One-loop holographic Weyl anomaly in six dimensions

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ABSTRACT: We compute $\mathcal{O}(1)$ corrections to the holographic Weyl anomaly for six-dimensional $\mathcal{N} = (1, 0)$ and $(2, 0)$ theories using the functional Schrödinger method that is conjectured to work for supersymmetric theories on Ricci-flat backgrounds. We show that these corrections vanish for long representations of the $\mathcal{N} = (1, 0)$ theory, and we obtain an expression for $\delta(c - a)$ for short representations with maximum spin two. We also confirm that the one-loop corrections to the $\mathcal{N} = (2, 0)$ M5-brane theory are equal and opposite to the anomaly for the free tensor multiplet. Finally, we discuss the possibility of extending the results to encompass multiplets with spins greater than two.

KEYWORDS: AdS-CFT Correspondence, Anomalies in Field and String Theories, Conformal Field Theory

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1 Introduction

The trace anomalies for conformal field theories provide an important set of quantities that characterize the theory. In two dimensions, Cardy’s formula [1] demonstrates that the central charge c is a reliable measure of the degrees of freedom. Furthermore, its physical implication can be seen from the Zamolodchikov c -theorem [2] which states that an effective c function can be defined that is monotonically decreasing along renormalization group flows to the infrared. While the picture is perhaps the clearest in two dimensions, recent work extending these results to higher dimensional CFTs has further emphasized the importance of trace anomalies in more general situations.

The AdS/CFT correspondence provides an ideal framework for investigating various anomalies, as they may often be reliably computed on both sides of the strong/weak coupling duality. Such calculations can provide a test of the AdS/CFT correspondence and

can also yield additional insights on strongly coupled CFTs. Here we focus on the conformal anomaly, which measures the change in the partition function that results from a Weyl scaling of the metric, $\delta g_{\mu\nu} = 2\delta\sigma g_{\mu\nu}$. In particular, for a partition function given by

$$Z = \int \mathcal{D}\phi \exp(-S[\phi]), \quad (1.1)$$

we define the anomaly $\mathcal{A}$ by

$$\delta \log Z = - \int d^d x \sqrt{\det g} \delta\sigma \mathcal{A}. \quad (1.2)$$

From the holographic point of view, the leading order Weyl anomaly can be obtained from the regularized classical action [3]. In the $\text{AdS}_5/\text{CFT}_4$ case, this gives the familiar result

$$c = a = \frac{N^2}{4} \frac{\pi^3}{\text{vol}(\Sigma^5)}, \quad (1.3)$$

where IIB supergravity has been compactified on $\text{AdS}_5 \times \Sigma^5$. Additional corrections to the leading order expression may arise from higher derivative modifications to the supergravity action as well as from quantum (i.e. loop) effects.

Holographically, the log-divergent part of the one-loop effective action provides an $\mathcal{O}(1)$ correction to the Weyl anomaly coefficients. This was initially computed for the case of $\text{AdS}_5 \times S^5$ in [4–8], where it was observed that the leading order result (1.3) is shifted according to $N^2 \rightarrow N^2 - 1$, in agreement with expectations for $\text{SU}(N)$ gauge symmetry. More recently, the one-loop computation in AdS_5 has been extended to holographic field theories with reduced or even no supersymmetry [9–12].

The one-loop holographic computation is essentially a sum over contributions from all states in the spectrum of single-trace operators. Curiously, when arranged in terms of four-dimensional $\mathcal{N} = 1$ superconformal multiplets, the contribution from long multiplets vanish identically. As a result, only short representations contribute to the $\mathcal{O}(1)$ shift in a and c . This allows for a close connection between the central charges and the superconformal index which encodes knowledge of the shortened spectrum [13, 14] (see also [15]).

1.1 The six-dimensional Weyl anomaly

Here we wish to extend some of the holographic results for the Weyl anomaly coefficients to six dimensions. In general, the anomaly takes the form

$$(4\pi)^3 \mathcal{A} = (4\pi)^3 \langle T \rangle = -aE_6 + (c_1 I_1 + c_2 I_2 + c_3 I_3) + D_\mu J^\mu, \quad (1.4)$$

where E_6 refers to the Euler density, I_i are Weyl invariants and the final term is a non-universal total derivative. At leading order, Einstein gravity on AdS_7 gives a relation of the form [3, 16]

$$c_1 = 4c_2 = -12c_3 = 96a \sim \mathcal{O}(N^3), \quad (1.5)$$

which is the six-dimensional analog of (1.3). The relation between the c_i coefficients arises naturally in the holographic computation, and is further consistent with six-dimensional $(2, 0)$ superconformal invariance.

The most extensively studied $(2,0)$ theory of relevance is that of N coincident M5-branes, which is dual to supergravity on $\text{AdS}_7 \times S^4$. Here the conjectured expression for the central charges are [17–19]

$$a = -\frac{1}{288} \left(4N^3 - \frac{9}{4}N - \frac{7}{4} \right), \quad c = -\frac{1}{288} (4N^3 - 3N - 1), \quad (1.6)$$

where $c_1 = 4c_2 = -12c_3 = 96c$. The $\mathcal{O}(N)$ terms arise from R^4 corrections [17], while the $\mathcal{O}(1)$ terms arise at one-loop [18, 19]. The $\mathcal{O}(1)$ shift $\delta a = 7/1152$ was computed in [19] by evaluating the one-loop partition function on global (Euclidean) AdS_7 with S^6 boundary. However, the conjectured $\delta c = 1/288$ has not yet been directly computed, as the most straightforward computation of one-loop determinants involve highly symmetric spaces with conformally flat boundaries. In such cases, the Weyl invariants vanish, so no information is provided about the c_i coefficients.

An alternative approach to the computation of δa and δc was developed in [5–8] based on a functional Schrödinger approach. In this approach, the contribution of each state to the $\mathcal{O}(1)$ shift in the Weyl anomaly takes the form

$$\delta \mathcal{A} = -\frac{1}{2} \left(\Delta - \frac{d}{2} \right) b_d, \quad (1.7)$$

where Δ is the conformal dimension and b_d is the heat kernel coefficient for the corresponding AdS_{d+1} field when restricted to the d -dimensional boundary. In principle, since the six-dimensional b_6 coefficient may be computed on a general curved background, this allows for a full determination of not just the a coefficient but the c_i 's as well.

It has been argued in [12], however, that the expression (1.7) cannot in general be valid, as the contribution for a single field should have a more complicated dependence on the conformal dimension Δ . This can be seen explicitly in comparison with the expression for δa obtained directly from the one-loop determinant on global AdS. Curiously, however, when (1.7) is summed over the states of a complete supermultiplet, the resulting expression appears to be valid on Ricci-flat backgrounds as it passes all consistency checks and has the expected connection to the index [12, 13].

Another line of reasoning has been developed to determine the anomaly coefficients directly from the appropriate conformal higher spin operators on the boundary [20, 21]. In particular, it is argued that AdS fields with higher dimensions Δ correspond to boundary fields whose kinetic operators are greater than second order in derivatives. The factorization of these operators on Ricci-flat backgrounds may serve as a justification of the functional Schrödinger method presented in [5–8].

In this paper, we use (1.7) to compute the $\mathcal{O}(1)$ contribution to the holographic Weyl anomaly of $\mathcal{N} = (1,0)$ theories from maximum spin-2 multiplets in the bulk. Since we consider Ricci-flat backgrounds, we only obtain information on $\delta(c - a)$, where c is some linear combination of the Weyl coefficients (and reduces to the c defined above in the $\mathcal{N} = (2,0)$ case). This is similar to the $\text{AdS}_5/\text{CFT}_4$ case, where $b_4 \sim \delta(c - a) R_{\mu\nu\rho\sigma}^2$ on Ricci-flat backgrounds. As a consistency check, we find that $\delta(c - a)$ vanishes for long representations of $\mathcal{N} = (1,0)$ supersymmetry, as expected.

While we would ideally want an expression for the anomaly contribution from arbitrary higher-spin multiplets, this would require a better understanding of the heat kernel coefficients b_6 of higher-spin operators. This was worked out in [22, 23] for general spins in four dimensions. However, a similar expression is lacking in six dimensions. Nevertheless, knowledge of $\delta(c-a)$ for spins up to two is sufficient for computing the holographic Weyl anomaly for 11-dimensional supergravity on $\text{AdS}_7 \times S^4$. In this case, we sum the expression for $\delta(c-a)$ over the Kaluza-Klein spectrum and find the expected result $\delta(c-a) = -1/384$, in agreement with (1.6) and the original computation of [18].

2 The $\mathcal{O}(1)$ contribution to the holographic Weyl anomaly

As indicated above, the six-dimensional Weyl anomaly may be parameterized as

$$(4\pi)^3 \mathcal{A} = -aE_6 + (c_1 I_1 + c_2 I_2 + c_3 I_3) + D_\mu J^\mu, \quad (2.1)$$

where $E_6 = \epsilon_6 \epsilon_6 RRR$ is the six-dimensional Euler density, and the Weyl invariants are given by

$$\begin{aligned} I_1 &= C^a{}_{mn} C^m{}_{pq} C^p{}_{ab} C^q{}_{ab}, \\ I_2 &= C^{ab}{}_{mn} C^{mn}{}_{pq} C^{pq}{}_{ab}, \\ I_3 &= C^{mnpq} \square C_{mnpq} + \dots \end{aligned} \quad (2.2)$$

Superconformal invariance imposes additional constraints on the anomaly coefficients $\{a, c_i\}$. In particular, $\mathcal{N} = (1, 0)$ supersymmetry requires $c_1 - 2c_2 + 6c_3 = 0$, while $\mathcal{N} = (2, 0)$ supersymmetry gives an additional constraint $c_1 - 4c_2 = 0$. This suggests the parametrization

$$c_1 = 96(c + c' + c''), \quad c_2 = 24(c - c' + c''), \quad c_3 = -8(c + 3c' - 3c''), \quad (2.3)$$

or equivalently

$$c = \frac{c_2 - c_3}{32}, \quad c' = \frac{c_1 - 4c_2}{192}, \quad c'' = \frac{c_1 - 2c_2 + 6c_3}{192}. \quad (2.4)$$

This is designed so that c'' vanishes for superconformal theories, and additionally c' vanishes when there is extended $\mathcal{N} = (2, 0)$ supersymmetry. The coefficient c is chosen to allow for a quantity analogous to $c-a$ in four dimensions, as will become clear below.

The procedure we use to obtain the $\mathcal{O}(1)$ shift in the anomaly for $\mathcal{N} = (1, 0)$ theories is to sum the expression (1.7) over complete representations of the corresponding $\text{OSp}(8^*|2)$ supergroup. However, we first start with states in the bosonic subgroup $\text{OSp}(8^*|2) \supset \text{SO}(2) \times \text{SU}(4) \times \text{SU}(2)_R$ labeled by $D(\Delta, j_1, j_2, j_3)$ along with R -symmetry representation r . We thus have

$$\delta \mathcal{A}(\text{rep}) = -\frac{1}{2} \sum_{\text{rep}} (\Delta - 3) b_6(j_1, j_2, j_3). \quad (2.5)$$

In the following, we first work out the heat kernel coefficients $b_6(j_1, j_2, j_3)$ on a Ricci-flat background, and then perform the sum over complete supermultiplets with maximum spin two.

2.1 Heat kernel coefficients

For an operator $\Delta = -\nabla^2 - E$ where E is some endomorphism, the six-dimensional Seeley-DeWitt coefficient $b_6(\Delta)$ takes the form [16, 24]

$$b_6(\Delta) = \frac{1}{(4\pi)^3 7!} \text{Tr} \left[18A_1 + 17A_2 - 2A_3 - 4A_4 + 9A_5 + 28A_6 - 8A_7 + 24A_8 + 12A_9 \right. \\ \left. + \frac{35}{9}A_{10} - \frac{14}{3}A_{11} + \frac{14}{3}A_{12} - \frac{206}{9}A_{13} + \frac{64}{3}A_{14} - \frac{16}{3}A_{15} + \frac{44}{9}A_{16} + \frac{80}{9}A_{17} \right. \\ \left. + 14 \left(8V_1 + 2V_2 + 12V_3 - 12V_4 + 6V_5 - 4V_6 + 5V_7 + 6V_8 + 60V_9 + 30V_{10} \right. \right. \\ \left. \left. + 60V_{11} + 30V_{12} + 10V_{13} + 4V_{14} + 12V_{15} + 30V_{16} + 12V_{17} + 5V_{18} - 2V_{19} + 2V_{20} \right) \right]. \quad (2.6)$$

Here the A_a 's form a basis of curvature invariants [16, 25], and the V_a 's are built from the endomorphism E and the curvature F_{ij} of the connection [16]. In particular, while the coefficients of the A_a 's are universal, the V_a terms are specific to the representation.

We follow the conventions spelled out in appendix A of [16], which also give explicit expressions for the A_a 's and V_a 's. However, we are concerned with only the combinations which are non-vanishing on Ricci-flat backgrounds. These are

$$A_5 = (\nabla_i R_{abcd})^2, \quad A_9 = R_{abcd} \nabla^2 R^{abcd}, \quad A_{16} = R_{ab}{}^{cd} R_{cd}{}^{ef} R_{ef}{}^{ab}, \quad A_{17} = R_{aibj} R^{manb} R^i{}_m{}^j{}_n. \quad (2.7)$$

The full list of A_a 's, and expressions for the V_a 's are given in appendix A.

The invariants E_6 and I_1 , I_2 , and I_3 may be written in terms of the basis A_a functions. On a Ricci-flat background, they become

$$E_6 = 32A_{16} - 64A_{17}, \quad I_1 = -A_{17}, \quad I_2 = A_{16}, \quad I_3 = 3A_5 + 6A_9 + 2A_{16} + 8A_{17}. \quad (2.8)$$

As these quantities are not all independent, we will be unable to determine the individual central charges $\{a, c_i\}$ using only a Ricci-flat background. Note that we may construct two combinations that are total derivatives

$$D_1 = \nabla_a (R_{mnij} \nabla_a R_{mnij}) = A_5 + A_9, \\ D_2 = 2\nabla_a (R_{mnij} \nabla_m R_{anij}) = -A_5 + A_{16} + 4A_{17}. \quad (2.9)$$

This allows us to rewrite (2.8) in terms of the two invariants A_{16} and A_{17}

$$E_6 = 32A_{16} - 64A_{17}, \quad I_1 = -A_{17}, \quad I_2 = A_{16}, \quad I_3 = -A_{16} - 4A_{17} + 6D_1 - 3D_2. \quad (2.10)$$

On a Ricci-flat background, we have the relations $E_6 = 32(2I_1 + I_2)$ and $I_3 = 4I_1 - I_2$ up to a total derivative. As a result, the six-dimensional anomaly, (2.1), takes the form

$$(4\pi)^3 \mathcal{A} = 32(c - a)A_{16} - 64(c - a + 3c'')A_{17} + D_\mu J^\mu, \quad (2.11)$$

on Ricci-flat backgrounds. The implication of this expression is that we will only be able to obtain information on the $\mathcal{O}(1)$ contribution to $c - a$ and to c'' . Since the latter must vanish for superconformal theories, it will serve as a consistency check of our approach.

Field	SU(4) Rep	c_5	c_9	c_{16}	c_{17}	γ_{16}	γ_{17}
ϕ	$(0, 0, 0) = \mathbf{1}$	9	12	44/9	80/9	17/9	-28/9
ψ	$(1, 0, 0) = \mathbf{4}$	-20	-36	-202/9	-436/9	-58/9	140/9
A_μ	$(0, 1, 0) = \mathbf{6}$	-58	-96	-164/3	-344/3	-50/3	112/3
$C_{\mu\nu\rho}^+$	$(2, 0, 0) = \mathbf{10}$	174	456	-5608/9	26504/9	-8146/9	16352/9
Ψ_μ	$(1, 1, 0) = \mathbf{\overline{20}}$	292	828	3526/9	22012/9	-1298/9	2716/9
$B_{\mu\nu}$	$(1, 0, 1) = \mathbf{15}$	107	348	2992/3	-1616/3	2269/3	-4508/3
$G_{\mu\nu}$	$(0, 2, 0) = \mathbf{20'}$	544	1416	-1388/9	49984/9	-9236/9	18592/9

Table 1. Heat kernel coefficients $(4\pi)^3 7! b_6 = c_5 A_5 + c_9 A_9 + c_{16} A_{16} + c_{17} A_{17}$ for fields of spins up to two on a Ricci-flat background. In the last two columns, we tabulate γ_{16} and γ_{17} , where $(4\pi)^3 7! b_6 = \gamma_{16} A_{16} + \gamma_{17} A_{17} + D_\mu J^\mu$.

This leaves us with a holographic determination of $\delta(c - a)$, which may be combined with the result of [19] for the δa coefficient to extract both δc and δa . This, in principle, provides a complete determination of the $\mathcal{O}(1)$ shift in the holographic Weyl anomaly of $\mathcal{N} = (2, 0)$ theories. Unfortunately the additional anomaly coefficient $\delta c'$ for $\mathcal{N} = (1, 0)$ theories cannot be determined in this manner on Ricci-flat backgrounds.

Ideally, we would like to have an expression for the heat kernel coefficient $b_6(\Delta)$ for fields transforming in an arbitrary (j_1, j_2, j_3) representation of the six-dimensional SU(4) Euclidean rotation group. However, this requires understanding of arbitrary higher-spin Laplacians which currently eludes us. There is also some potential ambiguity in relating ‘on-shell’ states in AdS₇ to their corresponding boundary Laplacians in the functional Schrodinger approach of [5]. We thus restrict to spins up to two. The relevant b_6 coefficients evaluated on a Ricci-flat background are summarized in table 1. The coefficients for ϕ , ψ , A_μ and $B_{\mu\nu}$ were computed in [16], while the remaining ones are worked out in appendix A.

2.2 $\mathcal{N} = (1, 0)$ theory

We now turn to the superconformal theories, starting with the $\mathcal{N} = (1, 0)$ theory. We expect that the anomaly vanishes when summed over long representations, and we will see that this is indeed the case. The $\mathcal{N} = (1, 0)$ superconformal algebra is $\text{OSp}(8^*|2)$, with bosonic subgroup $\text{SO}(2, 6) \times \text{SU}(2)_R$. Here $\text{SO}(2, 6)$ is either the isometry group of AdS₇ or the six-dimensional conformal group. We label representations of $\text{OSp}(8^*|2) \supset \text{SO}(2, 6) \times \text{SU}(2)_R \supset \text{SO}(2) \times \text{SU}(4) \times \text{SU}(2)_R$ by conformal dimension Δ , SU(4) Dynkin labels (j_1, j_2, j_3) and $\text{SU}(2)_R$ Dynkin label k (so that SU(2) ‘spin’ is given by $k/2$).

Unitary irreducible representations of the $\mathcal{N} = (1, 0)$ theory have been studied and explicitly constructed in [26–30]. The theory has one regular and three isolated short

Level	SU(4)	$D[0, 0, 0; k]$ $\Delta = 2k$	$C[0, 0, 0; k]$ $\Delta = 2k + 2$	$B[0, 0, 0; k]$ $\Delta = 2k + 4$	$A[0, 0, 0; k]$ $\Delta = 2k + 6$	$L[0, 0, 0; k]$ $\Delta > 2k + 6$
Δ	1	k	k	k	k	k
$\Delta + \frac{1}{2}$	4	$k-1$	$k-1, k+1$	$k-1, k+1$	$k-1, k+1$	$k-1, k+1$
$\Delta + 1$	10 6	$k-2$	k $k-2, k$	k $k-2, k, k+2$	k $k-2, k, k+2$	k $k-2, k, k+2$
$\Delta + \frac{3}{2}$	$\overline{20}$ $\overline{4}$	$k-3$	$k-1$ $k-3, k-1$	$k-1, k+1$ $k-3, k-1, k+1$	$k-1, k+1$ $k-3, k-1, k+1, k+3$	$k-1, k+1$ $k-3, k-1, k+1, k+3$
$\Delta + 2$	$20'$ 15 1	$k-4$	$k-2$ $k-4, k-2$	k $k-2, k$ $k-4, k-2, k$	k $k-2, k, k+2$ $k-4, k-2, k, k+2$	k $k-2, k, k+2$ $k-4, k-2, k, k+2, k+4$
$\Delta + \frac{5}{2}$	20 4		$k-3$	$k-1$ $k-3, k-1$	$k-1, k+1$ $k-3, k-1, k+1$	$k-1, k+1$ $k-3, k-1, k+1, k+3$
$\Delta + 3$	$\overline{10}$ 6			$k-2$	k $k-2, k$	k $k-2, k, k+2$
$\Delta + \frac{7}{2}$	$\overline{4}$				$k-1$	$k-1, k+1$
$\Delta + 4$	1					k
Anomaly	$2^5 \cdot 6! \delta(c-a)$ $\delta c''$	1 0	$57 + 180k$ 0	$303 + 180k$ 0	-1 0	0 0

Table 2. The $\mathcal{N} = (1, 0)$ multiplets with maximum spin two, and corresponding holographic Weyl anomaly coefficients $\delta(c-a)$ and $\delta c''$. Here k is the $SU(2)_R$ Dynkin label (with spin $= k/2$). The shortening conditions correspond to those of (2.12), while the last column is the maximum spin-two long representation.

representations, given generically by

$$\begin{aligned}
 A[j_1, j_2, j_3; k] : \quad & \Delta = \frac{1}{2}(j_1 + 2j_2 + 3j_3) + 2k + 6, \\
 B[j_1, j_2, 0; k] : \quad & \Delta = \frac{1}{2}(j_1 + 2j_2) + 2k + 4, \\
 C[j_1, 0, 0; k] : \quad & \Delta = \frac{1}{2}j_1 + 2k + 2, \\
 D[0, 0, 0; k] : \quad & \Delta = 2k.
 \end{aligned} \tag{2.12}$$

For maximum spin two, however, we must restrict to $j_1 = j_2 = j_3 = 0$. In this case, it is a simple exercise to perform the sum (2.5) over the multiplet using the values of γ_{16} and γ_{17} given in table 1. Comparison with (2.11) then allows us to extract $\delta(c-a)$ and $\delta c''$. The results are summarized in table 2.

As a consistency check, we note that the anomaly coefficient c'' vanishes identically after summation over a complete multiplet. This is a requirement of supersymmetry, but is not manifest from the individual b_6 coefficients in table 1. We also see that the anomaly vanishes for the long representation, which agrees with expectations from the AdS_5 case [9, 12]. As for the non-vanishing contributions, note that $\delta(c-a)$ for the A and D type multiplets are equal and opposite. This must be the case, as $A[0, 0, 0; k]$ and $D[0, 0, 0; k+2]$ are “mirror shorts” that sum to become a long multiplet.

Multiplet	Δ	$2^5 \cdot 6! \delta a$	$2^5 \cdot 6! \delta(c - a)$
$L[0, 0, 0; k]$	$> 2k + 6$	0	0
$A[0, 0, 0; k]$	$2k + 6$	$10\Delta^2(\Delta^2 - 2) + \frac{11}{3}$	-1
$B[0, 0, 0; k]$	$2k + 4$	$-10(\Delta - \frac{2}{3})^2(3(\Delta - \frac{2}{3})^2 - 14) - \frac{530}{9}(\Delta - \frac{2}{3}) - \frac{419}{9}$	$90(\Delta - \frac{2}{3}) + 3$
$C[0, 0, 0; k]$	$2k + 2$	$10(\Delta - \frac{4}{3})^2(3(\Delta - \frac{4}{3})^2 - 14) - \frac{530}{9}(\Delta - \frac{4}{3}) + \frac{419}{9}$	$90(\Delta - \frac{4}{3}) - 3$
$D[0, 0, 0; k]$	$2k$	$-10(\Delta - 2)^2((\Delta - 2)^2 - 2) - \frac{11}{3}$	1

Table 3. Contribution to the Weyl anomaly coefficients δa and δc from maximum spin two multiplets for the $\mathcal{N} = (1, 0)$ theory. Here c is related to the conventional anomaly coefficients c_i according to (2.4). The δa coefficient is computed using the results of [19].

Finally, recall that the $\mathcal{N} = (1, 0)$ theory admits three independent anomaly coefficients, which we have parametrized as a , c and c' . Since we only consider Ricci-flat backgrounds, we have only been able to determine the difference $\delta(c - a)$. This may be combined with the holographic δa coefficient obtained in [19] to separate out the contributions to δa and δc . These results are presented in table 3. However, we are unable to determine $\delta c'$ unless we can move away from Ricci-flat backgrounds.

2.3 $\mathcal{N} = (2, 0)$ theory

We may perform the same computation for the $\mathcal{N} = (2, 0)$ theory, noting however that only the 1/2-BPS multiplets have spins less than or equal to two. In this case, the superconformal algebra decomposes as $\text{OSp}(8^*|4) \supset \text{SO}(2, 6) \times \text{Sp}(4)_R \supset \text{SO}(2) \times \text{SU}(4) \times \text{Sp}(4)_R$. The shortening conditions follow the same pattern as (2.12), however with extended R -symmetry [26–30]

$$\begin{aligned}
 A[j_1, j_2, j_3; k_1, k_2] : \quad & \Delta = \frac{1}{2}(j_1 + 2j_2 + 3j_3) + 2(k_1 + k_2) + 6, \\
 B[j_1, j_2, 0; k_1, k_2] : \quad & \Delta = \frac{1}{2}(j_1 + 2j_2) + 2(k_1 + k_2) + 4, \\
 C[j_1, 0, 0; k_1, k_2] : \quad & \Delta = \frac{1}{2}j_1 + 2(k_1 + k_2) + 2, \\
 D[0, 0, 0; k_1, k_2] : \quad & \Delta = 2(k_1 + k_2).
 \end{aligned} \tag{2.13}$$

Here (k_1, k_2) are Dynkin labels for $\text{Sp}(4)$, with $(1, 0)$ denoting the **4** and $(0, 1)$ denoting the **5**. For maximum spin two, we restrict to the 1/2-BPS multiplets $D[0, 0, 0; 0, k]$ with $\Delta = 2k$. (The case $k = 1$ is the free tensor multiplet, while $k = 2$ is the stress tensor multiplet.)

The holographic computation of $\delta(c - a)$ and $\delta c''$ for the $D[0, 0, 0; 0, k]$ multiplets are shown in table 4. The case $k \geq 4$ is generic, and we do not include $k = 1$, which is a supersingleton and would not appear in a holographic computation. The special case $k = 3$ fits into the generic pattern, and in fact so does $k = 2$, although it requires separate treatment because of the presence of massless modes. For $k = 2$, the states in $D[0, 0, 0; 0, 2]$

Δ	SU(4)	$D[0, 0, 0; 0, 2]$	$D[0, 0, 0; 0, 3]$	$D[0, 0, 0; 0, k \geq 4]$
$2k$	1	$(0, 2)$	$(0, 3)$	$(0, k)$
$2k + \frac{1}{2}$	4	$(1, 1)$	$(1, 2)$	$(1, k - 1)$
$2k + 1$	6	$(2, 0)$	$(2, 1)$	$(2, k - 2)$
	10	$(0, 1)$	$(0, 2)$	$(0, k - 1)$
$2k + \frac{3}{2}$	$\overline{\mathbf{20}}$	$(1, 0)$	$(1, 1)$	$(1, k - 2)$
	$\overline{\mathbf{4}}$		$(3, 0)$	$(3, k - 3)$
$2k + 2$	20'	$(0, 0)$	$(0, 1)$	$(0, k - 2)$
	15		$(2, 0)$	$(2, k - 3)$
	1			$(4, k - 4)$
$2k + \frac{5}{2}$	20		$(1, 0)$	$(1, k - 3)$
	4			$(3, k - 4)$
$2k + 3$	$\overline{\mathbf{10}}$		$(0, 0)$	$(0, k - 3)$
	6			$(2, k - 4)$
$2k + \frac{7}{2}$	$\overline{\mathbf{4}}$			$(1, k - 4)$
$2k + 4$	1			$(0, k - 4)$
Anomaly	$384\delta(c - a)$	13	37	$6k(k - 1) + 1$
	$\delta c''$	0	0	0

Table 4. The $\mathcal{N} = (2, 0)$ 1/2-BPS (maximum spin two) representation $D[0, 0, 0; 0, k]$ and corresponding holographic Weyl anomaly coefficients $\delta(c - a)$ and $\delta c''$. Entries are $\text{Sp}(4)_R$ representations specified by Dynkin labels (k_1, k_2) .

are

$$\begin{aligned}
 & D(4; 0, 0, 0)_{\mathbf{14}} + D\left(4\frac{1}{2}; 1, 0, 0\right)_{\mathbf{16}} + D(5; 0, 1, 0)_{\mathbf{10}} \\
 & + D(5; 2, 0, 0)_{\mathbf{5}} + D\left(5\frac{1}{2}; 1, 1, 0\right)_{\mathbf{4}} + D(6; 0, 2, 0)_{\mathbf{1}},
 \end{aligned} \tag{2.14}$$

where $D(\Delta; j_1, j_2, j_3)$ labels the $\text{SO}(2, 6)$ representation and the subscript labels the $\text{Sp}(4)_R$ representation. The massless vector, gravitino and graviton representations can be obtained from the corresponding massive representations by subtracting out null states according to

$$\begin{aligned}
 D(5; 0, 1, 0) &= D(5 + \epsilon; 0, 1, 0) - D(6; 0, 0, 0), \\
 D\left(5\frac{1}{2}; 1, 1, 0\right) &= D\left(5\frac{1}{2} + \epsilon; 1, 1, 0\right) - D\left(6\frac{1}{2}; 1, 0, 0\right), \\
 D(6; 0, 2, 0) &= D(6 + \epsilon; 0, 2, 0) - D(7; 0, 1, 0).
 \end{aligned} \tag{2.15}$$

(Note that the three-form, $D(5; 2, 0, 0)$, is massive, so no subtraction is required.) Taking these null states into account then gives the result $\delta(c - a) = 13/384$ for $k = 2$ shown in table 4.

Although $k = 2$ and $k = 3$ are special cases, the holographic anomaly coefficient $\delta(c - a) = (1/384)(6k(k-1)+1)$ is in fact universal. Combining this with $\delta a = -(7/1152)(6k(k-1)+1)$ obtained in [19] then allows us to separate out the individual coefficients

$$D[0, 0, 0; 0, k \geq 2] : \quad \delta a = -\frac{1}{288} \cdot \frac{7}{4} (6k(k-1) + 1), \quad \delta c = -\frac{1}{288} (6k(k-1) + 1). \quad (2.16)$$

As an application, consider the $\mathcal{N} = (2, 0)$ theory obtained by compactifying 11-dimensional supergravity on $\text{AdS}_7 \times S^4$. The Kaluza-Klein spectrum is simply

$$\oplus_{k \geq 2} D[0, 0, 0; 0, k], \quad (2.17)$$

where $k = 2$ corresponds to the ‘massless’ supergravity sector. The anomaly coefficients δa and δc may be computed by summing over the Kaluza-Klein levels

$$\delta a = -\frac{1}{288} \cdot \frac{7}{4} \sum_{k=2}^{\infty} (6k(k-1) + 1), \quad \delta c = -\frac{1}{288} \sum_{k=2}^{\infty} (6k(k-1) + 1). \quad (2.18)$$

Following [19], we regulate the sums using a hard cutoff. This amounts to setting $\sum_{k=1}^{\infty} k^n = 0$ for any $n \geq 0$. This implies $\sum_{k=2}^{\infty} f(k) = -f(1)$, where $f(k)$ is polynomial in k . As a result, the regulated anomaly for $\text{AdS}_7 \times S^4$ is

$$\delta a = \frac{1}{288} \cdot \frac{7}{4}, \quad \delta c = \frac{1}{288}. \quad (2.19)$$

This is equal and opposite to the result for the conformal anomaly of the free tensor multiplet computed in [17], and agrees with the $\mathcal{O}(1)$ contributions in (1.6).

3 Discussion

While the six-dimensional Weyl anomaly coefficients are conventionally parametrized as a , c_1 , c_2 and c_3 , we have found it convenient to use an alternate linear combination of the c_i ’s given in (2.4). Holographically, the leading order anomaly coefficients (assuming Einstein gravity in the bulk) satisfy the relation

$$c = a, \quad c' = 0, \quad c'' = 0. \quad (3.1)$$

(While $c'' = 0$ must hold for superconformal theories, this holographic result is independent of whether the theory is supersymmetric or not.) At the one-loop level, we have been able to extend this leading order result by computing $\delta(c-a)$ using the expression (1.7) obtained through a functional Schrödinger method [5–8].

It is reasonable to question whether the use of (1.7) is valid, as it disagrees with the direct computation of δa performed in [12, 19]. A quick way to see this is to note that δa in table 3 is a fourth order polynomial in Δ , while the result of summing (1.7) over a supermultiplet can be at most quadratic in Δ . (One power comes directly from (1.7), while another can arise from the dimension of the shortened representation.) If $\delta(c-a)$ was expected to be cubic or higher in Δ , then our result, as shown in the last column of

table 3, cannot possibly be correct. However, we now demonstrate that $c - a$ can be at most linear in Δ , which is consistent with application of (1.7).

To see this, recall that, in superconformal field theories, the stress tensor is contained in a multiplet of currents, so that there is a corresponding multiplet of anomalies. For $\mathcal{N} = (1, 0)$ theory, the ‘t Hooft anomalies are characterized by the anomaly polynomial

$$\mathcal{I}_8 = \frac{1}{4!} [\alpha c_2(R)^2 + \beta c_2(R) p_1(T) + \gamma p_1(T)^2 + \delta p_2(T)], \quad (3.2)$$

and the relation to the Weyl anomaly coefficients has recently been worked out [20, 31–33]

$$a = -\frac{1}{72} \left(\alpha - \beta + \gamma + \frac{3}{8} \delta \right), \quad c - a = -\frac{\delta}{192}, \quad c' = \frac{1}{432} \left(\beta - 2\gamma + \frac{1}{2} \delta \right). \quad (3.3)$$

Since α is the coefficient of the $[\text{SU}(2)_R]^4$ anomaly, it can be at most fifth power in Δ , where the extra power comes from the dimension of the representation. Similarly, β can be at most cubic in Δ , while γ and δ can be at most linear in Δ . This in turn demonstrates that a will be at most fifth power in Δ , c' will be at most cubic and $c - a$ will be at most linear. Thus the functional Schrödinger method is indeed compatible with $\delta(c - a)$. However, we also see this approach cannot be used to compute either δa alone or $\delta c'$. Thus, while it would be desirable to compute $\delta c'$ in these theories, we do not expect that it can be done using this approach.

While we have focused on short multiplets with spins ≤ 2 , it would be desirable to work more generally with higher-spin multiplets. To do so, we would need knowledge of the b_6 coefficients for arbitrary spin fields. This in turn depends on the form of the higher-spin Laplacian. In general, this depends on the bulk dynamics of the higher-spin field and the further restriction to the boundary following from the procedure of [5–8]. For higher-spin bosons, it is natural to take a bulk Laplacian of the form $\Delta = -\square - E$ with the endomorphism $E = \Sigma_{ab} R^{abcd} \Sigma_{cd}$, where Σ_{ab} are $\text{SU}(4)$ generators in the appropriate bosonic higher-spin representation. However, the situation is less clear for fermions. The natural generalization would be to simply take Σ_{ab} to be in a fermionic higher-spin representation. However, this does not agree with the square of the Dirac operator for ordinary spin-1/2 fermions. Nevertheless, it is possible that the use of a universal endomorphism term for bosons and fermions would be appropriate when tracing over supermultiplets. Along these lines, we have computed the b_6 coefficient for general higher-spin representations in appendix B.

Finally, part of our motivation for exploring the $\mathcal{O}(1)$ contributions to the holographic Weyl anomaly is to make a connection to the $\mathcal{N} = (1, 0)$ superconformal index. As in the $\text{AdS}_5/\text{CFT}_4$ case, the bulk one-loop corrections to the Weyl anomaly vanish for long representations, so it is natural to expect that these corrections can be obtained from the index. More generally, we anticipate that the one-loop matching between $\delta(c - a)$ and the index can be extended to the full set of anomaly coefficients a and c_i , even in theories without holographic duals. This would then open up a new path towards characterizing superconformal field theories in six dimensions.

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A Heat kernel for spins up to two

The Seeley-DeWitt coefficients $b_n(\Delta)$ depend on the field and the form of the second order operator Δ . In four dimensions, the appropriate operators for irreducible fields up to spin two are listed in [23]. Here we write down the analogous operators in six dimensions and compute the contribution of each to the anomaly.

We start with the basis of curvature invariants [16, 25]

$$\begin{aligned}
 A_1 &= \square^2 R, & A_2 &= (\nabla_a R)^2, & A_3 &= (\nabla_a R_{mn})^2, & A_4 &= \nabla_a R_{bm} \nabla^b R^{am}, & A_5 &= (\nabla_a R_{mnij})^2, \\
 A_6 &= R \square R, & A_7 &= R_{ab} \square R^{ab}, & A_8 &= R_{ab} \nabla_m \nabla^b R^{am}, & A_9 &= R_{abmn} \square R^{abmn}, & A_{10} &= R^3 \\
 A_{11} &= R R_{ab}^2, & A_{12} &= R R_{abmn}^2, & A_{13} &= R_a^m R_m^i R_i^a, & A_{14} &= R_{ab} R_{mn} R^{abmn}, \\
 A_{15} &= R_{ab} R^{amnl} R_{mn}^b, & A_{16} &= R_{ab}{}^{cd} R_{cd}{}^{ef} R_{ef}{}^{ab}, & A_{17} &= R_{aibj} R^{manb} R^i{}_m{}^j{}_n.
 \end{aligned} \tag{A.1}$$

The b_6 coefficient may be computed from the expression (2.6), where the V_a 's are given by

$$\begin{aligned}
 V_1 &= \nabla_k F_{ij} \nabla^k F^{ij}, & V_2 &= \nabla_j F_{ij} \nabla^k F^{ik}, & V_3 &= F_{ij} \square F^{ij}, & V_4 &= F_{ij} F^{jk} F_k^i, \\
 V_5 &= R_{mnij} F^{mn} F^{ij}, & V_6 &= R_{jk} F^{jn} F_n^k, & V_7 &= R F_{ij} F^{ij}, & V_8 &= \square^2 E, & V_9 &= E \square E, \\
 V_{10} &= \nabla_k E \nabla^k E, & V_{11} &= E^3, & V_{12} &= E F_{ij}^2, & V_{13} &= R \square E, & V_{14} &= R_{ij} \nabla^i \nabla^j E, \\
 V_{15} &= \nabla_k R \nabla^k E, & V_{16} &= E^2 R, & V_{17} &= E \square R, & V_{18} &= E R^2, \\
 V_{19} &= E R_{ij}^2, & V_{20} &= E R_{ijkl}^2.
 \end{aligned} \tag{A.2}$$

Here $\Delta = -\nabla^2 - E$ and F_{ij} is the curvature of the connection, $[\nabla_i, \nabla_j] = F_{ij}$. Below we present the V -terms for each field after tracing over the representation.

A.1 Conformally coupled scalar

The conformally coupled scalar has $E = -\frac{1}{5}R$ and $F_{ij} = 0$, so the V -terms are:

V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	V_9	V_{10}
0	0	0	0	0	0	0	$-\frac{1}{5}A_1$	$\frac{1}{25}A_6$	$\frac{1}{25}A_2$
V_{11}	V_{12}	V_{13}	V_{14}	V_{15}	V_{16}	V_{17}	V_{18}	V_{19}	V_{20}
$-\frac{1}{125}A_{10}$	0	$-\frac{1}{5}A_6$	$\frac{2}{5}(-A_8 + A_{13} - A_{14})$	$-\frac{1}{5}A_2$	$\frac{1}{25}A_{10}$	$-\frac{1}{5}A_6$	$-\frac{1}{5}A_{10}$	$-\frac{1}{5}A_{11}$	$-\frac{1}{5}A_{12}$

The b_6 coefficient is

$$\begin{aligned}
 b_6(\mathcal{O}) &= \frac{1}{(4\pi)^3 7!} \left[\frac{6}{5}A_1 + \frac{1}{5}A_2 - 2A_3 - 4A_4 + 9A_5 - 8A_7 + \frac{8}{5}A_8 + 12A_9 \right. \\
 &\quad \left. - \frac{7}{225}A_{10} + \frac{14}{15}A_{11} - \frac{14}{15}A_{12} - \frac{32}{45}A_{13} - \frac{16}{15}A_{14} - \frac{16}{3}A_{15} + \frac{44}{9}A_{16} + \frac{80}{9}A_{17} \right].
 \end{aligned} \tag{A.3}$$

A.2 Weyl fermion

The appropriate second order operator for the Dirac fermion may be obtained as the square of the Dirac operator:

$$\mathcal{O}\psi = -\square\psi + \frac{1}{4}R\psi. \quad (\text{A.4})$$

The endomorphism and curvature of the connection coincide with the result obtained in [16].

$$E = -\frac{1}{4}R, \quad F_{ij} = \frac{1}{4}R_{ijab}\gamma^{ab}. \quad (\text{A.5})$$

Then the V -terms contributing to the anomaly are (after tracing):

V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	V_9	V_{10}
$-\frac{1}{2}A_5$	$A_4 - A_3$	$-\frac{1}{2}A_9$	$\frac{1}{2}A_{17}$	$-\frac{1}{2}A_{16}$	$-\frac{1}{2}A_{15}$	$-\frac{1}{2}A_{12}$	$-A_1$	$\frac{1}{4}A_6$	$\frac{1}{4}A_2$
V_{11}	V_{12}	V_{13}	V_{14}	V_{15}	V_{16}	V_{17}	V_{18}	V_{19}	V_{20}
$-\frac{1}{16}A_{10}$	$\frac{1}{8}A_{12}$	$-A_6$	$-2(A_8 - A_{13} + A_{14})$	$-A_2$	$\frac{1}{4}A_{10}$	$-A_6$	$-A_{10}$	$-A_{11}$	$-A_{12}$

The b_6 coefficient is

$$b_6(\mathcal{O}) = \frac{4}{(4\pi)^{37}!} \left[-3A_1 + \frac{5}{4}A_2 - 9A_3 + 3A_4 - 5A_5 + \frac{7}{2}A_6 - 8A_7 - 4A_8 - 9A_9 \right. \\ \left. - \frac{35}{72}A_{10} + \frac{7}{3}A_{11} + \frac{49}{24}A_{12} + \frac{44}{9}A_{13} - \frac{20}{3}A_{14} + \frac{5}{3}A_{15} - \frac{101}{18}A_{16} - \frac{109}{9}A_{17} \right]. \quad (\text{A.6})$$

A.3 Vector

The $(0, 1, 0)$ vector representation of $SU(4)$ is a one form, so the correct Laplacian may be obtained by computing the Hodge-deRham operator $d\delta + \delta d$. We get

$$\mathcal{O}A_\mu = -\square A_\mu + R^\nu{}_\mu A_\nu. \quad (\text{A.7})$$

The endomorphism and curvature of the connection here are:

$$E^a{}_b = -R^a{}_b, \quad (F_{ij})^a{}_b = R^a{}_{bij}, \quad (\text{A.8})$$

so that (after tracing)

V_1	V_2	V_3	V_4	V_5	V_6	V_7	V_8	V_9	V_{10}
$-A_5$	$2(A_4 - A_3)$	$-A_9$	A_{17}	$-A_{16}$	$-A_{15}$	$-A_{12}$	$-A_1$	A_7	A_3
V_{11}	V_{12}	V_{13}	V_{14}	V_{15}	V_{16}	V_{17}	V_{18}	V_{19}	V_{20}
$-A_{13}$	A_{15}	$-A_6$	$-2(A_8 - A_{13} + A_{14})$	$-A_2$	A_{11}	$-A_6$	$-A_{10}$	$-A_{11}$	$-A_{12}$

and

$$b_6(\mathcal{O}) = \frac{1}{(4\pi)^{37}!} \left[24A_1 - 66A_2 + 3529A_3 + 32A_4 - 58A_5 - 140A_6 + 792A_7 - 32A_8 - 96A_9 \right. \\ \left. - \frac{140}{3}A_{10} + 420A_{11} - 70A_{12} - \frac{2600}{3}A_{13} + 16A_{14} + 444A_{15} - \frac{164}{3}A_{16} - \frac{344}{3}A_{17} \right]. \quad (\text{A.9})$$

A.4 Self-dual three-form

The field which transforms under the $(2, 0, 0)$ representation is the 10-component self-dual three-form. A three-index antisymmetric tensor has 20 components and the self-duality condition removes half of these. The operator acting on this field is

$$\begin{aligned} \mathcal{O}C_{\mu\nu\rho} = & -\square C_{\mu\nu\rho} + R_\mu{}^\lambda C_{\lambda\nu\rho} + R_\nu{}^\lambda C_{\mu\lambda\rho} + R_\rho{}^\lambda C_{\mu\nu\lambda} \\ & - R_{\mu\nu}{}^{\lambda\sigma} C_{\lambda\sigma\rho} - R_{\nu\rho}{}^{\lambda\sigma} C_{\mu\lambda\sigma} - R_{\rho\mu}{}^{\lambda\sigma} C_{\lambda\nu\sigma}. \end{aligned} \quad (\text{A.10})$$

This means that the endomorphism and connection curvature are given by

$$E_{abc}{}^{def} = -3R_{[a}{}^{[d}\delta_b^e\delta_{c]}^f] + 3R_{[ab}{}^{[de}\delta_{c]}^f], \quad (F_{ij})_{abc}{}^{def} = -3R_{ij[a}{}^{[d}\delta_b^e\delta_{c]}^f]. \quad (\text{A.11})$$

Then we can compute the relevant terms:

V_1	V_2	V_3	V_4	V_5
$-6A_5$	$12(A_4 - A_3)$	$-6A_9$	$6A_{17}$	$-6A_{16}$
V_6	V_7	V_8	V_9	V_{10}
$-6A_{15}$	$-6A_{12}$	$-6A_1$	$2A_6 - 2A_7 + 2A_9$	$2A_2 - 2A_3 + 2A_5$
V_{11}	V_{12}	V_{13}	V_{14}	V_{15}
$*$	$2A_{12} - 2A_{15} + 2A_{16}$	$-6A_6$	$-12(A_8 - A_{13} + A_{14})$	$-6A_2$
V_{16}	V_{17}	V_{18}	V_{19}	V_{20}
$2(A_{10} - A_{11} + A_{12})$	$-6A_6$	$-6A_{10}$	$-6A_{11}$	$-6A_{12}$

where $V_{11} = -A_{10} + 6A_{11} - 3A_{12} - 6A_{13} - 12A_{14} + 12A_{15} - 2A_{16} + 8A_{17}$. (Again, all V -terms are given after tracing over the representation.) So the b_6 coefficient is given by

$$\begin{aligned} b_6(\mathcal{O}) = \frac{1}{(4\pi)^{37!}} \bigg[& -144A_1 + 172A_2 - 1216A_3 + 256A_4 + 348A_5 + 392A_6 - 1840A_7 \\ & - 192A_8 + 912A_9 + \frac{3080}{9}A_{10} + \frac{12824}{3}A_{11} - \frac{4004}{3}A_{12} - \frac{43472}{9}A_{13} \\ & - \frac{30976}{3}A_{14} + \frac{28408}{3}A_{15} - \frac{11216}{9}A_{16} + \frac{53008}{9}A_{17} \bigg]. \end{aligned} \quad (\text{A.12})$$

The self-duality condition reduces each of these terms by a factor of two, reproducing the A_{16} and A_{17} terms found in table 1.

A.5 Gravitino

The gravitino with the gauge condition $\gamma^\mu\psi_\mu = 0$ corresponds to the $(1, 1, 0)$ representation. In this case the operator $\mathcal{O}$ is the square of the Rarita-Schwinger operator:

$$\mathcal{O}\psi_\mu = -\square\psi_\mu + \frac{1}{4}R\psi_\mu - \frac{1}{2}\gamma^\rho\gamma^\sigma R_{\rho\sigma\mu\nu}\psi^\nu. \quad (\text{A.13})$$

The endomorphism and connection curvature are given by

$$E_b{}^a = -\frac{1}{4}R\delta_b^a + \frac{1}{2}R_{cdb}{}^a\gamma^{cd}, \quad (F_{ij})_b{}^a = \frac{1}{4}R_{ijcd}\gamma^{cd}\delta_b^a + R_{ijb}{}^a, \quad (\text{A.14})$$

so (after tracing)

V_1	V_2	V_3	V_4	V_5
$-7A_{15}$	$-7A_{12}$	$-6A_1$	$\frac{3}{2}A_6 + 2A_9$	$\frac{3}{2}A_2 + 2A_5$
V_6	V_7	V_8	V_9	V_{10}
$-7A_5$	$14(A_4 - A_3)$	$-7A_9$	$7A_{17}$	$-7A_{16}$
V_{11}	V_{12}	V_{13}	V_{14}	V_{15}
$-\frac{3}{8}A_{10} - \frac{3}{2}A_{12} + 4A_{17}$	$\frac{7}{4}A_{12} + 2A_{16}$	$-6A_6$	$-12(A_8 - A_{13} + A_{14})$	$-6A_2$
V_{16}	V_{17}	V_{18}	V_{19}	V_{20}
$\frac{3}{2}A_{10} + 2A_{12}$	$-6A_6$	$-6A_{10}$	$-6A_{11}$	$-6A_{12}$

and

$$b_6(\mathcal{O}) = \frac{1}{(4\pi)^{37}!} \left[-60A_1 + 25A_2 - 404A_3 + 284A_4 + 292A_5 + 70A_6 - 160A_7 \right. \\ \left. - 80A_8 + 828A_9 - \frac{175}{18}A_{10} + \frac{140}{3}A_{11} - \frac{1435}{6}A_{12} - \frac{880}{9}A_{13} \right. \\ \left. - \frac{400}{3}A_{14} + \frac{772}{3}A_{15} + \frac{3526}{9}A_{16} + \frac{22012}{9}A_{17} \right]. \quad (\text{A.15})$$

A.6 Two-form

The adjoint representation $(1, 0, 1)$ corresponds to the two-form computed in [16].

$$\mathcal{O}B_{\mu\nu} = -\square B_{\mu\nu} + R_\mu^\lambda B_{\lambda\nu} - R_\nu^\lambda B_{\lambda\mu} - R_{\mu\nu}{}^{\rho\sigma} B_{\rho\sigma}. \quad (\text{A.16})$$

This means that the endomorphism and connection curvature are given by

$$E_{ab}{}^{cd} = -2R_{[a}^{[c}\delta_{b]}^{d]} + R_{ab}{}^{cd}, \quad (F_{ij})_{ab}{}^{cd} = 2R_{ij[a}{}^{[c}\delta_{b]}^{d]}, \quad (\text{A.17})$$

so (after tracing)

V_1	V_2	V_3	V_4	V_5
$-4A_5$	$8(A_4 - A_3)$	$-4A_9$	$4A_{17}$	$-4A_{16}$
V_6	V_7	V_8	V_9	V_{10}
$-4A_{15}$	$-4A_{12}$	$-4A_1$	$A_6 + A_9$	$A_2 + A_5$
V_{11}	V_{12}	V_{13}	V_{14}	V_{15}
$-3A_{11} + 4A_{13} + 6A_{14} - 6A_{15} + A_{16}$	$A_{12} + A_{16}$	$-4A_6$	$-8(A_8 - A_{13} + A_{14})$	$-4A_2$
V_{16}	V_{17}	V_{18}	V_{19}	V_{20}
$A_{10} + A_{12}$	$-4A_6$	$-4A_{10}$	$-4A_{11}$	$-4A_{12}$

and

$$b_6(\mathcal{O}) = \frac{1}{(4\pi)^{37!}} \left[-66A_1 + 3A_2 - 254A_3 + 164A_4 + 107A_5 + 28A_6 - 120A_7 - 88A_8 + 348A_9 \right. \\ \left. + \frac{595}{3}A_{10} - 2478A_{11} + 518A_{12} + \frac{10384}{3}A_{13} - 4912A_{14} - 4896A_{15} + \frac{2992}{3}A_{16} - \frac{1616}{3}A_{17} \right]. \quad (\text{A.18})$$

A.7 Graviton

The symmetric spin-two field is the $(0, 2, 0)$ representation. The appropriate kinetic operator is the Lichnerowicz operator [34]:

$$\mathcal{O}h_{\mu\nu} = -\square h_{\mu\nu} + R_\mu{}^\lambda h_{\lambda\nu} + R_\nu{}^\lambda h_{\lambda\mu} - 2R_{\mu\rho\nu\sigma}h^{\rho\sigma}. \quad (\text{A.19})$$

The endomorphism and connection are given by

$$E_{\mu\nu}^{\rho\sigma} = -2R_{\{\mu}{}^{\rho}{}_{\nu\}}{}^{\sigma\}} + R_\mu{}^\rho{}_\nu{}^\sigma + R_\mu{}^\sigma{}_\nu{}^\rho, \quad (F_{ab})_{\mu\nu}^{\rho\sigma} = 2R_{ab\{\mu}{}^{\rho}{}_{\nu\}}{}^{\sigma\}}. \quad (\text{A.20})$$

Then we can compute the relevant terms:

V_1	V_2	V_3	V_4	V_5
$-8A_5$	$16(A_4 - A_3)$	$-8A_9$	$8A_{17}$	$-8A_{16}$
V_6	V_7	V_8	V_9	V_{10}
$-8A_{15}$	$-8A_{12}$	$-8A_1$	$A_6 + 12A_7 + 3A_9$	$A_2 + 12A_3 + 3A_5$
V_{11}	V_{12}	V_{13}	V_{14}	V_{15}
$*$	$A_{12} + 12A_{15} + 3A_{16}$	$-8A_6$	$-16(A_8 - A_{13} + A_{14})$	$-8A_2$
V_{16}	V_{17}	V_{18}	V_{19}	V_{20}
$A_{10} + 12A_{11} + 3A_{12}$	$-8A_6$	$-8A_{10}$	$-8A_{11}$	$-8A_{12}$

where $V_{11} = -3A_{11} - 16A_{13} - 6A_{14} - 18A_{15} - A_{16} + 8A_{17}$, and these terms are given after tracing over the representation. The b_6 coefficient is

$$b_6(\mathcal{O}) = \frac{1}{(4\pi)^{37!}} \left[-312A_1 - 584A_2 - 4552A_3 + 368A_4 + 544A_5 - 1064A_6 + 9920A_7 \right. \\ \left. - 416A_8 + 1416A_9 - \frac{560}{9}A_{10} + \frac{7952}{3}A_{11} + \frac{2968}{3}A_{12} - \frac{117056}{9}A_{13} \right. \\ \left. - \frac{16528}{3}A_{14} - \frac{29216}{3}A_{15} - \frac{1388}{9}A_{16} + \frac{49984}{9}A_{17} \right]. \quad (\text{A.21})$$

B Heat kernel for general spins

We are interested in a general formula to compute the heat kernel coefficients for spins higher than two, analogous to the algorithm [23] in four dimensions. We consider fields transforming in an irreducible representation of the spacetime symmetry group that are acted on by a generalized second-order operator $\Delta = -\square - E$. In four dimensions, the

method of computing the heat kernels for general representations assumes that the endomorphism term E for fields transforming as (A, B) of $\text{SO}(4) \simeq \text{SU}(2)_L \times \text{SU}(2)_R$ is given by:

$$E = \Sigma_{ab} R^{abcd} \Sigma_{cd} \quad \text{or} \quad E = \frac{1}{A} \Sigma_{ab} R_+^{abcd} \Sigma_{cd}, \quad (\text{B.1})$$

for bosonic ($A + B = \text{integer}$) or fermionic ($A + B = \text{half-integer}$, $A > B$) representations, respectively. Here $R_+^{abcd} = \frac{1}{2}(R^{abcd} + R^{*abcd})$. This prescription is shown to be valid for fields up to spin two in four dimensions, and is conjectured to be the appropriate operator for general spins. In six dimensions, it appears that this prescription is reasonable for bosonic representations, but straightforward generalizations for fermions fail to reproduce the conventional endomorphism terms for the Weyl fermion and gravitino. So it remains unclear what endomorphism term is appropriate for general fermions. Below we use this method for bosonic representations to compute all the V terms, which are built out of the endomorphism E and the connection F_{ij} .

B.1 Tracing over generators

Computing the heat kernel using this method requires computing the trace of a number of generators; the most we will need is six, as $E^3 \sim \Sigma^6$. We perform these traces using the algorithm presented in [35], which requires expanding the trace into a sum of symmetric traces, and then writing each symmetric trace in a basis of orthogonal tensors and higher order Dynkin indices. For example, the trace of two generators of an irreducible representation is

$$\text{Tr}[T_R^A T_R^B] = I_2(R) g^{AB}. \quad (\text{B.2})$$

Here R refers to the representation, and the capital Roman letters $A, B, \dots = 1, 2, \dots, 15$ label the generators of $\text{SU}(4)$. Each $\text{SU}(4)$ index is interchangeable with a pair of antisymmetrized six-dimensional spacetime indices $\{\mu, \nu\}$.

If the number of generators is greater than two, we will first need to break the trace into a sum of symmetrized traces. For a trace of n generators, this is accomplished by writing out each of the $n!$ terms in the symmetrized trace, and then using commutation relations to return each term to the original order, plus a number of traces of lower numbers of generators. For example, we may look at the trace of six generators. First consider the symmetrized trace

$$\begin{aligned} & \text{STr}[T_A T_B T_C T_D T_E T_F] \\ &= \frac{1}{6!} (\text{Tr}[T_A T_B T_C T_D T_E T_F] + \text{Tr}[T_B T_A T_C T_D T_E T_F] + 718 \text{ more terms}). \end{aligned} \quad (\text{B.3})$$

Using the fact that $T_B T_A = [T_B, T_A] + T_A T_B$ and the algebra, we may rewrite this trace as

$$\begin{aligned} \text{STr}[T_A T_B T_C T_D T_E T_F] &= \frac{1}{6!} (\text{Tr}[T_A T_B T_C T_D T_E T_F] \\ &+ \text{Tr}[T_A T_B T_C T_D T_E T_F] + \text{Tr}[f_{BAX} T^X T_C T_D T_E T_F] + 718 \text{ more terms}). \end{aligned} \quad (\text{B.4})$$

This gives two factors of the non-symmetrized trace plus a term which has a trace over only five generators. Each of the other 718 terms may be dealt with in the same way: commute

the generators to put them in the order $(ABCDEF)$ and keep track of all of the traces over five generators which are picked up along the way. This adds $5 \cdot 5!$ terms with five generators. Using this and rearranging the trace and symmetric trace, we get the schematic relation

$$\text{Tr}[T_A T_B T_C T_D T_E T_F] = \text{STr}[T_A T_B T_C T_D T_E T_F] - \frac{1}{6!} \cdot 600 \text{Tr}[TTTTT]. \quad (\text{B.5})$$

Each of these five-generator traces may be treated the same way— they each yield a symmetric trace with five generators plus $4 \cdot 4!$ terms with a trace over four generators. Schematically, the trace may be expanded as

$$\begin{aligned} \text{Tr}[T_A T_B T_C T_D T_E T_F] & \quad (\text{B.6}) \\ &= \text{STr}[T_A T_B T_C T_D T_E T_F] - \frac{1}{6!} \left(600 \left(\text{STr}[TTTTT] - \frac{1}{5!} \cdot 96 \text{Tr}[TTTT] \right) \right), \end{aligned}$$

and so on, until the result is a sum of symmetric traces of 2, 3, 4, 5, and 6 generators. Clearly this computation is not tractable by hand. Using the XACT package for Mathematica, we calculated all the necessary terms. The symmetric traces over an odd number of generators cancel each other out (which appears to be a sort of generalization of Furry's theorem). The result of this procedure includes a symmetric trace over six generators and a large number of symmetric traces over four generators and two generators.

B.2 Orthogonal tensors

The symmetrized traces may be expanded in a set of orthogonal symmetric tensors. The two needed for this calculation are

$$\text{STr}[T^A T^B T^C T^D] = I_4(R) d_{\perp}^{ABCD} + I_{2,2}(R) (\delta^{AB} \delta^{CD} + \delta^{AC} \delta^{BD} + \delta^{AD} \delta^{BC}) / 3, \quad (\text{B.7})$$

and

$$\begin{aligned} \text{STr}[T^A T^B T^C T^D T^E T^F] &= I_6(R) d_{\perp}^{ABCDEF} + I_{4,2}(R) (d_{\perp}^{ABCD} \delta^{EF} + d_{\perp}^{ABCE} \delta^{DF} + \dots) / 15 \\ &+ I_{3,3}(R) (d_{\perp}^{ABC} d_{\perp}^{DEF} + d_{\perp}^{ABD} d_{\perp}^{CEF} + \dots) / 10 + I_{2,2,2}(R) (\delta^{AB} \delta^{CD} \delta^{EF} + \dots) / 15. \end{aligned} \quad (\text{B.8})$$

Note that $I_6 = 0$ for all representations of $\text{SU}(4)$. The tensors $d_{\perp}^{ABCD}$ and $d_{\perp}^{ABC}$ are fixed by the condition of orthogonality; $d_{\perp}^{ABC}$ is the six-dimensional epsilon tensor (recalling that $A = \{\mu_1 \nu_1\}$, etc.) The fourth order $d_{\perp}^{ABCD}$ may be expressed in terms of the six-dimensional metric — its terms include $g^{\mu_1 \nu_4} g^{\mu_2 \nu_3} g^{\mu_3 \nu_2} g^{\mu_4 \nu_1}$ and the other 47 ways of arranging the indices. The indices $I_{4,2}$, $I_{3,3}$, and $I_{2,2,2}$ are not unique; imposing orthogonality and other group-theoretic relations yields the system of equations (158)–(160) in [35]. Solving these allows $I_{4,2}$, $I_{3,3}$, and $I_{2,2,2}$ to be expressed in terms of the Dynkin indices I_4 , I_3 , and I_2 .

B.3 Dynkin indices

A representation R with Dynkin labels (a, b, c) has dimension

$$\text{Dim}_R(a, b, c) = \frac{1}{12} (a+1)(b+1)(c+1)(a+b+2)(b+c+2)(a+b+c+3). \quad (\text{B.9})$$

The Weyl character formula may be used to show that

$$I_2(a, b, c) = \frac{\text{Dim}_R}{60} (3a^2 + 2a(2b + c + 6) + 4b^2 + 4b(c + 4) + 3c(c + 4)). \quad (\text{B.10})$$

The third and fourth order generalization to this index were computed in [36], which finds

$$\begin{aligned} I_3(a, b, c) &= \frac{\text{Dim}_R}{120} (a - c)(a + c + 2)(a + 2b + c + 4) \\ I_4(a, b, c) &= \frac{\text{Dim}_R}{3360} (3a^4 + 8a^3b + 4a^3c + 24a^3 + 2a^2b^2 + 2a^2bc + 30a^2b \\ &\quad - 4a^2c^2 + 6a^2c + 54a^2 - 12ab^3 - 18ab^2c - 50ab^2 + 2abc^2 - 28abc \\ &\quad - 34ab + 4ac^3 + 6ac^2 - 2ac + 24a - 6b^4 - 12b^3c - 48b^3 + 2b^2c^2 \\ &\quad - 50b^2c - 122b^2 + 8bc^3 + 30bc^2 - 34bc - 104b + 3c^4 + 24c^3 + 54c^2 + 24c). \end{aligned} \quad (\text{B.11})$$

B.4 Results

As the trace of each of the V_a coefficients may be reduced to a trace of generators variously contracted with the Riemann tensor, this method will allow each of them to be computed. The entire list of traced coefficients is presented here:

$$\begin{aligned} V_1 &= -\frac{A_5}{2} I_2, & V_2 &= (A_4 - A_3) I_2, & V_3 &= -\frac{A_9}{2} I_2, & V_4 &= \frac{A_{17}}{2} I_2 \\ V_5 &= -\frac{A_{16}}{2} I_2, & V_6 &= -\frac{A_{15}}{2} I_2, & V_7 &= -\frac{A_{12}}{2} I_2, & V_8 &= -\frac{A_1}{2} I_2, \\ V_9 &= \left(-\frac{A_6}{51} + \frac{A_7}{6} - \frac{25A_9}{204} \right) I_2 + \left(\frac{15A_6}{68} + \frac{15A_9}{34} \right) \frac{I_2^2}{\text{Dim}_R} + \left(\frac{11A_6}{51} - \frac{4A_7}{3} + \frac{5A_9}{51} \right) I_4, \\ V_{10} &= \left(-\frac{A_2}{51} + \frac{A_3}{6} - \frac{25A_5}{204} \right) I_2 + \left(\frac{15A_2}{68} + \frac{15A_5}{34} \right) \frac{I_2^2}{\text{Dim}_R} + \left(\frac{11A_2}{51} - \frac{4A_3}{3} + \frac{5A_5}{51} \right) I_4, \\ V_{11} &= \left(\frac{A_{10}}{612} - \frac{11A_{11}}{357} - \frac{3A_{12}}{238} - \frac{55A_{13}}{2142} + \frac{151A_{14}}{714} + \frac{3A_{15}}{34} - \frac{383A_{16}}{4284} - \frac{338A_{17}}{1071} \right) I_2 \\ &\quad + \left(\frac{5A_{10}}{136} - \frac{375A_{11}}{952} + \frac{1095A_{12}}{3808} + \frac{115A_{13}}{476} + \frac{345A_{14}}{952} - \frac{165A_{15}}{136} + \frac{325A_{16}}{476} + \frac{725A_{17}}{952} \right) \frac{I_2^2}{\text{Dim}_R} \\ &\quad + \left(\frac{10A_{10}}{153} - \frac{41A_{11}}{51} + \frac{6A_{12}}{17} + \frac{280A_{13}}{153} + \frac{38A_{14}}{51} - \frac{42A_{15}}{17} + \frac{43A_{16}}{153} - \frac{8A_{17}}{153} \right) I_4 \\ &\quad + \left(-\frac{5A_{10}}{68} - \frac{165A_{11}}{952} - \frac{1845A_{12}}{3808} + \frac{115A_{13}}{476} + \frac{345A_{14}}{952} - \frac{45A_{15}}{136} - \frac{305A_{16}}{476} - \frac{115A_{17}}{952} \right) \frac{I_2^3}{\text{Dim}_R^2} \\ &\quad + \left(-\frac{7A_{10}}{24} + \frac{209A_{11}}{56} - \frac{183A_{12}}{224} - \frac{437A_{13}}{84} - \frac{437A_{14}}{56} + \frac{57A_{15}}{8} - \frac{101A_{16}}{84} + \frac{437A_{17}}{168} \right) \frac{I_2^3}{\text{Dim}_R} \\ &\quad + \left(-\frac{13A_{10}}{102} + \frac{4A_{11}}{17} - \frac{12A_{12}}{17} + \frac{76A_{13}}{51} + \frac{38A_{14}}{17} + \frac{54A_{15}}{17} - \frac{2A_{16}}{51} - \frac{38A_{17}}{51} \right) \frac{I_2 I_4}{\text{Dim}_R}, \\ V_{12} &= \left(-\frac{A_{12}}{51} + \frac{A_{15}}{6} - \frac{25A_{16}}{204} \right) I_2 + \left(\frac{15A_{12}}{68} + \frac{15A_{16}}{34} \right) \frac{I_2^2}{\text{Dim}_R} + \left(\frac{11A_{12}}{51} - \frac{4A_{15}}{3} + \frac{5A_{16}}{51} \right) I_4, \\ V_{13} &= -\frac{A_6}{2} I_2, & V_{14} &= -(A_8 - A_{13} + A_{14}) I_2, & V_{15} &= -\frac{A_2}{2} I_2, \end{aligned}$$

$$\begin{aligned}
V_{16} &= \left(-\frac{A_{10}}{51} + \frac{A_{11}}{6} - \frac{25A_{12}}{204} \right) I_2 + \left(\frac{15A_{10}}{68} + \frac{15A_{12}}{34} \right) \frac{I_2^2}{\text{Dim}_R} + \left(\frac{11A_{10}}{51} - \frac{4A_{11}}{3} + \frac{5A_{12}}{51} \right) I_4, \\
V_{17} &= -\frac{A_6}{2} I_2, \quad V_{18} = -\frac{A_{10}}{2} I_2, \quad V_{19} = -\frac{A_{11}}{2} I_2, \quad V_{20} = -\frac{A_{12}}{2} I_2.
\end{aligned} \tag{B.12}$$

Since these expressions pertain to an endomorphism of the form $E = \Sigma_{ab} R^{abcd} \Sigma_{cd}$, where Σ_{ab} are $\text{SU}(4)$ generators in an arbitrary representation specified by Dynkin labels (a, b, c) , we refer to this as the “group theory method” for determining the heat kernel coefficients.

Now that the V_a ’s are known, we may compute the b_6 coefficient using the group theory method. We present the coefficient for a representation R on Ricci-flat backgrounds:

$$\begin{aligned}
b_6(R) \Big|_{R_{ab}=0} &= \frac{1}{(4\pi)^3 7!} \left[A_5 \left(\frac{3150I_2^2}{17\text{Dim}_R} + 9\text{Dim}_R - \frac{1827I_2}{17} + \frac{700I_4}{17} \right) \right. \\
&\quad + A_9 \left(\frac{6300I_2^2}{17\text{Dim}_R} + 12\text{Dim}_R - \frac{3178I_2}{17} + \frac{1400I_4}{17} \right) \\
&\quad + A_{16} \left(-\frac{9150I_2^3}{17\text{Dim}_R^2} + \frac{12900I_2^2}{17\text{Dim}_R} - \frac{560I_2I_4}{17\text{Dim}_R} - \frac{1010I_3^2}{\text{Dim}_R} + \frac{44\text{Dim}_R}{9} - \frac{8597I_2}{51} + \frac{14140I_4}{51} \right) \\
&\quad \left. + A_{17} \left(-\frac{1725I_2^3}{17\text{Dim}_R^2} + \frac{10875I_2^2}{17\text{Dim}_R} - \frac{10640I_2I_4}{17\text{Dim}_R} + \frac{2185I_3^2}{\text{Dim}_R} + \frac{80\text{Dim}_R}{9} - \frac{17804I_2}{51} - \frac{2240I_4}{51} \right) \right].
\end{aligned} \tag{B.13}$$

In general, the full b_6 coefficients obtained by the group theory method do not match the expressions (A.6) and (A.15), for the fermion and gravitino, respectively, as the group theory method does not correspond to the square of the Dirac operator when acting on fermions. This indicates that some modification may be necessary for fermionic representations, as was already noted in the four-dimensional case [23]. Curiously, however, this mismatch disappears when restricted to Ricci-flat backgrounds. This suggests that (B.13) may potentially be valid for fermions as well as bosons. If this were true, we could then derive a general expression for $\delta(c - a)$ for arbitrary higher spin supermultiplets.

Finally, we find that the expression $\delta\mathcal{A}$ in (2.5) vanishes on arbitrary (ie not just Ricci-flat) backgrounds for long multiplets using the group theory method for the heat kernel. This is in contrast to the conventional method where the fermions are treated by squaring the Dirac operator. There, $\delta\mathcal{A}$ for long multiplets only vanished on Ricci-flat backgrounds, but was otherwise non-vanishing on more general backgrounds. This complete vanishing of $\delta\mathcal{A}$ for long multiplets is consistent with expectations from $\text{AdS}_5/\text{CFT}_4$ [9, 12], and lends credibility to the idea that the group theory method may yield the correct expression for $\delta\mathcal{A}$ for general spins.

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