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Unparticle decay of neutrinos and its possible signatures at Km^2 detector for (3+1) flavour framework

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Abstract: We consider a scenario where ultra high energy neutrinos undergo unparticle decay during its passage from its cosmological source to Earth. The idea of unparticle had been first proposed by Georgi by considering the possible existence of an unknown scale invariant sector at high energies and the unparticles in this sector manifest itself below a dimensional transmutation scale $\Lambda_{\mathcal{U}}$. We then explore the possible signature of such decaying neutrinos to unparticles at a square kilometer detector such as IceCube.

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Contents

1 Introduction

Almost a decade back Georgi [1, 2] proposed the probable existence of a scale invariant sector. At a very high energy scale this scale invariance sector and the Standard Model (SM) sector may coexist and the fields of these two sectors can interact via a mediator messenger field of mass scale $M_{\mathcal{U}}$. This is the connector sector [3]. At low energies however, the scale invariance is manifestly broken since SM particles have masses. At a scale below $M_{\mathcal{U}}$ such interactions are suppressed by inverse powers of $M_{\mathcal{U}}$ and the effective theory at low energy can be expressed by a non-renormalizable operator. It is also to be noted that in a scale invariant scenario the particle masses are zero and in the real world, the scale invariance is manifestly broken. It is observed by Georgi [1, 2] that at low energies such a scale invariance sector of scale dimension $d_{\mathcal{U}}$ manifests itself as non-integral number $d_{\mathcal{U}}$ of massless invisible particles called “unparticles”.

It is to be noted that in 4-D Quantum Field Theory (QFT), the conformal invariance is broken by renormalization group effects. But such a conformal invariance in 4-D can be described by a vector like non Abelian gauge theory studied by Banks and Zaks (BZ) [4]. In this theory the scale invariant sector can flow to low energies with nontrivial infrared fixed points and the theory may be extended to low energy. Following Georgi's proposal, the interaction operator $\mathcal{O}_{\rm BZ}$ for the BZ fields with the operator $\mathcal{O}_{\rm SM}$ for SM fields can generically be represented by $\mathcal{O}_{\rm BZ}\mathcal{O}_{\rm SM}/(M_{\mathcal{U}}^k), k > 0$. In a massless non abelian gauge theory, the radiative corrections in the scale invariant sector induce dimensional transmutation [5] at another energy scale. As a result, another scale $\Lambda_{\mathcal{U}}$ appears and Georgi argued [1, 2] that below this scale the BZ field and field operator $\mathcal{O}_{\rm BZ}$ matches onto the unparticle operator $\mathcal{O}_{\mathcal{U}}$ with non-integral scaling dimension $d_{\mathcal{U}}$. Thus below $\Lambda_{\mathcal{U}}$, one has new low energy operator of the form $C_{\mathcal{O}_{\mathcal{U}}}\Lambda_{\mathcal{U}}^{d_{BZ}-d_{\mathcal{U}}}\mathcal{O}_{\rm SM}\mathcal{O}_{\mathcal{U}}/(M_{\mathcal{U}}^k)$, where $C_{\mathcal{O}_{\mathcal{U}}}$ is to be fixed from the matching conditions of BZ operator $\mathcal{O}_{\rm BZ}$ onto the unparticle operator $\mathcal{O}_{\mathcal{U}}$. In this operator $d_{\rm BZ}$ denotes the scaling dimension of the operator $\mathcal{O}_{\rm BZ}$. Since at low energies BZ fields decouple from the SM fields, the infrared fixed points of the unparticles will remain unaffected by the couplings of the unparticle and the SM particles.

The unparticle physics gives rise to rich phenemenology of many unexpected processes. Several authors in the literatures used the concept of unparticles in a wide range of particle physics issues. For example Kikuchi and Okada [6] addressed the unparticle couplings with Higgs and gauge bosons. The interactions of unpartilces with SM particles are addressed by various other authors [7–30]. The issues of dark matter and dark energy is discussed in the unparticle framework in the works of refs. [31–35]. We consider the unparticle decay of neutrinos and explore its consequences for Ultra High Energy (UHE) neutrinos from a distant Gamma Ray Bursts (GRBs). For this case, the decay length should be ∼ tens of Mpc for such decay is to be significant. Here we investigate the unparticle decay of neutrinos along with the mass flavour suppression due to passage of such UHE neutrinos from a distant GRB to an Earth bound detector such as IceCube [36]. We also consider a four flavour scenario for the neutrino species where we assume a 4th sterile species along with the usual 3 active neutrinos. The analyses of the data for reactor neutrinos, considering the existence of a fourth sterile neutrino along with three active neutrinos from the experiments (short and long baselines) such as MINOS [37]–[48], Daya Bay [48]–[55], Bugey [56] etc have given bounds on the mixing parameters for active sterile mixing. We calculate the neutrino induced muon yield in such a scenario at a square kilometer detector such as IceCube.

The paper is organised as follows. A brief account of the formalism of UHE neutrinos, which decay to unparticle from a single GRB is discussed in section 2. We have considered three active and one sterile neutrinos ((3+1) framework) in the present work. Section 2 is divided into two subsections. In subsection 2.1 we address the expression for the neutrino spectrum on reaching the Earth from a single GRB in the absence of decay or oscillations, while the form of the UHE neutrino fluxes, considering the unparticle decay phenomenon, from a single GRB at redshift z is furnished in subsubsection 2.1.1. In subsection 2.2 we describe the analytical expressions for the total number of neutrino induced muons from a point like source such as a single GRB at a square kilometer detector such as IceCube. The calculational results of the yield of secondary muons in different scenarios are given in section 3. Finally in section 4 we give a brief summary and discussions.

2 Formalism

2.1 UHE neutrino fluxes from a single GRB with neutrino decay to unparticles

Gamma-Ray Bursts (GRBs) [57] are some of the most energetic events in the Universe. We have considered the relativistically expanding fireball model, which is one of the few models that has been put forth to explain why GRBs tend to have such high energy levels. In this model, the Fermi mechanism in shocks developing in the GRB outflow can accelerate protons to energies as high as 10^{20} eV. These highly energetic accelerated protons interact with photons via a cosmic beam dump process inside the fireball and the pions are produced through these interactions. In our work we consider the UHE neutrinos which are produced by the decay of these pions and the decay process is $\pi^+ \rightarrow \mu^+ + \nu_\mu$, which is followed by the muons decaying to $\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$.

There are some parameters, which are required to calculate the GRB neutrino spectrum, like Lorentz factor Γ (Γ plays an important role in the neutrino production mechanism of the GRB), neutrino break energy $E_\nu^{\rm brk}$, observed photon spectral break energy $E_{\gamma,{\rm MeV}}^{\rm brk}$, the total amount of energy released at the time of neutrino emission $E_{\rm GRB}$ ($E_{\rm GRB} = 10^{53}$erg, which is 10% of the fireball photon energy), the wind variability time t_ν, redshift distance of GRB from the observer (z) and the wind luminosity L_w ($\simeq 10^{53}$ erg/sec) [58, 59].

The neutrino spectrum of the GRB [58–60] can be written as

$$\frac{dN_\nu}{dE_\nu^s} = N \times \min\left(1, \frac{E_\nu^s}{E_\nu^{\rm brk}}\right)\frac{1}{{E_\nu^s}^2} \ . \tag{2.1}$$

In the above, N represents the normalization constant and E_ν^s is the neutrino energy. The neutrino apectrum break energy $E_\nu^{\rm brk}$ can be expressed in terms of the Lorentz boost factor (Γ) and the photon spectral break energy ($E_{\gamma,{\rm MeV}}^{\rm brk}$).

$$E_\nu^{\rm brk} \approx 10^6 \frac{\Gamma_{2.5}^2}{E_{\gamma,{\rm MeV}}^{\rm brk}} {\rm GeV} \ , \tag{2.2}$$

where $\Gamma_{2.5} = \Gamma/10^{2.5}$. The normalization constant (N), which is mentioned in eq. (2.1), is given by

$$N = \frac{E_{\rm GRB}}{1 + \ln({\rm E}_{\nu\max}^{\rm s}/{\rm E}_\nu^{\rm brk})} \ . \tag{2.3}$$

The lower and the upper cut-off energy of the neutrino spectrum are denoted by $E_{\nu\min}^s$ and $E_{\nu\max}^s$ respectively.

At a particular distance of the GRB from the observer (z), the relation between the observed neutrino energy $E_\nu^{\rm obs}$ and the actual energy of neutrino at the source E_ν^s is given as

$$E_\nu^{\rm obs} = \frac{E_\nu^s}{(1+z)} \ . \tag{2.4}$$

Likewise for the upper cut-off energy of the source eq. (2.4) can be written as

$$E_{\nu\max}^{\rm obs} = \frac{E_{\nu\max}^s}{(1+z)} \ . \tag{2.5}$$

Thus in the absence of decay or oscillation the neutrino spectrum on reaching the Earth from a GRB at redshift z takes the form.

$$\frac{dN_\nu}{dE_\nu^{\rm obs}} = \frac{dN_\nu}{dE_\nu^s}\frac{1}{4\pi L^2(z)}(1+z) \ . \tag{2.6}$$

In the absence of CP violation $\mathcal{F}(E_\nu^s) = \frac{dN_\nu}{dE_\nu^s} = \frac{dN_{\nu+\bar{\nu}}}{dE_\nu^s}$. The spectra for neutrinos will be $0.5\mathcal{F}(E_\nu^s)$.

Now the neutrinos are produced in the GRB process in the proportion

$$\nu_e : \nu_\mu : \nu_\tau : \nu_s = 1 : 2 : 0 : 0 \ . \tag{2.7}$$

Therefore

$$\phi^s_{\nu_e} = \frac{1}{6}\mathcal{F}(E^s_\nu)\ ,\phi^s_{\nu_\mu} = \frac{2}{6}\mathcal{F}(E^s_\nu) = 2\phi^s_{\nu_e}\ ,\phi^s_{\nu_\tau} = 0\ ,\phi^s_{\nu_s} = 0\ , \tag{2.8}$$

where $\phi^s_{\nu_e}$, $\phi^s_{\nu_\mu}$, $\phi^s_{\nu_\tau}$ and $\phi^s_{\nu_s}$ are the fluxes of ν_e, ν_μ, ν_τ and ν_s at source repectively.

In eq. (2.6), $L(z)$ denotes the distance of the source (at a redshift z), which can be expressed as [61]

$$L(z) = \frac{c}{H_0}\int_0^z \frac{dz'}{(1+z')^2\sqrt{\Omega_\Lambda + \Omega_m(1+z')^3}}\ . \tag{2.9}$$

$\Omega_\Lambda + \Omega_m = 1$ for spatially flat Universe, where Ω_Λ is the contribution of dark energy density in units of the critical energy density of the Universe and Ω_m represents the contribution of the matter to the energy density of the Universe in units of critical density. In what follows in this paper we will henceforth use the symbol E_ν to denote $E_\nu^{\rm obs}$.

In eq. (2.9), c and H_0 denote respectively the speed of the light and the Hubble constant in the present epoch. The values of the constants which we have used in our calculations are $\Omega_\Lambda = 0.68$, $\Omega_m = 0.3$ and $H_0 = 67.8$ Km sec^{-1} Mpc^{-1}.

2.1.1 Unparticle decay of GRB neutrinos

After the Georgi's "Unparticle" proposal, extensive studies to investigate the unparticle phenomenology have been explored in the literature. Unparticle physics is a speculative theory that conjectures a form of matter that cannot be explained in terms of particles using the Standard Model (SM) of particle physics, because its components are scale invariant. So the interaction between the unparticle and SM particles is speculative in nature. The presence of this unparticle operator can effect the processes, which are all measured in experiments. Some processes where the invisible unparticles ($\mathcal{U}$) has been considered as the final state are (1) the top quark decay $\tau \to u + \mathcal{U}$ [1], (2) the electron-positron annihilation $e^+ + e^- \to \gamma + \mathcal{U}$, (3) the hadronic processes such as $q + q \to g + \mathcal{U}$ [2, 3] etc.

In the present work we consider a decay phenomenon , where neutrino having mass eigenstate ν_j decays to the invisible unparticle ($\mathcal{U}$) [62] and another light neutrino with mass eigenstate ν_i.

$$\nu_j \to \mathcal{U} + \nu_i\ . \tag{2.10}$$

The effective lagrangian for the above mentioned process takes the following form in the low energy regime.

$$\mathcal{L} = \frac{\lambda^{\alpha\beta}_\nu}{\Lambda_\mathcal{U}^{d_\mathcal{U}-1}}\bar{\nu}_\alpha\nu_\beta\mathcal{O}_\mathcal{U}\ , \tag{2.11}$$

where $\alpha, \beta = e, \mu, \tau, s$ are the flavour indices, $d_\mathcal{U}$ is the scaling dimension of the scalar unpartcile operator $\mathcal{O}_\mathcal{U}$. $\Lambda_\mathcal{U}$ and $\lambda^{\alpha\beta}_\nu$ indicate the dimension transmutation scale at which the scale invariance sets in and the relevant coupling constant respectively. From eq. (2.11). note that a heavier neutrino decays into a lighter neutrino and an unparticle.

The neutrino mass and flavour eigenstates are related through

$$|\nu_i\rangle = \sum_\alpha U^*_{\alpha i}|\nu_\alpha\rangle\ , \tag{2.12}$$

where $U_{\alpha i}$ are the elements of the Pontecorvo - Maki - Nakagawa - Sakata (PMNS) [63] mixing matrix. Working in the neutrino mass eigenstate basis is more convenient than the flavour eigenstate. So in this mass basis we can write the interaction term between neutrinos and the unparticles as $\lambda_\nu^{ij}\bar{\nu}_i\nu_j\mathcal{O}_\mathcal{U}/\Lambda_\mathcal{U}^{d_\mathcal{U}-1}$, where λ_ν^{ij} is the coupling constant in the mass eigenstate i, j.

Now the above mentioned coupling constant can be expressed as

$$\lambda_\nu^{ij} = \sum_{\alpha,\beta} U^*_{\alpha i}\lambda_\nu^{\alpha\beta}U_{\beta j} \ . \tag{2.13}$$

The total decay rate Γ_j or equivalently the lifetime of neutrino $\tau_\mathcal{U} = 1/\Gamma_j$ is the most relevant quantity for the decay process $\nu_j \to \mathcal{U} + \nu_i$ [62]. The lifetime $\tau_\mathcal{U}$ can be written as

$$\frac{\tau_\mathcal{U}}{m_j} = \frac{16\pi^2 d_\mathcal{U}(d_\mathcal{U}^2 - 1)}{A_d|\lambda_\nu^{ij}|^2}\left(\frac{\Lambda_\mathcal{U}^2}{m_j^2}\right)^{d_\mathcal{U}-1}\frac{1}{m_j^2} \ , \tag{2.14}$$

where m_j is the mass of the decaying neutrino.

The normalization constant [1] in the above equation (eq. (2.14)) is defined as

$$A_d = \frac{16\pi^{5/2}}{(2\pi)^{2d_\mathcal{U}}}\frac{\Gamma(d_\mathcal{U} + 1/2)}{\Gamma(d_\mathcal{U} - 1)\Gamma(2d_\mathcal{U})} \ . \tag{2.15}$$

In the decay process for the four flavour scenario the lightest mass state $|\nu_1\rangle$ is stable, because it does not decay and all other states $|\nu_2\rangle, |\nu_3\rangle$ and $|\nu_4\rangle$ are unstable. Also note that due to the astronomical baseline $L(z)$ for these neutrinos the oscillatory part $(\sin^2\left(\frac{\Delta m^2 L(z)}{4E_\nu}\right)$, Δm^2 being the mass square difference of any two neutrinos) is averaged out to half and the flavour oscillation probability (from flavour α to a flavour β) is reduced to $P_{\nu_\alpha\to\nu_\beta} = \sum_j |U_{\alpha j}|^2|U_{\beta j}|^2$, j being the mass index. In the absence of the decay therefore the neutrino flux at the detector for UHE neutrinos from distant GRB can be computed as $\phi_{\nu_\alpha}^{\text{detector}}(E_\nu) = \sum_i\sum_\beta[\phi^s_{\nu_\beta}|U_{\beta i}|^2|U_{\alpha i}|^2]a_1$, while in the presence of neutrino decay (in the present work we consider the unparticle decay of neutrinos) the expression for the neutrino flux at the detector is given as

$$\phi_{\nu_\alpha}^{\text{detector}}(E_\nu) = \sum_i\sum_\beta[\phi^s_{\nu_\beta}|U_{\beta i}|^2|U_{\alpha i}|^2 exp(-4\pi L(z)/(\lambda_d)_i)]a_1 \ , \tag{2.16}$$

where $L(z) = \frac{c}{H_0}\int_0^z\frac{dz'}{(1+z')^2\sqrt{\Omega_\Lambda+\Omega_m(1+z')^3}}$ and $a_1 = \frac{1}{4\pi L^2(z)}(1+z)$ and the decay length $(\lambda_d)_i = 4\pi\frac{E_\nu}{\alpha_i}$ with $\alpha_i = \frac{m_i}{\tau_\mathcal{U}}$. In eq. (2.16) α, β indicate the flavour indices and i is defined as mass index, $L(z)$ is the baseline length, $U_{\alpha i}$ etc. denote the elements of PMNS matrix. For the 4-flavour scenario, where an extra sterile neutrino ν_s is introduced along with the usual three families of active neutrinos ν_e, ν_μ and ν_τ, the elements of the PMNS mixing matrix can be generated by successive rotations (R) (in terms of six mixing angles $\theta_{14}, \theta_{24}, \theta_{34}, \theta_{13}, \theta_{12}, \theta_{23}$) as [65]

$$\tilde{U}_{(4\times4)} = R_{34}(\theta_{34})R_{24}(\theta_{24})R_{14}(\theta_{14})R_{23}(\theta_{23})R_{13}(\theta_{13})R_{12}(\theta_{12}) \ , \tag{2.17}$$

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where we consider that there is no CP violation in the neutrino sector and hence the CP phases are omitted for simplicity. For the 4-flavour framework the successive rotation terms (R) in eq. (2.17) can be expressed as

$$R_{34}(\theta_{34}) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & c_{34} & s_{34} \\ 0 & 0 & -s_{34} & c_{34} \end{pmatrix}, \qquad R_{24}(\theta_{24}) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{24} & 0 & s_{24} \\ 0 & 0 & 1 & 0 \\ 0 & -s_{24} & 0 & c_{24} \end{pmatrix},$$

$$R_{14}(\theta_{14}) = \begin{pmatrix} c_{14} & 0 & 0 & s_{14} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ -s_{14} & 0 & 0 & c_{14} \end{pmatrix}, \qquad R_{12}(\theta_{12}) = \begin{pmatrix} c_{12} & s_{12} & 0 & 0 \\ -s_{12} & c_{12} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

$$R_{13}(\theta_{13}) = \begin{pmatrix} c_{13} & 0 & s_{13} & 0 \\ 0 & 1 & 0 & 0 \\ -s_{13} & 0 & c_{13} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \qquad R_{23}(\theta_{23}) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{23} & s_{23} & 0 \\ 0 & -s_{23} & c_{23} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{2.18}$$

For the 4-flavour scenario (the minimal extension of 3 flavour case by a sterile neutrino) the PMNS matrix can be wriiten as

$$\tilde{U}_{(4\times 4)} = \begin{pmatrix} c_{14} & 0 & 0 & s_{14} \\ -s_{14}s_{24} & c_{24} & 0 & c_{14}s_{24} \\ -c_{24}s_{14}s_{34} & -s_{24}s_{34} & c_{34} & c_{14}c_{24}s_{34} \\ -c_{24}s_{14}c_{34} & -s_{24}c_{34} & -s_{34} & c_{14}c_{24}c_{34} \end{pmatrix} \times \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & 0 \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & 0 \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{2.19}$$

$$= \begin{pmatrix} c_{14}U_{e1} & c_{14}U_{e2} & c_{14}U_{e3} & s_{14} \\ -s_{14}s_{24}U_{e1} + c_{24}U_{\mu 1} & -s_{14}s_{24}U_{e2} + c_{24}U_{\mu 2} & -s_{14}s_{24}U_{e3} + c_{24}U_{\mu 3} & c_{14}s_{24} \\ \begin{array}{c} -c_{24}s_{14}s_{34}U_{e1} \\ -s24s34U_{\mu 1} \\ +c_{34}U_{\tau 1} \end{array} & \begin{array}{c} -c_{24}s_{14}s_{34}U_{e2} \\ -s24s34U_{\mu 2} \\ +c_{34}U_{\tau 2} \end{array} & \begin{array}{c} -c_{24}s_{14}s_{34}U_{e3} \\ -s24s34U_{\mu 3} \\ +c_{34}U_{\tau 3} \end{array} & c_{14}c_{24}s_{34} \\ \begin{array}{c} -c_{24}c_{34}s_{14}U_{e1} \\ -s_{24}c_{34}U_{\mu 1} \\ -s_{34}U_{\tau 1} \end{array} & \begin{array}{c} -c_{24}c_{34}s_{14}U_{e2} \\ -s_{24}c_{34}U_{\mu 2} \\ -s_{34}U_{\tau 2} \end{array} & \begin{array}{c} -c_{24}c_{34}s_{14}U_{e3} \\ -s_{24}c_{34}U_{\mu 3} \\ -s_{34}U_{\tau 3} \end{array} & c_{!4}c_{24}c_{34} \end{pmatrix}, \tag{2.20}$$

where $U_{\alpha i}$ represents the matrix elements of 3-flavour neutrino mixing matrix U, which is given as

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}s_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{pmatrix}. \tag{2.21}$$

In eq. (2.16), ϕ_{ν_α} represents the fluxes of ν_α and $\phi^s_{\nu_\beta}$ is the fluxes of neutrinos having flavour β at the source. The decay length $((\lambda_d)_i)$ in the eq. (2.16) can be expressed as

$$(\lambda_d)_i = 4\pi\frac{E_\nu}{\alpha_i} = 2.5\ \text{Km}\ \frac{E_\nu}{\text{GeV}}\frac{\text{eV}^2}{\alpha_i}\ , \tag{2.22}$$

where α_i is defined as $m_i/\tau_\mathcal{U}$, $\tau_\mathcal{U}$ being the neutrino decay lifetime.[1] Eq. (2.22) shows that the decay length $((\lambda_d)_i)$ is a function of neutrino energy (E).

Applying the equation eq. (2.8) and by considering the condition that the lightest mass state $|\nu_1\rangle$ is stable we can write the flux of neutrino flavours for four flavour cases on reaching the Earth as [66]–[68] (by assuming all the fluxes in terms of $\phi^s_{\nu_e}$ and using the unitarity condition $\sum_i U_{\alpha i}U^*_{\beta i} = \delta_{\alpha\beta}$).

$$\begin{aligned}
\phi^{\text{detector}}_{\nu_e} = {} & ([|\,\tilde U_{e1}\,|^2(1+|\,\tilde U_{\mu1}\,|^2-|\,\tilde U_{\tau1}\,|^2-|\,\tilde U_{s1}\,|^2)\\
& +|\,\tilde U_{e2}\,|^2(1+|\,\tilde U_{\mu2}\,|^2-|\,\tilde U_{\tau2}\,|^2-|\,\tilde U_{s2}\,|^2)\exp(-4\pi L(z)/(\lambda_d)_2)\\
& +|\,\tilde U_{e3}\,|^2(1+|\,\tilde U_{\mu3}\,|^2-|\,\tilde U_{\tau3}\,|^2-|\,\tilde U_{s3}\,|^2)\exp(-4\pi L(z)/(\lambda_d)_3)\\
& +|\,\tilde U_{e4}\,|^2(1+|\,\tilde U_{\mu4}\,|^2-|\,\tilde U_{\tau4}\,|^2-|\,\tilde U_{s4}\,|^2)\exp(-4\pi L(z)/(\lambda_d)_4)]\phi^s_{\nu_e})a_1\ ,\\
\phi^{\text{detector}}_{\nu_\mu} = {} & ([|\,\tilde U_{\mu1}\,|^2(1+|\,\tilde U_{\mu1}\,|^2-|\,\tilde U_{\tau1}\,|^2-|\,\tilde U_{s1}\,|^2)\\
& +|\,\tilde U_{\mu2}\,|^2(1+|\,\tilde U_{\mu2}\,|^2-|\,\tilde U_{\tau2}\,|^2-|\,\tilde U_{s2}\,|^2)\exp(-4\pi L(z)/(\lambda_d)_2)\\
& +|\,\tilde U_{\mu3}\,|^2(1+|\,\tilde U_{\mu3}\,|^2-|\,\tilde U_{\tau3}\,|^2-|\,\tilde U_{s3}\,|^2)\exp(-4\pi L(z)/(\lambda_d)_3)\\
& +|\,\tilde U_{\mu4}\,|^2(1+|\,\tilde U_{\mu4}\,|^2-|\,\tilde U_{\tau4}\,|^2-|\,\tilde U_{s4}\,|^2)\exp(-4\pi L(z)/(\lambda_d)_4)]\phi^s_{\nu_e})a_1\ ,\\
\phi^{\text{detector}}_{\nu_\tau} = {} & ([|\,\tilde U_{\tau1}\,|^2(1+|\,\tilde U_{\mu1}\,|^2-|\,\tilde U_{\tau1}\,|^2-|\,\tilde U_{s1}\,|^2)\\
& +|\,\tilde U_{\tau2}\,|^2(1+|\,\tilde U_{\mu2}\,|^2-|\,\tilde U_{\tau2}\,|^2-|\,\tilde U_{s2}\,|^2)\exp(-4\pi L(z)/(\lambda_d)_2)\\
& +|\,\tilde U_{\tau3}\,|^2(1+|\,\tilde U_{\mu3}\,|^2-|\,\tilde U_{\tau3}\,|^2-|\,\tilde U_{s3}\,|^2)\exp(-4\pi L(z)/(\lambda_d)_3)\\
& +|\,\tilde U_{\tau4}\,|^2(1+|\,\tilde U_{\mu4}\,|^2-|\,\tilde U_{\tau4}\,|^2-|\,\tilde U_{s4}\,|^2)\exp(-4\pi L(z)/(\lambda_d)_4)]\phi^s_{\nu_e})a_1\ ,\\
\phi^{\text{detector}}_{\nu_s} = {} & ([|\,\tilde U_{s1}\,|^2(1+|\,\tilde U_{\mu1}\,|^2-|\,\tilde U_{\tau1}\,|^2-|\,\tilde U_{s1}\,|^2)\\
& +|\,\tilde U_{s2}\,|^2(1+|\,\tilde U_{\mu2}\,|^2-|\,\tilde U_{\tau2}\,|^2-|\,\tilde U_{s2}\,|^2)\exp(-4\pi L(z)/(\lambda_d)_2)\\
& +|\,\tilde U_{s3}\,|^2(1+|\,\tilde U_{\mu3}\,|^2-|\,\tilde U_{\tau3}\,|^2-|\,\tilde U_{s3}\,|^2)\exp(-4\pi L(z)/(\lambda_d)_3)\\
& +|\,\tilde U_{s4}\,|^2(1+|\,\tilde U_{\mu4}\,|^2-|\,\tilde U_{\tau4}\,|^2-|\,\tilde U_{s4}\,|^2)\exp(-4\pi L(z)/(\lambda_d)_4)]\phi^s_{\nu_e})a_1\ .
\end{aligned} \tag{2.23}$$

In the above eqs. (2.23) $\phi^{\text{detector}}_{\nu_\alpha}$ represents the neutrino fluxes for four flavour cases on reaching the Earth.

In case of $L(z) \gg \lambda_d$, eq. (2.16) is then reduced to

$$\phi^{\text{detector}}_{\nu_\alpha}(E_\nu)(\text{no decay}) = \sum_{i(stable),\beta}(\phi^s_{\nu_\beta}|U_{\beta i}|^2|U_{\alpha i}|^2)a_1\ . \tag{2.24}$$

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[1] $\lambda_d = \frac{4\pi E_\nu}{\alpha_i}$, $\alpha_i = \frac{m_i}{\tau_\mathcal{U}}$; m in eV, E_ν in GeV, $\tau_\mathcal{U}$ is decay time. In natural units $\lambda_d \equiv 4\pi E_\nu[\text{GeV}] \times \frac{1}{\alpha_i} \times \frac{1}{[\text{GeV}]} \times \frac{1}{[\text{eV}]} = 4\pi\left(\frac{E_\nu}{\text{GeV}}\right)[\text{GeV}] \times \frac{1}{10^9} \times \frac{1}{[\text{eV}]^2} \times \left(\frac{\text{eV}^2}{\alpha_i}\right) = 4\pi\left(\frac{E_\nu}{\text{GeV}}\right)\left(\frac{\text{eV}^2}{\alpha_i}\right) \times \frac{[\text{GeV}][\text{GeV}]}{[\text{eV}]^2} \times 10^{-9} \times \frac{1}{[\text{GeV}]}$ $= 4\pi\left(\frac{E_\nu}{\text{GeV}}\right)\left(\frac{\text{eV}^2}{\alpha_i}\right) \times \frac{[\text{eV}]^2}{[\text{eV}]^2} \times \frac{10^9}{[\text{GeV}]} = \frac{4\pi}{5.6}\ \text{Km}\left(\frac{E_\nu}{\text{GeV}}\right)\left(\frac{\text{eV}^2}{\alpha_i}\right) = 2.5\ \text{Km}\left(\frac{E_\nu}{\text{GeV}}\right)\left(\frac{\text{eV}^2}{\alpha_i}\right)$.

Eq. (2.24) indicates that with the condition $L(z) \gg \lambda_d$, the decay term is removed because the neutrino decay is completed by the time it reaches the Earth. So only the stable state $|\nu_1\rangle$ exists. So the flavour ratio in 4-flavour scenario in this case is changed to $|U_{e1}|^2 : |U_{\mu 1}|^2 : |U_{\tau 1}|^2 : |U_{s1}|^2$ [64, 69, 70]. But when the decay length is close to the baseline length ($\lambda_d \sim L(z)$), then we cannot wash out the neutrino decay effect. Therefore the exponential term survives in eqs. (2.23) and the baseline length ($L(z)$) plays an important role. In such cases, considering GRB neutrino fluxes at a fixed redshift (z) is useful to explore the neutrino decay effects.

2.2 Detection of UHE neutrinos from a single GRB

Upward going muons [71] are produced by the interactions , which are weak in nature, of ν_μ or $\bar{\nu}_\mu$ with the rock surrounding the Super-K detector. While muons from interactions above the detector cannot be sorted out from the continuous rain of muons created in cosmic ray showers in the atmosphere above the mountain, muons coming from below can only be due to neutrino (ν_μ) charge current interactions ($\nu_\mu + N \to \mu + X$), since cosmic ray muons cannot make it through from the other side of the Earth. Looking upward going muons is the most encouraging way to detect the UHE neutrinos.

The secondary muon yields from the GRB neutrinos can be detected in a detector of unit area above a threshold energy $E_{\rm th}$ is given by [60, 72–74]

$$S = \int_{E_{\rm th}}^{E_{\nu {\rm max}}} dE_\nu \phi_{\nu_\alpha}^{\rm detector} P_{\rm shadow}(E_\nu) P_\mu(E_\nu, E_{\rm th}) \ , \tag{2.25}$$

where $P_{\rm shadow}(E_\nu)$ represents the probability that a neutrino reaches the terrestrial detector such as IceCube being unabsorbed by the Earth. We can express this shadow factor in terms of the energy dependent neutrino-nucleon interaction length $L_{\rm int}(E_\nu)$ in the Earth and the effective path length $X(\theta_z)$ (θ_z is fixed for a particular single GRB). Thus $P_{\rm shadow}(E_\nu)$ takes the form.

$$P_{\rm shadow} = \exp[-X(\theta_z)/L_{\rm int}(E_\nu)] \ , \tag{2.26}$$

where $L_{\rm int}(E_\nu)$ is given by

$$L_{\rm int}(E_\nu) = \frac{1}{\sigma_{\rm tot}(E_\nu) N_A} \ . \tag{2.27}$$

In the above, N_A is the Avogadro number ($N_A = 6.023 \times 10^{23}\ {\rm mol}^{-1} = 6.023 \times 10^{23}\ {\rm cm}^{-3}$) and $\sigma_{\rm tot}$ denotes the total cross-section (= charge current cross-section ($\sigma_{\rm CC}$) + neutral current cross-section ($\sigma_{\rm NC}$)) for neutrino absorptions.

The effective path length $X(\theta_z)$ (gm/cm^2) can be written as

$$X(\theta_z) = \int \rho(r(\theta_z, l)) dl \ . \tag{2.28}$$

We have considered Earth as a spherically symmetric ball having a dense inner and outer core and a lower mantle of medium density. So in eq. (2.28) $\rho(r(\theta_z, l))$ (l is the neutrino path length entering into the Earth) represents the matter density profile inside the Earth, which can be expressed by the Preliminary Earth Model (PREM) [75].

The probability $P_\mu(E_\nu, E_{\rm th})$ that a neutrino induced muon reaching the detector with an energy above $E_{\rm th}$ can be written as

$$P_\mu(E_\nu, E_{\rm th}) = N_A \sigma_{\rm cc}(E_\nu)\langle R(E_\mu; E_{\rm th})\rangle \ , \tag{2.29}$$

where the average muon range in the rock $\langle R(E_\mu; E_{\rm th})\rangle$ is given by

$$\langle R(E_\mu; E_{\rm th})\rangle = \frac{1}{\sigma_{\rm CC}} \int_0^{(1-E_{\rm th}/E_\nu)} dy R(E_\nu(1-y); E_{\rm th}) \times \frac{d\sigma_{\rm CC}(E_\nu, y)}{dy} \ , \tag{2.30}$$

where $y = (E_\nu - E_\mu)/E_\nu$ represents the fraction of energy loss by a neutrino of energy E_ν in the production of a secondary muons having energy E_μ. We can replace $E_\nu(1-y)$ by E_μ in the integrand of eq. (2.30). So now the muon range $R(E_\mu; E_{\rm th})$ can be expressed as

$$R(E_\mu, E_{\rm th}) = \int_{E_{\rm th}}^{E_\mu} \frac{dE_\mu}{\langle dE_\mu/dX\rangle} \simeq \frac{1}{\beta} \ln\left(\frac{\alpha + \beta \mathrm{E}_\mu}{\alpha + \beta \mathrm{E}_{\rm th}}\right) \ . \tag{2.31}$$

The average energy loss of muon with energy E_μ is given as [73, 74]

$$\left\langle \frac{dE_\mu}{dX} \right\rangle = -\alpha - \beta E_\mu \ . \tag{2.32}$$

The values of the constants α and β in eq. (2.32), which we have considered in our calculations are

$$\begin{aligned} \alpha &= 2.033 + 0.077\ \ln[E_\mu(\text{GeV})] \times 10^3\ \text{GeV cm}^2\ \text{gm}^{-1} \ , \\ \beta &= 2.033 + 0.077\ \ln[E_\mu(\text{GeV})] \times 10^{-6}\ \text{GeV cm}^2\ \text{gm}^{-1} \ , \end{aligned} \tag{2.33}$$

for $E_\mu \ \leq \ 10^6$ GeV [76] and otherwise [59]

$$\begin{aligned} \alpha &= 2.033 \times 10^{-3}\ \text{GeV cm}^2\ \text{gm}^{-1} \, , \\ \beta &= 3.9 \times 10^{-6}\ \text{GeV cm}^2\ \text{gm}^{-1} \, . \end{aligned} \tag{2.34}$$

In the case of detecting muon events at a 1 Km^2 detector such as IceCube the flux $\phi_{\nu_\alpha}^{\rm detector}$ in eq. (2.25) is replaced by $\phi_{\nu_\mu}^{\rm detector}$ in eq. (2.23).

Cosmic tau neutrinos undergo charge current deep inelastic scattering with nuclei of the detector material and produces hadronic shower as well as tau lepton ($\nu_\tau + N \to \tau + X$). After traversing some distances, which is proportional to the energy of tau lepton, τ decays into ν_τ (having diminished energy) and in this process a second hadronic shower is induced. These whole double shower processes are introduced as a double bang event. The detection of these tau leptons, which are regenerated in the lollipop event, is very much complicated due to its noninteracting nature with the other particles as they lose energy very fast. The only possible way of the detection of tau leptons other than double bang event is the production of muons via the decay channel $\nu_\tau \to \tau \to \nu_{\bar{\mu}} \mu \nu_\tau$ with probability 0.18 [77, 78]. The number of such muon events can be computed by solving numerically eqs. (2.25)–(2.34) and it is needless to say that $\phi_{\nu_\alpha}^{\rm detector}$ in eq. (2.25) is equivalent to $\phi_{\nu_\tau}^{\rm detector}$ (eq. (2.23)).

3 Calculations and results

In this section we explore the effect on a flux of neutrinos of different flavours on reaching the Earth from a distant astrophysical source, in case such neutrinos undergo unparticle decay along with the usual mass flavour oscillations. For this purpose we consider a specific example of ultra high energy neutrinos from a single GRB and its detection at a kilometer square Cherenkov detector such as IceCube. We also assume the existence of a 4th sterile neutrino in addition to the usual three active flavour neutrinos (ν_e, ν_μ and ν_τ).

The expression for the final flux for a neutrino flavour α on reaching the Earth is given in eq. (2.16) along with eqs. (2.20)–(2.23) (section 2.1.1). It is to be noted that the decay part $(\exp(-4\pi L(z)/(\lambda_d)_i)$ for a neutrino mass eigenstate $|\nu_i\rangle$ will be meaningful and significant for the baseline length $L(z) \sim (\lambda_d)_i$, the decay length. This decay length depends on the neutrino-unparticle coupling $|\lambda_\nu^{ij}|$, the non-integral scaling dimension $d_\mathcal{U}$, the dimensional transmutation scale $\Lambda_\mathcal{U}$ etc.

The neutrino flux from a single GRB is calculated using eqs. (2.1)–(2.9) in section 2.1. We have considered a GRB of energy $E_{\rm GRB} = 10^{53}$ GeV at a redshift $z = 0.1$ for the present calculations. The measure of distance (eq. (2.9)) corresponding to the chosen redshift is computed as 10^{15} km from the Earth where the values of cosmological parameters $\Omega_\Lambda = 0.68$ and $\Omega_m = 0.3$ are adopted from PLANCK 2015 data [79]. The break energy $E_\nu^{\rm brk}$ is obtained using eq. (2.2) with the value of photon spectrum break energy $E_\gamma^{\rm brk}$ adopted from table 1 of ref. [58] for the Lorentz boost factor $\Gamma = 50.12$. We have considered the current best fit values for three neutrino mixing angles ($\theta_{12} = 33.48^\circ, \theta_{23} = 45^\circ$ and $\theta_{13} = 8.5^\circ$). The following four flavour analysis of different experimental group such as MINOS, Daya Bay, Bugey, NOvA [38, 48, 56, 80–84] suggest some limits on four flavour mixing angles ($\theta_{14}, \theta_{24}, \theta_{34}$). For $\Delta m_{41}^2 = 0.5$ eV^2 NOvA [81] gives the upper limits on θ_{24} and θ_{34} as $\theta_{24} \leq 20.8^\circ$ and $\theta_{34} \leq 31.2^\circ$. For the same value of Δm_{41}^2 the upper limits on θ_{24} and θ_{34} obtained from MINOS [38] are $\theta_{24} \leq 7.3^\circ$ and $\theta_{34} \leq 26.6^\circ$. However IceCube - DeepCore [85] experimental results have proposed that $\theta_{24} \leq 19.4^\circ$ and $\theta_{34} \leq 22.8^\circ$ for $\Delta m_{41}^2 = 1$ eV^2. The limits on θ_{14} are chosen as $1^\circ \leq \theta_{14} \leq 4^\circ$ in the range 0.2 eV^2 $< \Delta m_{41}^2 <$ 2 eV^2, which is consistent with the observational results from the combined experimental analysis by MINOS, Daya Bay and Bugey-3 [48]. By considering the above mentioned limits on four flavour mixing angles we have taken θ_{14}, θ_{24} and θ_{34} as $3^\circ, 5^\circ$ and 20° respectively for our calculations. It is to be noted that in the four flavour neutrino decay framework the normal hierarchy is evident as we have already discussed in section 2.1.1 that $|\nu_2\rangle, |\nu_3\rangle$ and $|\nu_4\rangle$, considering as unstable states, are subjected to undergo unparticle decay while only $|\nu_1\rangle$ is stable. In our calculations we consider m_2 and m_3 as $\sqrt{\Delta m_{32}^2}$ and $\sqrt{2.0 \times \Delta m_{32}^2}$, where $\Delta m_{32}^2 = m_3^2 - m_2^2$ (normal hierarchy)[2] and $\Delta m_{32}^2 = 2.4 \times 10^{-3}$ eV^2 (from atmospheric neutrino oscillation) respectively. The value of m_4 is estimated from $m_4 = \sqrt{\Delta m_{41}^2}$, where Δm_{41}^2 lies within the range 0.2 eV^2 $< \Delta m_{41}^2 <$ 2 eV^2. By using eqs. (2.10)–(2.22) we now calculate the relevant neutrino flux from a single GRB reaching the detector with or without unparticle decay.

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[2] $m_2 \simeq \sqrt{\Delta m_{32}^2},\ m_3 \simeq \sqrt{\Delta m_{32}^2 + m_2^2} = \sqrt{\Delta m_{32}^2 + \Delta m_{32}^2} = \sqrt{2\Delta m_{32}^2}$.

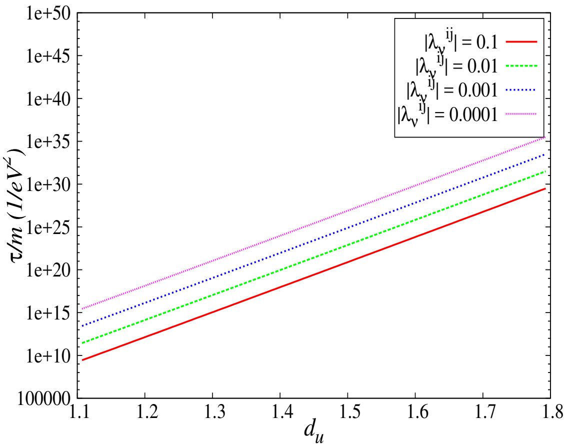

Figure 1. The variations of the neutrino decay lifetime (τ/m) with the unparticle dimension ($d_{\mathcal{U}}$) are shown for four different values (0.1, 0.01, 0.001, 0.0001) of couplings $|\lambda_\nu^{ij}|$.

The upgoing secondary muon yield from ν_μ in an Earth bound detector can be computed by using eqs. (2.25)–(2.34). We have considered a kilometer square detector such as IceCube for our present calculations in case the neutrinos undergo unparticle decay. Note that we consider UHE neutrinos from a single GRB here. Therefore its directionality of the neutrino beam with respect to the detector is fixed.

The effect of unparticle decay is characterised mainly by the three parameters namely, the neutrino-unparticle coupling $|\lambda_\nu^{ij}|$, the fractional dimension of unparticle ($d_{\mathcal{U}}$) and the transmutation scale $\Lambda_{\mathcal{U}}$. The scale $\Lambda_{\mathcal{U}}$ is fixed at 1 TeV for the present calculations. The effect of unparticle parameters $d_{\mathcal{U}}$ and $|\lambda_{\mathcal{U}}^{ij}|$ are varied to study how they affect the various quantities that can be measured at the detector.

We show the variations of decay life time of neutrino in terms of $\tau/m(=\tau_{\mathcal{U}}/m_j)$ for different fixed values of $|\lambda_\nu^{ij}|$ with the unparticle dimension $d_{\mathcal{U}}$ in figure 1. The plots clearly indicate the increasing nature of τ/m with the increase of $d_{\mathcal{U}}$, which is manifested in eq. (2.14) along with eq. (2.15). Figure 1 also reflects the fact that τ/m decreases with the reducing values of $|\lambda_\nu^{ij}|$ (eq. (2.14)).

Figure 2 shows the variations of neutrino induced muons at a square kilometer detector such as IceCube considered here for neutrinos from different single GRBs at varied redshifts (z). We have shown the results for three fixed values of $|\lambda_\nu^{ij}|$ as well as for no decay case. All the plots in figure 2 exhibit decrease of neutrino induced muons with increasing z (the distance of the GRBs from the observer) as is evident from eqs. (2.6) and (2.9). It is to be noted that the decrease of the coupling $|\lambda_\nu^{ij}|$ causes the decay length λ_d to increase and therefore the depletion of the neutrino flux (and hence the induced muon yield) will be effective for neutrinos from GRBs at larger distances or redshifts. For example in figure 2, when $|\lambda_\nu^{ij}| = 0.0001$ the decay effect is significant for a GRB with $z \sim 0.1$ whereas for $|\lambda_\nu^{ij}| = 0.001$ the depletion due to decay is evident for neutrinos from a nearer GRB with $z \sim 0.001$.

This is to be mentioned that the nature of the plots in figure 2 can be understood from the nature of the variation of the factor $a_1(=\frac{1}{4\pi L^2(z)}(1+z))$ with z. This is demonstrated

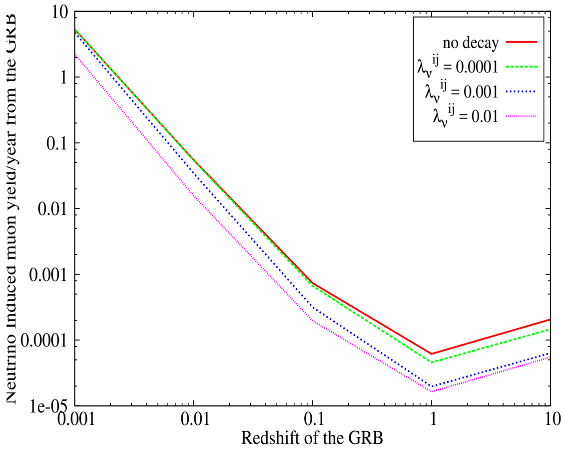

Figure 2. Variations of the neutrino induced muons per year from the GRB with different redshifts (z) for three different values of $|\lambda_\nu^{ij}|$ as well as for no decay case at a fixed zenith angle ($\theta_z = 160°$). See text for details.

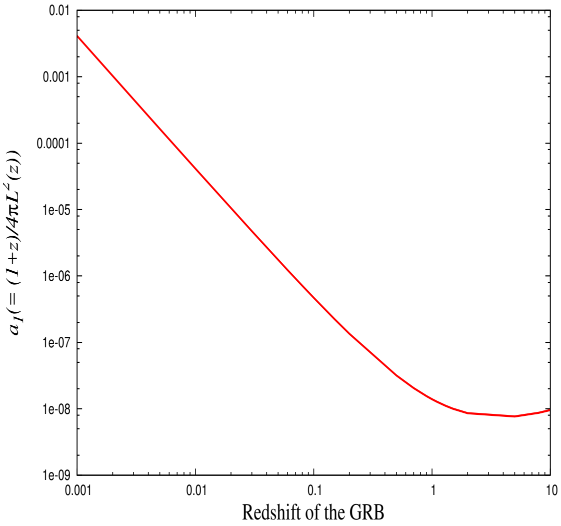

Figure 3. Variations of the factor $a_1(= \frac{1}{4\pi L^2(z)}(1+z))$ with different redshifts (z). See text for details.

in figure 3. It is to be noted from figure 3 that around $z = 1$ there is a change in the nature of the plot which is reflected in the results shown in figure 2.

In figures 4(a), 4(b) the effects of the unparticle parameters ($d_\mathcal{U}$ and $|\lambda_\nu^{ij}|$) on the unparticle decay of neutrinos are shown. Comparisons are also made with the cases when only mass-flavour oscillations are considered. Because of very long baseline the mass flavour oscillations effect all the neutrino fluxes will be manifested only through an overall depletion of the flux depending on just the neutrino mixing angles. The variations of the neutrno induced muon yields at the detector considered with the unparticle dimension $d_\mathcal{U}$ for different fixed values of $|\lambda_\nu^{ij}|$ are shown in figure 4(a). The results with only mass flavour oscillations (no unparticle decay) are also shown for comparison. All the calculations are made for

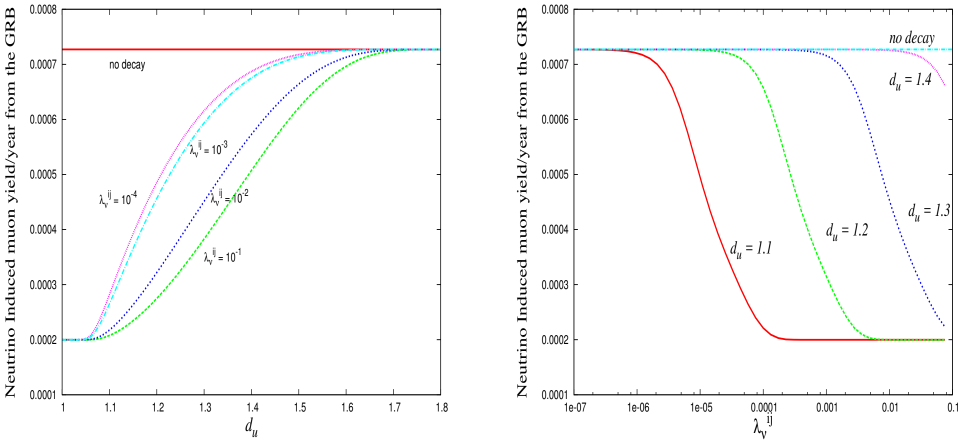

Figure 4. The variations of the neutrino induced upward going muons per year from the GRB with (a) different values of $d_\mathcal{U}$ for four different fixed values of $|\lambda_\nu^{ij}|$ as well as for the mass flavour case (no decay case), (b) different values of $|\lambda_\nu^{ij}|$ for four different fixed values of the unparticle dimension $d_\mathcal{U}$ (1.1, 1.2, 1.3, 1.4) and in addition for no decay case. See text for details.

θ_{14}	θ_{24}	θ_{34}	$d_\mathcal{U}$	$\lvert\lambda_\nu^{ij}\rvert$	R
3°	5°	20°	1.2	10^{-4}	0.9
			1.3	10^{-2}	0.6

Table 1. The ratio of muon yields at IceCube for UHE neutrinos from a GRB with and without unparticle decay in a four neutrino framework. See text for details.

θ_{14}	θ_{24}	θ_{34}	$d_\mathcal{U}$	$\lvert\lambda_\nu^{ij}\rvert$	$\phi_{\nu_e} : \phi_{\nu_\mu} : \phi_{\nu_\tau}$ (with decay and oscillation)	$\phi_{\nu_e} : \phi_{\nu_\mu} : \phi_{\nu_\tau}$ no decay (only oscillation)
3°	5°	20°	1.2	10^{-4}	0.349 : 0.621 : 0.489	0.449 : 0.979 : 0.834
			1.3	10^{-2}	0.344 : 0.265 : 9.5×10^{-2}	

Table 2. Same as table 1 but for the flux ratio for each flavour of active neutrinos in 4-flavour framework.

UHE neutrinos from a GRB at $z = 0.1$ and at a zenith angle $\theta_z = 160°$. The decay effect is evident in figure 4(a) as the muon yield depletes by $\sim 70\%$ from what is expected for only the mass-flavour case. It can also be noted from figure 4(a) that higher the value of the coupling for unparticle decay of neutrinos, higher is the unparticle dimension at which the decay effect starts showing up. Since, here we consider a single GRB at a fixed redshift, the baseline length $L(z)$ is fixed. Therefore the exponential decay term $\exp(-L(z)/(\lambda_d)_i)$ depends only on the decay length $(\lambda_d)_i$. As the decay length depends on $\frac{\tau}{m}$ (eq. (2.14)) which in turn is a function of both $d_\mathcal{U}$ and $|\lambda_\nu^{ij}|$, the nature of the plots in figure 4(a) varies accordingly. Similar trends can also be seen when the neutrino induced muons are plotted with $|\lambda_\nu^{ij}|$ for different fixed values of $d_\mathcal{U}$ (figure 4(b)).

In order to quantify the possible effect on UHE neutrino signal at IceCube in case the UHE neutrinos from a distant GRB undergo unparticle decay we define a ratio R of the expected muon yields per year with and without unparticle decay in addition to the 4-flavour oscillations as

$$R = \frac{\mathrm{Rate}_{(\text{with decay})}}{\mathrm{Rate}_{(\text{no decay})}} \ . \tag{3.1}$$

The results are given in table 1 for a fixed set of values of the mixing angles θ_{14}, θ_{24}, θ_{34} and for two sets of values of decay parameters namely $d_{\mathcal{U}}$, $|\lambda_\nu^{ij}|$. It can be seen from table 1 that for the decay parameter set $d_{\mathcal{U}} = 1.3$ and $|\lambda_\nu^{ij}| = 10^{-2}$ which may cause considerable decay (figure 4), this ratio R can be depleted to as low as a value of 0.6. We also estimate the flavour ratio for 3 active flavours in the present 4-flavour neutrino scenario for the cases when these neutrinos undergo both oscillations and decay and when these neutrinos suffer only four flavour oscillations during its passage from the source GRB to the Earth. We have adopted same sets of values for the mixing angles and the decay parameters as chosen for table 1. The results are shown in table 2. It is seen from table 2 that when $d_{\mathcal{U}} = 1.3$ and $|\lambda_\nu^{ij}| = 10^{-2}$ (decay is considerably strong (figure 4)) the flux for flavour ν_e suffers least depletion while the fluxes of ν_μ and ν_τ are considerably diminished. This is because of the fact that the $|\nu_1\rangle$ component in the flavour eigenstate $|\nu_e\rangle$ is dominant and $|\nu_2\rangle$ is the lightest component that undergoes no decay. The minimal change of ϕ_{ν_e} with decay in comparison to the case without decay (only 4-flavour oscillations) is due to the decay of small $|\nu_2\rangle$ and $|\nu_3\rangle$ components in $|\nu_e\rangle$. The depletion of ϕ_{ν_μ} and ϕ_{ν_τ} in comparison to the no decay case (only oscillation) can also be similarly understood (in $|\nu_\mu\rangle$, $|\nu_2\rangle$ component is dominant while in $|\nu_\tau\rangle$ the dominant component is $|\nu_3\rangle$).

4 Summary and discussions

In this work we have explored the possibility of unparticle decay of Ultra High Energy (UHE) neutrinos from a distant single GRB and its consequences on the neutrino induced muon yields at a kilometer square detector. The concept of unparticles first proposed by Georgi from the consideration of the presence of a yet unseen scale invariant sector which may be present in the four dimensions with non-renormalizable interactions with Standad Model particles. The "particles" in this scale invariant sector are termed as "unparticles". The unparticle scenario and its interaction with SM particles such as neutrinos are expressed by an effective lagrangian, which is expressed in terms of the effective couplings ($\lambda_\nu^{\alpha\beta}$, where α, β are the flavour indices) between neutrinos ($\nu_{\alpha,\beta}$) and the scalar unparticle operator ($\mathcal{O}_{\mathcal{U}}$), the scaling dimension ($d_{\mathcal{U}}$) and the dimension transmutaion scale ($\Lambda_{\mathcal{U}}$). In the case of the neutrino unparticle interaction, heavier neutrinos become unstable and can decay into the unparticles and lighter neutrinos. In the present work in order to explore the unparticle decay process we have considered the UHE neutrino signatures obtained from GRB events for a 3+1 neutrino framework. We estimate how the effect of an unparticle decay of neutrinos in addition to the mass-flavour oscillations can change the secondary muon yields from GRB neutrinos at a 1 Km^2 detector such as IceCube for a four flavour scenario. The advantage of choosing UHE neutrinos from GRB is that the oscillatory part is averaged out due to their astronomical baslines ($\Delta m^2 L/E \gg 1$). In the present work we consider the neutrino fluxes from a point like source such as a single GRB. We calculate the muon yield in such a scenario where both unparticle decay and flavour oscillation (suppression) is considered. We also investigate the effect of fractional unparticle dimension $d_{\mathcal{U}}$ as also the coupling $|\lambda_\nu^{ij}|$ on the muon yield and compare them with the case where only flavour suppression (without an unparticle decay) is considered. It is observed that the effect of unparticle decay considerably affects the muon yield. Also as is clear from table 2, the tau neutrino flux would be affected substantially if the neutrinos undergo unparticle decay in comparison to the case when only oscillation is effective without any decay. This is a representative calculation to demonstrate the unparticle decay neutrinos can indeed affect the neutrino flux from distant sources such as GRBs.

Finally, it is to be mentioned that recently on Sept. 22, 2017 IceCube detected a muon event from a muon neutrino, the positional determination of the source of which coincides with the counterpart flaring event from the Blazar TXS0506+056 detected by Fermi gamma ray telescope [86, 87]. Consequently, Gao et al. [88] proposed a Hybrid model that incorporates both leptonic and hadronic contributions to explain the observed energy flux for this Blazar event as well as the observed neutrino event. Work is in progress to incorporate the predicted neutrino flux from TXS0506+056 event in the unparticle decay formalism considered in the present work and the results will be presented in a different publication.

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