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QCD calculations of $B \to \pi, K$ form factors with higher-twist corrections

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Abstract: We update QCD calculations of $B \to \pi, K$ form factors at large hadronic recoil by including the subleading-power corrections from the higher-twist B-meson light-cone distribution amplitudes (LCDAs) up to the twist-six accuracy and the strange-quark mass effects at leading-power in Λ/m_b from the twist-two B-meson LCDA $\phi_B^+(\omega,\mu)$. The higher-twist corrections from both the two-particle and three-particle B-meson LCDAs are computed from the light-cone QCD sum rules (LCSR) at tree level. In particular, we construct the local duality model for the twist-five and -six B-meson LCDAs, in agreement with the corresponding asymptotic behaviours at small quark and gluon momenta, employing the method of QCD sum rules in heavy quark effective theory at leading order in α_s. The strange quark mass effects in semileptonic $B \to K$ form factors yield the leading-power contribution in the heavy quark expansion, consistent with the power-counting analysis in soft-collinear effective theory, and they are also computed from the LCSR approach due to the appearance of the rapidity singularities. We demonstrate explicitly that the SU(3)-flavour symmetry breaking effects between $B \to \pi$ and $B \to K$ form factors, free of the power suppression in Λ/m_b, are suppressed by a factor of $\alpha_s(\sqrt{m_b\Lambda})$ in perturbative expansion, and they also respect the large-recoil symmetry relations of the heavy-to-light form factors at least at one-loop accuracy. An exploratory analysis of the obtained sum rules for $B \to \pi, K$ form factors with two distinct models for the B-meson LCDAs indicates that the dominant higher-twist corrections are from the Wandzura-Wilczek part of the two-particle LCDA of twist five $g_B^-(\omega,\mu)$ instead of the three-particle B-meson LCDAs. The resulting SU(3)-flavour symmetry violation effects of $B \to \pi, K$ form factors turn out to be insensitive to the non-perturbative models of B-meson LCDAs. We further explore the phenomenological aspects of the semileptonic $B \to \pi\ell\nu$ decays and the rare exclusive processes $B \to K\nu\nu$, including the determination of the CKM matrix element $|V_{ub}|$, the normalized differential q^2 distributions and precision observables defined by the ratios of branching fractions for the above-mentioned two channels in the same intervals of q^2.

Keywords: NLO Computations, QCD Phenomenology

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Contents

1 Introduction

Precision calculations of the semileptonic $B \to \pi, K$ form factors are of essential importance for the determination of the CKM matrix element $|V_{ub}|$ exclusively and the theory description of the flavour-changing neutral current process $B \to K\ell\ell$ in QCD. At small hadronic recoil the Lattice QCD calculations of these form factors have been performed in [1–3], using the gauge-field ensembles with (2+1)-flavour lattice configurations. Diverse QCD techniques for computing the heavy-to-light form factors at large hadronic recoil have been developed with distinct theory assumptions and approximations. Factorization properties of exclusive $B \to \pi, K$ form factors at large recoil have been extensively explored in the framework of soft-collinear effective theory (SCET) [4–7], leading to the QCD factorization formulae at leading power in the heavy quark expansion

$$f^i_{B\to M}(E) = C_i(E)\, \xi_a(E) + \int d\tau\, C_i^{(B1)}(E,\tau)\, \Xi_a(\tau, E)\,, \tag{1.1}$$

$$\Xi_a(\tau, E) = \frac{1}{4} \int_0^\infty d\omega \int_0^1 dv\, J_a(\tau; v, \omega)\, \tilde{f}_B\, \phi_B^+(\omega)\, f_M\, \phi_M(v)\,. \tag{1.2}$$

The perturbative matching coefficients $C_i(E)$ and $C_i^{(B1)}(E,\tau)$ have been computed at one loop [4, 8, 9], and at two loops (only for $C_i(E)$) [10–14]. The jet functions J_a from matching the $\mathrm{SCET_I}$ matrix elements of the B-type operators onto $\mathrm{SCET_{II}}$ have been also determined at the one-loop accuracy [9, 15, 16]. However, the soft-collinear factorization for the $\mathrm{SCET_I}$ matrix elements $\xi_a(E)$ cannot be achieved due to the emergence of end-point divergences, whose regularizations will introduce an unwanted connection between the soft functions and the collinear functions (see [17] for more discussions). In this respect, the method of QCD sum rules on the light cone (LCSR) can be applied to evaluate the non-perturbative form factors $\xi_a(E)$ directly by introducing the parton-hadron duality ansatz, and the rapidity divergences appearing in the soft-collinear factorization are effectively regularized by the instinct sum rule parameters. As a matter of fact, the heavy-to-light B-meson form factors $f^i_{B\to M}(E)$ themselves have been widely investigated in the context of the LCSR approach with the light-meson light-cone distribution amplitudes (LCDAs) [18–20] and with the B-meson LCDAs [21–27] (see [28–30] for further extensions to the heavy-baryon decay form factors). An alternative approach to investigate the heavy-to-light form factors is based upon the transverse-momentum dependent (TMD) factorization [31, 32] with the assumption that the soft contribution does not contribute at leading power in Λ/m_b. Technically, the rapidity divergences appearing in SCET are regularized by the intrinsic momenta of the partons participating the hard reactions [33–41]. However, a rigorous proof of the TMD factorization for the hard exclusive processes is still not available due to the absence of a definite power counting scheme for the intrinsic momenta [42, 43].

Inspired by the experimental advances for precision measurements of the semileptonic $B \to \pi \ell \nu$ decays as well as the electroweak penguin B-meson decays from Belle II [44], we attempt to improve the theory predictions for $B \to \pi, K$ form factors from the LCSR approach with the B-meson LCDAs presented in [24, 25], where the next-to-leading-logarithmic (NLL) resummation improved sum rules for the leading-power contributions were derived by applying QCD factorization for the corresponding vacuum-to-B-meson correlation function and the dispersion relation technique. We summarize the main new ingredients of the present paper in the following.

- We compute the subleading-power contributions to the semileptonic $B \to \pi, K$ form factors from both the two-particle and three-particle higher-twist B-meson LCDAs with the LCSR method at the twist-six accuracy. To this end, we adopt a complete parametrization of the three-particle B-meson LCDAs proposed in [45], which introduces eight independent invariant functions in the light-cone limit (see [46] for the original incomplete parametrization).

- We construct the local duality model for the twist-five and -six B-meson LCDAs $\Phi_5(\omega_1, \omega_2, \mu)$, $\Psi_5(\omega_1, \omega_2, \mu)$, $\tilde{\Psi}_5(\omega_1, \omega_2, \mu)$ and $\Phi_6(\omega_1, \omega_2, \mu)$ employing the method of QCD sum rules at tree level. The local duality model for the two-particle twist-five B-meson LCDA $\hat{g}_B^-(\omega, \mu)$ will be further derived with QCD equations of motion (EOM). We verify explicitly that the obtained model for the higher-twist B-meson LCDAs is consistent with the corresponding asymptotic behaviours at small quark and gluon momenta, which can be inferred from the renormalization group (RG) equations of the corresponding light-ray operators at leading logarithmic (LL) accuracy.

- We derive the leading-power contribution to $B \to K$ form factors from the strange-quark mass effect applying the LCSR approach at next-to-leading-order (NLO) in α_s. Our computation supports the early observation based upon the power-counting analysis in SCET [47] that the SU(3)-flavour symmetry violation between $B \to \pi$ and $B \to K$ form factors is not suppressed in the heavy quark limit and the strange-quark mass effect will not give rise to the large-recoil symmetry breaking of the heavy-to-light B-meson form factors.

This paper is structured as follows. We present QCD factorization formulae of the leading-twist contributions to the vacuum-to-B-meson correlation function with an interpolating current for the light pseudoscalar meson at one loop in section 2, where the new jet function generated by the non-vanishing strange-quark mass is derived with the method of regions [48] and the NLL resummation improved sum rules for $B \to \pi, K$ form factors are further obtained at leading-twist approximation. A detailed calculation of the higher-twist contributions to the semileptonic B-meson form factors from the LCSR method at tree level is presented in section 3, where the power counting of both the two-particle and three-particle subleading-power contributions is further discussed. Inspecting the correlation functions of the light-ray operators defining the higher-twist B-meson LCDAs and the suitable local currents in heavy-quark effective theory (HQET), we derive the QCD sum rules for the twist-five and -six B-meson LCDAs at leading-order (LO) in α_s in section 4, where the local duality model for these distribution amplitudes is obtained by taking the limit $\omega_M \to \infty$. We explore the phenomenological implications of the new sum rules for $B \to \pi, K$ form factors with distinct models of the B-meson LCDAs in section 5, including the numerical impacts of the higher-twist corrections, the model dependence of the SU(3)-flavour symmetry breaking effects, a comparison of the large-recoil symmetry violation with that predicted from the QCD factorization approach [49], the determination of the CKM matrix element $|V_{ub}|$, and the differential q^2 distributions of $B \to \pi \ell \nu$ and $B \to K \nu \nu$. We will conclude in section 6 with a summary of our main observations and perspectives on the future developments.

2 The leading-twist contributions to the LCSR at $\mathcal{O}(\alpha_s)$

Following the procedure presented in [25], the sum rules for $B \to \pi, K$ form factors can be constructed from the following vacuum-to-B-meson correlation function

$$\begin{aligned}\Pi_\mu(n \cdot p, \bar{n} \cdot p) &= \int d^4x \; e^{ip\cdot x} \langle 0|T\left\{\bar{d}(x) \not{n} \, \gamma_5 \, q(x), \; \bar{q}(0)\, \Gamma_\mu \, b(0)\right\}|\bar{B}(p+q)\rangle \\ &= \begin{cases} \Pi(n \cdot p, \bar{n} \cdot p)\, n_\mu + \widetilde{\Pi}(n \cdot p, \bar{n} \cdot p)\, \bar{n}_\mu \,, & \Gamma_\mu = \gamma_\mu \\ \Pi_T(n \cdot p, \bar{n} \cdot p) \left[n_\mu - \frac{n\cdot q}{m_B}\, \bar{n}_\mu\right] , & \Gamma_\mu = \sigma_{\mu\nu}\, q^\mu \end{cases}\end{aligned} \tag{2.1}$$

for the two different $b \to q$ weak currents in QCD, where the light pseudoscalar meson is interpolated by an axial-vector current carrying the four-momentum p and $p + q \equiv m_B\, v$ indicates the four-momentum of the B meson. We further introduce two light-cone vectors n_μ and $\bar{n}_\mu$ satisfying $n^2 = \bar{n}^2 = 0$ and $n \cdot \bar{n} = 2$, and employ the following power counting scheme

$$n \cdot p \sim \mathcal{O}(m_B)\,, \qquad \bar{n} \cdot p \sim m_s \sim \mathcal{O}(\Lambda)\,. \tag{2.2}$$

Applying the method of regions one can establish QCD factorization formulae for the correlation function (2.1) at leading power in Λ/m_b

$$\begin{aligned}
\Pi &= \tilde{f}_B(\mu)\, m_B \sum_{k=\pm} C^{(k)}(n\cdot p,\mu) \int_0^\infty \frac{d\omega}{\omega-\bar{n}\cdot p}\, J^{(k)}\left(\frac{\mu^2}{n\cdot p\,\omega},\frac{\omega}{\bar{n}\cdot p}\right)\, \phi_B^k(\omega,\mu)\,,\\
\widetilde{\Pi} &= \tilde{f}_B(\mu)\, m_B \sum_{k=\pm} \widetilde{C}^{(k)}(n\cdot p,\mu) \int_0^\infty \frac{d\omega}{\omega-\bar{n}\cdot p}\, \widetilde{J}^{(k)}\left(\frac{\mu^2}{n\cdot p\,\omega},\frac{\omega}{\bar{n}\cdot p}\right)\, \phi_B^k(\omega,\mu)\,, \qquad (2.3)\\
\Pi_T &= -\frac{i}{2}\, \tilde{f}_B(\mu)\, m_B^2 \sum_{k=\pm} C_T^{(k)}(n\cdot p,\mu,\nu) \int_0^\infty \frac{d\omega}{\omega-\bar{n}\cdot p}\, J_T^{(k)}\left(\frac{\mu^2}{n\cdot p\,\omega},\frac{\omega}{\bar{n}\cdot p}\right)\, \phi_B^k(\omega,\mu)\,.
\end{aligned}$$

The B-meson LCDAs in coordinate space are defined by the renormalized matrix element of the following light-cone operator in HQET [50]

$$\begin{aligned}
&\langle 0|\left(\bar{d}\,Y_s\right)_\beta(\tau\,\bar{n})\left(Y_s^\dagger\, h_v\right)_\alpha(0)|\bar{B}(v)\rangle \qquad (2.4)\\
&\quad = -\frac{i\tilde{f}_B(\mu)\,m_B}{4}\left\{\frac{1+\not{v}}{2}\left[2\,\tilde{\phi}_B^+(\tau,\mu)+\left(\tilde{\phi}_B^-(\tau,\mu)-\tilde{\phi}_B^+(\tau,\mu)\right)\not{\bar{n}}\right]\gamma_5\right\}_{\alpha\beta},
\end{aligned}$$

where the soft Wilson line is given by

$$Y_s(\tau\,\bar{n}) = \mathrm{P}\left\{\mathrm{Exp}\left[i\,g_s\int_{-\infty}^{\tau} dx\,\bar{n}\cdot A_s(x\,\bar{n})\right]\right\}. \qquad (2.5)$$

The renormalization-scale dependent HQET decay constant $\tilde{f}_B(\mu)$ can be expressed in terms of the QCD decay constant f_B

$$\tilde{f}_B(\mu) = \left\{1-\frac{\alpha_s(\mu)\,C_F}{4\,\pi}\left[3\,\ln\frac{\mu}{m_b}+2\right]\right\}^{-1} f_B\,. \qquad (2.6)$$

We can further determine the renormalized hard functions and jet functions entering the factorization formulae (2.3) at the one-loop accuracy

$$\begin{aligned}
C^{(+)} &= \widetilde{C}^{(+)} = C_T^{(+)} = 1\,, \qquad C^{(-)} = \frac{\alpha_s\,C_F}{4\pi}\,\frac{1}{\bar{r}}\left[1+\frac{r}{\bar{r}}\ln r\right],\\
\widetilde{C}^{(-)} &= 1-\frac{\alpha_s\,C_F}{4\pi}\left[2\ln^2\frac{\mu}{n\cdot p}+5\ln\frac{\mu}{m_b}-\ln^2 r-2\,\mathrm{Li}_2\left(-\frac{\bar{r}}{r}\right)+\frac{2-r}{r-1}\ln r+\frac{\pi^2}{12}+5\right],\\
C_T^{(-)} &= 1+\frac{\alpha_s\,C_F}{4\pi}\left[-2\ln\frac{\nu}{m_b}-2\ln^2\frac{\mu}{n\cdot p}-5\ln\frac{\mu}{n\cdot p}-2\,\mathrm{Li}_2(1-r)-\frac{3-r}{1-r}\ln r-\frac{\pi^2}{12}-6\right],\\
J^{(+)} &= \frac{\alpha_s\,C_F}{4\pi}\left(1-\frac{\bar{n}\cdot p}{\omega}\right)\ln\left(1-\frac{\omega}{\bar{n}\cdot p}\right),\\
\widetilde{J}^{(+)} &= \frac{\alpha_s\,C_F}{4\pi}\left[r\left(1-\frac{\bar{n}\cdot p}{\omega}\right)+\frac{m_q}{\omega}\right]\ln\left(1-\frac{\omega}{\bar{n}\cdot p}\right),\\
J_T^{(+)} &= \frac{\alpha_s\,C_F}{4\pi}\left[-\left(1-\frac{\bar{n}\cdot p}{\omega}\right)+\frac{m_q}{\omega}\right]\ln\left(1-\frac{\omega}{\bar{n}\cdot p}\right),\\
J^{(-)} &= 1\,,\\
\widetilde{J}^{(-)} &= J_T^{(-)} = 1+\frac{\alpha_s\,C_F}{4\pi}\left[\ln^2\frac{\mu^2}{n\cdot p(\omega-\bar{n}\cdot p)}-2\ln\frac{\bar{n}\cdot p-\omega}{\bar{n}\cdot p}\ln\frac{\mu^2}{n\cdot p(\omega-\bar{n}\cdot p)}\right.\\
&\qquad\left.-\ln^2\frac{\bar{n}\cdot p-\omega}{\bar{n}\cdot p}-\left(1+\frac{2\bar{n}\cdot p}{\omega}\right)\ln\frac{\bar{n}\cdot p-\omega}{\bar{n}\cdot p}-\frac{\pi^2}{6}-1\right], \qquad (2.7)
\end{aligned}$$

where ν refers to the renormalization scale of the QCD tensor current and we have also introduced the conventions

$$r = n \cdot p / m_b \,, \qquad \bar{r} = 1 - r. \tag{2.8}$$

Several remarks on the resulting perturbative matching coefficients are in order.

- The hard matching coefficients appearing in the QCD factorization formulae for the vacuum-to-B-meson correlation function (2.3) are apparently identical to the short-distance functions from representing the corresponding QCD weak currents in $\mathrm{SCET_I}$. Employing the perturbative matching for heavy-to-light currents displayed in [4]

$$\begin{aligned} \bar{q}\,\gamma_\mu\, b &\to [C_4\, \bar{n}_\mu + C_5\, v_\mu]\, \bar{\xi}_{\bar{n}}\, W_{hc}\, Y_s^\dagger\, h_v + \ldots, \\ \bar{q}\,\sigma_{\mu\nu}\, q^\nu\, b &\to (-i)\, C_{11}\, (v_\mu \bar{n}_\nu - v_\nu \bar{n}_\mu)\, q^\nu\, \bar{\xi}_{\bar{n}}\, W_{hc}\, Y_s^\dagger\, h_v + \ldots\,, \end{aligned} \tag{2.9}$$

  one can readily determine the following relations for the hard functions

$$C^{(-)} = \frac{1}{2}\, C_5\,, \qquad \widetilde{C}^{(-)} = C_4 + \frac{1}{2}\, C_5\,, \qquad C_T^{(-)} = C_{11}\,, \tag{2.10}$$

  which can be further verified by comparing the explicit expressions of C_4, C_5 and C_{11} obtained in [4] with the results presented in (2.7).

- The nonvanishing light-quark mass gives rise to the leading-power contribution to the jet functions in the heavy quark expansion, which is independent of the Dirac structures of the QCD weak currents. Our calculation supports the power counting analysis for the light-quark mass effects in semileptonic B-meson decay form factors at large recoil in the framework of SCET [47]. Inspecting the diagrammatical representation of the two-particle contributions to correlation function (2.1) at NLO in QCD, we observe that the above-mentioned SU(3)-flavour symmetry breaking effect solely comes from the QCD correction to the light-pseudoscalar-meson vertex diagram (see figure 2(b) of [25]). It immediately follows that the light-quark-mass dependent jet function entering the factorization formulae (2.3) is universal for the vacuum-to-B-meson correlation functions with different weak currents.

We will proceed to perform the summation of parametrically large logarithms appearing in the factorization formulae (2.3) employing the RG equations in momentum space. Taking the factorization scale μ as a hard-collinear scale $\mu_{hc} \sim \sqrt{\Lambda\, m_b}$ and solving the evolution equations at the NLL accuracy leads to

$$\begin{aligned} \widetilde{C}^{(-)}(n \cdot p, \mu) &= U_1(n \cdot p, \mu_{h1}, \mu)\; \widetilde{C}^{(-)}(n \cdot p, \mu_{h1})\,, \\ C_T^{(-)}(n \cdot p, \mu, \nu) &= U_1(n \cdot p, \mu_{h1}, \mu)\; U_3(\nu_h, \nu)\; C_T^{(-)}(n \cdot p, \mu_{h1}, \nu_h)\,, \\ \tilde{f}_B(\mu) &= U_2(\mu_{h2}, \mu)\, \tilde{f}_B(\mu_{h2})\;, \end{aligned} \tag{2.11}$$

where the explicit expressions of the evolution functions U_1 and U_2 can be found in [51, 52] and the QCD evolution factor $U_3(\nu_h, \nu)$ is given by

$$
\begin{aligned}
U_3(\nu_h,\nu) &= \mathrm{Exp}\left[\int_{\alpha_s(\nu_h)}^{\alpha_s(\nu)} d\alpha_s\, \frac{\gamma_T(\alpha_s)}{\beta(\alpha_s)}\right] \\
&= z^{-\frac{\gamma_T^{(0)}}{2\beta_0}} \left[1 + \frac{\alpha_s(\nu_h)}{4\pi} \left(\frac{\gamma_T^{(1)}}{2\,\beta_0} - \frac{\gamma_T^{(0)}\,\beta_1}{2\,\beta_0^2}\right)(1-z) + \mathcal{O}(\alpha_s^2)\right], \qquad (2.12)
\end{aligned}
$$

with $z = \alpha_s(\nu)/\alpha_s(\nu_h)$. The anomalous dimension $\gamma_T(\alpha_s)$ for the tensor current at the two-loop accuracy is [14]

$$
\begin{aligned}
\gamma_T(\alpha_s) &= \sum_{n=0}^{\infty} \left(\frac{\alpha_s(\mu)}{4\,\pi}\right)^{n+1} \gamma_T^{(n)}, \qquad \gamma_T^{(0)} = -2\,C_F, \\
\gamma_T^{(1)} &= C_F\left[19\,C_F - \frac{257}{9}\,C_A + \frac{52}{9}\,n_f\,T_F\right]. \qquad (2.13)
\end{aligned}
$$

Since the hard-collinear scale μ_{hc} is quite close to the soft scale μ_0 of the B-meson LCDAs numerically and the two-loop evolution equations of the two-particle B-meson distribution amplitudes $\phi_B^{\pm}(\omega,\mu)$ are not available yet, we will not perform the NLL resummmation for the logarithms of μ/μ_0 due to the RG evolution of $\phi_B^{\pm}(\omega,\mu)$ (see also [51]). It is then straightforward to write down the (partial) NLL resummation improved QCD factorization formulae for the correlation function (2.1)

$$
\begin{aligned}
\Pi &= \left[U_2(\mu_{h2},\mu)\,\tilde{f}_B(\mu_{h2})\right] m_B \left\{\int_0^\infty \frac{d\omega}{\omega - \bar{n}\cdot p}\, J^{(+)}\left(\frac{\mu^2}{n\cdot p\,\omega}, \frac{\omega}{\bar{n}\cdot p}\right) \phi_B^+(\omega,\mu) \right.\\
&\qquad \left. + C^{(-)}(n\cdot p,\mu) \int_0^\infty \frac{d\omega}{\omega - \bar{n}\cdot p}\, \phi_B^-(\omega,\mu)\right\}, \\
\widetilde{\Pi} &= \left[U_2(\mu_{h2},\mu)\,\tilde{f}_B(\mu_{h2})\right] m_B \left\{\int_0^\infty \frac{d\omega}{\omega - \bar{n}\cdot p}\, \widetilde{J}^{(+)}\left(\frac{\mu^2}{n\cdot p\,\omega}, \frac{\omega}{\bar{n}\cdot p}\right) \phi_B^+(\omega,\mu) \right.\\
&\quad \left. + \left[U_1(n\cdot p,\mu_{h1},\mu)\,\widetilde{C}^{(-)}(n\cdot p,\mu_{h1})\right] \int_0^\infty \frac{d\omega}{\omega - \bar{n}\cdot p}\, \widetilde{J}^{(-)}\left(\frac{\mu^2}{n\cdot p\,\omega}, \frac{\omega}{\bar{n}\cdot p}\right) \phi_B^-(\omega,\mu)\right\}, \\
\Pi_T &= -\frac{i}{2}\left[U_2(\mu_{h2},\mu)\,\tilde{f}_B(\mu_{h2})\right] m_B^2 \left\{\int_0^\infty \frac{d\omega}{\omega - \bar{n}\cdot p}\, J_T^{(+)}\left(\frac{\mu^2}{n\cdot p\,\omega}, \frac{\omega}{\bar{n}\cdot p}\right) \phi_B^+(\omega,\mu) \right.\\
&\qquad + \left[U_1(n\cdot p,\mu_{h1},\mu)\,U_3(\nu_h,\nu)\,C_T^{(-)}(n\cdot p,\mu_{h1},\nu_h)\right] \\
&\qquad \left. \times \int_0^\infty \frac{d\omega}{\omega - \bar{n}\cdot p}\, J_T^{(-)}\left(\frac{\mu^2}{n\cdot p\,\omega}, \frac{\omega}{\bar{n}\cdot p}\right) \phi_B^-(\omega,\mu)\right\}. \qquad (2.14)
\end{aligned}
$$

Employing the standard definitions for $B \to P$ form factors (with $P = \pi,\ K$) and the decay constant of the pseudoscalar meson

$$
\begin{aligned}
\langle P(p)|\bar{q}\,\gamma_\mu\, b|\bar{B}(p+q)\rangle &= f_{B\to P}^{+}(q^2)\left[2p+q - \frac{m_B^2 - m_P^2}{q^2}\,q\right]_\mu + f_{B\to P}^{0}(q^2)\,\frac{m_B^2 - m_P^2}{q^2}\,q_\mu, \\
\langle P(p)|\bar{q}\,\sigma_{\mu\nu}\,q^\nu\, b|\bar{B}(p+q)\rangle &= i\,\frac{f_{B\to P}^{T}(q^2)}{m_B + m_P}\left[q^2\,(2p+q)_\mu - (m_B^2 - m_P^2)\,q_\mu\right], \\
\langle 0|\bar{d}\,\not{\bar{n}}\,\gamma_5\, q|P(p)\rangle &= i\,n\cdot p\, f_P, \qquad (2.15)
\end{aligned}
$$

we can readily derive the hadronic representations of the correlation function (2.1)

$$
\begin{aligned}
\Pi_{\mu,V}(n\cdot p,\bar n\cdot p) &= \frac{f_P\, m_B}{2\,(m_P^2/n\cdot p-\bar n\cdot p)}\bigg\{\bar n_\mu\, \left[\frac{n\cdot p}{m_B}\, f^{+}_{B\to P}(q^2)+f^{0}_{B\to P}(q^2)\right] \\
&\qquad + n_\mu\, \frac{m_B}{n\cdot p - m_B}\, \left[\frac{n\cdot p}{m_B}\, f^{+}_{B\to P}(q^2)-f^{0}_{B\to P}(q^2)\right]\bigg\} \\
&\quad + \int_{\omega_s}^{+\infty} \frac{d\omega'}{\omega'-\bar n\cdot p - i0}\, \left[\rho^h_{V,1}(\omega', n\cdot p)\, n_\mu\, + \rho^h_{V,2}(\omega', n\cdot p)\, \bar n_\mu\right], \\
\Pi_{\mu,T}(n\cdot p,\bar n\cdot p) &= -i\, \frac{f_P\, n\cdot p}{2\,(m_P^2/n\cdot p-\bar n\cdot p)}\, \frac{m_B^2}{m_B+m_P}\, \left[n_\mu - \frac{n\cdot q}{m_B}\,\bar n_\mu\right]\, f^{T}_{B\to P}(q^2) \\
&\quad + \int_{\omega_s}^{+\infty} \frac{d\omega'}{\omega'-\bar n\cdot p - i0}\, \left[n_\mu - \frac{n\cdot q}{m_B}\,\bar n_\mu\right]\, \rho^h_T(\omega', n\cdot p)\,,
\end{aligned} \tag{2.16}
$$

where $\Pi_{\mu,V}$ and $\Pi_{\mu,T}$ correspond to $\Gamma_\mu=\gamma_\mu$ and $\Gamma_\mu=\sigma_{\mu\nu}\,q^\nu$ for the Dirac structure of the weak current $\bar q(0)\,\Gamma_\mu\, b(0)$ in the definition (2.1), respectively. Matching the hadronic dispersion relations (2.16) and the resummation improved factorization formulae (2.14) with the aid of the parton-hadron duality ansatz and implementing the Borel transformation with respect to the variable $\bar n\cdot p\to\omega_M$ gives rise to the NLL LCSR for $B\to P$ form factors at leading power in the heavy quark expansion

$$
\begin{aligned}
& f_P\, \exp\left[-\frac{m_P^2}{n\cdot p\,\omega_M}\right]\, \left\{\frac{n\cdot p}{m_B}\, f^{+,\,2\mathrm{PNLL}}_{B\to P}(q^2)\,,\ \ f^{0,\,2\mathrm{PNLL}}_{B\to P}(q^2)\right\} \\
&= \left[U_2(\mu_{h2},\mu)\,\tilde f_B(\mu_{h2})\right]\, \int_0^{\omega_s} d\omega'\, e^{-\omega'/\omega_M} \\
&\quad \times\bigg\{\widetilde\phi^{+}_{B,\,\mathrm{eff}}(\omega',\mu)+\left[U_1(n\cdot p,\mu_{h1},\mu)\,\widetilde C^{(-)}(n\cdot p,\mu_{h1})\right]\,\widetilde\phi^{-}_{B,\,\mathrm{eff}}(\omega',\mu) \\
&\qquad \pm\frac{n\cdot p-m_B}{m_B}\,\left[\phi^{+}_{B,\,\mathrm{eff}}(\omega',\mu)+C^{(-)}(n\cdot p,\mu_{h1})\,\phi^{-}_{B,\,\mathrm{eff}}(\omega',\mu)\right]\bigg\}, \\
& f_P\, \exp\left[-\frac{m_P^2}{n\cdot p\,\omega_M}\right]\, \frac{n\cdot p}{m_B+m_P}\, f^{T,\,2\mathrm{PNLL}}_{B\to P}(q^2) \\
&= \left[U_2(\mu_{h2},\mu)\,\tilde f_B(\mu_{h2})\right]\, \int_0^{\omega_s} d\omega'\, e^{-\omega'/\omega_M} \\
&\quad \times\bigg\{\widehat\phi^{+}_{B,\,\mathrm{eff}}(\omega',\mu)+\left[U_1(n\cdot p,\mu_{h1},\mu)\,U_3(\nu_h,\nu)\,C^{(-)}_T(n\cdot p,\mu_{h1},\nu_h)\right]\,\widetilde\phi^{-}_{B,\,\mathrm{eff}}(\omega',\mu)\bigg\},
\end{aligned} \tag{2.17}
$$

where we have defined the effective B-meson "distribution amplitudes" for brevity

$$
\begin{aligned}
\widetilde\phi^{+}_{B,\,\mathrm{eff}}(\omega',\mu) &= \frac{\alpha_s C_F}{4\pi}\left[r\int_{\omega'}^{\infty}d\omega\,\frac{\phi^+_B(\omega,\mu)}{\omega}-m_q\int_{\omega'}^{\infty}d\omega\,\ln\left(\frac{\omega-\omega'}{\omega'}\right)\,\frac{d}{d\omega}\,\frac{\phi^+_B(\omega,\mu)}{\omega}\right], \\
\widetilde\phi^{-}_{B,\,\mathrm{eff}}(\omega',\mu) &= \phi^-_B(\omega',\mu)+\frac{\alpha_s C_F}{4\pi}\bigg\{\int_0^{\omega'}d\omega\,\left[\frac{2}{\omega-\omega'}\,\left(\ln\frac{\mu^2}{n\cdot p\omega'}-2\ln\frac{\omega'-\omega}{\omega'}\right)\right]_{\oplus}\phi^-_B(\omega,\mu) \\
&\quad -\int_{\omega'}^{\infty}d\omega\,\left[\ln^2\frac{\mu^2}{n\cdot p\omega'}-\left(2\ln\frac{\mu^2}{n\cdot p\omega'}+3\right)\,\ln\frac{\omega-\omega'}{\omega'}+2\ln\frac{\omega}{\omega'}+\frac{\pi^2}{6}-1\right] \\
&\quad \times\frac{d\phi^-_B(\omega,\mu)}{d\omega}\bigg\},
\end{aligned}
$$

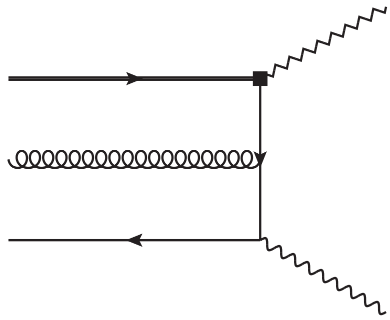

Figure 1. Diagrammatical representation of the three-particle higher-twist corrections to the vacuum-to-B-meson correlation function (2.1). The square box indicates the insertion of the weak vertex $\bar{q}\,\Gamma_\mu\, b$, and the waveline represents the interpolating current $\bar{d}\not{n}\,\gamma_5\; q$ for the light-pseudoscalar meson.

$$\begin{aligned}
\phi^{+}_{B,\mathrm{eff}}(\omega',\mu) &= \frac{\alpha_s\,C_F}{4\,\pi}\int_{\omega'}^{\infty} d\omega\,\frac{\phi^{+}_{B}(\omega,\mu)}{\omega}\,, \qquad \phi^{-}_{B,\mathrm{eff}}(\omega',\mu)=\phi^{-}_{B}(\omega',\mu)\,,\\
\widehat{\phi}^{+}_{B,\mathrm{eff}}(\omega',\mu) &= \frac{\alpha_s\,C_F}{4\,\pi}\left[-\int_{\omega'}^{\infty} d\omega\,\frac{\phi^{+}_{B}(\omega,\mu)}{\omega} - m_q\int_{\omega'}^{\infty} d\omega\,\ln\left(\frac{\omega-\omega'}{\omega'}\right)\,\frac{d}{d\omega}\,\frac{\phi^{+}_{B}(\omega,\mu)}{\omega}\right]. \quad (2.18)
\end{aligned}$$

The plus function entering (2.18) is further given by

$$\int_0^{\infty} d\omega\,\left[f(\omega,\omega')\right]_{\oplus}\, g(\omega) = \int_0^{\infty} d\omega\, f(\omega,\omega')\,\left[g(\omega)-g(\omega')\right]\,. \tag{2.19}$$

It is evident that the light-quark-mass dependent effects respect the large-recoil symmetry relations for the soft contribution to the heavy-to-light form factors.

3 The higher-twist contributions to the LCSR

We turn to compute the higher-twist corrections to $B\to\pi, K$ form factors from both the two-particle and three-particle B-meson LCDAs employing a complete parametrization of the corresponding three-particle light-cone matrix element and the EOM constraints of the higher-twist LCDAs presented in [45]. To achieve this goal, we make use of the light-cone expansion of the quark propagator in the background gluon field [53]

$$\langle 0|\mathrm{T}\,\{\bar{q}(x), q(0)\}|0\rangle \supset i\,g_s\int_0^{\infty}\frac{d^4k}{(2\pi)^4}\,e^{-i\,k\cdot x}\int_0^1 du\,\left[\frac{u\,x_\mu\,\gamma_\nu}{k^2-m_q^2}-\frac{(\not{k}+m_q)\,\sigma_{\mu\nu}}{2\,(k^2-m_q^2)^2}\right]\,G^{\mu\nu}(u\,x)\,, \tag{3.1}$$

where we only keep the one-gluon part without the covariant derivative of the $G_{\mu\nu}$ terms. Evaluating the tree-level diagram displayed in figure 1, it is straightforward to derive the

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three-particle higher-twist corrections to the vacuum-to-B-meson correlation function (2.1)

$$
\begin{aligned}
\Pi^{(3P)}_{\mu,V}(n\cdot p,\bar n\cdot p) &= -\frac{\tilde f_B(\mu)\,m_B}{n\cdot p}\int_0^\infty d\omega_1\int_0^\infty d\omega_2\int_0^1 du\,\frac{1}{[\bar n\cdot p-\omega_1-u\,\omega_2]^2}\\
&\quad\times\Big\{\bar n_\mu\left[\rho^{(3P)}_{\bar n,\rm LP}(u,\omega_1,\omega_2,\mu)+\frac{m_q}{n\cdot p}\rho^{(3P)}_{\bar n,\rm NLP}(u,\omega_1,\omega_2,\mu)\right]\\
&\qquad+n_\mu\left[\rho^{(3P)}_{n,\rm LP}(u,\omega_1,\omega_2,\mu)+\frac{m_q}{n\cdot p}\rho^{(3P)}_{n,\rm NLP}(u,\omega_1,\omega_2,\mu)\right]\Big\},\\
\Pi^{(3P)}_{\mu,T}(n\cdot p,\bar n\cdot p) &= \frac{i}{2}\,\frac{\tilde f_B(\mu)\,m_B^2}{n\cdot p}\left[n_\mu-\frac{n\cdot q}{m_B}\bar n_\mu\right]\int_0^\infty d\omega_1\int_0^\infty d\omega_2\int_0^1 du\,\frac{1}{[\bar n\cdot p-\omega_1-u\,\omega_2]^2}\\
&\quad\times\Big\{\rho^{(3P)}_{T,\rm LP}(u,\omega_1,\omega_2,\mu)+\frac{m_q}{n\cdot p}\rho^{(3P)}_{T,\rm NLP}(u,\omega_1,\omega_2,\mu)\Big\},
\end{aligned}
\tag{3.2}
$$

where we have taken into account the SU(3)-flavour symmetry breaking effect due to the light-quark mass. In contrast to the two-particle contributions to the correlation function (2.1), the light-quark mass dependent terms of the three-particle corrections at LO in α_s are suppressed by one power of Λ/m_b. The explicit expressions of $\rho^{(3P)}_{i,\rm LP}$ and $\rho^{(3P)}_{i,\rm NLP}$ $(i=n\,,\bar n\,,T)$ are given by

$$
\begin{aligned}
\rho^{(3P)}_{\bar n,\rm LP} &= (1-2\,u)\,\left[X_A-\Psi_A-2\,Y_A\right]-\tilde X_A-\Psi_V+2\,\tilde Y_A\,,\\
\rho^{(3P)}_{\bar n,\rm NLP} &= 2\,\left[\Psi_A-\Psi_V\right]+4\,\left[W+Y_A+\tilde Y_A-2\,Z\right]\,,\\
\rho^{(3P)}_{n,\rm LP} &= 2\,(u-1)\,\left(\Psi_A+\Psi_V\right)\,,\\
\rho^{(3P)}_{n,\rm NLP} &= \left(\Psi_A-\Psi_V\right)-\left[X_A+\tilde X_A-2\,Y_A-2\,\tilde Y_A\right]\,,\\
\rho^{(3P)}_{T,\rm LP} &= (1-2\,u)\,\left(\Psi_V+X_A-2\,Y_A\right)+\Psi_A-\tilde X_A+2\,\tilde Y_A\,,\\
\rho^{(3P)}_{T,\rm NLP} &= \left(\Psi_A-\Psi_V+X_A+\tilde X_A\right)+2\,\left[2\,W+Y_A+\tilde Y_A-4\,Z\right]\,,
\end{aligned}
\tag{3.3}
$$

where we have suppressed the arguments of the three-particle B-meson LCDAs for brevity. To obtain such three-particle corrections to the correlation function (2.1), we have adopted the following decomposition of the light-cone matrix element in HQET [45]

$$
\begin{aligned}
&\langle 0|\bar q_\alpha(z_1\,\bar n)\,g_s\,G_{\mu\nu}(z_2\,\bar n)\,h_{v\,\beta}(0)|\bar B(v)\rangle\\
&=\frac{\tilde f_B(\mu)\,m_B}{4}\Big[(1+\not v)\Big\{(v_\mu\gamma_\nu-v_\nu\gamma_\mu)\,\left[\Psi_A(z_1,z_2,\mu)-\Psi_V(z_1,z_2,\mu)\right]-i\,\sigma_{\mu\nu}\,\Psi_V(z_1,z_2,\mu)\\
&\qquad-(\bar n_\mu\,v_\nu-\bar n_\nu\,v_\mu)\,X_A(z_1,z_2,\mu)+(\bar n_\mu\,\gamma_\nu-\bar n_\nu\,\gamma_\mu)\,\left[W(z_1,z_2,\mu)+Y_A(z_1,z_2,\mu)\right]\\
&\qquad+i\,\epsilon_{\mu\nu\alpha\beta}\,\bar n^\alpha\,v^\beta\,\gamma_5\,\tilde X_A(z_1,z_2,\mu)-i\,\epsilon_{\mu\nu\alpha\beta}\,\bar n^\alpha\,\gamma^\beta\,\gamma_5\,\tilde Y_A(z_1,z_2,\mu)\\
&\qquad-(\bar n_\mu\,v_\nu-\bar n_\nu\,v_\mu)\,\not\bar n\,W(z_1,z_2,\mu)+(\bar n_\mu\,\gamma_\nu-\bar n_\nu\,\gamma_\mu)\,\not\bar n\,Z(z_1,z_2,\mu)\Big\}\gamma_5\Big]_{\beta\alpha}\,,
\end{aligned}
\tag{3.4}
$$

where we have neglected the soft Wilson lines to restore the gauge invariance of the light-ray operator and our convention corresponds to $\epsilon_{0123}=-1$. As emphasized in [45], the higher-twist two-particle B-meson LCDAs due to nonvanishing quark transverse momentum can be expressed in terms of the three-particle configurations with the exact EOM, and they must be taken into account simultaneously for consistency. Including the light-cone correction terms up to the $\mathcal{O}(x^2)$ accuracy, the two-particle renormalized light-cone matrix element (2.4) can be parameterized as follows [45]

$$
\begin{aligned}
&\langle 0|\left(\bar{d}\, Y_s\right)_\beta(x)\,\left(Y_s^\dagger\, h_v\right)_\alpha(0)|\bar{B}(v)\rangle \\
&= -\frac{i\tilde{f}_B(\mu)\, m_B}{4}\int_0^\infty d\omega\, e^{-i\,\omega\, v\cdot x}\left\{\frac{1+\not{v}}{2}\left[2\,\left(\phi_B^+(\omega,\mu)+x^2\, g_B^+(\omega,\mu)\right)\right.\right. \\
&\quad \left.\left. -\frac{1}{v\cdot x}\,\left[\left(\phi_B^+(\omega,\mu)-\phi_B^-(\omega,\mu)\right)+x^2\,\left(g_B^+(\omega,\mu)-g_B^-(\omega,\mu)\right)\right]\,\not{x}\right]\gamma_5\right\}_{\alpha\beta}, \qquad (3.5)
\end{aligned}
$$

where the two new distribution amplitudes g_B^+ and g_B^- are of twist-four and -five, respectively. Applying the operator identities between the two-body and three-body light-cone operators leads to the nontrivial relations of B-meson LCDAs in the momentum space

$$
\begin{aligned}
-\omega\,\frac{d}{d\omega}\,\phi_B^-(\omega,\mu) &= \phi_B^+(\omega,\mu)-2\int_0^\infty\frac{d\omega_2}{\omega_2^2}\,\Phi_3(\omega,\omega_2,\mu)+2\int_0^\omega\frac{d\omega_2}{\omega_2^2}\,\Phi_3(\omega-\omega_2,\omega_2,\mu) \\
&\quad +2\int_0^\omega\frac{d\omega_2}{\omega_2}\,\frac{d}{d\omega}\,\Phi_3(\omega-\omega_2,\omega_2,\mu)\,, \qquad (3.6) \\
-2\,\frac{d^2}{d\omega^2}\,g_B^+(\omega,\mu) &= \left[\frac{3}{2}+(\omega-\bar{\Lambda})\,\frac{d}{d\omega}\right]\,\phi_B^+(\omega,\mu)-\frac{1}{2}\,\phi_B^-(\omega,\mu)+\int_0^\infty\frac{d\omega_2}{\omega_2}\,\frac{d}{d\omega}\,\Psi_4(\omega,\omega_2,\mu) \\
&\quad -\int_0^\infty\frac{d\omega_2}{\omega_2^2}\,\Psi_4(\omega,\omega_2,\mu)+\int_0^\omega\frac{d\omega_2}{\omega_2^2}\,\Psi_4(\omega-\omega_2,\omega_2,\mu)\,, \qquad (3.7) \\
-2\,\frac{d^2}{d\omega^2}\,g_B^-(\omega,\mu) &= \left[\frac{3}{2}+(\omega-\bar{\Lambda})\,\frac{d}{d\omega}\right]\,\phi_B^-(\omega,\mu)-\frac{1}{2}\,\phi_B^+(\omega,\mu)+\int_0^\infty\frac{d\omega_2}{\omega_2}\,\frac{d}{d\omega}\,\Psi_5(\omega,\omega_2,\mu) \\
&\quad -\int_0^\infty\frac{d\omega_2}{\omega_2^2}\,\Psi_5(\omega,\omega_2,\mu)+\int_0^\omega\frac{d\omega_2}{\omega_2^2}\,\Psi_5(\omega-\omega_2,\omega_2,\mu)\,, \qquad (3.8) \\
\phi_B^-(\omega,\mu) &= \left(2\,\bar{\Lambda}-\omega\right)\,\frac{d\phi_B^+(\omega,\mu)}{d\omega}-2\int_0^\infty\frac{d\omega_2}{\omega_2^2}\,\Phi_4(\omega,\omega_2,\mu) \\
&\quad +2\int_0^\omega\frac{d\omega_2}{\omega_2}\,\left(\frac{d}{d\,\omega_2}+\frac{d}{d\,\omega}\right)\,\Phi_4(\omega-\omega_2,\omega_2,\mu) \qquad (3.9) \\
&\quad +2\int_0^\omega\frac{d\omega_2}{\omega_2}\,\frac{d}{d\omega}\,\Psi_4(\omega-\omega_2,\omega_2,\mu)-2\int_0^\infty\frac{d\omega_2}{\omega_2}\,\frac{d}{d\omega}\,\Psi_4(\omega,\omega_2,\mu)\,,
\end{aligned}
$$

which can be obtained from the Fourier transformation of the coordinate-space representations obtained in [45]. Here, the HQET paremater $\bar{\Lambda}$ corresponds to the mass of the light degrees of freedom in the heavy meson [54]

$$\bar{\Lambda} = m_B - m_b\,. \qquad (3.10)$$

We mention in passing that the classical EOM (3.6)–(3.9) are not consistent with the RG equations of light-cone operators in HQET and it remains interesting to derive the above-mentioned EOM at one-loop accuracy for the precision calculations of higher-twist corrections in the future. For the tree-level calculations of higher-twist contributions presented in this work, it is necessary to implement the classical EOM constraints for the construction of the twist-five and -six B-meson LCDAs for consistency. We have also introduced the three-particle B-meson LCDAs of definite twist

$$
\begin{aligned}
\Phi_3(\omega_1,\omega_2,\mu) &= \Psi_A(\omega_1,\omega_2,\mu)-\Psi_V(\omega_1,\omega_2,\mu)\,,\\
\Phi_4(\omega_1,\omega_2,\mu) &= \Psi_A(\omega_1,\omega_2,\mu)+\Psi_V(\omega_1,\omega_2,\mu)\,,\\
\Psi_4(\omega_1,\omega_2,\mu) &= \Psi_A(\omega_1,\omega_2,\mu)+X_A(\omega_1,\omega_2,\mu)\,,\\
\tilde{\Psi}_4(\omega_1,\omega_2,\mu) &= \Psi_V(\omega_1,\omega_2,\mu)-\tilde{X}_A(\omega_1,\omega_2,\mu)\,,\\
\Phi_5(\omega_1,\omega_2,\mu) &= \Psi_A(\omega_1,\omega_2,\mu)+\Psi_V(\omega_1,\omega_2,\mu)+2\,\left[Y_A-\tilde{Y}_A+W\right](\omega_1,\omega_2,\mu)\,,\\
\Psi_5(\omega_1,\omega_2,\mu) &= -\Psi_A(\omega_1,\omega_2,\mu)+X_A(\omega_1,\omega_2,\mu)-2\,Y_A(\omega_1,\omega_2,\mu)\,,\\
\tilde{\Psi}_5(\omega_1,\omega_2,\mu) &= -\Psi_V(\omega_1,\omega_2,\mu)-\tilde{X}_A(\omega_1,\omega_2,\mu)+2\,\tilde{Y}_A(\omega_1,\omega_2,\mu)\,,\\
\Phi_6(\omega_1,\omega_2,\mu) &= \Psi_A(\omega_1,\omega_2,\mu)-\Psi_V(\omega_1,\omega_2,\mu)+2\,\left[Y_A+\tilde{Y}_A+W-2\,Z\right](\omega_1,\omega_2,\mu)\,.
\end{aligned}
\tag{3.11}
$$

We are now ready to derive the two-particle higher-twist corrections to the vacuum-to-B-meson correlation function (2.1) at tree level

$$
\begin{aligned}
\Pi_{\mu,V}^{\rm 2PHT} &= -4\,\frac{\tilde{f}_B(\mu)\,m_B}{n\cdot p}\,\bar{n}_\mu\left\{-\frac{1}{2}\int_0^\infty d\omega_1\int_0^\infty d\omega_2\int_0^1 du\,\frac{\bar{u}\,\Psi_5(\omega_1,\omega_2,\mu)}{(\bar{n}\cdot p-\omega_1-u\,\omega_2)^2}\right.\\
&\qquad\left.+\int_0^\infty\frac{d\omega}{(\bar{n}\cdot p-\omega)^2}\,\hat{g}_B^-(\omega,\mu)\right\},\\
\Pi_{\mu,T}^{\rm 2PHT} &= 2\,i\,\frac{\tilde{f}_B(\mu)\,m_B^2}{n\cdot p}\,\left[n_\mu-\frac{n\cdot q}{m_B}\,\bar{n}_\mu\right]\left\{-\frac{1}{2}\int_0^\infty d\omega_1\int_0^\infty d\omega_2\int_0^1 du\,\frac{\bar{u}\,\Psi_5(\omega_1,\omega_2,\mu)}{(\bar{n}\cdot p-\omega_1-u\,\omega_2)^2}\right.\\
&\qquad\left.+\int_0^\infty\frac{d\omega}{(\bar{n}\cdot p-\omega)^2}\,\hat{g}_B^-(\omega,\mu)\right\},
\end{aligned}
\tag{3.12}
$$

where we have introduced the convention

$$
\hat{g}_B^-(\omega,\mu)=\frac{1}{4}\int_\omega^\infty d\rho\left\{(\rho-\omega)\,\left[\phi_B^+(\rho)-\phi_B^-(\rho)\right]-2\,(\bar{\Lambda}-\rho)\,\phi_B^-(\rho)\right\}. \tag{3.13}
$$

Adding up the two-particle and three-particle higher-twist corrections at tree level together and implementing the standard strategy to construct the sum rules for heavy-to-light form factors gives rise to the following expressions

$$
\begin{aligned}
&\frac{f_P\,n\cdot p}{2}\,\exp\left[-\frac{m_P^2}{n\cdot p\,\omega_M}\right]\,\left[f_{B\to P}^{+,\,\rm HT}(q^2)+\frac{m_B}{n\cdot p}\,f_{B\to P}^{0,\,\rm HT}(q^2)\right]\\
&=-\frac{\tilde{f}_B(\mu)\,m_B}{n\cdot p}\left\{e^{-\omega_s/\omega_M}\,H_{\bar{n},\rm LP}^{\rm 2PHT}(\omega_s,\mu)+\int_0^{\omega_s}d\omega'\,\frac{1}{\omega_M}\,e^{-\omega'/\omega_M}\,H_{\bar{n},\rm LP}^{\rm 2PHT}(\omega',\mu)\right.\\
&\qquad+\int_0^{\omega_s}d\omega_1\int_{\omega_s-\omega_1}^\infty\frac{d\omega_2}{\omega_2}\,e^{-\omega_s/\omega_M}\left[H_{\bar{n},\rm LP}^{\rm 3PHT}\left(\frac{\omega_s-\omega_1}{\omega_2},\omega_1,\omega_2,\mu\right)\right.\\
&\qquad\qquad\left.+\frac{m_q}{n\cdot p}\,H_{\bar{n},\rm NLP}^{\rm 3PHT}\left(\frac{\omega_s-\omega_1}{\omega_2},\omega_1,\omega_2,\mu\right)\right]\\
&\quad+\int_0^{\omega_s}d\omega'\int_0^{\omega'}d\omega_1\int_{\omega'-\omega_1}^\infty\frac{d\omega_2}{\omega_2}\,\frac{1}{\omega_M}\,e^{-\omega'/\omega_M}\left[H_{\bar{n},\rm LP}^{\rm 3PHT}\left(\frac{\omega'-\omega_1}{\omega_2},\omega_1,\omega_2,\mu\right)\right.\\
&\qquad\qquad\left.\left.+\frac{m_q}{n\cdot p}\,H_{\bar{n},\rm NLP}^{\rm 3PHT}\left(\frac{\omega'-\omega_1}{\omega_2},\omega_1,\omega_2,\mu\right)\right]\right\},
\end{aligned}
\tag{3.14}
$$

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$$
\begin{aligned}
&\frac{f_P\, n\cdot p}{2}\,\exp\left[-\frac{m_P^2}{n\cdot p\,\omega_M}\right]\,\frac{m_B}{n\cdot p-m_B}\,\left[f_{B\to P}^{+,\,\mathrm{HT}}(q^2)-\frac{m_B}{n\cdot p}\,f_{B\to P}^{0,\,\mathrm{HT}}(q^2)\right]\\
&=-\frac{\tilde f_B(\mu)\,m_B}{n\cdot p}\left\{\int_0^{\omega_s}d\omega_1\int_{\omega_s-\omega_1}^{\infty}\frac{d\omega_2}{\omega_2}\,e^{-\omega_s/\omega_M}\left[H_{n,\mathrm{LP}}^{\mathrm{3PHT}}\left(\frac{\omega_s-\omega_1}{\omega_2},\omega_1,\omega_2,\mu\right)\right.\right.\\
&\qquad\qquad\left.+\frac{m_q}{n\cdot p}\,H_{n,\mathrm{NLP}}^{\mathrm{3PHT}}\left(\frac{\omega_s-\omega_1}{\omega_2},\omega_1,\omega_2,\mu\right)\right]\\
&\quad+\int_0^{\omega_s}d\omega'\int_0^{\omega'}d\omega_1\int_{\omega'-\omega_1}^{\infty}\frac{d\omega_2}{\omega_2}\,\frac{1}{\omega_M}\,e^{-\omega'/\omega_M}\left[H_{n,\mathrm{LP}}^{\mathrm{3PHT}}\left(\frac{\omega'-\omega_1}{\omega_2},\omega_1,\omega_2,\mu\right)\right.\\
&\qquad\qquad\left.\left.+\frac{m_q}{n\cdot p}\,H_{n,\mathrm{NLP}}^{\mathrm{3PHT}}\left(\frac{\omega'-\omega_1}{\omega_2},\omega_1,\omega_2,\mu\right)\right]\right\}, \qquad (3.15)\\
&f_P\, n\cdot p\exp\left[-\frac{m_P^2}{n\cdot p\,\omega_M}\right]\,f_{B\to P}^{T,\,\mathrm{HT}}(q^2)\\
&=-\frac{\tilde f_B(\mu)\,(m_B+m_P)}{n\cdot p}\left\{e^{-\omega_s/\omega_M}\,H_{T,\mathrm{LP}}^{\mathrm{2PHT}}(\omega_s,\mu)+\int_0^{\omega_s}d\omega'\,\frac{1}{\omega_M}\,e^{-\omega'/\omega_M}\,H_{T,\mathrm{LP}}^{\mathrm{2PHT}}(\omega',\mu)\right.\\
&\qquad+\int_0^{\omega_s}d\omega_1\int_{\omega_s-\omega_1}^{\infty}\frac{d\omega_2}{\omega_2}\,e^{-\omega_s/\omega_M}\left[H_{T,\mathrm{LP}}^{\mathrm{3PHT}}\left(\frac{\omega_s-\omega_1}{\omega_2},\omega_1,\omega_2,\mu\right)\right.\\
&\qquad\qquad\left.+\frac{m_q}{n\cdot p}\,H_{T,\mathrm{NLP}}^{\mathrm{3PHT}}\left(\frac{\omega_s-\omega_1}{\omega_2},\omega_1,\omega_2,\mu\right)\right]\\
&\quad+\int_0^{\omega_s}d\omega'\int_0^{\omega'}d\omega_1\int_{\omega'-\omega_1}^{\infty}\frac{d\omega_2}{\omega_2}\,\frac{1}{\omega_M}\,e^{-\omega'/\omega_M}\left[H_{T,\mathrm{LP}}^{\mathrm{3PHT}}\left(\frac{\omega'-\omega_1}{\omega_2},\omega_1,\omega_2,\mu\right)\right.\\
&\qquad\qquad\left.\left.+\frac{m_q}{n\cdot p}\,H_{T,\mathrm{NLP}}^{\mathrm{3PHT}}\left(\frac{\omega'-\omega_1}{\omega_2},\omega_1,\omega_2,\mu\right)\right]\right\}, \qquad (3.16)
\end{aligned}
$$

where the nonvanishing spectral functions $H_{i,\mathrm{LP}}^{\mathrm{2PHT}}$ and $H_{i,\mathrm{(N)LP}}^{\mathrm{3PHT}}$ ($i=n,\,\bar n,\,T$) are given by

$$
\begin{aligned}
H_{\bar n,\mathrm{LP}}^{\mathrm{2PHT}}(\omega,\mu)&=H_{T,\mathrm{LP}}^{\mathrm{2PHT}}(\omega,\mu)=4\,\hat g_B^-(\omega,\mu)\,,\\
H_{n,\mathrm{LP}}^{\mathrm{3PHT}}(u,\omega_1,\omega_2,\mu)&=2\,(u-1)\,\Phi_4(\omega_1,\omega_2,\mu)\,,\\
H_{n,\mathrm{NLP}}^{\mathrm{3PHT}}(u,\omega_1,\omega_2,\mu)&=\tilde\Psi_5(\omega_1,\omega_2,\mu)-\Psi_5(\omega_1,\omega_2,\mu)\,,\\
H_{\bar n,\mathrm{LP}}^{\mathrm{3PHT}}(u,\omega_1,\omega_2,\mu)&=\tilde\Psi_5(\omega_1,\omega_2,\mu)-\Psi_5(\omega_1,\omega_2,\mu)\,,\\
H_{\bar n,\mathrm{NLP}}^{\mathrm{3PHT}}(u,\omega_1,\omega_2,\mu)&=2\,\Phi_6(\omega_1,\omega_2,\mu)\,,\\
H_{T,\mathrm{LP}}^{\mathrm{3PHT}}(u,\omega_1,\omega_2,\mu)&=2\,(1-u)\,\Phi_4(\omega_1,\omega_2,\mu)-\Psi_5(\omega_1,\omega_2,\mu)+\tilde\Psi_5(\omega_1,\omega_2,\mu)\,,\\
H_{T,\mathrm{NLP}}^{\mathrm{3PHT}}(u,\omega_1,\omega_2,\mu)&=\Psi_5(\omega_1,\omega_2,\mu)-\tilde\Psi_5(\omega_1,\omega_2,\mu)+2\,\Phi_6(\omega_1,\omega_2,\mu)\,. \qquad (3.17)
\end{aligned}
$$

It is evident that the two-particle higher-twist corrections preserve the large-recoil symmetry relations of $B\to P$ form factors and the three-particle higher-twist contributions violate such relations already at tree level (see [55] for a similar observation in the context of the $B\to\gamma\ell\nu$ decays). Employing the power counting scheme for the Borel mass ω_M and the threshold parameter ω_s [25]

$$
\omega_s\sim\omega_M\sim\mathcal{O}(\Lambda^2/m_b)\,, \qquad (3.18)
$$

we can identify the scaling behaviours of the higher-twist corrections to $B\to\pi,K$ form

LCDAs	Γ_1	Γ_2
$\Phi_5(\omega_1, \omega_2, \mu)$	$n^\beta \, \not{\bar{n}} \, \gamma_\perp^\alpha \, \gamma_5$	$\sigma^{\rho\lambda} \, \gamma_5$
$\Psi_5(\omega_1, \omega_2, \mu)$	$n^\alpha \, \bar{n}^\beta \, \not{n} \, \gamma_5$	$\sigma^{\rho\lambda} \, \gamma_5$
$\tilde{\Psi}_5(\omega_1, \omega_2, \mu)$	$-\frac{i}{2} \, \epsilon^{\mu\nu\alpha\beta} \, n_\mu \, \bar{n}_\nu \, \not{n}$	$\sigma^{\rho\lambda} \, \gamma_5$
$\Phi_6(\omega_1, \omega_2, \mu)$	$n^\beta \, \not{n} \, \gamma_\perp^\alpha \, \gamma_5$	$n^\lambda \, \not{n} \, \gamma_\perp^\rho \, \gamma_5$

Table 1. The Dirac structures of the interpolating currents entering the correlation function (4.1) and the corresponding three-particle B-meson LCDAs.

factors

$$f_{B\to P}^{+,\,\mathrm{HT}}(q^2) \sim f_{B\to P}^{0,\,\mathrm{HT}}(q^2) \sim f_{B\to P}^{T,\,\mathrm{HT}}(q^2) \sim \mathcal{O}\left(\frac{\Lambda}{m_b}\right)^{5/2} \tag{3.19}$$

in the heavy quark limit, which is suppressed by one power of Λ/m_b compared with the leading-twist contribution in (2.17).

Collecting different pieces together, the final expressions for the LCSR of $B \to \pi, K$ form factors at large hadronic recoil can be written as

$$\begin{aligned}
f_{B\to P}^{+}(q^2) &= f_{B\to P}^{+,\mathrm{2PNLL}}(q^2) + f_{B\to P}^{+,\,\mathrm{2PHT}}(q^2) + f_{B\to P}^{+,\,\mathrm{3PHT}}(q^2)\,, \\
f_{B\to P}^{0}(q^2) &= f_{B\to P}^{0,\mathrm{2PNLL}}(q^2) + f_{B\to P}^{0,\,\mathrm{2PHT}}(q^2) + f_{B\to P}^{0,\,\mathrm{3PHT}}(q^2)\,, \\
f_{B\to P}^{T}(q^2) &= f_{B\to P}^{T,\mathrm{2PNLL}}(q^2) + f_{B\to P}^{T,\,\mathrm{2PHT}}(q^2) + f_{B\to P}^{T,\,\mathrm{3PHT}}(q^2)\,,
\end{aligned} \tag{3.20}$$

where the manifest expressions of $f_{B\to P}^{i,\mathrm{2PNLL}}(q^2)$ ($i = +,\, 0,\, T$) including the light-quark mass effect can be found in (2.17), and the higher-twist corrections $f_{B\to P}^{i,\,\mathrm{2PHT}}(q^2)$ and $f_{B\to P}^{i,\,\mathrm{3PHT}}(q^2)$ can be extracted from (3.14), (3.15) and (3.16).

4 QCD sum rules for the higher-twist B-meson LCDAs

The objective of this section is to construct a realistic model for the twist-five and -six B-meson LCDAs consistent with the corresponding asymptotic behaviour at small quark and gluon momenta, employing the method of QCD sum rules [50, 56]. We introduce the correlation function with two HQET currents

$$\begin{aligned}
F(\omega, z_1, z_2) = i \int d^4 y \, e^{-i\,\omega\,y} \, &\langle 0 | T\{\bar{q}(z_1 \bar{n}) \, Y_s(z_1 \bar{n}, z_2 \bar{n}) \, g_s \, G_{\alpha\beta}(z_2 \bar{n}) \, Y_s(z_2 \bar{n}, 0) \, \Gamma_1 \, h_v(0)\,, \\
&\bar{h}_v(y \bar{n}) \, g_s \, G_{\rho\lambda}(y \bar{n}) \, \Gamma_2 \, q(y \bar{n})\} | 0 \rangle\,,
\end{aligned} \tag{4.1}$$

where the Dirac matrices Γ_1 and Γ_2 of the interpolating currents are specified in table 1.

Employing the HQET parametrization for the local matrix element with the EOM constraints for both the heavy and light quarks [50]

$$\begin{aligned}
\langle 0 | \bar{q} \, g_s \, G_{\mu\nu} \, \Gamma \, h_v | \bar{B}(v) \rangle = -\frac{\tilde{f}_B(\mu) \, m_B}{6} \bigg\{ &i \, \lambda_H^2 \mathrm{Tr}\left[\gamma_5 \, \Gamma \, \frac{1+\not{v}}{2} \, \sigma_{\mu\nu}\right] \\
&+ (\lambda_H^2 - \lambda_E^2) \, \mathrm{Tr}\left[\gamma_5 \, \Gamma \, \frac{1+\not{v}}{2} \, (v_\mu \, \gamma_\nu - v_\nu \, \gamma_\mu)\right] \bigg\}\,,
\end{aligned} \tag{4.2}$$

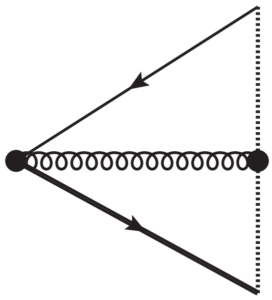

Figure 2. The leading-power contribution to the correlation function (4.1) with two HQET currents at LO in α_s.

and comparing (4.2) with the definitions of the three-particle B-meson LCDAs (3.4) with the aid of (3.11) leads to the normalization conditions

$$\begin{aligned}
\Phi_5(z_1 = z_2 = 0, \mu) &= \int_0^\infty d\omega_1 \int_0^\infty d\omega_2 \ \Phi_5(\omega_1, \omega_2, \mu) = \frac{\lambda_E^2 + \lambda_H^2}{3} \,, \\
\Psi_5(z_1 = z_2 = 0, \mu) &= \int_0^\infty d\omega_1 \int_0^\infty d\omega_2 \ \Psi_5(\omega_1, \omega_2, \mu) = -\frac{\lambda_E^2}{3} \,, \\
\tilde{\Psi}_5(z_1 = z_2 = 0, \mu) &= \int_0^\infty d\omega_1 \int_0^\infty d\omega_2 \ \tilde{\Psi}_5(\omega_1, \omega_2, \mu) = -\frac{\lambda_H^2}{3} \,, \\
\Phi_6(z_1 = z_2 = 0, \mu) &= \int_0^\infty d\omega_1 \int_0^\infty d\omega_2 \ \Phi_6(\omega_1, \omega_2, \mu) = \frac{\lambda_E^2 - \lambda_H^2}{3} \,.
\end{aligned} \tag{4.3}$$

Here, we have implicitly assumed that the integrations of three-particle B-meson LCDAs over quark and gluon momenta at a low renormalization scale are finite. This assumption cannot hold true generally after taking into account perturbative QCD corrections. The hadronic representation of the HQET correlation function (4.1) can be written as

$$\begin{aligned}
F(\omega, z_1, z_2) = & \frac{1}{2\,(\bar{\Lambda} - \omega)\, m_B} \langle 0 | \bar{q}(z_1 \bar{n})\, Y_s(z_1 \bar{n}, z_2 \bar{n})\, g_s\, G_{\alpha\beta}(z_2 \bar{n})\, Y_s(z_2 \bar{n}, 0)\, \Gamma_1\, h_v(0) | \bar{B}(v) \rangle \\
& \times \langle \bar{B}(v) | \bar{h}_v(0)\, g_s\, G_{\rho\lambda}(0)\, \Gamma_2\, q(0) | 0 \rangle + \ldots \,,
\end{aligned} \tag{4.4}$$

where the ellipses indicate the contributions from the higher resonances and continuum states.

Evaluating the perturbative diagram displayed in figure 2 and applying the HQET Feynman rules, we can readily derive the leading-power contribution to the correlation function (4.1) at tree level

$$\begin{aligned}
F(\omega, z_1, z_2) = & \, g_s^2\, C_F\, N_c \int \frac{d^4 k_1}{(2\pi)^4} \int \frac{d^4 k_3}{(2\pi)^4}\, e^{i\, \bar{n} \cdot k_1\, z_1}\, e^{i\, \bar{n} \cdot k_3\, z_2}\, \mathrm{Tr} \left[\not{k}_1\, \Gamma_1\, \frac{1 + \not{v}}{2}\, \Gamma_2 \right] \\
& \times \frac{1}{[k_1^2 + i0][v \cdot (k_1 + k_3) + \omega + i0][k_3^2 + i0]} \\
& \times \left[k_{3\,\alpha}\, k_{3\,\rho}\, g_{\beta\lambda} - k_{3\,\alpha}\, k_{3\,\lambda}\, g_{\beta\rho} - k_{3\,\beta}\, k_{3\,\rho}\, g_{\alpha\lambda} + k_{3\,\beta}\, k_{3\,\lambda}\, g_{\alpha\rho} \right] .
\end{aligned} \tag{4.5}$$

Performing the loop momentum integration in (4.5) and matching the two different representations of the correlation function (4.1) with the parton-hadron duality ansatz, we obtain the QCD sum rules for the three-particle higher-twist B-meson LCDAs

$$
\begin{aligned}
&[\tilde{f}_B(\mu)]^2\, m_B\, (\lambda_H^2+\lambda_E^2)\, \Phi_5(\omega_1,\omega_2,\mu) \\
&\qquad = -\frac{g_s^2\, C_F\, N_c}{96\,\pi^4} \int_{\frac{\omega_1+\omega_2}{2}}^{\omega_0} ds \exp\left[\frac{\bar{\Lambda}-s}{\omega_M}\right] \omega_1\, (\omega_1+\omega_2-2\,s)^3\, \theta(2\,s-\omega_1-\omega_2)\,, \\
&[\tilde{f}_B(\mu)]^2\, m_B\, (\lambda_H^2+\lambda_E^2)\, \Psi_5(\omega_1,\omega_2,\mu) \\
&\qquad = \frac{g_s^2\, C_F\, N_c}{192\,\pi^4} \int_{\frac{\omega_1+\omega_2}{2}}^{\omega_0} ds \exp\left[\frac{\bar{\Lambda}-s}{\omega_M}\right] \omega_2\, (\omega_1+\omega_2-2\,s)^3\, \theta(2\,s-\omega_1-\omega_2)\,, \\
&[\tilde{f}_B(\mu)]^2\, m_B\, (\lambda_H^2+\lambda_E^2)\, \tilde{\Psi}_5(\omega_1,\omega_2,\mu) \\
&\qquad = \frac{g_s^2\, C_F\, N_c}{192\,\pi^4} \int_{\frac{\omega_1+\omega_2}{2}}^{\omega_0} ds \exp\left[\frac{\bar{\Lambda}-s}{\omega_M}\right] \omega_2\, (\omega_1+\omega_2-2\,s)^3\, \theta(2\,s-\omega_1-\omega_2)\,, \\
&[\tilde{f}_B(\mu)]^2\, m_B\, (\lambda_E^2-\lambda_H^2)\, \Phi_6(\omega_1,\omega_2,\mu) \\
&\qquad = \frac{g_s^2\, C_F\, N_c}{128\,\pi^4} \int_{\frac{\omega_1+\omega_2}{2}}^{\omega_0} ds \exp\left[\frac{\bar{\Lambda}-s}{\omega_M}\right] (\omega_1+\omega_2-2\,s)^4\, \theta(2\,s-\omega_1-\omega_2)\,, \qquad (4.6)
\end{aligned}
$$

where the Borel transformation with respect to the variable ω have been implemented to suppress the higher-order nonperturbative corrections and minimize the model dependence on the continuum contributions. For the phenomenological applications, we first suggest the local duality model for the three-particle B-meson LCDAs by taking the limit $\omega_M \to \infty$ of the obtained QCD sum rules (4.6)

$$
\begin{aligned}
\Phi_5^{\rm LD}(\omega_1,\omega_2,\mu) &= \frac{35}{64}\,(\lambda_E^2+\lambda_H^2)\, \frac{\omega_1}{\omega_0^7}\, (2\,\omega_0-\omega_1-\omega_2)^4\, \theta(2\,\omega_0-\omega_1-\omega_2)\,, \\
\Psi_5^{\rm LD}(\omega_1,\omega_2,\mu) &= -\frac{35}{64}\,\lambda_E^2\, \frac{\omega_2}{\omega_0^7}\, (2\,\omega_0-\omega_1-\omega_2)^4\, \theta(2\,\omega_0-\omega_1-\omega_2)\,, \\
\tilde{\Psi}_5^{\rm LD}(\omega_1,\omega_2,\mu) &= -\frac{35}{64}\,\lambda_H^2\, \frac{\omega_2}{\omega_0^7}\, (2\,\omega_0-\omega_1-\omega_2)^4\, \theta(2\,\omega_0-\omega_1-\omega_2)\,, \\
\Phi_6^{\rm LD}(\omega_1,\omega_2,\mu) &= \frac{7}{64}\,(\lambda_E^2-\lambda_H^2)\, \frac{1}{\omega_0^7}\, (2\,\omega_0-\omega_1-\omega_2)^5\, \theta(2\,\omega_0-\omega_1-\omega_2)\,. \qquad (4.7)
\end{aligned}
$$

It is then straightforward to verify that the asymptotic behaviour of the twist-five and -six B-meson LCDAs from the local duality model (4.7)

$$
\Phi_5(\omega_1,\omega_2,\mu) \sim \omega_1\,, \qquad \Psi_5(\omega_1,\omega_2,\mu) \sim \tilde{\Psi}_5(\omega_1,\omega_2,\mu) \sim \omega_2\,, \qquad \Phi_6(\omega_1,\omega_2,\mu) \sim 1\,, \quad (4.8)
$$

in agreement with predictions from the RG equations at one loop [57]. We further present the local duality model for the remaining two-particle and three-particle B-meson LCDAs constructed in [45]

$$
\begin{aligned}
\phi_B^{+,\mathrm{LD}}(\omega,\mu) &= \frac{5}{8\,\omega_0^5}\,\omega(2\,\omega_0-\omega)^3\,\theta(2\,\omega_0-\omega)\,,\\
\phi_B^{-,\mathrm{LD}}(\omega,\mu) &= \frac{5(2\,\omega_0-\omega)^2}{192\,\omega_0^5}\left\{6\,(2\,\omega_0-\omega)^2-\frac{7\,(\lambda_E^2-\lambda_H^2)}{\omega_0^2}\,(15\,\omega^2-20\,\omega\,\omega_0+4\,\omega_0^2)\right\}\\
&\quad\times\theta(2\,\omega_0-\omega)\,,\\
\Phi_3^{\mathrm{LD}}(\omega_1,\omega_2,\mu) &= \frac{105\,(\lambda_E^2-\lambda_H^2)}{8\,\omega_0^7}\,\omega_1\,\omega_2^2\left(\omega_0-\frac{\omega_1+\omega_2}{2}\right)^2\theta(2\,\omega_0-\omega_1-\omega_2)\,,\\
\Phi_4^{\mathrm{LD}}(\omega_1,\omega_2,\mu) &= \frac{35\,(\lambda_E^2+\lambda_H^2)}{4\,\omega_0^7}\,\omega_2^2\left(\omega_0-\frac{\omega_1+\omega_2}{2}\right)^3\theta(2\,\omega_0-\omega_1-\omega_2)\,,\\
\Psi_4^{\mathrm{LD}}(\omega_1,\omega_2,\mu) &= \frac{35\,\lambda_E^2}{2\,\omega_0^7}\,\omega_1\,\omega_2\left(\omega_0-\frac{\omega_1+\omega_2}{2}\right)^3\theta(2\,\omega_0-\omega_1-\omega_2)\,,\\
\tilde{\Psi}_4^{\mathrm{LD}}(\omega_1,\omega_2,\mu) &= \frac{35\,\lambda_H^2}{2\,\omega_0^7}\,\omega_1\,\omega_2\left(\omega_0-\frac{\omega_1+\omega_2}{2}\right)^3\theta(2\,\omega_0-\omega_1-\omega_2)\,,
\end{aligned}
\tag{4.9}
$$

from which we can further derive the corresponding model for the "effective" distribution amplitude defined in (3.13)

$$
\begin{aligned}
\hat{g}_B^{-,\mathrm{LD}}(\omega,\mu) = \frac{\omega\,(2\omega_0-\omega)^3}{\omega_0^5}&\left\{\frac{5}{256}\,(2\omega_0-\omega)^2-\frac{35\,(\lambda_E^2-\lambda_H^2)}{1536}\left[4-12\left(\frac{\omega}{\omega_0}\right)+11\left(\frac{\omega}{\omega_0}\right)^2\right]\right\}\\
&\times\theta(2\omega_0-\omega)\,.
\end{aligned}
\tag{4.10}
$$

Applying the EOM constraint between the leading-twist and the higher-twist B-meson LCDAs (3.9), the HQET parameters entering the local duality model for the B-meson LCDAs must satisfy the following relations [45]

$$
\omega_0=\frac{5}{2}\,\lambda_B=2\,\bar{\Lambda}\,,\qquad 3\,\omega_0^2=14\,(2\,\lambda_E^2+\lambda_H^2)\,.
\tag{4.11}
$$

An alternative model for the twist-five and -six B-meson LCDAs consistent with the asymptotic behaviours (4.8) and the normalization conditions (4.3) can be constructed by implementing an exponential falloff at large quark and gluon momenta

$$
\begin{aligned}
\Phi_5^{\exp}(\omega_1,\omega_2,\mu) &= \frac{\lambda_E^2+\lambda_H^2}{3\,\omega_0^3}\,\omega_1\,e^{-(\omega_1+\omega_2)/\omega_0}\,,\\
\Psi_5^{\exp}(\omega_1,\omega_2,\mu) &= -\frac{\lambda_E^2}{3\,\omega_0^3}\,\omega_2\,e^{-(\omega_1+\omega_2)/\omega_0}\,,\\
\tilde{\Psi}_5^{\exp}(\omega_1,\omega_2,\mu) &= -\frac{\lambda_H^2}{3\,\omega_0^3}\,\omega_2\,e^{-(\omega_1+\omega_2)/\omega_0}\,,\\
\Phi_6^{\exp}(\omega_1,\omega_2,\mu) &= \frac{\lambda_E^2-\lambda_H^2}{3\,\omega_0^2}\,e^{-(\omega_1+\omega_2)/\omega_0}\,.
\end{aligned}
\tag{4.12}
$$

We further collect the exponential model for the remaining LCDAs obtained in [45]

$$
\begin{aligned}
\phi_B^{+,\,\exp}(\omega,\mu) &= \frac{\omega}{\omega_0^2}\, e^{-\omega/\omega_0}\,, \\
\phi_B^{-,\,\exp}(\omega,\mu) &= \frac{1}{\omega_0}\, e^{-\omega/\omega_0} - \frac{\lambda_E^2-\lambda_H^2}{9\,\omega_0^3}\left[1-2\left(\frac{\omega}{\omega_0}\right)+\frac{1}{2}\left(\frac{\omega}{\omega_0}\right)^2\right] e^{-\omega/\omega_0}\,, \\
\Phi_3^{\exp}(\omega_1,\omega_2,\mu) &= \frac{\lambda_E^2-\lambda_H^2}{6\,\omega_0^5}\,\omega_1\,\omega_2^2\, e^{-(\omega_1+\omega_2)/\omega_0}\,, \\
\Phi_4^{\exp}(\omega_1,\omega_2,\mu) &= \frac{\lambda_E^2+\lambda_H^2}{6\,\omega_0^4}\,\omega_2^2\, e^{-(\omega_1+\omega_2)/\omega_0}\,, \\
\Psi_4^{\exp}(\omega_1,\omega_2,\mu) &= \frac{\lambda_E^2}{3\,\omega_0^4}\,\omega_1\,\omega_2\, e^{-(\omega_1+\omega_2)/\omega_0}\,, \\
\tilde{\Psi}_4^{\exp}(\omega_1,\omega_2,\mu) &= \frac{\lambda_H^2}{3\,\omega_0^4}\,\omega_1\,\omega_2\, e^{-(\omega_1+\omega_2)/\omega_0}\,,
\end{aligned} \tag{4.13}
$$

which imply the following expression for the two-particle twist-five LCDA

$$
\hat{g}_B^{-,\,\exp}(\omega,\mu) = \omega\left\{\frac{3}{4}-\frac{\lambda_E^2-\lambda_H^2}{12\,\omega_0^2}\left[1-\left(\frac{\omega}{\omega_0}\right)+\frac{1}{3}\left(\frac{\omega}{\omega_0}\right)^2\right]\right\} e^{-\omega/\omega_0}\,. \tag{4.14}
$$

Implementing the EOM constraint (3.9) for the exponential model leads to

$$
\omega_0 = \lambda_B = \frac{2}{3}\,\bar{\Lambda}\,, \qquad 2\,\bar{\Lambda}^2 = 2\,\lambda_E^2 + \lambda_H^2\,. \tag{4.15}
$$

It is worthwhile to point that the HQET relations (4.11) and (4.15) are derived from the classical EOM with the assumption that the first two moments of the leading-twist B-meson LCDA $\phi_B^+(\omega,\mu)$ are finite. Apparently, perturbative QCD corrections to the B-meson LCDAs will violate such tree-level relations in a nontrivial way (see [58] for further discussion).

5 Numerical analysis

The purpose of this section is to explore phenomenological implications of the newly derived sum rules (3.20) for $B \to \pi, K$ form factors with the subleading-twist corrections. We will place particular attention to the normalized differential q^2 distributions of $B \to \pi \ell \nu_\ell$ ($\ell = \mu\,,\tau$), the determination of the CKM matrix element $|V_{ub}|$ and the differential branching fractions of the rare exclusive $B \to K \nu \nu$ decays.

5.1 Theory inputs

We will proceed by specifying the theory inputs entering the LCSR for $B \to \pi, K$ form factors, including the shape parameters of B-meson LCDAs, the intrinsic sum rule parameters and the decay constants of the B-meson and the light pseudoscalar mesons. Due to the EOM constraints (4.11) and (4.15), only two of the three HQET parameters $\lambda_B(\mu)$, $\lambda_E^2(\mu)$ and $\lambda_H^2(\mu)$ appearing in the B-meson LCDAs are independent of each other. As observed in [45], the ratio $R(\mu) = \lambda_E^2(\mu)/\lambda_H^2(\mu)$ estimated from the QCD sum rule approach [50, 59] is insensitive to the perturbative QCD corrections and the higher-order nonperturbative QCD corrections. We will therefore take $\lambda_B(\mu)$ and $R(\mu)$ as free parameters in the numerical analysis. The renormalization scale dependence of the inverse moment $\lambda_B(\mu)$

$$\lambda_B(\mu) = \lambda_B(\mu_0) \left\{ 1 + \frac{\alpha_s(\mu_0)\, C_F}{4\,\pi} \ln \frac{\mu}{\mu_0} \left[2 - 2 \ln \frac{\mu}{\mu_0} - 4\, \sigma_1(\mu_0) \right] + \mathcal{O}(\alpha_s^2) \right\}^{-1} \tag{5.1}$$

can be obtained from the Lange-Neubert evolution equation of $\phi_B^+(\omega, \mu)$ [60]. We employ the definition of the inverse-logarithmic moment

$$\sigma_1(\mu) = \lambda_B(\mu) \int_0^\infty \frac{d\omega}{\omega} \ln \frac{\mu}{\omega}\, \phi_B^+(\omega, \mu) \tag{5.2}$$

and adopt the interval $\sigma_1(\mu_0) = 1.4 \pm 0.4$ from the QCD sum rule calculation for $\mu_0 = 1\,\mathrm{GeV}$ [56]. The ratio $R(\mu_0) = 0.5 \pm 0.1$ based upon the nonperturbative QCD computations [50, 59] will be taken in the subsequent calculations.

Implementing the standard procedure for the determinations of the internal sum rule parameters as discussed in [25] gives rise to

$$\begin{aligned} M^2 &= n \cdot p\, \omega_M = (1.25 \pm 0.25)\,\mathrm{GeV}^2\,, \qquad s_0^\pi = n \cdot p\, \omega_s^\pi = (0.70 \pm 0.05)\,\mathrm{GeV}^2\,, \\ s_0^K &= n \cdot p\, \omega_s^K = (1.05 \pm 0.05)\,\mathrm{GeV}^2\,, \end{aligned} \tag{5.3}$$

in agreement with the values used for the LCSR of the pion-photon form factor [61] and for the two-point QCD sum rules of the kaon decay constant [62].

By virtue of the matching relation (2.6), the HQET decay constant $\tilde{f}_B(\mu)$ will be related to the QCD decay constant f_B, for which we will take the averaged Lattice results $f_B = (192.0 \pm 4.3)\,\mathrm{MeV}$ [63] with $N_f = 2 + 1$. In addition, the QCD decay constants of the light pseudoscalar mesons

$$f_\pi = (130.2 \pm 1.7)\,\mathrm{MeV}\,, \qquad f_K = (155.6 \pm 0.4)\,\mathrm{MeV} \tag{5.4}$$

are borrowed from the Particle Data Group (PDG) [64], which differ slightly from the Flavour Lattice Averaging Group (FLAG) values [63] mainly due to the different treatments of theory uncertainties and correlations.

The masses of the light quarks in the $\overline{\mathrm{MS}}$ scheme summarized in PDG [64]

$$\begin{aligned} m_u(2\,\mathrm{GeV}) &= (2.15 \pm 0.15)\,\mathrm{MeV}\,, \qquad m_d(2\,\mathrm{GeV}) = (4.70 \pm 0.20)\,\mathrm{MeV}\,, \\ m_s(2\,\mathrm{GeV}) &= (93.8 \pm 1.5 \pm 1.9)\,\mathrm{MeV}\,, \end{aligned} \tag{5.5}$$

will be employed in the following. We further take the numerical values of the $\overline{\mathrm{MS}}$ bottom quark mass determined from non-relativistic sum rules [65] (see [66] for independent determinations from relativistic sum rules with similar results)

$$\overline{m_b}(\overline{m_b}) = (4.193^{+0.022}_{-0.035})\,\mathrm{GeV}\,. \tag{5.6}$$

Following the discussion presented in [25], the factorization scale entering the leading-twist LCSR for $B \to \pi, K$ form factors at NLL will be varied in the interval $1\,\mathrm{GeV} \le \mu \le 2\,\mathrm{GeV}$ around the default value $\mu = 1.5\,\mathrm{GeV}$. The hard scales μ_{h1} and μ_{h2} as well as the QCD renormalization scale for the tensor current ν_h will be taken as $\mu_{h1} = \mu_{h2} = \nu_h = m_b$ with the variation in the range $[m_b/2, 2\, m_b]$.

5.2 Predictions for $B \to \pi, K$ form factors

We will proceed to investigate the numerical impacts of the higher-twist corrections and the SU(3)-flavour symmetry breaking effects computed from the method of LCSR. Prior to presenting the breakdown of the distinct terms contributing to the semileptonic $B \to \pi, K$ decay form factors, we need to determine the inverse moment $\lambda_B(\mu_0)$ of the leading-twist B-meson LCDA $\phi_B^+(\omega, \mu)$. Despite of the numerous studies of $\lambda_B(\mu_0)$ with the direct nonperturbative calculations [56] and the indirect determinations from measurements of the partial branching fractions of $B \to \gamma \ell \nu$ [52, 55, 67–69], the current constraints of $\lambda_B(\mu_0)$ are still far from satisfactory due to the systematic uncertainty of the direct QCD approach and the sensitivity of the $B \to \gamma$ form factors to the shape of $\phi_B^+(\omega, \mu)$ at small ω. Following the strategy displayed in [25], we will match our calculations for the vector $B \to \pi$ form factor at $q^2 = 0$ from the LCSR with B-meson LCDAs with the independent predictions $f^+_{B\to\pi}(q^2 = 0) = 0.28 \pm 0.03$ from the LCSR with pion LCDAs including the higher-twist corrections up to twist-four accuracy [70] (see [71, 72] for slightly different values). Performing such matching procedure we obtain

$$\lambda_B(\mu_0) = \begin{cases} 285^{+27}_{-23} \text{ MeV}\,, & \text{(Exponential Model)} \\ \\ 286^{+26}_{-22} \text{ MeV}\,, & \text{(Local Duality Model)} \end{cases} \tag{5.7}$$

which correspond to the following intervals of the HQET parameter $\bar{\Lambda}$

$$\bar{\Lambda} = \begin{cases} 428^{+41}_{-35} \text{ MeV}\,, & \text{(Exponential Model)} \\ \\ 358^{+33}_{-28} \text{ MeV}\,. & \text{(Local Duality Model)} \end{cases} \tag{5.8}$$

It is apparent that the determined values of $\lambda_B(\mu_0)$ for the considered two models of B-meson LCDAs are practically identical, which can be understood from the fact that the small ω behaviours of the above-mentioned two models are very similar to each other (albeit with the rather different high-energy behaviours) as observed in [45]. The determined values of $\lambda_B(\mu_0)$ (5.7) differ from the previous interval presented in [25], where only the leading-power two-particle contributions to the sum rules were taken into account at NLL. It is interesting to point out that the determined values of $\bar{\Lambda}$ displayed in (5.8) are comparable to the expected intervals $\bar{\Lambda} = (480 \pm 100)\,\text{MeV}$ with the pole mass scheme for the bottom quark. For the illustration purpose, we will adopt the exponential model for B-meson LCDAs as our default choice and the theory uncertainty due to the model dependence of these distribution amplitudes will be included in the final predictions for $B \to \pi, K$ form factors.

We first display the breakdown of distinct pieces contributing to the LCSR of the vector $B \to \pi$ form factor at $0 \leq q^2 \leq 8\,\text{GeV}^2$ in figure 3. It is evident that higher-twist corrections to the $B \to \pi$ form factor $f^+_{B\to\pi}(q^2)$ are dominated by the two-particle

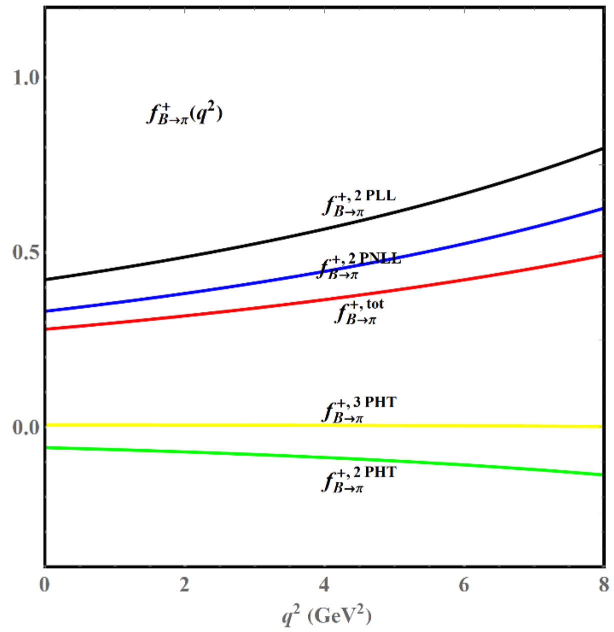

Figure 3. The momentum-transfer dependence of the vector $B \to \pi$ form factor from the leading-power contribution at LL ($f^{+,\,\mathrm{2PLL}}_{B\to\pi}$, black), the leading-power contribution at NLL ($f^{+,\,\mathrm{2PNLL}}_{B\to\pi}$, blue), the two-particle higher-twist correction ($f^{+,\,\mathrm{2PHT}}_{B\to\pi}$, green), and the three-particle higher-twist correction ($f^{+,\,\mathrm{3PHT}}_{B\to\pi}$, yellow).

twist-five contribution from $\hat{g}^-_B(\omega,\mu)$, which can shift the leading-power prediction by an amount of approximately $(20 \sim 30)\%$. The three-particle higher-twist contribution only generates a minor impact on the theory prediction of $f^+_{B\to\pi}(q^2)$ and numerically $\mathcal{O}(2\%)$. We further observe that the NLL QCD correction to the leading-power contribution can yield approximately $\mathcal{O}(20\%)$ reduction of the corresponding LL QCD prediction. We have also verified that such observations also hold true for the momentum-transfer dependence of the scalar and tensor $B \to \pi$ form factors at large hadronic recoil. The SU(3)-flavour symmetry breaking effects between the $B \to \pi$ and $B \to K$ form factors

$$R^i_{\mathrm{SU(3)}}(q^2) = \frac{f^i_{B\to K}(q^2)}{f^i_{B\to\pi}(q^2)}\,, \qquad (\text{with } i = +,\, 0,\, T) \tag{5.9}$$

which originate from the nonvanishing strange-quark mass, from the discrepancy between the threshold parameters for the pion and kaon channels and from the difference between the decay constants f_π and f_K are presented in figure 4. It can be observed that our predictions for the SU(3)-flavour symmetry breaking effects are in good agreement with that obtained from the LCSR with the light-meson LCDAs [72], but are somewhat smaller than the previous calculations [73].

We are now in a position to discuss the large-recoil symmetry breaking effects of $B \to P$ form factors due to both the perturbative QCD corrections at leading power in $1/m_b$ and the subleading power soft contributions from the three-particle higher-twist B-meson LCDAs. To compare our predictions with the perturbative calculations from the

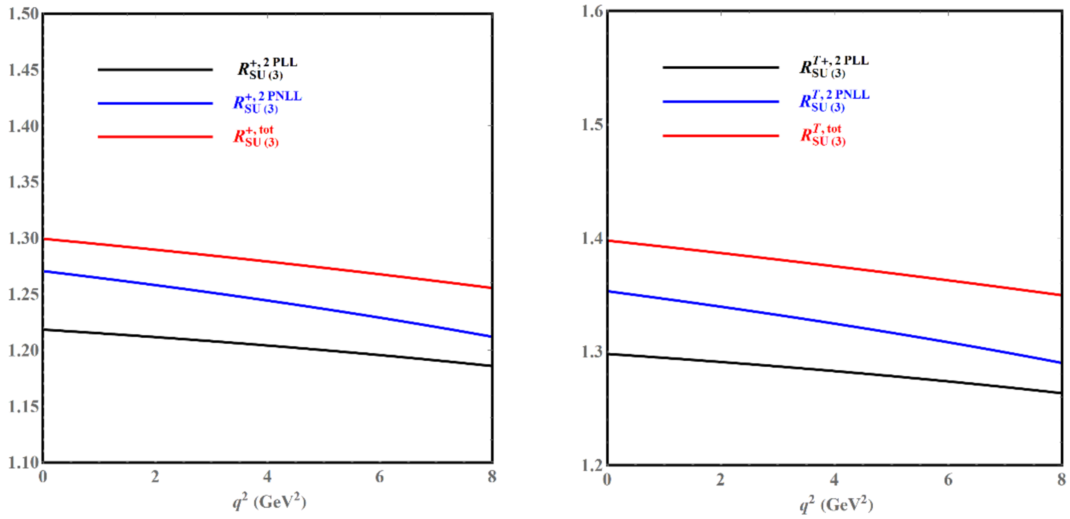

Figure 4. The SU(3)-flavour symmetry breaking effects between $B \to \pi$ and $B \to K$ form factors predicted from the LCSR with B-meson LCDAs. The momentum-transfer dependence of the ratio $R^0_{\mathrm{SU(3)}}(q^2)$ behaves in a similar way to $R^+_{\mathrm{SU(3)}}(q^2)$ at $0 \leq q^2 \leq 8\,\mathrm{GeV}^2$ and we will therefore not display this ratio for brevity.

QCD factorization approach, we collect the factorization formulae for the heavy-to-light B-meson form factors in the heavy quark limit at one loop [49] (see [14, 16] for further improvement)

$$\begin{aligned} f^0_{B\to P}(q^2) &= \frac{n\cdot p}{m_B}\, f^+_{B\to P}(q^2) \left[1 + \frac{\alpha_s C_F}{2\pi}\left(1 - \frac{n\cdot p}{n\cdot p - m_B}\ln\frac{n\cdot p}{m_B}\right)\right] \\ &\quad + \frac{m_B - n\cdot p}{n\cdot p}\,\frac{\alpha_s C_F}{4\pi}\,\frac{8\pi^2 f_B f_P}{N_c m_B}\int_0^1 du\,\frac{\phi_P(u,\mu)}{\bar u}\int_0^\infty d\omega\,\frac{\phi^+_B(\omega,\mu)}{\omega}, \quad (5.10) \\ f^T_{B\to P}(q^2) &= \frac{m_B + m_P}{m_B}\, f^+_{B\to P}(q^2)\left[1 + \frac{\alpha_s C_F}{4\pi}\left(\ln\frac{m_b^2}{\mu^2} + 2\,\frac{n\cdot p}{n\cdot p - m_B}\ln\frac{n\cdot p}{m_B}\right)\right] \\ &\quad - \frac{m_B + m_P}{n\cdot p}\,\frac{\alpha_s C_F}{4\pi}\,\frac{8\pi^2 f_B f_P}{N_c m_B}\int_0^1 du\,\frac{\phi_P(u,\mu)}{\bar u}\int_0^\infty d\omega\,\frac{\phi^+_B(\omega,\mu)}{\omega}, \quad (5.11) \end{aligned}$$

where $\phi_P(u,\mu)$ is the twist-two pseudoscalar-meson LCDA.

Introducing the form-factor ratios for the semileptonic $B \to \pi$ decays

$$R^{0\,+}_{B\to\pi}(q^2) = \frac{m_B}{n\cdot p}\,\frac{f^0_{B\to\pi}(q^2)}{f^+_{B\to\pi}(q^2)}\,, \qquad R^{T\,+}_{B\to\pi}(q^2) = \frac{m_B}{m_B + m_\pi}\,\frac{f^T_{B\to\pi}(q^2)}{f^+_{B\to\pi}(q^2)}\,, \quad (5.12)$$

we present the theory predictions for these ratios from both our calculations and the QCD factorization results in figure 5. It is evident that both the magnitude and sign of the symmetry-breaking corrections computed from the two QCD methods are consistent with each other and our predictions for the large-recoil symmetry violations are generally larger than the previous LCSR computations with pion LCDAs [74].

To understand the model dependence of our predictions on the B-meson LCDAs, we display in figure 6 the obtained $B \to \pi, K$ form factors from both the exponential and

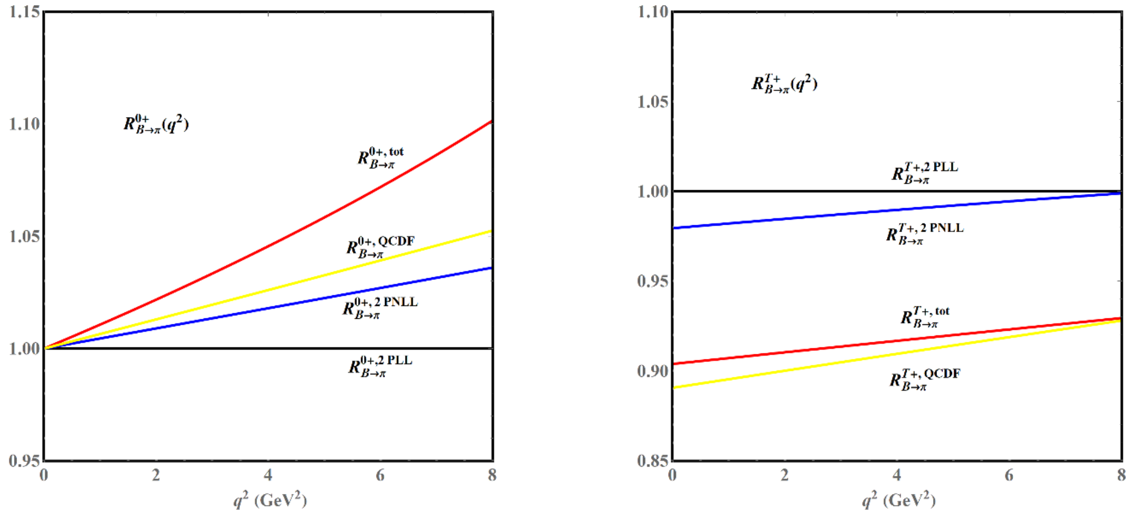

Figure 5. The large-recoil symmetry breaking effects of $B \to \pi$ form factors computed from the LCSR approach at LL accuracy ($R^{i+,\mathrm{2PLL}}_{B\to\pi}$, black), at NLL accuracy ($R^{i+,\mathrm{2PNLL}}_{B\to\pi}$, blue), and from the QCD factorization approach ($R^{i+,\mathrm{QCDF}}_{B\to\pi}$, yellow). The complete LCSR predictions for $R^{i+}_{B\to\pi}$ ($i = 0,\, T$) with the higher-twist B-meson LCDA corrections at tree level are represented by the red curves.

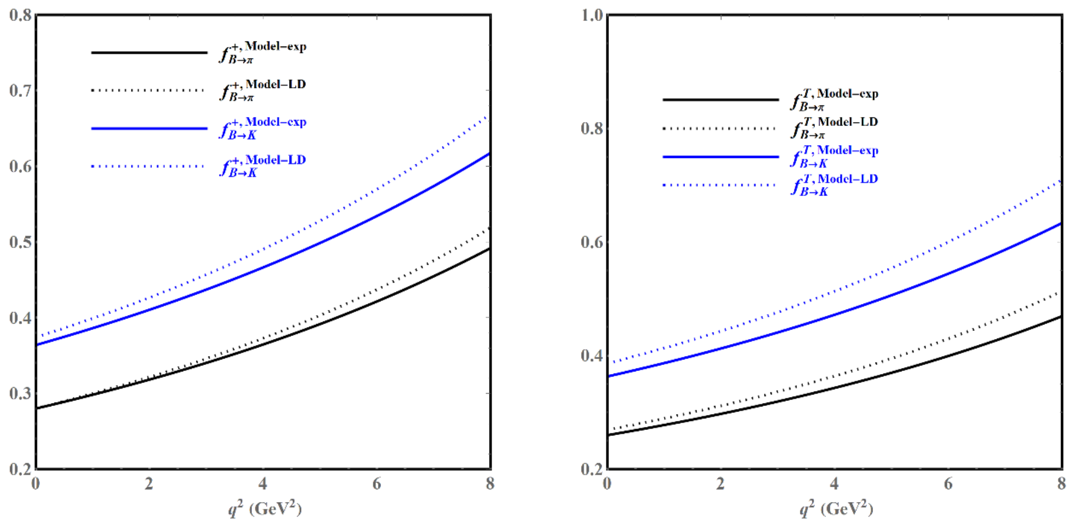

Figure 6. Dependence of the $B \to \pi, K$ form factors on the nonperturbative models of B-meson LCDAs at $0 \le q^2 \le 8\,\mathrm{GeV}^2$. The observed pattern for the scalar form factors $f^0_{B\to\pi}(q^2)$ and $f^0_{B\to K}(q^2)$, in analogy to the corresponding behaviours for the vector form factors, are not presented here for brevity.

the local duality models as a function of the momentum transfer q^2. Taking into account the fact that the vector $B \to \pi$ form factor at $q^2 = 0$ has been adjusted to reproduce the values from the pion LCSR, our main prediction is the momentum-transfer dependence of $B \to \pi, K$ form factors, which turns out to be insensitive to the specific models of B-meson LCDAs (see also [25] for a similar observation).

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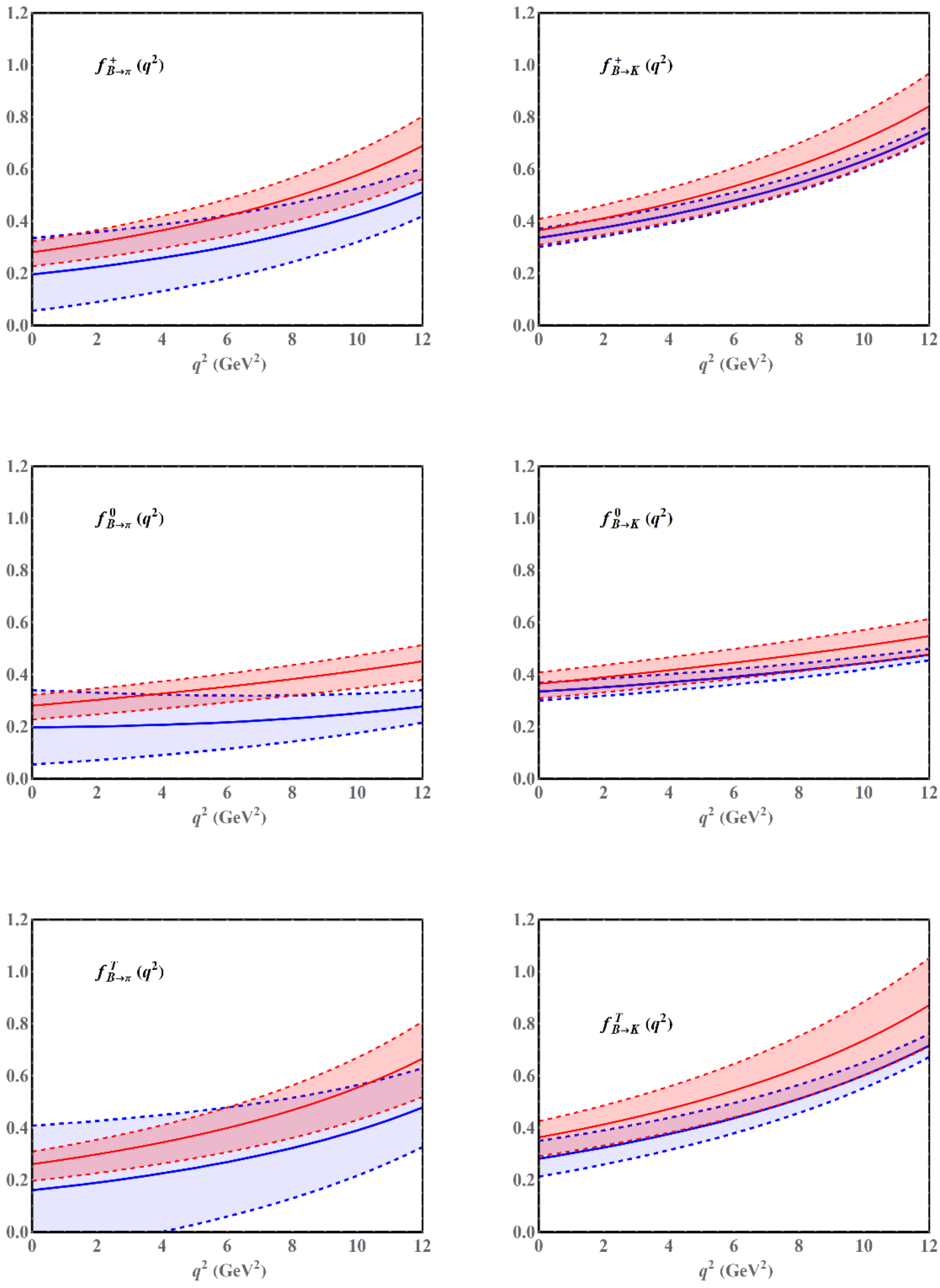

Figure 7. The momentum-transfer dependence of $B \to \pi, K$ form factors predicted from the B-meson LCSR are displayed with the pink bands. For a comparison, we also present the Lattice QCD calculations from Fermilab/MILC Collaborations [1–3] with an extrapolation to small q^2 in terms of the z-series expansion (5.15) as indicated by the blue bands.

Parameters	Central value	λ_B	σ_1	μ	ν	M^2	s_0	$\phi_B^{\pm}(\omega)$
$f^{+,0}_{B\to\pi}(0)$	0.280	${}^{-0.030}_{+0.031}$	${}^{-0.012}_{+0.013}$	${}^{+0.000}_{-0.032}$	-	${}^{+0.012}_{-0.017}$	${}^{+0.014}_{-0.014}$	-
$b^{+}_{1,\pi}$	-2.77	${}^{+0.05}_{-0.02}$	${}^{+0.02}_{-0.01}$	${}^{+0.09}_{-0.16}$	-	${}^{+0.02}_{-0.03}$	${}^{+0.07}_{-0.07}$	${}^{+0.00}_{-0.64}$
$b^{0}_{1,\pi}$	-4.88	${}^{-0.10}_{+0.11}$	${}^{-0.04}_{+0.04}$	${}^{+0.17}_{-0.61}$	-	${}^{+0.04}_{-0.06}$	${}^{+0.11}_{-0.11}$	${}^{+0.00}_{-0.37}$
$f^{T}_{B\to\pi}(0)$	0.260	${}^{-0.031}_{+0.031}$	${}^{-0.013}_{+0.013}$	${}^{+0.000}_{-0.044}$	${}^{-0.017}_{+0.025}$	${}^{+0.011}_{-0.016}$	${}^{+0.013}_{-0.014}$	${}^{+0.009}_{-0.000}$
$b^{T}_{1,\pi}$	-3.14	${}^{+0.05}_{-0.02}$	${}^{+0.02}_{-0.01}$	${}^{+0.21}_{-0.57}$	${}^{+0.05}_{-0.06}$	${}^{+0.02}_{-0.03}$	${}^{+0.07}_{-0.07}$	${}^{+0.00}_{-0.67}$
$f^{+,0}_{B\to K}(0)$	0.364	${}^{-0.035}_{+0.034}$	${}^{-0.014}_{+0.014}$	${}^{+0.000}_{-0.032}$	-	${}^{+0.010}_{-0.014}$	${}^{+0.008}_{-0.009}$	${}^{+0.010}_{-0.000}$
$b^{+}_{1,K}$	-3.04	${}^{+0.02}_{-0.00}$	${}^{+0.00}_{-0.00}$	${}^{+0.04}_{-0.06}$	-	${}^{+0.05}_{-0.07}$	${}^{+0.05}_{-0.06}$	${}^{+0.00}_{-0.76}$
$b^{0}_{1,K}$	-4.56	${}^{-0.13}_{+0.14}$	${}^{-0.06}_{+0.06}$	${}^{+0.08}_{-0.41}$	-	${}^{+0.07}_{-0.10}$	${}^{+0.07}_{-0.08}$	${}^{+0.00}_{-0.42}$
$f^{T}_{B\to K}(0)$	0.363	${}^{-0.038}_{+0.038}$	${}^{-0.016}_{+0.016}$	${}^{+0.000}_{-0.048}$	${}^{-0.022}_{+0.033}$	${}^{+0.011}_{-0.014}$	${}^{+0.009}_{-0.009}$	${}^{+0.023}_{-0.000}$
$b^{T}_{1,K}$	-3.47	${}^{+0.02}_{-0.00}$	${}^{-0.00}_{+0.01}$	${}^{+0.16}_{-0.46}$	${}^{+0.05}_{-0.06}$	${}^{+0.05}_{-0.07}$	${}^{+0.05}_{-0.06}$	${}^{+0.00}_{-0.79}$

Table 2. Theory predictions for the shape parameters and the normalizations of $B \to \pi, K$ form factors at $q^2 = 0$ entering the z expansion (5.15) with the dominant uncertainties from variations of different input parameters.

As already discussed in [22, 25], the light-cone operator product expansion (OPE) of the vacuum-to-B-meson correlation function (2.1) can be verified only at large hadronic recoil. We will extrapolate the LCSR predictions of $B \to \pi, K$ form factors at $q^2 \leq 8\,\mathrm{GeV}^2$ to the full kinematic region by applying the z-series expansion, where the entire cut q^2-plane is mapped onto the unit disk $|z(q^2,\, t_0)| \leq 1$ with the conformal transformation

$$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}\,. \tag{5.13}$$

Here, $t_+ = (m_B + m_P)^2$ is determined by the threshold of the lowest continuum state which can be generated by the weak transition currents in QCD. The auxiliary parameter t_0 determining the q^2 point to be mapped onto the origin of the complex z plane will be further taken as [72]

$$t_0 = (m_B + m_P)\,(\sqrt{m_B} + \sqrt{m_P})^2\,. \tag{5.14}$$

For concreteness, we will adopt the simplified series expansion for $B \to P$ form factors originally proposed in [75] (see [76] for an alternative parametrization)

$$\begin{aligned}
f^{+,T}_{B\to P}(q^2) &= \frac{f^{+,T}_{B\to P}(0)}{1 - q^2/m^2_{B^*_{(s)}}} \left\{ 1 + \sum_{k=1}^{N-1} b^{+,T}_{k,P} \left(z(q^2,\, t_0)^k - z(0,\, t_0)^k \right.\right. \\
&\qquad\qquad \left.\left. - (-1)^{N-k}\, \frac{k}{N}\, \left[z(q^2,\, t_0)^N - z(0,\, t_0)^N \right] \right) \right\}, \\
f^0_{B\to P}(q^2) &= f^0_{B\to P}(0) \left\{ 1 + \sum_{k=1}^{N} b^0_{k,P} \left(z(q^2,\, t_0)^k - z(0,\, t_0)^k \right) \right\}, \qquad (5.15)
\end{aligned}$$

where the threshold behaviour at $q^2 = t_+$ has been implemented and we will truncate the z-series at $N = 2$ for the vector (tensor) form factors and at $N = 1$ for the scalar form factors.

Matching the B-meson LCSR calculations in the kinematic region $-6\,\mathrm{GeV}^2 \leq q^2 \leq 8\,\mathrm{GeV}^2$ with the z-series expansion (5.15) leads to our main predictions for the q^2 dependence of $B \to \pi, K$ form factors displayed in figure 7, where the theory uncertainties due to varying different input parameters discussed in section 5.1 are also included. We further display the low q^2-extrapolation of the Lattice QCD predictions from Fermilab/MILC Collaborations [1–3] in the same figure for a comparison. While we find a fair agreement of the two calculations, our predictions for the three $B \to \pi$ form factors are more precise than the corresponding Lattice QCD results. The resulting shape parameters and the normalizations of $B \to \pi, K$ form factors at $q^2 = 0$ entering the z-series (5.15) are collected in table 2 with the numerically important uncertainties. We can readily observe that the dominant theory uncertainties for the form factors at $q^2 = 0$ originate from the variations of the inverse moment $\lambda_B(\mu_0)$, while the model dependence of the B-meson LCDAs leads to the most significant errors for the shape parameters $b^i_{1,P}$ ($i = +,\, 0,\, T$). In particular, the tensor $B \to \pi, K$ form factors appear to suffer from larger uncertainties compared with the corresponding vector and scalar form factors, due to the sizeable errors from variations of the QCD renormalization scale of the tensor current.

5.3 Phenomenological aspects of $B \to \pi l \nu$

Having at our disposal the theory predictions for $B \to \pi$ form factors, we proceed to explore phenomenological aspects of the semileptonic $B \to \pi \ell \nu$ decays, which serves as the golden channel for the determination of the CKM matrix element $|V_{ub}|$ exclusively (see [44] for the future advances of precision measurements of Belle II). It is straightforward to write down the differential decay rate for $B \to \pi \ell \nu$

$$\frac{d\,\Gamma(B \to \pi \ell \nu)}{dq^2} = \frac{G_F^2\,|V_{ub}|^2}{192\,\pi^3\,m_B^3}\,\lambda^{3/2}(m_B^2, m_\pi^2, q^2)\left(1-\frac{m_l^2}{q^2}\right)^2\left(1+\frac{m_l^2}{2\,q^2}\right)\Bigg[|f^+_{B\to\pi}(q^2)|^2 + \frac{3\,m_l^2\,(m_B^2-m_\pi^2)^2}{\lambda(m_B^2, m_\pi^2, q^2)\,(m_l^2+2\,q^2)}\,|f^0_{B\to\pi}(q^2)|^2\Bigg], \quad (5.16)$$

where $\lambda(a,b,c) = a^2+b^2+c^2-2\,ab-2\,ac-2\,bc$.

Following the strategy presented in [70], the extraction of $|V_{ub}|$ can be achieved by introducing the following quantity

$$\Delta\zeta_\ell(q_1^2, q_2^2) = \frac{1}{|V_{ub}|^2}\int_{q_1^2}^{q_2^2} dq^2\,\frac{d\,\Gamma(B\to\pi\ell\nu)}{dq^2}\,. \quad (5.17)$$

Employing the predicted $B \to \pi$ form factor $f^+_{B\to\pi}(q^2)$ from the B-meson LCSR with an extrapolation toward large q^2, we obtain

$$\begin{aligned}\Delta\zeta_\mu(0, 12\,\mathrm{GeV}^2) &= \left(5.17\,{}^{+1.23}_{-1.07}\,\Big|_{\lambda_B}\,{}^{+0.50}_{-0.44}\,\Big|_{\sigma_1}\,{}^{+0.44}_{-0.59}\,\Big|_{M^2}\,{}^{+0.49}_{-0.47}\,\Big|_{s_0}\,{}^{+0.38}_{-0.00}\,\Big|_{\phi_B^\pm}\right)\ \mathrm{ps}^{-1} \\ &= 5.17^{+1.65}_{-1.85}\ \mathrm{ps}^{-1}\,,\end{aligned} \quad (5.18)$$

where the negligibly small uncertainties due to variations of the remaining parameters (not explicitly displayed here) are also taken into account in the total uncertainty. Taking advantage of the experimental measurements of $B \to \pi\mu\nu_\mu$ from BaBar and Belle Collaborations [77, 78], the CKM matrix element $|V_{ub}|$ is determined as

$$|V_{ub}| = \left(3.23\,{}^{+0.66}_{-0.48}\Big|_{\mathrm{th.}}\,{}^{+0.11}_{-0.11}\Big|_{\mathrm{exp.}}\right)\times 10^{-3}\,, \quad (5.19)$$

which are in agreement with the averaged exclusive determinations presented in PDG [64] and the previous LCSR calculations [25, 70], but are significantly lower than the averaged inclusive determinations reported in [64]

$$|V_{ub}|_{\mathrm{inc.}} = \left(4.49 \pm 0.15\,{}^{+0.16}_{-0.17} \pm 0.17\right)\times 10^{-3}\,. \quad (5.20)$$

We further display in figure 8 our predictions for the normalized differential q^2 distributions of $B \to \pi\ell\nu$ ($\ell = \mu\,,\tau$) in the whole kinematical region, where the experimental measurements from BaBar and Belle Collaborations are also displayed for a comparison. On account of the substantial cancellation of theory uncertainties between the differential and the total decay rates of $B \to \pi\ell\nu$, the momentum-transfer dependence of the normalized differential distributions suffers from much less uncertainty than the semileptonic $B \to \pi$ form factors shown in figure 7. The future precision measurements of $B \to \pi\ell\nu$ from Belle II (with remarkably high accuracy of $\mathcal{O}(1.4\,\%)$ [44]) will be helpful to distinguish the theory predictions based upon the distinct LCSR methods presented in figure 8.

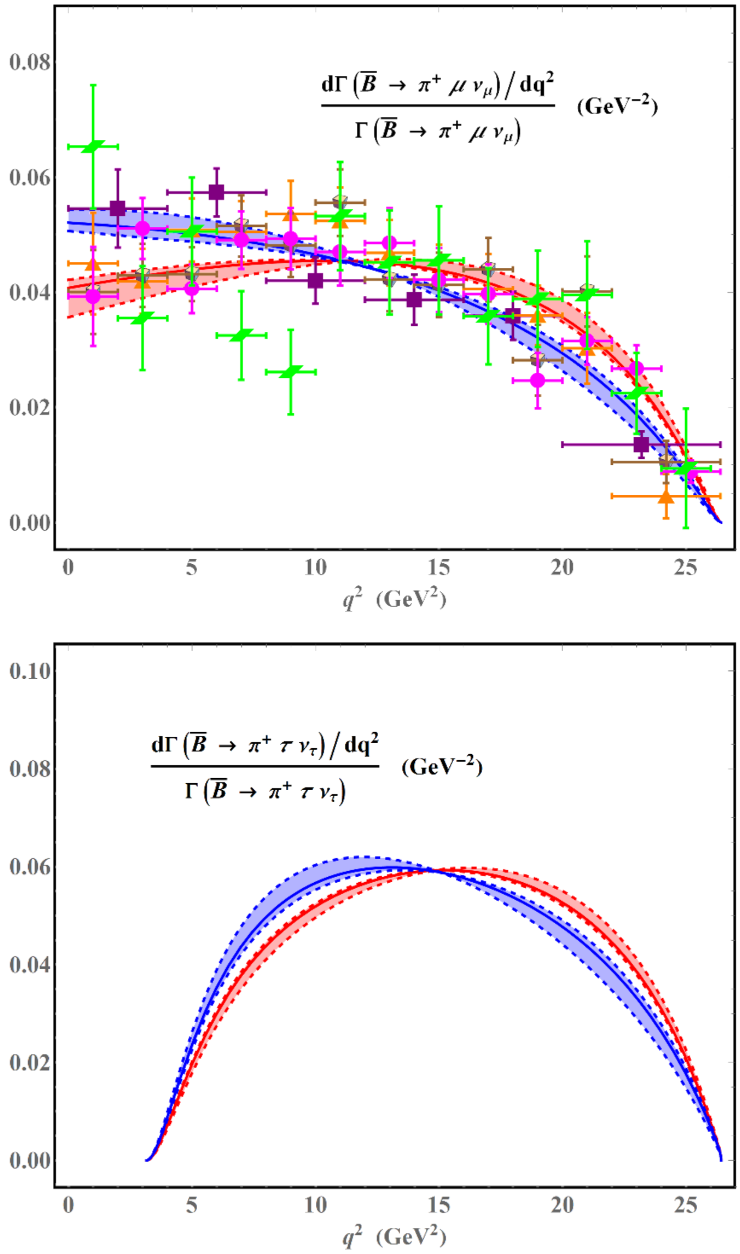

Figure 8. The normalized differential q^2 distributions of the semileptonic $B \to \pi \ell \nu_\ell$ ($\ell = \mu\,, \tau$) decays with the form factors computed from the B-meson LCSR with an extrapolation to the whole kinematical region (pink bands) and from the pion LCSR with the z-series expansion (blue bands). We also present the experimental data points $B \to \pi \mu \nu_\mu$ from [79] (purple squares), [80] (orange triangles), [77] (brown hexahedrons), [81] (magenta circles) and [78] (green parallelograms).

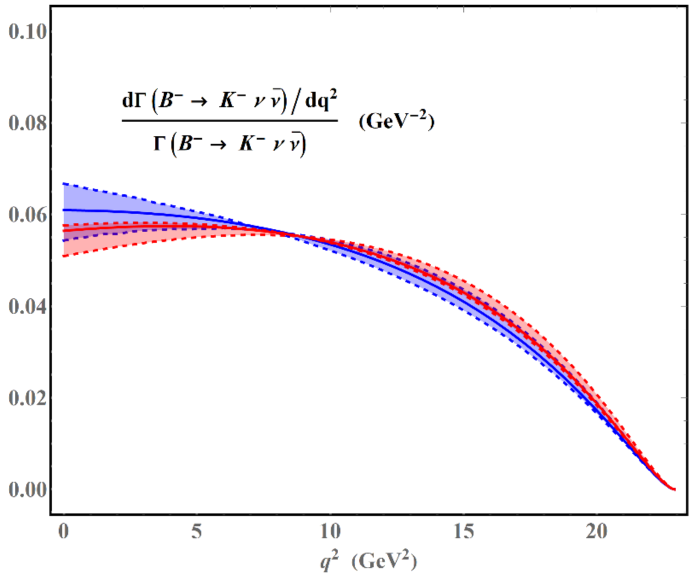

Figure 9. The normalized differential q^2 distribution of $B \to K\nu\nu$ computed with the form factors from the B-meson LCSR (pink band) and the Lattice QCD simulations [3] (blue band).

5.4 Phenomenological aspects of $B \to K\nu\nu$

The information of $B \to K$ form factors enables us to investigate the rare exclusive $B \to K\nu\nu$ decays induced by the flavour-changing neutral current $b \to s\nu\nu$. An important advantage of such process over the more complicated $B \to K^{(*)}\ell\ell$ decays [82–86] lies in the fact that the strong interaction dynamics of $B \to K\nu\nu$ is completely encoded in the semileptonic B-meson form factors. It is straightforward to write down the differential decay rate for $B \to K\nu\nu$

$$\frac{d\,\Gamma(B \to K\nu\nu)}{dq^2} = \frac{G_F^2\,\alpha_{\rm em}^2}{256\,\pi^5}\frac{\lambda^{3/2}(m_B^2, m_K^2, q^2)}{m_B^2\,\sin^4\theta_W}\,|V_{tb}\,V_{ts}^*|^2\left[X_t\left(\frac{m_t^2}{m_W^2}, \frac{m_H^2}{m_t^2}, \sin\theta_W, \mu\right)\right]^2 \times |f^+_{B\to K}(q^2)|^2\,, \tag{5.21}$$

where the short-distance Wilson coefficient X_t has been computed at NLO in QCD [87–89] and at two loops in the electroweak Standard Model (SM) [90]. For the numerical analysis, the intervals of various electroweak parameters entering (5.21) will be taken from [90]. Our prediction for the normalized differential q^2 distribution of $B \to K\nu\nu$ with the vector $B \to K$ form factor computed from the B-meson LCSR is presented in figure 9, where the theory results with the Lattice QCD form factor [3] are also displayed.

$[q_1^2,\, q_2^2]$ (in GeV^2)	$10^6 \times \Delta\mathcal{BR}(q_1^2, q_2^2)$	$10^2 \times R_{K\pi}(q_1^2, q_2^2)$
$[0.0, 1.0]$	$0.34^{+0.09}_{-0.10}$	$5.33^{+0.60}_{-0.39}$
$[1.0, 2.5]$	$0.52^{+0.13}_{-0.15}$	$5.27^{+0.59}_{-0.39}$
$[2.5, 4.0]$	$0.52^{+0.14}_{-0.15}$	$5.18^{+0.57}_{-0.38}$
$[4.0, 6.0]$	$0.69^{+0.19}_{-0.20}$	$5.06^{+0.55}_{-0.38}$
$[6.0, 8.0]$	$0.68^{+0.19}_{-0.20}$	$4.90^{+0.53}_{-0.37}$
$[0.0, 8.0]$	$2.75^{+0.64}_{-0.81}$	$5.11^{+0.56}_{-0.38}$
$[0, (m_B - m_K)^2]$	$6.02^{+1.68}_{-1.76}$	$4.06^{+0.39}_{-0.30}$

Table 3. Theory predictions for the partial branching fractions of $B \to K\nu\nu$ and the binned distributions of the precision observable $R_{K\pi}(q_1^2, q_2^2)$ with $B \to \pi, K$ form factors computed from the B-meson LCSR and the z-series expansion.

To facilitate the comparison with the future Belle II data, we further introduce the partial branching fraction of $B \to K\nu\nu$

$$\Delta\mathcal{BR}(q_1^2, q_2^2) = \tau_{B^0} \int_{q_1^2}^{q_2^2} dq^2 \, \frac{d\Gamma(B \to K\nu\nu)}{dq^2} \,, \tag{5.22}$$

whose predictions for the selected q^2 bins are collected in table 3. Our results for the integrated branching fraction $\Delta\mathcal{BR}(0, (m_B - m_K)^2) = \left(6.02^{+1.68}_{-1.76}\right) \times 10^{-6}$ are larger than the previous calculations [91], where the authors employed the rather small numbers of $f_{BK}^{+}(q^2 = 0) = 0.304 \pm 0.042$, but they are still far below the experimental upper bound from BaBar Collaboration [92]. Given the sizeable uncertainties for the predicted partial branching fractions of $B \to K\nu\nu$, we suggest to consider the ratio of the partial branching fractions for $B \to K\nu\nu$ and $B \to \pi\mu\nu_\mu$

$$R_{K\pi}(q_1^2, q_2^2) = \frac{\int_{q_1^2}^{q_2^2} dq^2 \, d\Gamma(B \to K\nu\nu)/dq^2}{\int_{q_1^2}^{q_2^2} dq^2 \, d\Gamma(B \to \pi\mu\nu_\mu)/dq^2} \,, \tag{5.23}$$

where the theory uncertainties due to the model-dependence of the B-meson LCDAs are expected to be reduced significantly. Our predictions for this ratio collected in table 3 imply that the theory precision of $R_{K\pi}$ is approximately improved by a factor of three, when compared with that of the partial branching fraction of $B \to K\nu\nu$.

6 Summary

In this paper we have computed the SU(3)-flavour symmetry breaking effects between $B \to \pi$ and $B \to K$ form factors at large recoil from the LCSR with B-meson LCDAs. It has been explicitly shown that the strange-quark-mass induced corrections are not suppressed by Λ/m_b in the heavy quark expansion and they also preserve the large-recoil symmetry relations of $B \to P$ form factors. We further evaluated the higher-twist corrections to the semileptonic B-meson decay form factors from both the two-particle and three-particle B-meson LCDAs with a complete parametrization of the corresponding light-cone matrix elements. In particular, we constructed an alternative model for the three-particle twist-five and twist-six B-meson LCDAs employing the method of QCD sum rules. The asymptotic behaviours of these higher-twist LCDAs in HQET at small quark and gluon momenta from the resulting local duality model are consistent with that determined from the conformal spins of the relevant fields. It is interesting to observe that the two-particle higher-twist corrections from the twist-five LCDA $\hat{g}_B^-(\omega,\mu)$ satisfy the symmetry relations of the soft form factors, while the three-particle higher-twist contributions violate such relations already at tree level.

Inspecting the obtained sum rules for $B \to \pi, K$ form factors numerically, we observed that the dominant higher-twist corrections come from the two-particle B-meson LCDA effects instead of the three-particle contributions. Our predictions for the SU(3)-flavour symmetry violations are in nice agreement with that obtained from the recent calculations with the light-meson LCSR approach. Applying the z-series parametrization, the improved LCSR results of $B \to \pi, K$ form factors were extrapolated to the whole kinematical region and compared with the Lattice QCD determinations. Having in our hands the theory predictions for these form factors, we computed the quantity $\Delta\zeta_\mu(0, 12\,\mathrm{GeV}^2)$ for the semileptonic $B \to \pi\mu\nu_\mu$ decay in (5.18), from which the CKM matrix element $|V_{ub}| = \left(3.23\,^{+0.66}_{-0.48}\big|_{\rm th.}\,^{+0.11}_{-0.11}\big|_{\rm exp.}\right) \times 10^{-3}$ was determined at the accuracy of $\mathcal{O}\,(20\,\%)$. The most significant theory uncertainty was identified to be generated by the variations of the inverse moment $\lambda_B(\mu_0)$. Employing our results for the vector $B \to K$ form factor, we proceeded to compute the normalized differential q^2 distributions of the rare exclusive $B \to K\nu\nu$ decays, which are expected to be well measured (approximately $9\,\%$ accuracy) at SuperKEKB with the design luminosity forty times larger than that of KEKB. In order to reduce the theory uncertainties, we constructed precision observables defined by the ratio of the partial branching fractions of $B \to K\nu\nu$ and $B \to \pi\ell\nu$.

Further improvements of the theory predictions for the heavy-to-light B-meson form factors can be made in distinct directions. First, it would be interesting to improve the considered models for the higher-twist B-meson LCDAs by taking into account the large-momentum behaviours from perturbative QCD analysis. To this end, the classical EOM relations between the leading-twist and higher-twist LCDAs displayed in (3.6)–(3.9) also need to be extended to the one-loop level. Second, computing perturbative corrections to the higher-twist contributions in $B \to \pi, K$ form factors is of both technical and conceptual importance for understanding factorization properties of the exclusive semileptonic B-meson decays. The one-loop evolution equations of the higher-twist B-meson LCDAs at twist-six accuracy will be essential to such analysis. Third, improving the unitary bounds for the z-series parametrizations of $B \to \pi, K$ form factors will be helpful to constrain the momentum-transfer dependence of these form factors (see [93] for further discussions on $B \to D$ form factors).

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