi}, & \text{tr}(R_5 S_5) &= 8R^{\mu\nu} R_{\mu\nu} - 24(\bar{F}^{\mu\nu} \bar{F}_{\mu\nu})^2, \\ \text{tr}(R_3 S_1) &= -4R_{\mu\nu\theta\phi} \bar{F}^{\mu\nu} \bar{F}^{\theta\phi}, & \text{tr}(R_6 S_6) &= 10R_{\mu\nu\theta\phi} \bar{F}^{\mu\nu} \bar{F}^{\theta\phi} - 10R^{\mu\nu} R_{\mu\nu}, \\ \text{tr}(R_2 S_3) &= 2R^{\mu\nu} R_{\mu\nu}, & \text{tr}(R_7 S_7) &= 10R_{\mu\nu\theta\phi} \bar{F}^{\mu\nu} \bar{F}^{\theta\phi} - 10R^{\mu\nu} R_{\mu\nu}. \\ \text{tr}(R_3 S_2) &= 2R^{\mu\nu} R_{\mu\nu}, \end{aligned} \quad (\text{A.13})$$

Equations (A.9) to (A.13) gives

$$\begin{aligned} \text{tr}((\Omega_{\alpha\beta})_{\psi\mu}^{\psi\nu} (\Omega^{\alpha\beta})_{\psi\nu}^{\psi\mu}) &= -6R^{\mu\nu\theta\phi} R_{\mu\nu\theta\phi} - 8R_{\mu\nu\theta\phi} \bar{F}^{\mu\nu} \bar{F}^{\theta\phi} \\ &\quad + 10R^{\mu\nu} R_{\mu\nu} - 36(\bar{F}^{\mu\nu} \bar{F}_{\mu\nu})^2, \\ \text{tr}((\Omega_{\alpha\beta})_{\lambda}^{\lambda} (\Omega^{\alpha\beta})_{\lambda}^{\lambda}) &= -\frac{1}{2}R^{\mu\nu\theta\phi} R_{\mu\nu\theta\phi} + 4R^{\mu\nu} R_{\mu\nu} - 24(\bar{F}^{\mu\nu} \bar{F}_{\mu\nu})^2, \\ \text{tr}((\Omega_{\alpha\beta})_{\psi\mu}^{\lambda} (\Omega^{\alpha\beta})_{\lambda}^{\psi\mu}) &= 10R_{\mu\nu\theta\phi} \bar{F}^{\mu\nu} \bar{F}^{\theta\phi} - 10R^{\mu\nu} R_{\mu\nu}, \\ \text{tr}((\Omega_{\alpha\beta})_{\lambda}^{\psi\nu} (\Omega^{\alpha\beta})_{\psi\nu}^{\lambda}) &= 10R_{\mu\nu\theta\phi} \bar{F}^{\mu\nu} \bar{F}^{\theta\phi} - 10R^{\mu\nu} R_{\mu\nu}. \end{aligned} \quad (\text{A.14})$$

Finally, from (A.8) and (A.14), we have obtained  $\text{tr}(\Omega_{\alpha\beta} \Omega^{\alpha\beta})$  (4.15).

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