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Erratum: AdS_2 holographic dictionary

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We provide a correction to the argument leading to the Schwarzian derivative effective action for running dilaton solutions in [1]. The two paragraphs below replace the corresponding ones on p. 18 and 19 of [1]. All results remain unaffected.

Generating functional. The renormalized one-point functions in eq. (3.14) of [1] can be expressed as

$$\mathcal{T}=\frac{\delta S_{\rm ren}}{\delta\alpha},\quad \mathcal{O}_\psi=\frac{\beta}{\alpha}\frac{\delta S_{\rm ren}}{\delta\beta},\quad \mathcal{J}^t=-\frac{1}{\alpha}\frac{\delta S_{\rm ren}}{\delta\mu}\,, \tag{1}$$

in terms of the renormalized on-shell action

$$S_{\rm ren}[\alpha,\beta,\mu]=-\frac{L}{2\kappa_2^2}\int \mathrm{d}t\left(\frac{m\alpha}{\beta}+\frac{\beta'^2}{\beta\alpha}+\frac{2\mu Q}{L}\right)+S_{\rm global}\,, \tag{2}$$

which is identified with the generating function of connected correlation functions in the dual theory. Since (2) is obtained by functionally integrating the relations (1), it is determined only up to an integration 'constant' $S_{\rm global}$, which is independent of the local sources $\alpha(t)$, $\beta(t)$ and $\mu(t)$. As indicated, $S_{\rm global}$ captures global properties of the theory

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and can be determined by evaluating explicitly the renormalized on-shell action.[1] Like the one-point functions in eq. (3.14) of [1], (2) is exact in the sources $\alpha(t)$, $\beta(t)$ and $\mu(t)$, although the fact that S_{global} has not been determined allows us to add an arbitrary total derivative term in this expression. In particular, successively differentiating (2) or the one-point functions in eq. (3.14) of [1] with respect to the sources $\alpha(t)$, $\beta(t)$ and $\mu(t)$ one can evaluate any n-point correlation function of the operators $\mathcal{T}$, $\mathcal{O}_\psi$ and $\mathcal{J}^t$ in the dual theory.

Effective action and the Schwarzian derivative. As we anticipated in section 2 of [1], the sources $\alpha(t)$, $\beta(t)$ and $\mu(t)$ are locally pure gauge. In particular, the number of independent source components coincides with the number of independent local parameters of bulk diffeomorphisms and U(1) gauge transformations that preserve the Fefferman-Graham gauge in eq. (2.2) of [1], which are discussed in detail in section 5 of [1]. Exponentiating the infinitesimal transformations in eq. (5.7) of [1] for running dilaton solutions with respect to the local parameters $\sigma(t)$ and $\varphi(t)$ we can express the local sources in terms of the parameters of local symmetry transformations as

$$\alpha = e^{\sigma}\left(1+\varepsilon'+\varepsilon\sigma'\right)+\mathcal{O}(\varepsilon^2), \quad \beta = e^{\sigma}\left(1+\varepsilon\sigma'\right)+\mathcal{O}(\varepsilon^2), \quad \mu = \varphi'+\varepsilon'\varphi'+\varepsilon\varphi''+\mathcal{O}(\varepsilon^2)\,, \tag{4}$$

where as above the primes $'$ denote a derivative with respect to t. Inserting this form of the sources in (2) and absorbing total derivative terms in S_{global} we obtain

$$S_{\text{ren}} = \frac{L}{\kappa_2^2}\int \mathrm{d}t\Big(\{\tau,t\}-m/2\Big)+S_{\text{global}}, \qquad \sigma = \log\tau', \tag{5}$$

where $\tau(t)$ is a 'dynamical time' [3] and

$$\{\tau,t\} = \frac{\tau'''}{\tau'}-\frac{3}{2}\frac{\tau''^2}{\tau'^2}\,, \tag{6}$$

denotes the Schwarzian derivative. This form of the effective action arises in the infrared limit of the Sachdev-Ye-Kitaev model [4, 5] and is a key piece of evidence for the holographic identification of this model with AdS_2 dilaton gravity [3, 6, 7]. In terms of σ the action (5) is just the Liouville action in one dimension, and once S_{global} is taken into account, it corresponds to a circle reduction of the Takhtajan-Zograf Liouville action obtained in [2]

[1] With the counterterm in eq. (3.11) of [1] the expression for the renormalized on-shell action is

$$S_{\text{ren}} = \frac{1}{2\kappa_2^2}\int dt\left(\lim_{u_o\to\infty}\left[\partial_u(\sqrt{-\gamma}\,e^{-\psi})-\frac{2}{L}\sqrt{-\gamma}\,(1-u_oL\Box_t)\,e^{-\psi}\right]\Big|_{u_o}+\left(e^{-\psi}\partial_u\sqrt{-\gamma}-\sqrt{-\gamma}\,\partial_u e^{-\psi}\right)\Big|_{u_+}\right), \tag{3}$$

where u_+ is the minimum value of the radial coordinate and u_o is the radial cutoff. In order to compute the global part S_{global} in (2), both u_+ and u_o need to be expressed in terms of a suitable 'uniformizing' Liouville field as is done in the case of pure AdS_3 gravity in [2]. In fact, as we demonstrate in section 4 of [1], the running dilaton solutions of the action in eq. (1.1) of [1] correspond to a circle reduction of pure AdS_3 gravity, and hence, the renormalized on-shell action (2) can also be obtained by a circle reduction of the Takhtajan-Zograf Liouville action obtained in [2]. In particular, S_{global} corresponds to the circle reduction of the global terms in the Takhtajan-Zograf action. However, here we are interested in the local part of the on-shell action, which, as we shall see momentarily, is related to the universal local part of the Liouville action and can be expressed in terms of a Schwarzian derivative.

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for pure AdS_3 gravity. Note that it only depends on the local parameter $\sigma(t)$, or $\tau(t)$, but not on $\varepsilon(t)$ and $\varphi(t)$. This reflects the fact that the Ward identities in eq. (3.15) of [1] are non-anomalous, while the trace Ward identity in eq. (3.16), which corresponds to the transformation of S_{ren} under $\sigma(t)$ transformations, is anomalous. We therefore see that the appearance of the Schwarzian derivative is a manifestation of the conformal anomaly, exactly as in the case of conformal field theories in two dimensions.

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