

The Effective Field Theory of nonsingular cosmology

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ABSTRACT: In this paper, we explore the nonsingular cosmology within the framework of the Effective Field Theory (EFT) of cosmological perturbations. Due to the recently proved no-go theorem, any nonsingular cosmological models based on the cubic Galileon suffer from pathologies. We show how the EFT could help us clarify the origin of the no-go theorem, and offer us solutions to break the no-go. Particularly, we point out that the gradient instability can be removed by using some spatial derivative operators in EFT. Based on the EFT description, we obtain a realistic healthy nonsingular cosmological model, and show the perturbation spectrum can be consistent with the observations.

KEYWORDS: Spacetime Singularities, Classical Theories of Gravity, Cosmology of Theories beyond the SM

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1 Introduction

In the modern era of Cosmology, theories of Hot Big Bang (HBB) and Inflation have achieved great success, and thus have long been regarded as the standard paradigm of the early universe. However, the inflation still suffers from the cosmological singularity problem [1, 2], unless it was preceded by a bounce [3–7] or a Genesis phase [8–10]. It is exciting to study classical nonsingular cosmology, such as bounce universe models [11, 12], Genesis models [13–15], slow expansion models [16–19], since we might get classical nonsingular cosmology without begging the details of the unknown UV-complete gravity theory.

One of the most exciting endeavors in this area is to build nonsingular cosmological models with the field theories which can violate the Null Energy Condition (NEC) [20]. Usually the violation of NEC may lead to the ghost instability [21, 22]. This problem can be solved if one considers the so-called Galileon theory [23] or its generalizations (such as the Horndeski theory [24, 25] and its beyond [26]). Making use of the simplest cubic Galileon, many heuristic nonsingular cosmological models have been constructed, e.g. [13, 17, 18, 27, 28]. However, it seems quite difficult to avoid the gradient instability problem, which indicates a negative sound speed squared [7, 9, 29, 30] and thus leads to an exponential growth of the perturbation [31, 32].

Recently, Libanov, Mironov and Rubakov (LMR) have proved a no-go theorem, which shows that healthy nonsingular cosmological models based on the cubic Galileon does not exist [33]. Hereafter, it was generalized with an additional scalar in ref. [34] or with the full Horndeski theory in ref. [35]. However, Ijjas and Steinhardt claimed that there exists a loophole in the proof of ref. [35] (which was also noticed by the author of ref. [35]), and they can even reconstruct a fully stable classical bounce [36] throughout the whole evolution by using the “inverse method” [37]. However, we believe that this relevant issue still needs to be studied further.

Prior to LMR’s work, studies were also made along other lines. The danger of $c_s^2 < 0$ is mainly attributed to the exponential growth of the amplitude of short wavelength modes. In [31, 32], it was argued that the strong coupling scale during $c_s^2 < 0$ is low so that the dangerous short wavelength modes lie outside the range of the validity of the effective theory, thus can be disregarded. However, this argument begs unknown strong coupling physics, which actually makes the “classical nonsingular” bounce loose sense. What is the effective theory of nonsingular cosmology is a significant issue. It is interesting to notice that some spatial covariant operators also help to remove the gradient instability [7, 9, 10].

The Effective Field Theory (EFT) of cosmological perturbations is extremely powerful and has been widely used to study inflation [38, 39] and dark energy [40–42]. It offers a unifying platform to deal with the cosmological perturbations of all kinds of theories, such as the Horndeski theory and its beyond, the Horava gravity [43], and the spatial covariant gravity [44, 45]. In the following context, we will see that it is also a powerful tool for studying nonsingular cosmology.

In this paper, we will explore how to build healthy nonsingular cosmological models within the framework of EFT. Practically, in section 2, based on EFT, we clarify how to understand the no-go theorem and how to avoid it. We find that some effective operators can play significant role in building nonsingular cosmological models without pathologies. In section 3, we study the evolution of primordial perturbation in nonsingular models with these corresponding effective operators, and find the perturbation spectrum can be consistent with the observations. In section 4, we present a realistic healthy nonsingular bounce model by introducing an effective operator of $R^{(3)}\delta g^{00}$. Finally, we conclude in section 5.

Note added. After our paper appeared in arXiv, nearly simultaneously Creminelli sent us their draft (the preprint [46]), which overlaps substantially with ours.

2 The framework of EFT and the no-go theorem

We consider the metric in the ADM form:

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt), \tag{2.1}$$

where N and N^i are the lapse function and shift vector, and h_{ij} is the 3-dimensional spatial metric.

With the spirit of the EFT of cosmological perturbation [38, 40, 43], we write down the EFT action for nonsingular cosmological models

$$\begin{aligned} S = \int d^4x \sqrt{-g} & \left[\frac{M_p^2}{2} f(t) R - \Lambda(t) - c(t) g^{00} \right. \\ & + \frac{M_2^4(t)}{2} (\delta g^{00})^2 - \frac{m_3^3(t)}{2} \delta K \delta g^{00} - m_4^2(t) (\delta K^2 - \delta K_{\mu\nu} \delta K^{\mu\nu}) + \frac{\tilde{m}_4^2(t)}{2} R^{(3)} \delta g^{00} \\ & - \bar{m}_4^2(t) \delta K^2 + \frac{\bar{m}_5(t)}{2} R^{(3)} \delta K + \frac{\bar{\lambda}(t)}{2} (R^{(3)})^2 + \dots \\ & \left. - \frac{\tilde{\lambda}(t)}{M_p^2} \nabla_i R^{(3)} \nabla^i R^{(3)} + \dots \right], \tag{2.2} \end{aligned}$$

where we turn off the accelerator vectors a_i in [43] for simplicity. We assume the matter part is minimally coupled to field so that the expansion or contraction of the background with respect to physical rulers is unambiguous. The first line describes the background of our model, while the rest is for perturbations. One is also allowed to contain terms such as $R^{(3)}_{\mu\nu}R^{(3)\mu\nu}$ and $\nabla_i R^{(3)}_{jk}\nabla^i R^{(3)jk}$, which we don't bother to write them explicitly and just put them into the ellipsis. All the coefficients are allowed to vary with t , with the dimension $[m_i] = 1$, $[\lambda_i] = 0$, so as to make the action dimensionless. Moreover, in this action we define $\delta K_{\mu\nu} = K_{\mu\nu} - HH_{\mu\nu}$, $\delta K = K - 3H$, with the induced metric $H_{\mu\nu} \equiv g_{\mu\nu} + n_\mu n_\nu$ and the normal vector is defined as $n_\mu \equiv (-N, 0, 0, 0)$.

It is rather straightforward to fix the relations among the functions $f(t)$, $c(t)$ and $\Lambda(t)$, which is in the background part. Varying the first line of action eq. (2.2) with respect to N and a , one can get the two equations:

$$3M_p^2[f(t)H^2 + \dot{f}(t)H] = c(t) + \Lambda(t), \quad (2.3)$$

$$-M_p^2[2f(t)\dot{H} + 3f(t)H^2 + 2\dot{f}(t)H + \ddot{f}(t)] = c(t) - \Lambda(t). \quad (2.4)$$

For the minimal coupling theories where $f(t) = 1$, these are nothing but the Friedmann equations, thus we have $c(t) = -M_p^2\dot{H}$ and $\Lambda(t) = M_p^2(\dot{H} + 3H^2)$. The $c(t)$ and $\Lambda(t)$ have the same expressions as those in the EFT of inflation, however, to have a non-singular scenario a crucial condition must be satisfied, i.e., the violation of NEC. That means $c(t)$ must be negative at least for a while. Since the NEC will finally be restored in the expanding universe, we conclude for the EFT of nonsingular cosmology, $c(t)$ must be a function that can pass the zero boundary. For the case with non-minimal coupling, $f(t)$ is nontrivial, then a more complicated constraint will be imposed on $c(t)$ and $\Lambda(t)$.

2.1 The no-go theorem

It is straightforward to derive the quadratic action of scalar and tensor perturbation from eq. (2.2). We give some main steps of the derivation in appendix A and just write down the results here. Under the unitary gauge, the quadratic action of scalar perturbation is

$$S_\zeta^{(2)} = \int d^4x a^3 \left[c_1 \dot{\zeta}^2 - \left(\frac{\dot{c}_3}{a} - c_2 \right) \frac{(\partial\zeta)^2}{a^2} + \frac{c_4}{a^4} (\partial^2\zeta)^2 - \frac{16\tilde{\lambda}(t)}{M_p^2 a^6} (\partial^3\zeta)^2 \right], \quad (2.5)$$

where we have left the expressions of c_i in appendix A since they are complicated (except for the c_2 , which has a quite simple expression as $c_2 = M_p^2 f(t)$). The sound speed squared reads

$$c_s^2 = \left(\frac{\dot{c}_3}{a} - c_2 \right) / c_1. \quad (2.6)$$

The conditions to avoid the ghost instability and the gradient instability are

$$c_1 > 0, \quad \dot{c}_3 - a c_2 > 0. \quad (2.7)$$

Moreover, the quadratic action of tensor perturbation from eq. (2.2) is

$$S_\gamma^{(2)} = \frac{M_p^2}{8} \int d^4x a^3 Q_T \left[\dot{\gamma}_{ij}^2 - c_T^2 \frac{(\partial_k \gamma_{ij})^2}{a^2} \right], \quad (2.8)$$

where

$$Q_T = f + 2 \left(\frac{m_4}{M_p} \right)^2, \quad c_T^2 = \frac{f}{Q_T}. \quad (2.9)$$

To avoid the ghost and gradient instability for tensor modes, we need $Q_T > 0$ and $c_T^2 > 0$, respectively.

We begin with $\dot{c}_3 - ac_2 > 0$, which indicates

$$c_3|_{t_f} - c_3|_{t_i} > \int_{t_i}^{t_f} ac_2 dt = M_p^2 \int_{t_i}^{t_f} af(t) dt. \quad (2.10)$$

This expression is the key inequality to clarify the no-go theorem. This inequality turns out to be remarkably general since it is correct not only for the Horndeski theory, but also for these theories beyond the Horndeski. As matter of fact, by mapping the cubic Galileon to the EFT [41], eq. (2.10) will lead to the key inequality used to prove the LMR no-go theorem [33] (see the following part of this subsection); and by mapping the whole Horndeski theory to the EFT [41], eq. (2.10) will produce the key inequality in Kobayashi's paper [35].

Now let's consider the cubic Galileon $\mathcal{L}_2 + \mathcal{L}_3$ with $f(t) = 1$, eq. (2.10) reads

$$c_3|_{t_f} - c_3|_{t_i} > \int_{t_i}^{t_f} ac_2 dt = M_p^2 \int_{t_i}^{t_f} a dt, \quad (2.11)$$

and according to the appendix A, we find

$$c_3 = \frac{2aM_p^4}{2HM_p^2 - m_3^3} = \frac{aM_p^2}{\gamma}, \quad (2.12)$$

where $\gamma = H - (1/2)m_3^3/M_p^2$. We see from eq. (2.10) that c_3 is increased with time. Supposing $c_3|_{t_i} < 0$, from

$$c_3|_{t_f} > c_3|_{t_i} + M_p^2 \int_{t_i}^{t_f} a dt \quad (2.13)$$

we can tell that $c_3|_{t_f}$ will finally be larger than zero, thus c_3 must equal to zero at sometime t with $t_i < t < t_f$, making γ blows away. Therefore the gradient instability cannot be avoided. The remaining case is that c_3 be always positive. However, from

$$c_3|_{t_f} - M_p^2 \int_{t_i}^{t_f} a dt > c_3|_{t_i} \quad (2.14)$$

and let $t_i \rightarrow -\infty$, we see this is impossible in a similar manner. So we have reformulated the LMR no-go theorem [33] for the cubic Galileon in the framework of EFT, which indicates the pathologies in nonsingular cosmological models based on the cubic Galileon are inevitable.

2.2 How to evade the no-go theorem within the framework of EFT

Recently, the no-go proof has been extended to the full Horndeski theory by T. Kobayashi [35]. However, it seems that this no-go theorem might be broken if the integral $\int_{t_i}^{t_f} a f dt$ is not divergent.¹ Very recently, A. Ijjas and P. J. Steinhardt found a fully stable bounce by keeping the integral $\int_{t_i}^{t_f} a f dt$ convergent [36, 37]. In this section, we discuss how to avoid the no-go theorem within the framework of EFT eq. (2.2), while we assume $\int_{t_i}^{t_f} a f dt$ is divergent and $Q_T > 0$ throughout (see [36] for the cases $\int_{t_i}^{t_f} a f dt$ is convergent or $Q_T = 0$ at some time), which actually indicates that we have to go beyond Horndeski theory.

We firstly consider the addition of the effective operator $R^{(3)}\delta g^{00}$ to the cubic Galileon. It gives a contribution with $(\partial\zeta)^2 \sim k^2\zeta_k^2$ to the scalar perturbation, while does not change the tensor perturbation at quadratic order. The EFT action is written as:

$$S_{\text{eff}} = \int d^4x \sqrt{-g} \left[\frac{M_p^2}{2} R - \Lambda(t) - c(t)g^{00} + \frac{M_2^4(t)}{2} (\delta g^{00})^2 - \frac{m_3^3(t)}{2} \delta K \delta g^{00} + \frac{\tilde{m}_4^2(t)}{2} R^{(3)} \delta g^{00} \right], \quad (2.15)$$

here we have set $f(t) = 1$, the coefficients $c(t)$, $\Lambda(t)$, $M_2^4(t)$ and $m_3^3(t)$ can be found by requiring that they have the same time-dependent behaviors as in the cubic Galileon $\mathcal{L}_2 + \mathcal{L}_3$. The existence of the last $\tilde{m}_4^2(t)$ term indicates this model eq. (2.15) goes beyond the Horndeski. Note, the dynamical equation for the true degree of freedom is still second order, thus the $\tilde{m}_4^2(t)$ term here, as well as the higher order spacial derivative terms $(R^{(3)})^2$ and $\nabla_i R^{(3)} \nabla^i R^{(3)}$ used below, does not introduce the Ostrogradski instability (see, e.g., [41]). According to the appendix A, we have

$$c_3 = \frac{aM_p^2}{\gamma} \left(1 + \frac{2\tilde{m}_4^2}{M_p^2} \right), \quad (2.16)$$

with $\gamma = H - (1/2)m_3^3/M_p^2$. Again with eq. (2.13), suppose $c_3|_{t_i} < 0$, since the integral $\int_{t_i}^{t_f} a dt$ diverges, eventually we have $c_3|_{t_f} > 0$, thus c_3 must cross 0 at sometime t with $t_i < t < t_f$. However, if at that time we have $2\tilde{m}_4^2/M_p^2$ cross -1 , the c_3 will cross 0 naturally without the divergence of γ . So the no-go behavior can be avoided, and notice for eq. (2.15), since $m_4^2(t) = 0$ and $Q_T = 1$, the tensor perturbation will be healthy. Generally, we could set the effective operator $\tilde{m}_4^2 R^{(3)} \delta g^{00}/2$ to be dominated only when we meet $c_s^2 < 0$, thus it just modifies the sound speed squared during this time, see section 4 for details.

We can further add the term $-m_4^2(t) (\delta K^2 - \delta K_{\mu\nu} \delta K^{\mu\nu})$ into the effective action eq. (2.15), then c_3 changes to be

$$c_3 = \frac{aM_p^2 Q_T}{\gamma} \left(1 + \frac{2\tilde{m}_4^2}{M_p^2} \right) \quad (2.17)$$

with $\gamma = H \left(1 + \frac{2m_4^2}{M_p^2} \right) - (1/2)m_3^3/M_p^2$. Generally, we may find those coefficients with $2\tilde{m}_4^2/M_p^2$ crossing -1 and $m_4^2(t) \neq \tilde{m}_4^2(t)$ which goes beyond Horndeski, to make c_3 cross 0 while $Q_T \neq 0$ and γ won't blow up. The case of Q_T/γ crosses 0 is discussed in [36] with \mathcal{L}_4 .

¹Note that in EFT description of Horndeski theory, we have $f = 2[G_4 - X(\ddot{\phi}G_{5,X} + G_{5,\phi})]$ [41, 43].

However, when $m_4^2(t) = \tilde{m}_4^2(t)$, such as for the case of Horndeski theory,

$$c_3 = \frac{aM_p^2}{\gamma} Q_T^2 \tag{2.18}$$

crosses 0 suggests the no-go behavior must happen unless the integral $\int_{t_i}^{t_f} adt$ in eq. (2.10) is convergent or Q_T^2/γ crosses 0. Obviously, this argument also applies to a general \mathcal{L}_4 with time-dependent $f(t)$, as has been argued by T. Kobayashi [35] (see also [36]).

Furthermore, let's consider the effective operators $(R^{(3)})^2$ and $\nabla_i R^{(3)} \nabla^i R^{(3)}$, which will give contributions to higher order spatial derivatives with $k^4 \zeta_k^2$ and $k^6 \zeta_k^2$. As the operator $(R^{(3)})^2$ has been applied to the nonsingular cosmology in [9], here we'd like to take the following nonsingular model

$$S_{\text{eff}} = \int d^4x \sqrt{-g} \left[\frac{M_p^2}{2} R - \Lambda(t) - c(t) g^{00} + \frac{M_2^4(t)}{2} (\delta g^{00})^2 - \frac{m_3^3(t)}{2} \delta K \delta g^{00} - \frac{\tilde{\lambda}(t)}{M_p^2} \nabla_i R^{(3)} \nabla^i R^{(3)} \right] \tag{2.19}$$

with $f(t) = 1$, and the coefficients $c(t)$, $\Lambda(t)$, $M_2^4(t)$ and $m_3^3(t)$ are taken according to the EFT mapping for the cubic Galileon $\mathcal{L}_2 + \mathcal{L}_3$ [41]. Then we have an effective sound speed squared with

$$c_{s,\text{eff}}^2(k) = c_s^2 + \frac{32\tilde{\lambda}}{M_p^2 a^2 z^2} k^4, \tag{2.20}$$

where $z = \sqrt{2a^2 c_1}$ and c_s^2 are given by eq. (2.6).

From the equation of motion of ζ eq. (3.1), we see that if these effective operators with higher order spatial derivatives have not been included, we'll have a solution of $\zeta \sim e^{-i\sqrt{c_s^2} k \Delta\tau}$ which indicates an exponential growth when $c_s^2 < 0$. However, the growth turns out to be negligible for the perturbation modes with $k\Delta\tau \ll 1$, and can be quite dangerous for the modes with $k\Delta\tau \gg 1$ [31, 32]. So we may specify $\tilde{\lambda}(t)$ to make $c_{s,\text{eff}}^2(k) \sim c_s^2$ for the modes with $k\Delta\tau \ll 1$, while make $c_{s,\text{eff}}^2(k)$ modified to be positive for the modes with $k\Delta\tau \gg 1$. Then such kind of exponential growth of ζ due to $c_s^2 < 0$ can be removed.

3 Primordial perturbation spectrum within the framework of EFT

In the last section we have presented how to evade the no-go theorem within framework of EFT, i.e., by adding the effective operators such as $R^{(3)} \delta g^{00}$, $(R^{(3)})^2$, $\nabla_i R^{(3)} \nabla^i R^{(3)}$, to the original nonsingular cosmological models based on the cubic Galileon. One might ask if the perturbation spectrum will be modified due to these operators. In this section, we study the perturbation evolution in detail, and show that the predictions can be consistent with the observations.

The equation of motion of ζ is

$$u'' + \left(c_{s,\text{eff}}^2(k) k^2 - \frac{z''}{z} \right) u = 0, \tag{3.1}$$

where

$$c_{s,\text{eff}}^2(k) = c_s^2 - \frac{2c_4}{z^2}k^2 + \frac{32\tilde{\lambda}}{M_p^2 a^2 z^2}k^4 \quad (3.2)$$

with $u = z\zeta$, $z = \sqrt{2a^2 c_1}$ and c_s^2 is given by eq. (2.6), the prime denotes the derivative with respect to the conformal time $\tau = \int dt/a$.

To study the evolution of the primordial perturbation concretely, let's consider a bounce inflation background in this section. We can define the ‘‘bouncing phase’’ as the time interval during which the NEC is violated, i.e., $\dot{H} > 0$. At the beginning time τ_{B-} and the ending time τ_{B+} of the ‘‘bouncing phase’’ we have $\dot{H} = 0$, while before the beginning time and after the ending time, the NEC is restored and thus leads to $\dot{H} < 0$.

By adding the effective operators like $R^{(3)}\delta g^{00}$, $(R^{(3)})^2$ or $\nabla_i R^{(3)}\nabla^i R^{(3)}$ to the original G-bounce models [27, 47], within the framework of EFT the whole Lagrangian tends to be like the ones in eq. (2.15) or eq. (2.19). To cure the gradient instability problem in these models, we can set the corresponding operators to be dominated only during the duration $\Delta\tau = \tau_{B+} - \tau_{B-}$ of the bouncing phase. Thus eq. (3.1) can be written as

$$u'' + \left(k^2 - \frac{z''}{z}\right)u = 0, \quad (\tau < \tau_{B-}, \tau > \tau_{B+}), \quad (3.3)$$

$$u'' + A^{2n}k^{2n}u = 0, \quad (\tau_{B-} < \tau < \tau_{B+}), \quad (3.4)$$

where $n \geq 1$, and the corresponding operators will respectively contribute $\sim k^2, k^4, k^6$ corrections to the equation of motion. To be rigorous, all the coefficients A^{2n} of the k^{2n} terms should be time-dependent. However, here we set A^{2n} constant for simplicity.

During the contracting phase $\tau < \tau_{B-}$, the background can be parameterized as

$$a_c = a_{B-} \left(\frac{\tau - \tilde{\tau}_{B-}}{\tau_{B-} - \tilde{\tau}_{B-}} \right)^{\frac{1}{\epsilon_c - 1}}, \quad (3.5)$$

where $\tilde{\tau}_{B-} = \tau_{B-} - [(\epsilon_c - 1)\mathcal{H}_{B-}]^{-1}$, and $\epsilon_c > 3$ is a constant. Thus we have

$$\frac{a_c''}{a_c} = \frac{\nu_c^2 - \frac{1}{4}}{(\tau - \tilde{\tau}_{B-})^2}, \quad (3.6)$$

where $\nu_c = 1/2 - \frac{1}{\epsilon_c - 1}$. The solution of eq. (3.3) can be given as

$$u_c = \frac{\sqrt{\pi}}{2} \sqrt{|\tau - \tilde{\tau}_{B-}|} \left[c_{1,1} H_{\nu_c}^{(1)}(k|\tau - \tilde{\tau}_{B-}|) + c_{1,2} H_{\nu_c}^{(2)}(k|\tau - \tilde{\tau}_{B-}|) \right], \quad (3.7)$$

where $H_{\nu}^{(1)}$ and $H_{\nu}^{(2)}$ are the ν -th order Hankel function of the first and the second kind.

Initially, the perturbations are deep inside the horizon. The initial condition can be taken as $u \sim \frac{1}{\sqrt{2k}} e^{-ik\tau}$, thus

$$c_{1,1} = 1, \quad c_{1,2} = 0. \quad (3.8)$$

During the bouncing phase $\tau_{B-} < \tau < \tau_{B+}$, the solution of eq. (3.4) is

$$u_b = c_{2,1} \cdot e^{iA^n k^n (\tau - \tau_B)} + c_{2,2} \cdot e^{-iA^n k^n (\tau - \tau_B)}, \quad (3.9)$$

where $c_{2,1}$ and $c_{2,2}$ are determined by the evolution of the contracting phase. By considering the effective operators, the effective sound speed squared $c_{s,\text{eff}}^2 > 0$ for short wavelength perturbation modes, thus there won't be any dangerous growths of the curvature perturbation ζ .

During the inflation $\tau > \tau_{B+}$, the background can be parameterized as

$$a_e = a_{B+} \left(\frac{\tau - \tilde{\tau}_{B+}}{\tau_{B+} - \tilde{\tau}_{B+}} \right)^{\frac{1}{\epsilon_e - 1}}, \quad (3.10)$$

where $\tilde{\tau}_{B+} = \tau_{B+} - [(\epsilon_e - 1)\mathcal{H}_{B+}]^{-1}$. So we have

$$\frac{a_e''}{a_e} = \frac{\nu_e^2 - \frac{1}{4}}{(\tau - \tilde{\tau}_{B+})^2}, \quad (3.11)$$

where $\nu_e = 1/2 - \frac{1}{\epsilon_e - 1}$. The solution of eq. (3.3) can be given as

$$u_e = \frac{\sqrt{\pi}}{2} \sqrt{|\tau - \tilde{\tau}_{B+}|} \left[c_{3,1} H_{\nu_e}^{(1)}(k|\tau - \tilde{\tau}_{B+}|) + c_{3,2} H_{\nu_e}^{(2)}(k|\tau - \tilde{\tau}_{B+}|) \right]. \quad (3.12)$$

The power spectrum is calculated as

$$P_\zeta = P_\zeta^{\text{inf}} \cdot |c_{3,1} - c_{3,2}|^2. \quad (3.13)$$

The information of the evolution history of the universe and the contributions of the EFT operators are encoded in $c_{3,1}$ and $c_{3,2}$. Though we work with bounce inflation scenario, actually, our result is also applicable to the bounce scenario, as will be seen.

By requiring the continuity of u and u' at the matching surfaces, we obtain

$$\begin{pmatrix} c_{3,1} \\ c_{3,2} \end{pmatrix} = \mathcal{M}^{(3,2)} \times \mathcal{M}^{(2,1)} \times \begin{pmatrix} c_{1,1} \\ c_{1,2} \end{pmatrix}, \quad (3.14)$$

where the components of the matrix $\mathcal{M}^{(2,1)}$ are

$$\begin{aligned} \mathcal{M}_{11}^{(2,1)} &= \frac{e^{idA^n k^n} \sqrt{\pi}}{8A^n k^n \sqrt{\hat{\mathcal{H}}}} \left[ikH_{\nu_c-1}^{(1)} \left(\frac{k}{\hat{\mathcal{H}}} \right) - ikH_{\nu_c+1}^{(1)} \left(\frac{k}{\hat{\mathcal{H}}} \right) + (2A^n k^n + i\hat{\mathcal{H}}) H_{\nu_c}^{(1)} \left(\frac{k}{\hat{\mathcal{H}}} \right) \right], \\ \mathcal{M}_{12}^{(2,1)} &= \frac{e^{idA^n k^n} \sqrt{\pi}}{8A^n k^n \sqrt{\hat{\mathcal{H}}}} \left[ikH_{\nu_c-1}^{(2)} \left(\frac{k}{\hat{\mathcal{H}}} \right) - ikH_{\nu_c+1}^{(2)} \left(\frac{k}{\hat{\mathcal{H}}} \right) + (2A^n k^n + i\hat{\mathcal{H}}) H_{\nu_c}^{(2)} \left(\frac{k}{\hat{\mathcal{H}}} \right) \right], \\ \mathcal{M}_{21}^{(2,1)} &= \frac{e^{-idA^n k^n} \sqrt{\pi}}{8A^n k^n \sqrt{\hat{\mathcal{H}}}} \left[-ikH_{\nu_c-1}^{(1)} \left(\frac{k}{\hat{\mathcal{H}}} \right) + ikH_{\nu_c+1}^{(1)} \left(\frac{k}{\hat{\mathcal{H}}} \right) + (2A^n k^n - i\hat{\mathcal{H}}) H_{\nu_c}^{(1)} \left(\frac{k}{\hat{\mathcal{H}}} \right) \right], \\ \mathcal{M}_{22}^{(2,1)} &= \frac{e^{-idA^n k^n} \sqrt{\pi}}{8A^n k^n \sqrt{\hat{\mathcal{H}}}} \left[-ikH_{\nu_c-1}^{(2)} \left(\frac{k}{\hat{\mathcal{H}}} \right) + ikH_{\nu_c+1}^{(2)} \left(\frac{k}{\hat{\mathcal{H}}} \right) + (2A^n k^n - i\hat{\mathcal{H}}) H_{\nu_c}^{(2)} \left(\frac{k}{\hat{\mathcal{H}}} \right) \right], \end{aligned}$$

and the components of matrix $\mathcal{M}^{(3,2)}$ are

$$\begin{aligned}\mathcal{M}_{11}^{(3,2)} &= -\frac{ie^{idA^n k^n} \sqrt{\pi}}{4\sqrt{\mathcal{H}_{B+}}} \left[-2kH_{\nu_e-1}^{(2)} \left(\frac{k}{\mathcal{H}_{B+}} \right) + \left(-2iA^n k^n + (2\nu_e - 1)\mathcal{H}_{B+} \right) H_{\nu_e}^{(2)} \left(\frac{k}{\mathcal{H}_{B+}} \right) \right], \\ \mathcal{M}_{12}^{(3,2)} &= \frac{e^{-idA^n k^n} \sqrt{\pi}}{4\sqrt{\mathcal{H}_{B+}}} \left[2ikH_{\nu_e-1}^{(2)} \left(\frac{k}{\mathcal{H}_{B+}} \right) + \left(2A^n k^n - i(2\nu_e - 1)\mathcal{H}_{B+} \right) H_{\nu_e}^{(2)} \left(\frac{k}{\mathcal{H}_{B+}} \right) \right], \\ \mathcal{M}_{21}^{(3,2)} &= \frac{e^{idA^n k^n} \sqrt{\pi}}{4\sqrt{\mathcal{H}_{B+}}} \left[-2ikH_{\nu_e-1}^{(1)} \left(\frac{k}{\mathcal{H}_{B+}} \right) + \left(2A^n k^n + i(2\nu_e - 1)\mathcal{H}_{B+} \right) H_{\nu_e}^{(1)} \left(\frac{k}{\mathcal{H}_{B+}} \right) \right], \\ \mathcal{M}_{22}^{(3,2)} &= \frac{ie^{-idA^n k^n} \sqrt{\pi}}{4\sqrt{\mathcal{H}_{B+}}} \left[-2kH_{\nu_e-1}^{(1)} \left(\frac{k}{\mathcal{H}_{B+}} \right) + \left(2iA^n k^n + (2\nu_e - 1)\mathcal{H}_{B+} \right) H_{\nu_e}^{(1)} \left(\frac{k}{\mathcal{H}_{B+}} \right) \right]\end{aligned}$$

with $d = \tau_{B+} - \tau_B$, $\hat{\mathcal{H}} = (\epsilon_c - 1)\mathcal{H}_{B+}$, and \mathcal{H}_{B+} is the comoving Hubble parameter at τ_{B+} .

Considering the long wavelength limit, $k/\mathcal{H}_{B+} \ll 1$, we have

$$\begin{aligned}|c_{3,1} - c_{3,2}|^2 &\approx \frac{1}{9\pi} \left(\frac{k}{\mathcal{H}_{B+}} \right)^{\frac{2\epsilon_c}{\epsilon_c - 1}} (1 - 4d\mathcal{H}_{B+})^2 (2\epsilon_c - 2)^{\frac{2}{1-\epsilon_c}} \Gamma^2 \left(\frac{1}{2} + \frac{1}{1-\epsilon_c} \right) \\ &\sim \left(\frac{k}{\mathcal{H}_{B+}} \right)^{\frac{2\epsilon_c}{\epsilon_c - 1}}.\end{aligned}\quad (3.15)$$

In bounce scenario where the bounce is followed by the Hot Big-Bang expansion, P_ζ is given by eq. (3.15),

$$P_\zeta \sim |c_{3,1} - c_{3,2}|^2 \sim \left(\frac{k}{\mathcal{H}_{B+}} \right)^{\frac{2\epsilon_c}{\epsilon_c - 1}} \quad (3.16)$$

since the perturbation modes with $k/\mathcal{H}_{B+} > 1$ can be hardly produced during the expansion after the bounce. The result is consistent with that in ekpyrotic universe [48–50]. Thus the spectrum of primordial perturbations in bounce scenario is unaffected by the corresponding spatial derivative operators in eq. (2.2).

However, in the bounce inflation scenario, the perturbation modes with $k/\mathcal{H}_{B+} > 1$ will be produced during the inflation after the bounce. When we take the short wavelength limit, $k/\mathcal{H}_{B+} \gg 1$, the $|c_{3,1} - c_{3,2}|^2$ acquires drastic oscillation and even diverges when $k/\mathcal{H}_{B+} \rightarrow \infty$. Without making qualitative deviation, we have

$$|c_{3,1} - c_{3,2}|^2 \approx 1 + \left(\frac{k}{\mathcal{H}_{B+}} \right)^{2n-2} \frac{A^{2n}}{\mathcal{H}_{B+}^{2-2n}} \cos^2 \left(\frac{k}{\mathcal{H}_{B+}} \right) \sin^2(2dA^n k^n). \quad (3.17)$$

The “1” in right-hand side of eq. (3.17) actually stands for the terms $\sim k^0$, such as $\cos^2(2dA^n k^n)$. Here, we do not specify it, since it makes no qualitative difference when $k/\mathcal{H}_{B+} \gg 1$. In order to satisfy the observations, eq. (3.17) should be nearly scale invariant. Thus the operator $R^{(3)}\delta g^{00} \sim k^2$ in EFT eq. (2.2) is applicable, but the operators $(R^{(3)})^2 \sim k^4$ and $\nabla_i R^{(3)} \nabla^i R^{(3)} \sim k^6$ will make eq. (3.17) diverge, since

$$|c_{3,1} - c_{3,2}|^2 \sim \left(\frac{k}{\mathcal{H}_{B+}} \right)^{2n-2}, \quad (3.18)$$

for $k/\mathcal{H}_{B+} \gg 1$, which are unacceptable. This result could be general, though the drastic oscillations in eq. (3.17) might be attributed to the matching method and the oversimplified approximation we have used.

4 Application: constructing a healthy G-bounce inflation model

In this section we apply the effective operator $R^{(3)}\delta g^{00}$ to cure the gradient instability faced by the G-bounce inflation model proposed in [7] (see also [30]). The G-bounce inflation background was built by using the cubic Galileon, which can be written in the EFT language as

$$\begin{aligned}
 c(t) &= \frac{1}{2}\dot{\phi}_0^2 \left(\mathcal{K}(\phi) + \mathcal{T}\dot{\phi}_0^2 \right) + \frac{1}{2}\dot{\phi}_0^2 \left(-\ddot{\phi}_0 + 3H\dot{\phi}_0 \right) G_{3X} - \dot{\phi}_0^2 G_{3\phi}, \\
 \Lambda(t) &= \frac{1}{4}\mathcal{T}(\phi)\dot{\phi}_0^4 + V(\phi) + \frac{1}{2}\dot{\phi}_0^2 \left(\ddot{\phi}_0 + 3H\dot{\phi}_0 \right) G_{3X}, \\
 M_2^4 &= \frac{1}{2}\mathcal{T}(\phi)\dot{\phi}_0^4 + \frac{1}{4} \left(\ddot{\phi}_0 + 3H\dot{\phi}_0 \right) \dot{\phi}_0^2 G_{3X} + \frac{3}{4}H\dot{\phi}_0^5 G_{3XX} - \frac{1}{4}\dot{\phi}_0^4 G_{3X\phi}, \\
 m_3^3 &= \dot{\phi}_0^3 G_{3X},
 \end{aligned} \tag{4.1}$$

where

$$\begin{aligned}
 \mathcal{K}(\phi) &= 1 - 2k_0 \left[1 + 2\kappa_1 \left(\frac{\phi}{M_p} \right)^2 \right]^{-2}, \quad \mathcal{T}(\phi) = \frac{t_0}{M_p^4} \left[1 + 2\kappa_2 \left(\frac{\phi}{M_p} \right)^2 \right]^{-2}, \\
 G_3(\phi, X) &= \frac{\theta X}{M_p^3} \left[1 + 2\kappa_2 \left(\frac{\phi}{M_p} \right)^2 \right]^{-2},
 \end{aligned} \tag{4.2}$$

and

$$V(\phi) = -V_0 e^{\bar{c}\phi/M_p} \left[1 - \tanh \left(\lambda_1 \frac{\phi}{M_p} \right) \right] + \Lambda_{\text{inf}}^4 \left(1 - \frac{\phi^2}{v^2} \right)^2 \left[1 + \tanh \left(\lambda_2 \frac{\phi}{M_p} \right) \right] \tag{4.3}$$

such that $V = -V_0 e^{\bar{c}\phi}$ for $\phi \ll -M_p/\lambda_1$ (responsible for the ekpyrotic contraction), and is $V = \Lambda_{\text{inf}}^4 (1 - \frac{\phi^2}{v^2})^2$ for $\phi \gg M_p/\lambda_2$ (responsible for the inflation after bounce). Here $k_0, t_0, \theta, \kappa_1, \kappa_2, \lambda_1, \lambda_2, V_0, \bar{c}, \Lambda_{\text{inf}}$ and v are constants.

However, the bounce with the cubic Galileon is pathological due to the existence of the no-go theorem. Actually, the gradient instability exists since $c_s^2 < 0$ around the bounce [7, 30]. As has been argued in section 2.2, it can be avoided by introducing an effective operator $\frac{\tilde{m}_4^2}{2} R^{(3)}\delta g^{00}$. By doing so, c_s^2 is modified to

$$c_s^2 = \frac{c'_3 - a^2 M_p^2}{a^2 c_1}, \tag{4.4}$$

where

$$c_1 = \frac{M_p^2}{(2HM_p^2 - m_3^3)^2} (3m_3^6 + 4H^2 \epsilon M_p^4 + 8M_p^2 M_2^4), \tag{4.5}$$

$$c_3 = \frac{aM_p^2}{H - m_3^3/(2M_p^2)} \left(1 + 2\frac{\tilde{m}_4^2}{M_p^2} \right). \tag{4.6}$$

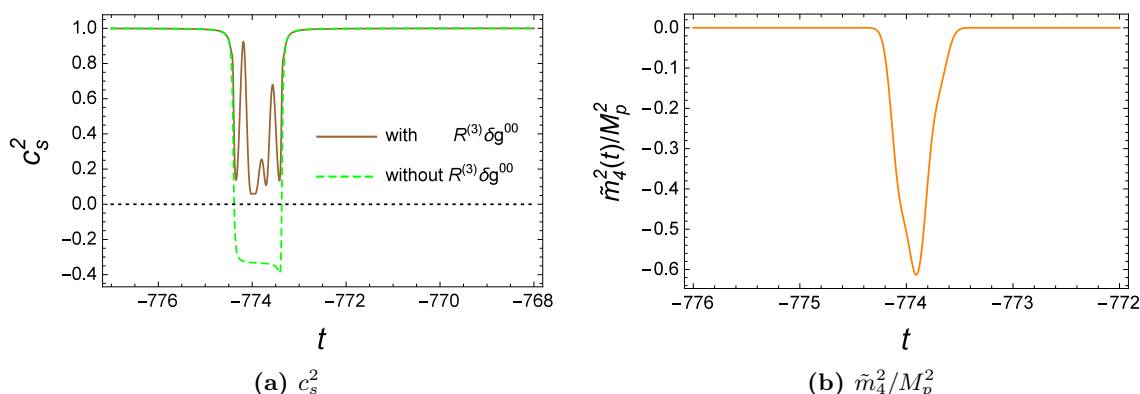


Figure 1. Left: the evolution of c_s^2 in G-bounce inflation model [7], right: the function of \tilde{m}_4^2 . We can see the c_s^2 can be modified to be larger than 0 by introducing the effective operator $\frac{\tilde{m}_4^2}{2}R^{(3)}\delta g^{00}$.

We are able to avoid the gradient instability by choosing a suitable $\tilde{m}_4^2(t)$. We have numerically calculated eqs. (4.1), see e.g., [7, 51], and plotted the evolution of c_s^2 in figure 1. The effect of $R^{(3)}\delta g^{00}$ on c_s^2 can be clearly seen in figure 1. Because c_1 is unaffected by \tilde{m}_4^2 , there is also no ghost instability, as demonstrated in [7]. Noting that the operator $\xi(t)R^{(3)}$ used in ref. [7] does not involve $R^{(3)}\delta g^{00}$.

5 Conclusion

Building classical nonsingular cosmological models is inspiring, since it offers us a self-consistent framework to deeply understand the physics of the primordial universe, even though we still don't know the complete theory of the quantum gravity. However, the popular nonsingular cosmological models based on the cubic Galileon are afflicted by the LMR no-go theorem, which means we have to go beyond the cubic Galileon to construct models without pathologies.

In this paper, we have explored the nonsingular cosmology within the framework of EFT. We have illustrated how to avoid the no-go theorem in theories beyond Horndeski, and pointed out how could the effective operators, such as $R^{(3)}\delta g^{00}$, $(R^{(3)})^2$ and $\nabla_i R^{(3)}\nabla^i R^{(3)}$, play significant roles in building healthy nonsingular cosmological models. We also have studied the perturbation evolution of these healthy models. We find that the spectrum of the primordial perturbation can be consistent with the observations.

We conclude that based on EFT, a fully healthy nonsingular bounce model can be built without begging any unknown physics. As an application of the EFT, we have presented a realistic healthy bounce inflation model by making use of the operator $R^{(3)}\delta g^{00}$. The study of classical nonsingular cosmology in the framework of EFT will be helpful for understanding the evolution and the gravity theory in the primordial universe.

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A The derivations of the quadratic actions for scalar and tensor perturbations

With the ADM line element given in eq. (2.1), we have

$$g_{\mu\nu} = \begin{pmatrix} N_k N^k - N^2 & N_j \\ N_i & h_{ij} \end{pmatrix}, \quad g^{\mu\nu} = \begin{pmatrix} -N^{-2} & \frac{N^j}{N^2} \\ \frac{N^i}{N^2} & h^{ij} - \frac{N^i N^j}{N^2} \end{pmatrix}, \quad (\text{A.1})$$

and $\sqrt{-g} = N\sqrt{h}$, where $N_i = h_{ij}N^j$, and the spatial indices are raised and lowered by the spatial metric h_{ij} . We can define the unit one-form tangent vector $n_\nu = n_0(dt/dx^\mu) = (-N, 0, 0, 0)$ and $n^\nu = g^{\mu\nu}n_\mu = (1/N, -N^i/N)$, which satisfies $n_\mu n^\mu = -1$. The induced 3-dimensional metric on the hypersurface is $H_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$, thus

$$H_{\mu\nu} = \begin{pmatrix} N_k N^k & N_j \\ N_i & h_{ij} \end{pmatrix}, \quad H^{\mu\nu} = \begin{pmatrix} 0 & 0 \\ 0 & h^{ij} \end{pmatrix}. \quad (\text{A.2})$$

Moreover, the extrinsic curvature on the hypersurface is

$$\begin{aligned} K_{\mu\nu} &\equiv \frac{1}{2}\mathcal{L}_n H_{\mu\nu} \\ &= \frac{1}{2N}(\dot{H}_{\mu\nu} - D_\mu N_\nu - D_\nu N_\mu), \end{aligned} \quad (\text{A.3})$$

where \mathcal{L}_n is the Lie derivative with respect to n^μ , and D_μ is the covariant derivative associate with $H_{\mu\nu}$. The Ricci scalar is decomposed as

$$R = R^{(3)} - K^2 + K_{\mu\nu}K^{\mu\nu} + 2\nabla_\mu(Kn^\mu - n^\nu\nabla_\nu n^\mu), \quad (\text{A.4})$$

where $R^{(3)}$ is the induced 3-dimensional Ricci scalar associated with $H_{\mu\nu}$. Note that in general, when there is a non-minimal coupling between the scalar field and R , the last term in eq. (A.4) cannot be discarded.

In action (2.2), we have defined

$$\delta g^{00} = g^{00} + 1, \quad (\text{A.5})$$

$$\delta K_{\mu\nu} = K_{\mu\nu} - H_{\mu\nu}H, \quad (\text{A.6})$$

$$\delta K^{\mu\nu} = K^{\mu\nu} - H^{\mu\nu}H, \quad (\text{A.7})$$

$$\delta K = \delta K_\mu^\mu = K_\mu^\mu - 3H. \quad (\text{A.8})$$

In the unitary gauge, we set

$$h_{ij} = a^2 e^{2\zeta} (e^\gamma)_{ij}, \quad \gamma_{ii} = 0 = \partial_i \gamma_{ij}. \quad (\text{A.9})$$

Moreover, N and N_i are expressed as $N = 1 + \alpha$ and $N_i = \partial_i \beta$. Then, it is straightforward to obtain

$$\delta g^{00} = 1 - \frac{1}{(1 + \alpha)^2}, \quad (\text{A.10})$$

$$R^{(3)} = -2a^{-2} e^{-2\zeta} \left[2\partial^2 \zeta + (\partial\zeta)^2 \right], \quad (\text{A.11})$$

$$\delta K_{ij} = \frac{1}{1 + \alpha} \left\{ a^2 (\dot{\zeta} - \alpha H) e^{2\zeta} \delta_{ij} - \partial_i \partial_j \beta + \partial_i \beta \partial_j \zeta + \partial_j \beta \partial_i \zeta - \partial_k \beta \partial_k \zeta \delta_{ij} \right\}, \quad (\text{A.12})$$

$$\delta K^{ij} = \frac{a^{-4} e^{-4\zeta}}{1 + \alpha} \left\{ a^2 (\dot{\zeta} - \alpha H) e^{2\zeta} \delta^{ij} - \partial_i \partial_j \beta + \partial_i \beta \partial_j \zeta + \partial_j \beta \partial_i \zeta - \partial_k \beta \partial_k \zeta \delta_{ij} \right\}, \quad (\text{A.13})$$

where $\partial^2 = \partial_i \partial_i$.

Substituting eqs. (A.10) to (A.13) into the action (2.2) and using the Hamiltonian constraints

$$\frac{\partial \mathcal{L}}{\partial \alpha} = 0, \quad \frac{\partial \mathcal{L}}{\partial (\partial^2 \beta)} = 0, \quad (\text{A.14})$$

we find

$$\alpha = A_1 \dot{\zeta} + A_2 \partial^2 \zeta, \quad \partial^2 \beta = B_1 \dot{\zeta} + B_2 \partial^2 \zeta, \quad (\text{A.15})$$

in which

$$\begin{aligned} A_1 &= \frac{2}{D} (f M_p^2 + 2m_4^2) \left[(2fH + \dot{f}) M_p^2 - m_3^3 + 4Hm_4^2 + 6H\bar{m}_4^2 \right], \\ A_2 &= \frac{2}{a^2 D} \left\{ M_p^2 \left[2f\bar{m}_4^2 + (2fH + \dot{f}) \bar{m}_5 \right] - (m_3^3 - 4Hm_4^2) \bar{m}_5 + 4\bar{m}_4^2 \tilde{m}_4^2 \right\}, \\ B_1 &= \frac{a^2}{D} \left\{ 3m_3^6 - 6\dot{f} m_3^3 M_p^2 + 8f M_2^4 M_p^2 + (4f^2 H^2 \epsilon + 2f \dot{f} H + 3\dot{f}^2 - 2f\ddot{f}) M_p^4 \right. \\ &\quad \left. + (4m_4^2 + 6\bar{m}_4^2) \left[4M_2^4 + (2fH^2 \epsilon + \dot{f}H - \ddot{f}) M_p^2 \right] \right\}, \\ B_2 &= \frac{2}{D} \left\{ \left[3Hm_3^3 + 4M_2^4 + (2fH^2 \epsilon - 2H\dot{f} - \ddot{f}) M_p^2 \right] \bar{m}_5 \right. \\ &\quad \left. - \left[(2fH + \dot{f}) M_p^2 - m_3^3 + 4Hm_4^2 + 6H\bar{m}_4^2 \right] (f M_p^2 + 2\tilde{m}_4^2) \right\}, \\ D &= \left[m_3^3 - 4Hm_4^2 - (2fH + \dot{f}) M_p^2 \right]^2 \\ &\quad + 2\bar{m}_4^2 \left[12H^2 m_4^2 + 4M_2^4 + (fH^2 (6 + 2\epsilon) + \dot{f}H - \ddot{f}) M_p^2 \right]. \end{aligned} \quad (\text{A.16})$$

Then, with eqs. (A.15), we obtain the quadratic action of scalar perturbation, which is displayed in eq. (2.5). Here, we write down the expressions of the coefficients in eq. (2.5):

$$\begin{aligned}
c_1 &= \frac{1}{D} (2m_4^2 + fM_p^2) \left\{ 3m_3^6 + 4f^2 H^2 \epsilon M_p^4 + 8M_2^4 (2m_4^2 + 3\bar{m}_4^2) \right. \\
&\quad + M_p^2 \left[-2\ddot{f} (2m_4^2 + 3\bar{m}_4^2) + \dot{f} \left(-6m_3^3 + 4Hm_4^2 + 3\dot{f}M_p^2 + 6H\bar{m}_4^2 \right) \right] \\
&\quad \left. + 2fM_p^2 \left[4M_2^4 - \ddot{f}M_p^2 + H \left(4H\epsilon m_4^2 + \dot{f}M_p^2 + 6H\epsilon\bar{m}_4^2 \right) \right] \right\}, \\
c_2 &= fM_p^2, \\
c_3 &= \frac{2a}{D} (2m_4^2 + fM_p^2) \left\{ 2f^2 HM_p^4 + \bar{m}_5 \left[(2H\dot{f} + \ddot{f})M_p^2 - 3Hm_3^3 - 4M_2^4 \right] \right. \\
&\quad + fM_p^2 \left[-m_3^3 + \dot{f}M_p^2 + 2H (2m_4^2 + 3\bar{m}_4^2 - H\epsilon\bar{m}_5 + 2\tilde{m}_4^2) \right] \\
&\quad \left. + \tilde{m}_4^2 \left(8Hm_4^2 - 2m_3^3 + 2\dot{f}M_p^2 + 12H\bar{m}_4^2 \right) \right\}, \\
c_4 &= \frac{2}{D} \left\{ 4\bar{\lambda}D + \left[12H^2 m_4^2 + 4M_2^4 + (H\dot{f} - \ddot{f}) M_p^2 \right] \bar{m}_5^2 \right. \\
&\quad - 2f^2 M_p^4 (\bar{m}_4^2 + 2H\bar{m}_5) + 4 \left(m_3^3 - 4Hm_4^2 - \dot{f}M_p^2 \right) \bar{m}_5 \tilde{m}_4^2 \\
&\quad + 2fM_p^2 \bar{m}_5 \left[m_3^3 - 4Hm_4^2 - \dot{f}M_p^2 + H^2(3 + \epsilon)\bar{m}_5 \right] \\
&\quad \left. - 8fM_p^2 (\bar{m}_4^2 + H\bar{m}_5) \tilde{m}_4^2 - 8\bar{m}_4^2 \tilde{m}_4^4 \right\}. \tag{A.17}
\end{aligned}$$

As for the tensorial part, we have $N = 1$, $N_i = 0$ and $\zeta = 0$. It is also straightforward to obtain

$$R^{(3)} = -\frac{1}{4} a^{-2} \gamma^{kl,i} \gamma_{kl,i} + \mathcal{O}(\gamma^3), \tag{A.18}$$

$$K_{ij} = a^2 \left[H\delta_{ij} + H\gamma_{ij} + \frac{1}{2} \dot{\gamma}_{ij} + \frac{1}{2} H\gamma_{ik} \gamma_j^k + \frac{1}{4} (\dot{\gamma}_{ik} \gamma_j^k + \gamma_{ik} \dot{\gamma}_j^k) \right] + \mathcal{O}(\gamma^3), \tag{A.19}$$

$$\delta K_{ij} = \frac{a^2}{2} \left[\dot{\gamma}_{ij} + \frac{1}{2} (\dot{\gamma}_{ik} \gamma_j^k + \gamma_{ik} \dot{\gamma}_j^k) \right] + \mathcal{O}(\gamma^3). \tag{A.20}$$

$$K^{ij} = a^{-2} \left[H\delta^{ij} + \frac{1}{2} \dot{\gamma}^{ij} - H\gamma^{ij} - \frac{1}{4} (\dot{\gamma}_i^j \gamma^{lj} + \gamma_i^j \dot{\gamma}^{lj}) + \frac{1}{2} H\gamma^{je} \gamma_e^i \right] + \mathcal{O}(\gamma^3), \tag{A.21}$$

$$\delta K^{ij} = \frac{a^{-2}}{2} \left[\dot{\gamma}^{ij} - \frac{1}{2} (\dot{\gamma}_i^j \gamma^{lj} + \gamma_i^j \dot{\gamma}^{lj}) \right] + \mathcal{O}(\gamma^3), \tag{A.22}$$

$$K = 3H + \mathcal{O}(\gamma^3). \tag{A.23}$$

Note that $\delta K = K - 3H$ contains only scalars up to the quadratic order, as well as the last term in eq. (A.4). Substituting the above results into action (2.2), we obtain the quadratic action of tensor perturbation, which is displayed in eq. (2.8).

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