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# Bulk viscous corrections to screening and damping in QCD at high temperatures

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**ABSTRACT:** Non-equilibrium corrections to the distribution functions of quarks and gluons in a hot and dense QCD medium modify the “hard thermal loops” (HTL). The HTLs determine the retarded, advanced, and symmetric (time-ordered) propagators for gluons with soft momenta as well as the Debye screening and Landau damping mass scales. We compute such corrections to a thermal as well as to a non-thermal fixed point. The screening and damping mass scales are sensitive to the bulk pressure and hence to (pseudo-) critical dynamical scaling of the bulk viscosity in the vicinity of a second-order critical point. This could be reflected in the properties of quarkonium bound states in the deconfined phase and in the dynamics of soft gluon fields.

**KEYWORDS:** Perturbative QCD, Quark-Gluon Plasma

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**1 Introduction**

In order to understand the physics of the quark-gluon plasma (QGP) generated in ultrarelativistic heavy-ion collisions a first step is to understand the dynamics of the high temperature phase of QCD. At extremely high temperatures  $T \gg \Lambda_{\text{QCD}}$  the system can be described as a weakly-interacting gas of quark and gluon quasiparticles which can be understood systematically using hard-thermal loop (HTL) resummation [1–4]. Such a picture has been quite successful in describing the thermodynamics of the QGP down to temperatures on the order of  $T \sim 250$  MeV, particularly when considering the various quark susceptibilities [5–7]. The resulting picture is one in which the QGP is comprised of quasiparticle-like excitations which experience Landau-damping in a similar manner as electromagnetic plasma waves. In the high-temperature equilibrium limit there is only one scale, the Debye mass  $m_D \sim gT$ , which enters the quark and gluon self energies. For both quarks and gluons, in equilibrium the retarded two-point function has two real time-like ( $\omega > k$ ) poles corresponding to propagating soft modes (plasmon/plasmino and transverse/longitudinal for quarks and gluons, respectively) and a space-like ( $\omega < k$ ) cut which results in Landau-damping of soft quark and gluon modes. Such modifications of the two-point functions are reflected in analogous hard-thermal-loop modifications to all soft  $n$ -point functions which must be taken into account in order to maintain the explicit gauge-invariance of the soft resummation program [4].

Knowledge of the HTL-resummed gluon self-energy allows one to compute quantities such as the inter-quark potential in the heavy quark limit [8–10]. The resulting heavy-quark potential is complex-valued, with the real part of the potential taking the form

of a Debye-screened Coulomb potential which reflects color-screening in the quark-gluon plasma (QGP) and the imaginary part being related to the in-medium decay width of heavy quark bound states. As interesting as this is in and of itself, if one is interested in QGP phenomenology, then one must incorporate non-equilibrium corrections to the heavy-quark potential. This requires input information about the analogously resummed non-equilibrium quark and gluon self-energies. In the early stages of a heavy-ion collision when the temperature is highest and the expansion is highly anisotropic the most important non-equilibrium correction in the QGP stems from finite shear viscosity of the plasma,  $\eta$ .

When  $\eta$  is non-zero, the rapid longitudinal expansion of the QGP created in relativistic heavy ion collisions results in anisotropies in the diagonal components of the energy-momentum tensor in the local rest frame and, in a kinetic-theory framework, this translates into momentum-space anisotropies in the quark and gluon distribution functions [11, 12]. As a result, one must revisit the calculation of the heavy-quark potential, taking into account these momentum-space anisotropies [13–15]. Such calculations have led to detailed phenomenological calculations of the expected level of heavy quarkonium suppression generated in high-energy heavy-ion collisions [16–19]. These papers have demonstrated that it is necessary to include the shear correction to the heavy-quark potential when computing heavy-quark suppression.

In recent years, attention has broadened to include and fit other transport coefficients in the QGP with the most obvious candidate being the bulk viscosity,  $\zeta$ . It has been demonstrated that self-consistent inclusion of bulk viscous effects improves the agreement of hydrodynamical model predictions with experimental data, see e.g. [20]. The bulk viscosity  $\zeta(T)$  in QCD at very high temperatures  $T \gg \Lambda_{\text{QCD}}$  has been computed to leading order in the coupling in ref. [21]. They find that it is very small indeed since  $\zeta$  is proportional to the *square* of the deviation from conformality given by the  $\beta$ -function. This leads to  $\zeta/\eta \sim \alpha_s^4$  (neglecting logarithms of the inverse coupling).

On the other hand, it is known from the lattice that the trace anomaly of QCD, expressed as energy density minus three times the pressure, grows large at  $T \sim \Lambda_{\text{QCD}}$  [22].<sup>1</sup> Thus, it has been suggested in the literature that the bulk viscosity to entropy density ratio should increase, too, as the temperature approaches the confinement-deconfinement temperature [23, 24]. In this paper we analyze the high-temperature weakly-coupled phase of the QGP and try to assess the impact of bulk-viscous corrections on the heavy-quark potential. While our weak-coupling analysis may not apply for  $T \simeq \Lambda_{\text{QCD}}$ , nevertheless it is clearly of interest to obtain a baseline expectation for bulk-viscous effects on screening and damping from (resummed) weakly coupled QCD.

Bulk viscous corrections are expected to grow large also in the vicinity of a second order critical point; this could be realized in hot QCD either by tuning of the quark masses [25, 26] or perhaps by introducing a baryon charge asymmetry [27–29]. Due to critical slowing down the bulk viscosity should diverge [30]  $\zeta \sim \xi^z$  where  $\xi \rightarrow \infty$  is the correlation length and  $z$  is a dynamical critical exponent. However, since the relaxation time in the critical region of the bulk pressure diverges as well, in heavy-ion collisions its magnitude relative to the ideal pressure should not be much greater than  $\sim 1$  [31].

<sup>1</sup>For recent results for  $N_f = 2 + 1$  QCD see for example ref. [22].

Additionally, we mention that there is shear-bulk coupling in non-conformal relativistic viscous hydrodynamics derived from kinetic theory via the 14 moment approximation [32]. Due to this coupling, a large shear pressure may induce significant bulk-viscous corrections, and possibly even invert their sign [33–35].

To compute the gluon self energy in the hard loop approximation we require the phase space distributions of the particles in the medium. In the local rest frame, we take them to be

$$f(\mathbf{p}) = f_{\text{id}}(p) + \delta_{\text{bulk}}f(p) + \delta_{\text{shear}}f(\mathbf{p}). \quad (1.1)$$

Here,  $f_{\text{id}}(p)$  is an isotropic reference distribution when non-equilibrium corrections are absent. This would normally correspond to thermal Fermi-Dirac or Bose-Einstein distributions, respectively, if the “ideal” reference is the thermal fixed point; see section 2. In section 3 on the other hand we shall choose a non-thermal fixed point parameterized by a mass-like (scalar field) distortion of the ideal gas distributions, with  $m \sim T$ .

The corrections  $\delta f$  in eq. (1.1) correspond to non-equilibrium corrections. We denote the isotropic correction  $\delta_{\text{bulk}}f(p)$  as a bulk-viscous correction while the anisotropic part  $\delta_{\text{shear}}f(\mathbf{p})$  is analogous to shear. However, we do not assume that these corrections are parametrically suppressed. The corrections to the real and imaginary parts of the HTL resummed gluon propagator due to  $\delta_{\text{shear}}f(\mathbf{p})$  have been worked out in refs. [13, 14]. Here, we focus on bulk viscous corrections instead. Unlike the thermal distribution functions non-equilibrium corrections are not universal and so we work out explicit expressions for two different examples in sections 2 and 3, respectively.

Throughout the manuscript we use natural units  $\hbar = c = k_B = 1$  and a “mostly minus” (+ − − −) metric. Capital letters denote four-momenta while lower caps letters are three-momenta.

## 2 Bulk-viscous corrections about a thermal fixed point

In this section we compute the temporal component of the gluon self energies for massless thermal particles. The retarded gluon self energy in the real time formalism is given by [36]<sup>2</sup>

$$\Pi_R(P) = \frac{2\pi N_f g^2}{(2\pi)^4} \int k dk d\Omega_k (f_F^+(\mathbf{k}) + f_F^-(\mathbf{k})) \frac{1 - (\hat{\mathbf{k}} \cdot \hat{\mathbf{p}})^2}{(\hat{\mathbf{k}} \cdot \hat{\mathbf{p}} + \frac{p_0 + i\epsilon}{p})^2}. \quad (2.1)$$

This expression accounts for the contribution due to  $N_f$  (massless) quark loops. Three-momenta with a hat denote unit vectors. The distribution function may have any non-equilibrium form so long as the dominant contribution is from hard loop momenta of order  $T$  so that the HTL approximation is applicable. In the thermal equilibrium case, the distribution function for (anti-)quarks with chemical potential  $\mu$  is given by

$$n_F^\pm(k) = \frac{1}{\exp[(k \mp \mu)/T] + 1}. \quad (2.2)$$

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<sup>2</sup>Formally, this expression can be obtained from the corresponding result in refs. [37, 38] at zero chemical potential by the simple replacement  $f(\mathbf{k}) \rightarrow (f^+(\mathbf{k}) + f^-(\mathbf{k}))/2$ . However, this is in general not true for the symmetric self energy. See appendix.

In the absence of non-equilibrium corrections the ideal distribution  $f_{\text{id}}(k)$  function is given by  $n_F^\pm(k)$  and we have

$$\Pi_R^{\text{id}}(P) = N_f \frac{g^2 T^2}{6} \left( 1 + \frac{3\tilde{\mu}^2}{\pi^2} \right) \left( \frac{p_0}{2p} \ln \frac{p_0 + p + i\epsilon}{p_0 - p + i\epsilon} - 1 \right), \quad (2.3)$$

where the dimensionless quantity  $\tilde{\mu}$  is defined as  $\tilde{\mu} \equiv \frac{\mu}{T}$ . The contribution due to a gluon loop has the same structure as eq. (2.1) but with  $f_{\text{id}}(k)$  now a Bose distribution. In equilibrium,

$$\Pi_R^{\text{id}}(P) = 2N_c \frac{g^2 T^2}{6} \left( \frac{p_0}{2p} \ln \frac{p_0 + p + i\epsilon}{p_0 - p + i\epsilon} - 1 \right). \quad (2.4)$$

The symmetric (time ordered) self energy due to  $N_f$  quark loops is given by<sup>3</sup>

$$\Pi_F(P) = 4iN_f g^2 \pi^2 \int \frac{k^2 dk}{(2\pi)^3} \sum_{i=\pm} f_F^i(k) (f_F^i(k) - 1) \frac{2}{p} \Theta(p^2 - p_0^2). \quad (2.5)$$

In equilibrium,

$$\Pi_F^{\text{id}}(P) = -2\pi i N_f \frac{g^2 T^2}{6} \left( 1 + \frac{3\tilde{\mu}^2}{\pi^2} \right) \frac{T}{p} \Theta(p^2 - p_0^2). \quad (2.6)$$

For a thermal gluon loop one replaces  $f_F^i(k)(1 - f_F^i(k))$  in eq. (2.5) by  $f_B(k)(1 + f_B(k))$  which leads to

$$\Pi_F^{\text{id}}(P) = -2\pi i 2N_c \frac{g^2 T^2}{6} \frac{T}{p} \Theta(p^2 - p_0^2). \quad (2.7)$$

From the above results we see that at the thermal fixed point the modification of the mass scales due to the quark-chemical potential  $\mu$  is exactly the same for both retarded (advanced) and symmetric gluon self energies.

We now determine the non-equilibrium corrections to the expressions above. We assume that the bulk viscous correction to the local thermal distribution function takes the form

$$\delta_{\text{bulk}} f(k) = \left( \frac{k}{T} \right)^a \Phi f_{\text{id}}(k) (1 \pm f_{\text{id}}(k)). \quad (2.8)$$

Here, the “+” sign is for a Bose distribution while the “−” sign applies in case of a Fermi distribution.  $\Phi$  is proportional to the bulk pressure (divided by the ideal pressure) and  $a$  is a constant. We require that  $a > 0$  to ensure that the dominant contribution to the retarded self energy is from hard (gluon) loop momenta,  $k \sim T$ . To see this note that in the massless limit the Bose distribution for  $k \ll T$  behaves as  $f_B(k) \sim T/k$  and so  $f_B(k)(1 + f_B(k)) \sim (T/k)^2$ . The “hard gluon loop” from eqs. (2.1), (2.9), with  $f(k)$  replaced by  $\delta_{\text{bulk}} f(k)$ , is insensitive to soft momenta  $k \ll T$  if  $a > 0$ . The bulk viscous correction to the symmetric self energy at  $\mathcal{O}(\Phi^2)$  involves the fourth power of the distribution function and so we have to impose a more stringent bound,  $a > 1/2$ , in order to employ the HTL approximation, see below. We note that these bounds on  $a$  correspond to the regime of applicability of HTL power counting but may in principle be violated in certain non-equilibrium scenarios.

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<sup>3</sup>See appendix for details.

We shall also assume that  $|\Phi| \gg g^2$  so that two-loop corrections to the gluon self energy are negligible. In fact, since (2.8) is an ad-hoc schematic model for the non-equilibrium correction we may assume that it applies even at  $|\Phi| \sim 1$ .

Since  $f_{\text{id}}(p) + \delta_{\text{bulk}}f(p)$  is isotropic eq. (2.1) simplifies to

$$\Pi_R(P) = \frac{2\pi N_f g^2}{(2\pi)^4} \int k dk (f_F^+(k) + f_F^-(k)) \int d\Omega_k \frac{1 - (\hat{\mathbf{k}} \cdot \hat{\mathbf{p}})^2}{\left(\hat{\mathbf{k}} \cdot \hat{\mathbf{p}} + \frac{p_0 + i\epsilon}{p}\right)^2}. \quad (2.9)$$

Hence, the dependence on the frequency  $p_0$  and on the momentum  $p$  is the same as in equilibrium, cf. eqs. (2.3), (2.4). Specifically, for our distribution function (2.8) this expression gives

$$\delta_{\text{bulk}}\Pi_R(P) = c_R^{(q)}(a, \tilde{\mu}) \Phi N_f \frac{g^2 T^2}{6} \left(1 + \frac{3\tilde{\mu}^2}{\pi^2}\right) \left(\frac{p_0}{2p} \ln \frac{p_0 + p + i\epsilon}{p_0 - p + i\epsilon} - 1\right). \quad (2.10)$$

A similar correction is obtained for the contribution due to a gluon loop,

$$\delta_{\text{bulk}}\Pi_R(P) = c_R^{(g)}(a) \Phi 2N_c \frac{g^2 T^2}{6} \left(\frac{p_0}{2p} \ln \frac{p_0 + p + i\epsilon}{p_0 - p + i\epsilon} - 1\right). \quad (2.11)$$

The dimensionless numbers  $c_R^{(q)}(a, \tilde{\mu})$  and  $c_R^{(g)}(a)$  are given by

$$c_R^{(q,g)}(a, \tilde{\mu}) = \frac{1}{\Phi} \frac{\int k dk \delta_{\text{bulk}}f(k)}{\int k dk f_{\text{id}}(k)} = \begin{cases} \frac{-6\Gamma(2+a)[\text{Li}_{(1+a)}(-e^{-\tilde{\mu}}) + \text{Li}_{(1+a)}(-e^{\tilde{\mu}})]}{\pi^2 + 3\tilde{\mu}^2} & (\text{fermion}), \\ \frac{6}{\pi^2} \Gamma(2+a) \zeta(1+a) & (\text{boson}), \end{cases} \quad (2.12)$$

where  $\text{Li}_n(z)$  denotes the polylogarithm function. In the limit of vanishing baryon charge,  $\mu \rightarrow 0$ , the above result for  $c_R^{(q)}(a, \tilde{\mu})$  reduces to

$$c_R^{(q)}(a, \tilde{\mu} = 0) = \frac{12}{\pi^2} (1 - 2^{-a}) \Gamma(2+a) \zeta(1+a). \quad (2.13)$$

In addition, in the special case where  $a = 1$ ,  $c_R^{(q)}(a, \tilde{\mu})$  becomes a  $\tilde{\mu}$ -independent constant

$$c_R^{(q)}(a = 1, \tilde{\mu}) = 2. \quad (2.14)$$

Numerical values at vanishing chemical potential are listed in table 1 for various values of the power  $a$  of momentum introduced in eq. (2.8). Note that both  $c_R^{(q)}(a, \tilde{\mu} = 0)$  and  $c_R^{(g)}(a)$  increase with  $a$  and that  $c_R^{(q)}(a, \tilde{\mu} = 0) \simeq 2c_R^{(g)}(a)$  for large values of  $a$ . That is, the correction to the quark loop contribution to screening is twice as large as the correction to the gluon loop if  $a \gg 1$ .

Hence, in all we find that bulk viscous corrections “shift” the Debye mass appearing in the retarded self energy by

$$\left(2N_c + N_f \left(1 + \frac{3\tilde{\mu}^2}{\pi^2}\right)\right) \frac{g^2 T^2}{6} \rightarrow \left(2N_c \left(1 + c_R^{(g)}(a) \Phi\right) + N_f \left(1 + \frac{3\tilde{\mu}^2}{\pi^2}\right) \left(1 + c_R^{(q)}(a, \tilde{\mu}) \Phi\right)\right) \frac{g^2 T^2}{6}. \quad (2.15)$$

$a$	1	2	3
$c_R^{(q)}(a)$	2	$\frac{54\zeta(3)}{\pi^2}$	$\frac{14\pi^2}{5}$
$c_F^{(q)}(a)$	3	$\frac{108\zeta(3)}{\pi^2}$	$7\pi^2$
$e^{(q)}(a)$	4	$\frac{13500\zeta(5)}{7\pi^4}$	$\frac{620\pi^2}{49}$
$c_R^{(g)}(a)$	2	$\frac{36\zeta(3)}{\pi^2}$	$\frac{8\pi^2}{5}$
$c_F^{(g)}(a)$	3	$\frac{72\zeta(3)}{\pi^2}$	$4\pi^2$
$e^{(g)}(a)$	4	$\frac{1800\zeta(5)}{\pi^4}$	$\frac{80\pi^2}{7}$

**Table 1.** The numerical coefficients  $c_R^{(q,g)}(a)$ ,  $c_F^{(q,g)}(a)$  and  $e^{(q,g)}(a)$  at  $\tilde{\mu} = 0$  for various values of the power  $a$  of momentum introduced in the bulk viscous corrections  $\delta_{\text{bulk}}f$  in eq. (2.8).

The isotropy of  $f_{\text{id}}(p) + \delta_{\text{bulk}}f(p)$  also implies that the dependence of the symmetric self energy on energy and momentum is the same as in equilibrium, eqs. (2.6), (2.7). The correction to the distribution function written in eq. (2.8) amounts to a shift of the mass scale. For the symmetric self energy to linear order in  $\Phi$  it is<sup>4</sup>

$$\left(2N_c + N_f \left(1 + \frac{3\tilde{\mu}^2}{\pi^2}\right)\right) \frac{g^2 T^2}{6} \rightarrow \tag{2.16}$$

$$m_{F,D}^2 + \delta m_{F,D}^2 = \left(2N_c \left(1 + c_F^{(g)}(a)\Phi\right) + N_f \left(1 + \frac{3\tilde{\mu}^2}{\pi^2}\right) \left(1 + c_F^{(q)}(a, \tilde{\mu})\Phi\right)\right) \frac{g^2 T^2}{6}.$$

Here,

$$c_F^{(q,g)}(a, \tilde{\mu}) = \frac{1}{\Phi} \frac{\int dk k^2 \delta_{\text{bulk}}f(k) [1 \pm 2f_{\text{id}}(k)]}{\int dk k^2 f_{\text{id}}(k) [1 \pm f_{\text{id}}(k)]}$$

$$= \begin{cases} \frac{-3\Gamma(3+a) [\text{Li}_{(1+a)}(-e^{-\tilde{\mu}}) + \text{Li}_{(1+a)}(-e^{\tilde{\mu}})]}{\pi^2 + 3\tilde{\mu}^2} & \text{(fermion)}, \\ \frac{3}{\pi^2} \Gamma(3+a) \zeta(1+a) & \text{(boson)}. \end{cases} \tag{2.17}$$

We also list the results for  $c_F^{(q)}(a, \tilde{\mu})$  for the two cases where  $\tilde{\mu} = 0$  or  $a = 1$

$$c_F^{(q)}(a, \tilde{\mu} = 0) = \frac{6}{\pi^2} (1 - 2^{-a}) \Gamma(3+a) \zeta(1+a),$$

$$c_F^{(q)}(a = 1, \tilde{\mu}) = 3. \tag{2.18}$$

If  $|\Phi| \sim 1$ , there are corrections at  $\mathcal{O}(\Phi^2)$  to  $\Pi_F^{\text{id}}(P)$  which are not negligible. The corresponding contributions to the Debye mass (divided by its ideal value  $m_{F,D}^2$ ) are given by

$$\begin{cases} \Phi^2 \frac{\Gamma(3+2a)}{2(\pi^2 + 3\tilde{\mu}^2)} [\text{Li}_{(2+2a)}(-e^{-\tilde{\mu}}) + \text{Li}_{(2+2a)}(-e^{\tilde{\mu}}) - \text{Li}_{2a}(-e^{-\tilde{\mu}}) - \text{Li}_{2a}(-e^{\tilde{\mu}})] & \text{(fermion)}, \\ \Phi^2 \frac{\Gamma(3+2a)}{2\pi^2} [\zeta(2a) - \zeta(2+2a)] & \text{(boson)}. \end{cases} \tag{2.19}$$

<sup>4</sup>Note that the mass scale obtained from hard thermal loops in equilibrium,  $\Phi = 0$ , is the same for the retarded (advanced) and symmetric self energies:  $m_{R,D}^2 = m_{F,D}^2$ .

Recall that at this order in  $\Phi$  the validity of the HTL approximation requires  $a > 1/2$  and so the  $\zeta$ -function is well defined.

### 3 Expansion about non-thermal fixed point

In this section we compute the temporal component of the gluon self energies using a non-equilibrium correction inspired by “anisotropic hydrodynamics” [39]. There the isotropic non-equilibrium distribution takes the form

$$f(p) = f_{\text{id}} \left( \frac{1}{T} \sqrt{p^2 + m^2} (1 + \tilde{\Phi}) \right) \quad (3.1)$$

$$\approx f_{\text{id}}(\tilde{p}) + \frac{m^2 \Phi}{2T \sqrt{p^2 + m^2}} f_{\text{id}}(\tilde{p}) (1 \pm f_{\text{id}}(\tilde{p})), \quad (3.2)$$

where  $\tilde{p} \equiv \frac{1}{T} \sqrt{p^2 + m^2}$ . Note that here the scale  $m$  is a scalar field expectation value introduced to skew the ideal distribution from the thermal fixed point (which would correspond to  $m = 0$ ). At weak coupling  $m$  does not correspond to the mass of the quasi-particles which must be obtained from their self energies, see below.

As already noted in the previous section we must have  $|\Phi| \gg g^2$  in order to be able to compute the gluon self energy at one loop order. The expansion of  $f(p)$  in powers of  $\Phi$  furthermore requires that  $|\Phi| \ll 1$ . This is not a fundamental requirement though, we expand in powers of  $\Phi$  only in order to be able to provide relatively simple analytic expressions.

Hence, in what follows we consider the bulk viscous correction

$$\delta_{\text{bulk}} f(p) = \frac{m^2 \Phi}{2T \sqrt{p^2 + m^2}} f_{\text{id}}(\tilde{p}) (1 \pm f_{\text{id}}(\tilde{p})). \quad (3.3)$$

It is straightforward to see that for this correction the gluon loop contribution to the self energies in the limit  $m^2 \ll T^2$  is not a hard thermal loop since it is not dominated by momenta  $k \sim T$ . We therefore restrict to  $m^2 \sim T^2$  (and greater) where the HTL approximation is applicable.

For  $\Phi = 0$  the retarded self energy becomes<sup>5</sup>

$$\Pi_R^{\text{id}}(P) = N_f \frac{g^2 T^2}{6} \left( 1 + \frac{3\tilde{\mu}^2}{\pi^2} \right) f_q(\tilde{m}, \tilde{\mu}) \left( \frac{p_0}{2p} \ln \frac{p_0 + p + i\epsilon}{p_0 - p + i\epsilon} - 1 \right), \quad (3.4)$$

with

$$f_q(\tilde{m}, \tilde{\mu}) \equiv 2 \left( 1 + \frac{-3\tilde{\mu}\tilde{m} + 3\tilde{m} \ln[(1 + e^{\tilde{\mu} + \tilde{m}})(1 + e^{\tilde{\mu} - \tilde{m}})] + 3[\text{Li}_2(-e^{\tilde{m} + \tilde{\mu}}) + \text{Li}_2(-e^{\tilde{m} - \tilde{\mu}})]}{\pi^2 + 3\tilde{\mu}^2} \right), \quad (3.5)$$

for the fermion loop and

$$\Pi_R^{\text{id}}(P) = 2N_c \frac{g^2 T^2}{6} f_g(\tilde{m}) \left( \frac{p_0}{2p} \ln \frac{p_0 + p + i\epsilon}{p_0 - p + i\epsilon} - 1 \right), \quad (3.6)$$

<sup>5</sup>In fact, the self energies corresponding to the distribution function eq. (3.1) can be obtained from eqs. (3.4), (3.6) for the retarded solution and from eqs. (3.10), (3.11) for the symmetric solution simply by replacing  $\tilde{m} \rightarrow \tilde{m} \sqrt{1 + \tilde{\Phi}} = \tilde{m} / \sqrt{1 - \tilde{\Phi}}$ .



with

$$f_g(\tilde{m}) \equiv \frac{3\tilde{m}^2 + 2\pi^2 - 6\tilde{m} \ln(-1 + e^{\tilde{m}}) - 6\text{Re}[\text{Li}_2(e^{\tilde{m}})]}{\pi^2}, \quad (3.7)$$

for the contribution from the boson loop. In the above equations,  $\tilde{m} \equiv \frac{m}{T}$ . Evidently, for  $m \sim T$  the quasi-particle masses are still of order  $gT$ .

The corrections to the self energies of order  $\Phi$  are given by

$$\begin{aligned} \delta_{\text{bulk}}\Pi_R(P) &= N_f \frac{g^2 T^2}{6} \Phi \frac{\tilde{m}^2}{\pi^2} \left( \frac{3}{e^{\tilde{m}+\tilde{\mu}} + 1} + \frac{3}{e^{\tilde{m}-\tilde{\mu}} + 1} \right) \left( \frac{p_0}{2p} \ln \frac{p_0 + p + i\epsilon}{p_0 - p + i\epsilon} - 1 \right), \\ \delta_{\text{bulk}}\Pi_R(P) &= 2N_c \frac{g^2 T^2}{6} \Phi \frac{3\tilde{m}^2/\pi^2}{e^{\tilde{m}} - 1} \left( \frac{p_0}{2p} \ln \frac{p_0 + p + i\epsilon}{p_0 - p + i\epsilon} - 1 \right). \end{aligned} \quad (3.8)$$

This corresponds to a shift of the screening mass to

$$\begin{aligned} m_{R,D}^2 + \delta m_{R,D}^2 &= \left[ N_f \left( \left( 1 + \frac{3\tilde{\mu}^2}{\pi^2} \right) f_q(\tilde{m}, \tilde{\mu}) + \Phi \frac{\tilde{m}^2}{\pi^2} \left( \frac{3}{e^{\tilde{m}+\tilde{\mu}} + 1} + \frac{3}{e^{\tilde{m}-\tilde{\mu}} + 1} \right) \right) \right. \\ &\quad \left. + 2N_c \left( f_g(\tilde{m}) + \Phi \frac{3\tilde{m}^2/\pi^2}{e^{\tilde{m}} - 1} \right) \right] \frac{g^2 T^2}{6}. \end{aligned} \quad (3.9)$$

The symmetric self energy is obtained from the general expression (2.5).<sup>6</sup> For  $\Phi = 0$ ,

$$\Pi_F^{\text{id}}(P) = -2\pi i N_f \frac{g^2 T^2}{6} \frac{T}{p} \frac{6\tilde{m}^2}{\pi^2} \Theta(p^2 - p_0^2) \sum_{n=1}^{\infty} (-1)^{n+1} K_2(\tilde{m}n) \cosh(\tilde{\mu}n), \quad (3.10)$$

for  $N_f$  fermion loops and

$$\Pi_F^{\text{id}}(P) = -2\pi i 2N_c \frac{g^2 T^2}{6} \frac{T}{p} \frac{3\tilde{m}^2}{\pi^2} \Theta(p^2 - p_0^2) \sum_{n=1}^{\infty} K_2(\tilde{m}n), \quad (3.11)$$

for the boson loops. As before  $\tilde{m} \equiv \frac{m}{T}$  is assumed to be of order 1 or greater.  $K_n(z)$  denotes the modified Bessel function of the second kind.

The corresponding bulk-viscous corrections are given by

$$\begin{aligned} \delta_{\text{bulk}}\Pi_F(P) &= -2\pi i N_f \frac{g^2 T^2}{6} \frac{T}{p} \frac{3\Phi\tilde{m}^3}{\pi^2} \Theta(p^2 - p_0^2) \sum_{n=1}^{\infty} n (-1)^{n+1} K_1(\tilde{m}n) \cosh(\tilde{\mu}n), \\ \delta_{\text{bulk}}\Pi_F(P) &= -2\pi i 2N_c \frac{g^2 T^2}{6} \frac{T}{p} \frac{3\Phi\tilde{m}^3}{2\pi^2} \Theta(p^2 - p_0^2) \sum_{n=1}^{\infty} n K_1(\tilde{m}n). \end{aligned} \quad (3.12)$$

Hence, the corrections have a different dependence on  $\tilde{m} \equiv \frac{m}{T}$  than the ‘‘ideal’’ contributions.

For the symmetric self energy, the bulk viscous correction also corresponds to a shift of the mass scale to

$$\begin{aligned} m_{F,D}^2 + \delta m_{F,D}^2 &= \left[ N_f \frac{6\tilde{m}^2}{\pi^2} \sum_{n=1}^{\infty} (-1)^{n+1} \cosh(\tilde{\mu}n) \left( K_2(\tilde{m}n) + \Phi \frac{\tilde{m}}{2} n K_1(\tilde{m}n) \right) \right. \\ &\quad \left. + 2N_c \frac{3\tilde{m}^2}{\pi^2} \sum_{n=1}^{\infty} \left( K_2(\tilde{m}n) + \Phi \frac{\tilde{m}}{2} n K_1(\tilde{m}n) \right) \right] \times \frac{g^2 T^2}{6}. \end{aligned} \quad (3.13)$$

<sup>6</sup>The explicit expressions for the Fermion contribution to  $\Pi_F$  given here apply when  $m > \mu$ . For  $m < \mu$  the symmetric self energy could be obtained by a numerical evaluation of eq. (2.5) with the appropriate distribution function.

Notice that in general the Debye mass obtained from the ideal distribution is different for the retarded (advanced) and symmetric solutions. Only at the thermal fixed point, i.e. in thermal equilibrium with  $m = 0$  as studied in the previous section, do we have  $m_{R,D}^2 = m_{F,D}^2 = (2N_c + N_f(1 + \frac{3\tilde{\mu}^2}{\pi^2}))\frac{g^2 T^2}{6}$ .

As before, we also give the  $\mathcal{O}(\Phi^2)$  contributions to the mass  $m_{F,D}^2$ :

$$\begin{cases} \Phi^2 N_f \frac{g^2 T^2}{6} \frac{\tilde{m}^4}{4\pi^2} \sum_{n=1}^{\infty} (-1)^n n(n+1)(n+2) \mathcal{G}(\tilde{m}, n) \cosh[\tilde{\mu}(n+1)] & \text{(fermion)}, \\ \Phi^2 2N_c \frac{g^2 T^2}{6} \frac{\tilde{m}^4}{8\pi^2} \sum_{n=1}^{\infty} n(n+1)(n+2) \mathcal{G}(\tilde{m}, n) & \text{(boson)}. \end{cases} \quad (3.14)$$

In the above expressions, the function  $\mathcal{G}(\tilde{m}, n)$  is defined as

$$\mathcal{G}(\tilde{m}, n) = \int_{\tilde{m}}^{\infty} \frac{\sqrt{t^2 - \tilde{m}^2}}{t} e^{-(n+1)t} dt. \quad (3.15)$$

#### 4 Bulk viscous correction to the gluon propagator

In this section we compute the HTL resummed propagator for longitudinal gluons. We employ Coulomb gauge with gauge parameter set to zero.

The system under consideration is still isotropic after including the bulk viscous correction to the distribution function. For such a system in Coulomb gauge, the temporal component of the resummed propagator is independent of the spatial components of the self energy and bare propagator and it can be determined through the following Schwinger-Dyson equation<sup>7</sup>

$$\tilde{D}_R^*(P) = D_R(P) + D_R(P) \tilde{\Pi}_R(P) \tilde{D}_R^*(P), \quad (4.1)$$

with

$$D_R(P) = D_A(P) = \frac{1}{p^2}, \quad (4.2)$$

the (temporal component of) the non-resummed real-time propagators for gluons. Here, a superscript star on propagators indicates a resummed propagator. We define  $\tilde{\Pi}_{R/A/F} \equiv \Pi_{R/A/F}^{\text{id}} + \delta_{\text{bulk}} \Pi_{R/A/F}$  and the same definition holds for  $\tilde{D}_{R/A/F}^*$ . Eq. (4.1) is solved by

$$\tilde{D}_R^*(P) = \frac{1}{p^2 - \tilde{\Pi}_R(P)} = \frac{1}{p^2 - \left( m_{R,D}^2 + \delta m_{R,D}^2 \right) \left( \frac{p_0}{2p} \ln \frac{p_0 + p + i\epsilon}{p_0 - p + i\epsilon} - 1 \right)}. \quad (4.3)$$

This propagator applies for momenta of order  $\sqrt{m_{R,D}^2 + \delta m_{R,D}^2}$  (or greater). If  $\delta_{\text{bulk}} \Pi_R \ll \Pi_R^{\text{id}}$  or  $|\Phi| \ll 1$  then bulk viscous corrections are small. On the other hand, for  $|\Phi| \sim 1$  the propagator resums insertions of  $\delta_{\text{bulk}} \Pi_R$  into each hard thermal loop. For  $|\Phi| = 0$ , one has the well known result

$$\tilde{D}_R^*(P) = \frac{1}{p^2 - m_{R,D}^2 \left( \frac{p_0}{2p} \ln \frac{p_0 + p + i\epsilon}{p_0 - p + i\epsilon} - 1 \right)}, \quad (4.4)$$

---

<sup>7</sup>Since only temporal components appear for all the propagators and self energies, we omit the superscript “00” in the following.

where  $m_{R,D}^2 = (2N_c + N_f)g^2T^2/6$  for massless particles in thermal equilibrium. For the model from section 3 which expands about a non-thermal distribution function the screening mass depends on the scale  $m$  and is given in eq. (3.9). The advanced propagator is obtained by the replacement  $i\epsilon \rightarrow -i\epsilon$ .

The symmetric (time ordered) resummed propagator is obtained from

$$\begin{aligned} \tilde{D}_F^*(P) &= (1 + 2\tilde{f}(p_0)) \operatorname{sgn}(p_0) [\tilde{D}_R^*(P) - \tilde{D}_A^*(P)] \\ &+ \tilde{D}_R^*(P) \{ \tilde{\Pi}_F(P) - [1 + 2\tilde{f}(p_0)] \operatorname{sgn}(p_0) [\tilde{\Pi}_R(P) - \tilde{\Pi}_A(P)] \} \tilde{D}_A^*(P), \end{aligned} \quad (4.5)$$

where  $\tilde{f}(p_0) \equiv f_{\text{id}}(p_0) + \delta_{\text{bulk}} f(p_0)$ .

In thermal equilibrium the KMS relation implies that  $\Pi_F^{\text{id}}(P) = [1 + 2f_{\text{id}}(p_0)] \operatorname{sgn}(p_0) [\Pi_R^{\text{id}}(P) - \Pi_A^{\text{id}}(P)]$  and so the second line on the r.h.s. of the previous equation vanishes.

Our model with  $m \sim T$  involves an ideal distribution corresponding to a non-thermal fixed point. In an unbroken theory such as QED or QCD the gauge bosons are massless and no mass appears in the bare propagators  $D(P)$ , cf. eq. (4.2). Indeed, the gluon self-energies derived in previous sections have been computed using massless propagators. Rather, the scalar mass-like scale  $m$  is merely a parameter which distorts the distribution function  $f_{\text{id}}(p_0)$  from the thermal fixed point and so the KMS relation for  $\Pi_F^{\text{id}}(P)$  does not apply. Hence, the second line in eq. (4.5) does not vanish in the ‘‘ideal limit’’  $\Phi \rightarrow 0$ . This can be checked easily by taking the limit  $p_0 \rightarrow 0$  of the expressions for  $\Pi_{R/A/F}^{\text{id}}$  given in section 3.

Using the identity

$$\tilde{D}_R^*(P) - \tilde{D}_A^*(P) = \tilde{D}_R^*(P) [\tilde{\Pi}_R(P) - \tilde{\Pi}_A(P)] \tilde{D}_A^*(P), \quad (4.6)$$

which follows from eq. (4.3) and an analogous expression for the resummed advanced propagator, eq. (4.5) can be simplified to

$$\tilde{D}_F^*(P) = \tilde{D}_R^*(P) \tilde{\Pi}_F(P) \tilde{D}_A^*(P). \quad (4.7)$$

eqs. (4.3) and (4.7) are the main results of this section. They are applicable for both models introduced above.

## 5 Static potential

In this section we apply the results obtained above to the QCD static potential at finite temperature. We define the static potential due to one gluon exchange through the Fourier transform of the physical ‘‘11’’ Schwinger-Keldysh component of the (longitudinal) gluon propagator in the static limit [14]:

$$\begin{aligned} V(\mathbf{r}) &= (ig)^2 C_F \int \frac{d^3\mathbf{p}}{(2\pi)^3} (e^{i\mathbf{p}\cdot\mathbf{r}} - 1) \left( \tilde{D}^*(p_0 = 0, \mathbf{p}) \right)_{11} \\ &= -g^2 C_F \int \frac{d^3\mathbf{p}}{(2\pi)^3} (e^{i\mathbf{p}\cdot\mathbf{r}} - 1) \frac{1}{2} \left( \tilde{D}_R^* + \tilde{D}_A^* + \tilde{D}_F^* \right) \\ &= -g^2 C_F \int \frac{d^3\mathbf{p}}{(2\pi)^3} (e^{i\mathbf{p}\cdot\mathbf{r}} - 1) \frac{1}{2} \left( \tilde{D}_R^* + \tilde{D}_A^* \right) \\ &\quad - g^2 C_F \int \frac{d^3\mathbf{p}}{(2\pi)^3} (e^{i\mathbf{p}\cdot\mathbf{r}} - 1) \frac{1}{2} \tilde{D}_F^*. \end{aligned} \quad (5.1)$$

We have taken the sources in the fundamental representation and subtracted an  $r$ -independent (self-energy) contribution. In the static limit,  $\frac{1}{2}(\tilde{D}_R^* + \tilde{D}_A^*) = \tilde{D}_R^* = \tilde{D}_A^*$ . The Fourier transform of this quantity gives the real part of the (screened) potential while its imaginary part, describing Landau damping [8–10], comes from the Fourier transform of the symmetric propagator.

In the limit  $p_0 \rightarrow 0$  the retarded or advanced self energies equal (minus) the square of the screening mass and so the Fourier transform of eq. (4.3) gives

$$\text{Re } V(r) = -\frac{g^2 C_F}{4\pi r} e^{-r \sqrt{m_{R,D}^2 + \delta m_{R,D}^2}} + 2F_Q(m_{R,D}). \quad (5.2)$$

Expressions for  $m_{R,D}^2 + \delta m_{R,D}^2$  have been given in eqs. (2.15), (3.9) above. This potential applies to distance scales of order  $1/m_{R,D}$  or less, where  $m_{R,D} = \sqrt{m_{R,D}^2 + \delta m_{R,D}^2}$ . Also, in eq. (5.2) we have restored the  $r$ -independent but  $T$ -dependent free energy contribution

$$2F_Q = g^2 C_F \int \frac{d^3 \mathbf{p}}{(2\pi)^3} (\tilde{D}_R^* - D_R) = -\frac{g^2 C_F}{4\pi} \sqrt{m_{R,D}^2 + \delta m_{R,D}^2}. \quad (5.3)$$

The imaginary part of the potential is given by<sup>8</sup>

$$\text{Im } V(r) = -\frac{g^2 C_F T}{4\pi} \frac{m_{F,D}^2 + \delta m_{F,D}^2}{m_{R,D}^2 + \delta m_{R,D}^2} \phi(\hat{r}), \quad (5.4)$$

where

$$\phi(\hat{r}) = 2 \int_0^\infty dz \frac{z}{(z^2 + 1)^2} \left[ 1 - \frac{\sin(z \hat{r})}{z \hat{r}} \right], \quad (5.5)$$

with  $\hat{r} \equiv r \sqrt{m_{R,D}^2 + \delta m_{R,D}^2}$ . The expressions for  $m_{F,D}^2 + \delta m_{F,D}^2$  have been given in eqs. (2.16), (3.13) above. For small  $\hat{r}$  the function  $\phi(\hat{r})$  is proportional to  $\hat{r}^2 \ln \hat{r}$ .

From the above results, we can conclude that since the bulk viscous corrections are isotropic the real part of the potential has the same structure as in the ideal case with  $m_{R,D}^2$  replaced by  $m_{R,D}^2 + \delta m_{R,D}^2$ . The imaginary part of the potential is multiplied by a factor  $\frac{m_{F,D}^2 + \delta m_{F,D}^2}{m_{R,D}^2 + \delta m_{R,D}^2}$  which equals 1 in thermal equilibrium.

## 6 Application to heavy-ion collisions: Landau matching

There are two sources of corrections when comparing an ideal to a viscous thermal medium. There are, of course, corrections to the hydrodynamic evolution equations as well as to the initial conditions.<sup>9</sup> Second, there are *explicit* corrections to observables such as the heavy quark potential considered above.

<sup>8</sup>The expression for  $\text{Im } V(r)$  at the thermal fixed point without non-equilibrium corrections was first derived in refs. [8–10] and generalized to anisotropic shear-viscous corrections in ref. [14]. For recent lattice measurements of the real and imaginary parts of the static potential in equilibrium, see ref. [40].

<sup>9</sup>For applications to heavy-ion collisions the initial condition for viscous hydrodynamics, e.g. the initial temperature etc., is adapted such as to reproduce the *measured* final state of the collision. For example, the observed charged hadron multiplicity constrains the entropy in the final state and so on.

In order not to mix these corrections one matches the 00-components of the energy momentum tensors of the ideal and viscous fluids, respectively, in the local rest frames. That is, in the local rest frame the non-equilibrium corrections should not contribute to the energy density. For “anisotropic hydrodynamics” the matching is done in ref. [39], here we focus on the model introduced in section 2 where the ideal distribution corresponds to the thermal fixed point. We also set  $\mu = 0$  for simplicity.

To match the energy densities we shift the temperature of the viscous medium to  $T'$  which is determined from<sup>10</sup>

$$\int \frac{d^3k}{(2\pi)^3} E_k f_{\text{id}}(k; T) = \int \frac{d^3k}{(2\pi)^3} E_k \tilde{f}(k; T'). \quad (6.1)$$

With the bulk viscous correction from eq. (2.8) this leads to

$$T^4 = T'^4 \left[ 1 + \Phi \frac{2(N_c^2 - 1)e^{(g)}(a) + 4N_f \frac{7}{8} e^{(q)}(a)}{2(N_c^2 - 1) + 4N_f \frac{7}{8}} \right]. \quad (6.2)$$

To invert this relation for simplicity we now assume that  $\Phi$  is a small parameter so that to linear order in  $\Phi$

$$T' \simeq T \left[ 1 - \frac{1}{4} \Phi \frac{2(N_c^2 - 1)e^{(g)}(a) + 4N_f \frac{7}{8} e^{(q)}(a)}{2(N_c^2 - 1) + 4N_f \frac{7}{8}} \right]. \quad (6.3)$$

The numbers  $e^{(g)}(a)$  and  $e^{(q)}(a)$  are defined as follows:

$$e^{(g)}(a) = \frac{1}{\Phi T'^4} \frac{30}{\pi^2} \int \frac{dk}{2\pi^2} k^3 \delta_{\text{bulk}} f(k; T'), \quad (6.4)$$

$$e^{(q)}(a) = \frac{1}{\Phi T'^4} \frac{8 \cdot 30}{7\pi^2} \int \frac{dk}{2\pi^2} k^3 \delta_{\text{bulk}} f(k; T'), \quad (6.5)$$

and have been listed (for  $a = 1, 2, 3$ ) in table 1.

The temperature  $T$  which appears in the gluon self energies and propagators, and in the static potential, should now be replaced by  $T'$  as given in eq. (6.3).

As an example, consider the case  $a = 1$  so that  $e^{(g)}(1) = e^{(q)}(1) = 4$ . We then obtain  $T'^2 \simeq T^2(1 - 2\Phi)$  if  $|\Phi| \ll 1$ . Since  $c_R^{(g)}(1) = c_R^{(q)}(1) = 2$  in all we find that the “shift” of the Debye mass appearing in the retarded self energy, eq. (2.15), cancels. Hence, in this case there are no *explicit* bulk viscous corrections to the retarded gluon self energy. It is only affected by the implicit change of initial conditions and hydrodynamic solution in the presence of a non-vanishing bulk viscosity. On the other hand, for the symmetric self energy  $c_F^{(g)}(1) = c_F^{(q)}(1) = 3$  and so there is an explicit correction

$$(2N_c + N_f) \frac{g^2 T^2}{6} \rightarrow (2N_c(1 + \Phi) + N_f(1 + \Phi)) \frac{g^2 T^2}{6}, \quad (6.6)$$

even after Landau matching has been performed. Since  $|\Phi| \sim \zeta$ , the correction inherits the dynamical critical scaling of the bulk viscosity in the vicinity of a second order critical point.

<sup>10</sup>Notice that for the quark contribution, there is a pre-factor  $2N_f$  counting the number of quarks. For the gluon contribution, the factor is  $N_c^2 - 1$ .

Finally, we write the correction to the pressure which for the model from section 2 is given by

$$\frac{\delta_{\text{bulk}p}}{p_{\text{id}}} = \Phi \left( \frac{T'}{T} \right)^4 \frac{2(N_c^2 - 1)e^{(g)}(a) + 4N_f \frac{7}{8} e^{(q)}(a)}{2(N_c^2 - 1) + 4N_f \frac{7}{8}}. \quad (6.7)$$

Generically one expects a negative bulk pressure, so  $\Phi < 0$ , unless its sign is reversed by shear-bulk coupling [33–35].

## 7 Summary and discussion

Non-equilibrium corrections to the distribution functions of quarks and gluons in a hot and dense QCD medium result in corrections to “hard thermal loops” (HTL) which define Debye screening and Landau damping. In this paper we have considered two different forms of bulk-viscous corrections to ideal distributions corresponding to either thermal distributions or to a non-thermal fixed point obtained by introducing a non-vanishing scalar field. We find that the gluon hard thermal loop is dominated by hard momenta provided that the bulk-viscous corrections are sufficiently suppressed (relative to Bose enhanced behavior  $f_B(k)(1 + f_B(k)) \sim (T/k)^2$ ) for momenta  $k \ll T$ ; for the quark loop this is ensured by Pauli blocking.

Our main result is that isotropic bulk-viscous corrections shift the screening and damping mass scales which appear in the retarded/advanced versus the symmetric gluon HTL self energies. The shift is different for the two types of self energies. For example, bulk-viscous corrections to the thermal fixed point lead to the replacement

$$\left[ 2N_c + N_f \left( 1 + \frac{3\tilde{\mu}^2}{\pi^2} \right) \right] \frac{g^2 T^2}{6} \rightarrow m_{R,D}^2 + \delta m_{R,D}^2 = \left[ 2N_c \left( 1 + c_R^{(g)} \Phi \right) + N_f \left( 1 + \frac{3\tilde{\mu}^2}{\pi^2} \right) \left( 1 + c_R^{(q)} \Phi \right) \right] \frac{g^2 T^2}{6}, \quad (7.1)$$

in the retarded self energy (screening mass), and to

$$\left[ 2N_c + N_f \left( 1 + \frac{3\tilde{\mu}^2}{\pi^2} \right) \right] \frac{g^2 T^2}{6} \rightarrow m_{F,D}^2 + \delta m_{F,D}^2 = \left[ 2N_c \left( 1 + c_F^{(g)} \Phi \right) + N_f \left( 1 + \frac{3\tilde{\mu}^2}{\pi^2} \right) \left( 1 + c_F^{(q)} \Phi \right) \right] \frac{g^2 T^2}{6}, \quad (7.2)$$

in the symmetric self energy (to linear order in  $\Phi$ ). Here,  $\tilde{\mu}$  is the quark-chemical potential divided by temperature  $T$  and  $\Phi$  is proportional to the bulk pressure divided by the ideal pressure. The  $c_{R/F}^{(q,g)}$  are coefficients which we computed (also see section 6 on how to determine the temperature of the non-equilibrium system).

In the absence of strong shear-bulk coupling, generically  $\Phi < 0$  which implies reduced screening and damping scales. In particular, at a second order critical point it is expected that, up to finite time and finite size effects, the bulk viscosity diverges  $\sim \xi^z$  as a power (dynamical critical exponent) of the correlation length. This reflects the coupling of the hydrodynamical modes, i.e. of fluctuations of conserved currents, to those of the light, slow order parameter. Our analysis shows that in general the screening and damping mass scales

are also sensitive to such increase of the bulk viscosity. This could reflect, for example, in the properties of quarkonium bound states measured in heavy-ion collisions [41, 42].

Non-equilibrium bulk-viscous corrections also affect the dynamics of high occupancy soft fields [43–45] which is given by the classical Yang-Mills equations,

$$D_\mu F^{\mu\nu} = 2(m_{R,D}^2 + \delta m_{R,D}^2) \int \frac{d^3v}{4\pi} v^\nu w(\mathbf{x}, \mathbf{v}). \quad (7.3)$$

$w(\mathbf{x}, \mathbf{v})$  describes color charge fluctuations due to hard particles. It satisfies

$$v^\mu D_\mu w(\mathbf{x}, \mathbf{v}) = \mathbf{v} \cdot \mathbf{E}. \quad (7.4)$$

eq. (7.3) involves the shifted screening mass squared  $m_{R,D}^2 + \delta m_{R,D}^2$ . If negative this would lead to instabilities of the soft gauge fields.

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## A Gluon self energies in the real time formalism

In this appendix, we employ the real time formalism of thermal field theory to compute the gluon self energies at one-loop order within the Hard Thermal Loop (HTL) approximation. In finite temperature field theory, the real time formalism is more appropriate when dealing with a non-equilibrium situation. The corresponding results at vanishing chemical potential  $\mu$  have already been obtained before; see, for example, ref. [14, 36–38]. Here, we recompute the gluon self energies at finite chemical potential by keeping track of the quark distribution function  $f_F^+(\mathbf{k})$  and the anti quark distribution function  $f_F^-(\mathbf{k})$  explicitly during the calculation.

In the Keldysh representation, only the symmetric component of the bare fermion propagators depends on the chemical potential. It reads

$$S_F(K) = -2\pi i \not{K} [1 - 2(\Theta(k_0)f_F^+ + \Theta(-k_0)f_F^-)] \delta(K^2). \quad (A.1)$$

The retarded/advanced components are given by

$$\begin{aligned} S_R(K) &= \frac{\not{K}}{K^2 + i \operatorname{sgn}(k_0) \epsilon}, \\ S_A(K) &= \frac{\not{K}}{K^2 - i \operatorname{sgn}(k_0) \epsilon}, \end{aligned} \quad (A.2)$$

where  $\operatorname{sgn}(x)$  is the sign function. We neglect the fermion mass in our calculation.

We only need to consider the Feynman diagram with a quark loop, since it is the only one at one-loop order that depends on the chemical potential. For the retarded/advanced gluon self-energy, the temporal component can be expressed as

$$\begin{aligned} \Pi_R(P) = & -iN_f g^2 \int \frac{d^4 K}{(2\pi)^4} (q_0 k_0 + \mathbf{q} \cdot \mathbf{k}) \left[ \tilde{\Delta}_F(Q) \tilde{\Delta}_R(K) + \tilde{\Delta}_A(Q) \tilde{\Delta}_F(K) \right. \\ & \left. + \tilde{\Delta}_A(Q) \tilde{\Delta}_A(K) + \tilde{\Delta}_R(Q) \tilde{\Delta}_R(K) \right]. \end{aligned} \quad (\text{A.3})$$

Here,  $S_{R/A/F}(K) \equiv K \tilde{\Delta}_{R/A/F}(K)$  and  $Q = K - P$ . The last two terms of the integrand vanish after integration over  $k_0$ . At vanishing chemical potential, one can show that the first two terms contribute equally to the final result by the substitution  $K \rightarrow -K + P$ .<sup>11</sup> However, this is no longer true when  $\mu \neq 0$ . With the same replacement, we can recombine the contributions from the first two terms and finally arrive at

$$\begin{aligned} \Pi_R(P) = & 2\pi N_f g^2 \int \frac{k dk d\Omega}{(2\pi)^4} (f_F^+(\mathbf{k}) + f_F^-(\mathbf{k})) \left[ \frac{2k^2 - p_0 k - \mathbf{k} \cdot \mathbf{p}}{P^2 - 2kp_0 + 2\mathbf{k} \cdot \mathbf{p} - i \operatorname{sgn}(k - p_0)\epsilon} \right. \\ & \left. + \frac{2k^2 + p_0 k - \mathbf{k} \cdot \mathbf{p}}{P^2 + 2kp_0 + 2\mathbf{k} \cdot \mathbf{p} - i \operatorname{sgn}(-k - p_0)\epsilon} \right]. \end{aligned} \quad (\text{A.4})$$

The remainder of the calculation is very similar to the  $\mu = 0$  case. In the HTL approximation, the leading contribution is given by eq. (2.1).

Furthermore, for the temporal component of the symmetric gluon self-energy, we have

$$\begin{aligned} \Pi_F(P) = & -iN_f g^2 \int \frac{d^4 K}{(2\pi)^4} (q_0 k_0 + \mathbf{q} \cdot \mathbf{k}) \left[ \tilde{\Delta}_F(Q) \tilde{\Delta}_F(K) - (\tilde{\Delta}_R(Q) - \tilde{\Delta}_A(Q)) \right. \\ & \left. \times (\tilde{\Delta}_R(K) - \tilde{\Delta}_A(K)) \right]. \end{aligned} \quad (\text{A.5})$$

Using the HTL approximation, a straightforward calculation leads to

$$\begin{aligned} \Pi_F(P) = & 4iN_f g^2 \pi^2 \int \frac{k^2 dk d\Omega}{(2\pi)^4} \frac{2}{p} \left[ f_F^+(\mathbf{k})(f_F^+(\mathbf{k}) - 1) \delta \left( \hat{\mathbf{k}} \cdot \hat{\mathbf{p}} - \frac{p_0}{p} \right) \right. \\ & \left. + f_F^-(\mathbf{k})(f_F^-(\mathbf{k}) - 1) \delta \left( \hat{\mathbf{k}} \cdot \hat{\mathbf{p}} + \frac{p_0}{p} \right) \right]. \end{aligned} \quad (\text{A.6})$$

If the distribution function only depends on the modulus of the momentum  $\mathbf{k}$ , we can simplify this further. Notice that although the arguments of the two delta functions are different that nevertheless they give the same contribution after integrating over  $d\Omega$ . Finally, we arrive at

$$\Pi_F(P) = 4iN_f g^2 \pi^2 \int \frac{k^2 dk}{(2\pi)^3} \frac{2}{p} \left[ f_F^+(k)(f_F^+(k) - 1) + f_F^-(k)(f_F^-(k) - 1) \right] \Theta(p^2 - p_0^2). \quad (\text{A.7})$$

For thermal equilibrium distributions we can rewrite the above equation as

$$\Pi_F(P) = 4iN_f g^2 T \pi^2 \int \frac{k^2 dk}{(2\pi)^3} \frac{2}{p} \left( \frac{dn_F^+(k)}{dk} + \frac{dn_F^-(k)}{dk} \right) \Theta(p^2 - p_0^2). \quad (\text{A.8})$$

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<sup>11</sup>For a non-equilibrium distribution this requires that  $f(\mathbf{k}) = f(-\mathbf{k})$ .



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