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The Euler characteristic correction to the Kähler potential — revisited

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Abstract: We confirm the leading α'^3 correction to the 4d, $\mathcal{N} = 1$ Kähler potential of type IIB orientifold compactifications, proportional to the Euler characteristic of the Calabi-Yau threefold (BBHL correction). We present the explicit solution for the α'^3-modified internal background metric in terms of the non-harmonic part of the third Chern form of the leading order Calabi-Yau manifold. The corrected internal manifold is almost Calabi-Yau and admits an SU(3) structure with non-vanishing torsion. We also find that the full ten-dimensional Einstein frame background metric is multiplied by a non-trivial Weyl factor. Performing a Kaluza-Klein reduction on the modified background we derive the α'^3-corrected kinetic terms for the dilaton and the Kähler deformations of the internal Calabi-Yau threefold for arbitrary $h^{1,1}$. We analyze these kinetic terms in the 4d, $\mathcal{N} = 2$ un-orientifolded theory, confirming the expected correction to the Kähler moduli space pre-potential, as well as in the 4d, $\mathcal{N} = 1$ orientifolded theory, thus determining the corrections to the Kähler potential and Kähler coordinates.

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1 Introduction

Supersymmetric flux compactifications of type IIB superstring theory constitute a vast and rich arena for the study of the implications of string dynamics on 4d physics. Comprehensive reviews on the subject include [1–5]. A common feature of several flux compactification scenarios is the crucial role played by α' and/or g_s corrections to the leading-order effective action. One of the most prominent perturbative corrections to the 4d low-energy effective action is the α'^3 correction to the Kähler potential proportional to the Euler characteristic of the Calabi-Yau threefold used in the compactification. This correction was first computed in a paper by Becker, Becker, Haack, and Louis (BBHL) [6]. It plays in essential role in the Large Volume Scenario for type IIB compactifications [7–9].

The 10d origin of the Euler characteristic correction to the 4d Kähler potential resides in the leading α' corrections to the bulk type IIB supergravity action. This is an eight-derivative coupling built with four Riemann tensors, accompanied by several additional couplings involving other type IIB fields as a consequence of $\mathrm{SL}(2,\mathbb{Z})$ symmetry. For the derivation and discussion of these couplings see e.g. [10–19]. The authors of [6] infer the corresponding correction to the 4d Kähler potential in a somewhat indirect way. Exploiting mirror symmetry, the starting point of their analysis are the results of [20, 21] about the corrected metric of the hypermultiplet moduli space of a Calabi-Yau type II compactification to four dimensions with $\mathcal{N}=2$ supersymmetry. Making use of the results of [22] these corrections are then reformulated in terms of 4d field variables whose 10d origin is manifest, in such a way that the orientifold projection to $\mathcal{N}=1$ supersymmetry can be performed.

The aim of this paper is to provide a direct derivation of the Euler characteristic correction to the 4d Kähler potential by means of a completely explicit Kaluza-Klein reduction of the relevant α'^3 couplings in the 10d bulk action. Not only is this approach more transparent, but it also presents other advantages. Firstly, we do not have to make any assumption about the superpotential or the scalar potential of the full 4d, $\mathcal{N}=1$ low-energy effective action, since our conclusions are entirely drawn from the examination of the kinetic terms for the dilaton and the Kähler moduli of the Calabi-Yau threefold for arbitrary $h^{1,1}$. Secondly, our approach allows us to derive both the correction to the 4d Kähler potential and the correction to the Kähler coordinates as a function of the Kähler moduli. Finally, in the process of the derivation one necessarily has to analyze the background solution and show how it gets modified by the α' corrections under examination. In particular, we are able to provide an explicit solution for the α'-corrected internal metric in terms of the non-harmonic part of the third Chern form c_3 of the leading order Calabi-Yau threefold background. The corrected metric is Kähler but not Ricci-flat. As a result it no longer has SU(3) holonomy, but rather an SU(3) structure with non-vanishing torsion. The corrected geometry fits in the framework of almost Calabi-Yau manifolds [23]. Similar results were derived in [24] in the case of M-theory compactifications on a Calabi-Yau fourfold. Additionally, we find that the 10d background metric is corrected by an overall Weyl factor at order α'^3, in analogy with the findings of the three-dimensional M-theory analysis of [24–28]. Throughout this paper, we let higher derivatives act in the internal space, working at two-derivative level in the external directions. Recently, α'^3 external four-derivative couplings between Kähler moduli and gravity were derived in [29] for the same class of type IIB orientifold setups considered in this work, with potential applications to Kähler moduli inflation studies.

The main results of our paper are summarized in equations (4.18) to (4.21). We reproduce the Euler characteristic correction to the Kähler potential originally found by BBHL [6]. We also compute the leading α'^3 corrections to the Kähler coordinates and the 4d axio-dilaton in terms of the Kähler moduli, and find that they are vanishing. Finally, we exclude the possibility that the effect of the correction to the Kähler potential can be undone by a Kähler coordinate redefinition, and we reformulate our findings in the formalism of linear multiplets [30–32] in order to elucidate the physical relevance of the correction. Let us remark that our analysis does not take into account explicitly localized sources of the $\mathcal{N}=1$ setup, such as seven-branes and orientifold planes.

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This paper is organized as follows. Section 2 is devoted to the computation of the relevant α' corrections to the 4d kinetic terms of the dilaton and Kähler moduli. In particular, in section 2.1 we present the relevant higher-derivative corrections to the 10d action of type IIB supergravity, in section 2.2 we discuss the background solutions of interest, while section 2.3 is devoted to the dimensional reduction to four dimensions. The results of the computation are then analyzed in section 3 in the context of the 4d, $\mathcal{N}=2$ effective theory obtained in the absence of orientifold planes and D-branes. We show that our findings are compatible with the expected perturbative correction to the prepotential for the Kähler moduli space metric. We then proceed in section 4 to analyze the 4d, $\mathcal{N}=1$ setup obtained after implementation of an orientifold projection. We verify that the α'-corrected kinetic terms of the dilaton and Kähler moduli can be written in terms of a corrected Kähler potential. We determine the latter, reproducing the correction of BBHL [6], and we identify the α'-corrected form of the Kähler coordinates. Section 5 summarizes our conclusions. Our conventions, together with useful identities and some technical material, can be found in the appendices.

2 Four-dimensional α'^3 Lagrangian

This section discusses the dimensional reduction of IIB supergravity action including a suitable class of eight-derivative corrections, on a Calabi-Yau threefold to four dimensions. In particular, we restrict our analysis to purely gravitational terms and dilaton terms. We fluctuate the background metric by Kähler deformations and focus on couplings that carry two external spacetime derivatives and are at most quadratic in the infinitesimal Kähler deformations. We first review the relevant eight-derivative α'^3 corrections to 10d type IIB supergravity and the supersymmetric background.

2.1 Type IIB higher-derivative action

The starting point of our analysis consists of the type IIB supergravity action including the leading order α'^3 eight-derivative correction built with four Riemann tensors [10–19]. For our purposes this gravitation coupling has to be supplemented with suitable dilaton couplings discussed below.

In order to set up our notation, let us first record the gravitational and axio-dilaton terms in the two-derivative type IIB supergravity in the Einstein frame,

$$S^{(2)}_{R,\tau} = \frac{1}{2\kappa_{10}^2}\int \left(R - \frac{1}{2\tau_2^2}\nabla_M\tau\nabla^M\overline{\tau}\right) *_{10} 1\,, \tag{2.1}$$

where $2\kappa_{10}^2 = (2\pi)^7\alpha'^4$, $M=0,\ldots,9$ is a 10d world index, and the axio-dilaton is defined as

$$\tau = \tau_1 + i\,\tau_2 = C_0 + i\,e^{-\hat{\phi}}\,. \tag{2.2}$$

The action (2.1) is invariant under the $\mathrm{SL}(2,\mathbb{Z})$ symmetry of type IIB superstring theory, under which the Einstein frame metric is invariant and the axio-dilaton transforms as

$$\tau' = \frac{a\,\tau + b}{c\,\tau + d}\,, \qquad \begin{pmatrix} a & b \\ c & d\end{pmatrix} \in \mathrm{SL}(2,\mathbb{Z})\,. \tag{2.3}$$

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Next, let us consider the α'^3 correction constructed with four Riemann tensors. It takes the form

$$S^{(8)}_{R^4} = \frac{1}{2\kappa_{10}^2}\cdot\frac{\alpha'^3}{3\cdot 2^{12}}\int f_0(\tau,\overline{\tau})\left(t_8t_8+\frac{1}{8}\epsilon_{10}\epsilon_{10}\right)R^4\ *_{10}1\,, \tag{2.4}$$

where the explicit tensor contractions are given by

$$\begin{aligned}\epsilon_{10}\epsilon_{10}R^4 &= \epsilon^{R_1R_2M_1\ldots M_8}\epsilon_{R_1R_2N_1\ldots N_8}R^{N_1N_2}{}_{M_1M_2}R^{N_3N_4}{}_{M_3M_4}R^{N_5N_6}{}_{M_5M_6}R^{N_7N_8}{}_{M_7M_8}\,,\\ t_8t_8R^4 &= t_8^{M_1\ldots M_8}t_{8\,N_1\ldots N_8}R^{N_1N_2}{}_{M_1M_2}R^{N_3N_4}{}_{M_3M_4}R^{N_5N_6}{}_{M_5M_6}R^{N_7N_8}{}_{M_7M_8}\,,\end{aligned} \tag{2.5}$$

and the tensor t_8 is defined in terms of the metric in the standard way [12]. We have also introduced the function $f_0(\tau,\overline{\tau})$, which is the non-holomorphic, $\mathrm{SL}(2,\mathbb{Z})$-invariant function defined as

$$f_0(\tau,\overline{\tau}) = \sum_{(m,n)\neq(0,0)}\frac{\tau_2^{3/2}}{|m+n\,\tau|^3}\ . \tag{2.6}$$

It is useful to note that in the large τ_2 limit, which corresponds to the small string coupling limit, we have

$$f_0(\tau,\overline{\tau}) = 2\zeta(3)\,\tau_2^{3/2}+\frac{2\pi^2}{3}\tau_2^{-1/2}+\mathcal{O}(e^{-2\pi\tau_2})\ . \tag{2.7}$$

Our analysis requires the consideration of additional higher-derivative terms involving gradients of the dilaton. Such terms can be obtained following the approach of [14, 17, 18], by replacing each occurrence of the Riemann tensor in (2.5) according to

$$R_{MN}{}^{PQ}\to R_{MN}{}^{PQ}+\tilde{c}_1 g_{[M}{}^{[P}\nabla_{N]}\nabla^{Q]}\hat{\phi}+\tilde{c}_2 g_{[M}{}^{[P}\nabla_{N]}\hat{\phi}\nabla^{Q]}\hat{\phi}+\tilde{c}_3 g_{[M}{}^{[P}g_{N]}{}^{Q]}\nabla_K\phi\nabla^K\hat{\phi}\,, \tag{2.8}$$

where antisymmetrizations are performed with weight one, and $\tilde{c}_1$, $\tilde{c}_2$, $\tilde{c}_3$ are real numerical coefficients that we leave unfixed for now.[1] In order to achieve $\mathrm{SL}(2,\mathbb{Z})$-invariance several other terms have to be added to the action [33, 34], but they are not relevant for our discussion.

In summary, the total action utilized in the analysis of the following sections is obtained by summing the two-derivative terms in (2.1) with the terms generated by the replacement rule (2.8) in the terms given in (2.5). We only keep terms which can correct the kinetic terms of the resulting 4d dilaton and discarding higher derivative terms thereof, as well as terms for the axion C_0. We also retain only the leading term in the large τ_2 expansion of $f_0(\tau,\overline{\tau})$, see (2.7). We thus obtain

$$\begin{aligned}S = \frac{1}{2\kappa_{10}^2}\int\Bigg[&R-\frac{1}{2}\nabla_M\phi\nabla^M\phi+\frac{\zeta(3)\alpha'^3}{3\cdot 2^{11}}\,e^{-\frac{3}{2}\phi}\left(t_8t_8+\frac{1}{8}\epsilon_{10}\epsilon_{10}\right)R^4\\ &+\frac{\zeta(3)\alpha'^3}{3\cdot 2^{11}}\,e^{-\frac{3}{2}\phi}\left(\tilde{c}_1\nabla^M\nabla_M\hat{\phi}\,f_1+\tilde{c}_2\,\nabla_M\hat{\phi}\,\nabla_N\hat{\phi}\,f_2^{MN}+\tilde{c}_3\,\nabla^M\hat{\phi}\,\nabla_M\hat{\phi}\,f_3\right)\Bigg]*_{10}1\,,\end{aligned} \tag{2.9}$$

[1] The analysis of the 4-point tree-level dilaton scattering amplitude gives the value $\tilde{c}_1=-1$ (see [14, 17, 18] and also [33, 34]), while $\tilde{c}_2$, $\tilde{c}_3$ cannot be fixed in this way. In what follows, however, we proceed treating all $\tilde{c}$ coefficients on the same footing.

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where the quantities f_1, f_2^{MN}, f_3 are homogeneous polynomials of degree three in the components of the Riemann tensor and are given explicitly in appendix B. For notational convenience let us introduce the dimensionful constant

$$\alpha = \frac{\zeta(3)\alpha'^3}{3 \cdot 2^{11}} \,, \tag{2.10}$$

which plays the role of the small expansion parameter relevant for our problem. Note that we adopt conventions in which the 10d metric components and the dilaton are dimensionless, while coordinates x^M have dimension of length.

2.2 Supersymmetric background

In this section we study how the supersymmetric Calabi-Yau background solutions of type IIB supergravity are modified in the presence of the higher-derivative terms in (2.9) and we present the relevant dilaton and Kähler fluctuations around the corrected background. Let us note that we cannot analyze directly the supersymmetry properties of the background, since the type IIB supersymmetry variations are not completely known at the required order in α', but we can give necessary conditions for the α' modification of the background by solving the equations of motion.

2.2.1 Corrections to the background

Our problem fits into the framework of supersymmetric flux compactifications of type IIB superstring on a Calabi-Yau threefold Y_3. Let us first recall some facts about these setups neglecting higher-derivative corrections to the 10d supergravity action [35]. For compactifications to flat four-dimensional spacetime the background metric has the form

$$ds_{10}^2 = e^{2A}\,\eta_{\mu\nu}dx^\mu dx^\nu + e^{-2A}\,g^{(0)}_{ab}dy^a dy^b \quad , \tag{2.11}$$

where $\mu,\nu = 0,1,2,3$ are 4d external spacetime world indices, x^μ are Cartesian coordinates, $\eta_{\mu\nu}$ is the Minkowski metric, $a,b = 1,\ldots 6$ are real internal world indices associated to the coordinates y^a, and $g^{(0)}_{ab}$ denotes the Ricci-flat metric on Y_3. The warp factor A only depends on the internal coordinates and also determines the background F_5 flux via

$$F_5 = (1 + *_{10})\, de^{4A} \wedge dx^0 \wedge dx^1 \wedge dx^3 \wedge dx^4 \,. \tag{2.12}$$

The F_5 flux has to obey the Bianchi identity

$$dF_5 = H_3 \wedge F_3 + \rho_6 \,, \tag{2.13}$$

where $H_3 = dB_2$ and $F_3 = dC_2 - C_0 dB_2$ are the usual NSNS and RR three-form field strengths and the six-form ρ_6 encodes the D3-brane charge density associated with possible localized sources. Integrating (2.13) on the internal manifold yields the D3-brane tadpole cancellation condition

$$\frac{1}{2\kappa_{10}^2 T_3}\int_{Y_3} H_3 \wedge F_3 + Q_{\rm D3} = 0 \,, \tag{2.14}$$

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where $T_3 = (2\pi)^{-3}(\alpha')^{-2}$ is the D3-brane tension and $Q_{\rm D3}$ is the total D3-brane charge, proportional to the integral of ρ_6.[2] If all local sources are removed, all fluxes have to vanish and the warp factor is necessarily trivial.

The inclusion of higher-derivative corrections to the type IIB bulk action induces modifications in the previous picture. In what follows we analyze the 10d equations of motion in presence of higher-derivative corrections of order $\alpha \propto \alpha'^3$ but without introducing any local source in the problem. Even though this setup yields an effective 4d, $\mathcal{N} = 2$ low-energy effective action, we argue that the following analysis is sufficient for the purpose of studying the Euler characteristic correction to the Kähler potential in the 4d, $\mathcal{N} = 1$ theory after performing the orientifold truncation in section 4.

We adopt the following α'-corrected Ansatz for the 10d background metric,

$$ds_{10}^2 = e^{\Phi}\left[e^{2A}\,\eta_{\mu\nu}dx^\mu dx^\nu + e^{-2A}\,g_{ab}dy^a dy^b\right]\,, \tag{2.15}$$

where

$$\begin{aligned}\Phi &= \alpha\,\Phi^{(1)} + \mathcal{O}(\alpha^2)\,,\\ A &= \alpha\,A^{(1)} + \mathcal{O}(\alpha^2)\,,\\ g_{ab} &= g^{(0)}_{ab} + \alpha\,g^{(1)}_{ab} + \mathcal{O}(\alpha^2)\,.\end{aligned} \tag{2.16}$$

The quantity Φ is an overall 10d Weyl rescaling of the metric, and it has been introduced in analogy with the analysis of [24]. This parametrization of the prefactors multiplying the internal and external metric is general and is useful for the following discussion. The zeroth-order metric is the Calabi-Yau Ricci-flat metric. Accordingly, Φ and A have no $\mathcal{O}(\alpha^0)$ term. All quantities depend exclusively on the internal coordinates in order not to break Poincaré invariance in the external directions. The corrections to the dilaton profile are parametrized as

$$\hat{\phi} = \phi_0 + \alpha\,\phi^{(1)} + \mathcal{O}(\alpha^2)\,, \tag{2.17}$$

where ϕ_0 is the constant uncorrected dilaton VEV.

The 10d Einstein equation at order α can be written in the form

$$0 = R^{(1)}_{MN} - \frac{1}{2}(R^{(1)}_{PQ}\,g^{(0)PQ})\,g^{(0)}_{MN} + \mathcal{T}_{MN}\,. \tag{2.18}$$

The first two terms capture the contribution coming from the two-derivative part of the Einstein equation evaluated on the α-corrected Ansatz (2.15). The symbol $R^{(1)}_{MN}$ is used to denote the $\mathcal{O}(\alpha)$ part of the 10d Ricci tensor computed using the metric (2.15), while $g^{(0)}_{MN}$ is used for the $\mathcal{O}(\alpha^0)$ part of (2.15). The quantity $\mathcal{T}_{MN}$ encodes all the contributions coming from the higher-derivative part of the Einstein equation, derived from (2.9), upon evaluation on the $\mathcal{O}(\alpha^0)$ part of the metric Ansatz (2.15). We find

$$\mathcal{T}_{\mu\nu} = 0\,, \qquad \mathcal{T}_{\mu a} = 0\,, \qquad \mathcal{T}_{ab} = 768\,\alpha\,e^{-\frac{3}{2}\phi_0}\,J^{(0)}{}_a{}^c J^{(0)}{}_b{}^d\,\nabla^{(0)}_c\nabla^{(0)}_d Q\,, \tag{2.19}$$

[2] Recall that $Q_{\rm D3}$ generically receives contributions not only from $D3$-branes and $O3$-planes, but also from higher-dimensional defects with flux- and/or geometry-induced $D3$-brane charge, such as $D7$-branes and $O7$-planes [35].

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where $\nabla^{(0)}$, $J^{(0)}$ denote the Levi-Civita connection and complex structure associated to the zeroth-order metric, respectively, and the quantity Q is the six-dimensional Euler density,

$$Q = \frac{1}{12}\left(R^{(0)}_{a_1a_2}{}^{a_3a_4} R^{(0)}_{a_3a_4}{}^{a_5a_6} R^{(0)}_{a_5a_6}{}^{a_1a_2} - 2R^{(0)a_2}_{a_1}{}_{b_1}{}^{b_2} R^{(0)a_3}_{a_2}{}_{b_2}{}^{b_3} R^{(0)a_1}_{a_3}{}_{b_3}{}^{b_1}\right) . \tag{2.20}$$

This object satisfies

$$Q = (2\pi)^3 \, *_6^{(0)} c_3^{(0)} , \qquad \int Q \, *_6^{(0)} 1 = (2\pi)^3 \chi , \tag{2.21}$$

where χ is the Euler characteristic of the internal space, $*_6^{(0)}$ is the Hodge star operator with respect to the zeroth-order metric, and c_3 is the third Chern form built from $g^{(0)}_{ab}$, defined in appendix A.

It is convenient to use holomorphic and antiholomorphic indices $m = 1, 2, 3$, $\bar{m} = \bar{1}, \bar{2}, \bar{3}$ associated to the zeroth-order complex structure, in such a way that $J^{(0)}{}_m{}^n = +i\delta_m{}^n$. One may then check that all components of the order-α Einstein's equation are solved by imposing

$$\Phi^{(1)} = -192\, e^{-\frac{3}{2}\phi_0}\, Q , \quad R^{(1)}_{m\bar{n}} = -1536\, e^{-\frac{3}{2}\phi_0}\, \nabla^{(0)}_m \nabla^{(0)}_{\bar{n}} Q , \quad R^{(1)}_{mn} = 0 , \quad R^{(1)}_{\bar{m}\bar{n}} = 0 , \quad A^{(1)} = 0 , \tag{2.22}$$

where $R^{(1)}$ denotes the linearized Ricci tensor of the metric correction $g^{(1)}$. Let us also point out that we can exhibit an explicit expression for $g^{(1)}$. To this end we start recalling that, as a consequence of the $\partial\bar{\partial}$-lemma, the $(3,3)$-form c_3 can be decomposed as

$$c_3 = Hc_3 + i\partial\bar{\partial}F , \tag{2.23}$$

where Hc_3 denotes the harmonic part of c_3 with respect to the zeroth-order metric and F is a suitable co-closed $(2,2)$-form.[3] We can express the Euler density Q in terms of F as

$$(2\pi)^{-3} Q = *_6^{(0)} Hc_3 + \frac{1}{2} \Delta^{(0)} \, *_6^{(0)} (J^{(0)} \wedge F) , \tag{2.24}$$

where $\Delta^{(0)} = 2g^{(0)m\bar{n}} \nabla^{(0)}_m \nabla^{(0)}_{\bar{n}}$ denotes the scalar Laplacian of the zeroth-order metric, and we exploited the co-closure of F. Let us also stress that the first term in (2.24) is constant on the threefold as a result of the harmonic projection. Utilizing (2.24) one can observe that the equations for $R^{(1)}$ in (2.22) can be solved by setting

$$g^{(1)}_{mn} = 0 , \quad g^{(1)}_{\bar{m}\bar{n}} = 0 , \quad g^{(1)}_{m\bar{n}} = -1536(2\pi)^3 \, e^{-\frac{3}{2}\phi_0} \, \nabla^{(0)}_m \nabla^{(0)}_{\bar{n}} \, *_6^{(0)} (J^{(0)} \wedge F) . \tag{2.25}$$

[3] The existence and co-closure of F can be seen by the following argument, similar to an argument in [36]. Let us first apply the Hodge decomposition theorem to the 6-form c_3 and write $c_3 = Hc_3 + db_5$ for a suitable globally defined 5-form b_5. Since the form db_5 is d-exact, ∂-closed, and $\bar{\partial}$-closed, the $\partial\bar{\partial}$-lemma ensures that it is $\partial\bar{\partial}$-exact, so that $d\beta_5 = \partial\bar{\partial} f_4$ for a globally defined 4-form f_4. We can now apply the Hodge decomposition theorem to f_4 and write $f_4 = Hf_4 + dg_3 + \delta g_5$, where δ denotes the codifferential and g_3, g_5 are a globally defined 3- and 5-form, respectively. We now note that Hf_4 is harmonic on a Kähler manifold and hence $\bar{\partial}$-closed, and that $\partial\bar{\partial} d = 0$. It follows that we can write $c_3 = Hc_3 + \partial\bar{\partial}\delta g_5$. We can thus set $F = -i\delta g_5$, which is co-closed because it is co-exact.

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As we can see, the corrected metric $g^{(0)}_{m\bar{n}} + \alpha g^{(1)}_{m\bar{n}}$ is still Kähler and belongs to the same Kähler class as $g^{(0)}_{m\bar{n}}$.

The dilaton also receives a correction sourced by the higher-derivative terms in (2.9). The relevant terms in the dilaton equation of motion yield the relation

$$0 = g^{(0)\bar{n}m}\nabla^{(0)}_m\nabla^{(0)}_{\bar{n}}\phi^{(1)} - 1152\, e^{-\frac{3}{2}\phi_0}\,\tilde{c}_1\, g^{(0)\bar{n}m}\nabla^{(0)}_m\nabla^{(0)}_{\bar{n}}Q\,, \tag{2.26}$$

and is solved by

$$\phi^{(1)} = 1152\,\tilde{c}_1\, e^{-\frac{3}{2}\phi_0}Q\ . \tag{2.27}$$

An integration constant has been set to zero because, up to $\mathcal{O}(\alpha^2)$ terms, it can always be reabsorbed in ϕ_0. Our findings about the modification of the background dilaton profile and internal metric are in line with previous work [13, 37–40].

In closing this section let us remark that a full determination of F_1, G_3, F_5 at order α would require a detailed knowledge of order-α corrections to their equations of motion, which are related to the $\mathrm{SL}(2,\mathbb{Z})$ completion of the higher-derivative terms recorded in (2.9). This investigation is beyond the scope of the present work. Since we focus on couplings involving the metric and the dilaton, however, we only need the expressions (2.22) and (2.27) for the order-α Weyl rescaling factor and dilaton correction.

2.2.2 SU(3) structure on the internal manifold

In this section we show how the SU(3) holonomy of the zeroth-order Calabi-Yau internal metric is deformed to a specific SU(3) structure of the α'^3-modified background. Our discussion applies to $g^{(0)}_{ab} + \alpha\, g^{(1)}_{ab}$ and does not involve the overall Weyl rescaling factor of the 10d metric.

The Kähler form and the $(3,0)$-form of the uncorrected Calabi-Yau metric are subject to α' corrections but remain nonetheless globally defined on the internal space. As a result, the structure group of the internal manifold is reduced from SO(6) to SU(3) even after taking corrections into account. This guarantees the existence of local SU(3) coframes, consisting of triplets e^I, $I = 1,2,3$, of complex one-forms together with their complex conjugates $\bar{e}^{\bar{I}}$, with the property that $\{e^I, \bar{e}^{\bar{I}}\}$ is a local basis of $T^*Y_3\otimes\mathbb{C}$ and on overlapping patches the transition functions for e^I take values in SU(3). We may express locally J and Ω, as well as the metric, in terms of the SU(3) coframe as

$$J = i\,\delta_{I\bar{J}}\, e^I\wedge\bar{e}^{\bar{J}}\,, \qquad \Omega = \frac{1}{3!}\,\epsilon_{IJK}\, e^I\wedge e^J\wedge e^K\,, \qquad ds^2 = 2\,\delta_{I\bar{J}}\, e^I\,\bar{e}^{\bar{J}}\,, \tag{2.28}$$

where $\epsilon_{123} = 1$. The coframe can be written as the sum of an uncorrected contribution and a correction,

$$e^I = e^{(0)I} + \alpha\, e^{(1)I}\ . \tag{2.29}$$

Since the uncorrected geometry is Kähler, we can adopt complex coordinates and choose the coframe $e^{(0)I}$ in such a way that its only non-zero components are ${e^{(0)I}}_m$, ${\bar{e}^{(0)\bar{I}}}_{\bar{n}}$.

The relevant correction to the SU(3) coframe takes the form

$$e^{(1)I} = -768(2\pi)^3\, e^{-\frac{3}{2}\phi_0}\,{e^{(0)I}}_n\,\nabla^{(0)}_m\nabla^{(0)n}\,{*}^{(0)}_6\,(J^{(0)}\wedge F)\,dz^m\ . \tag{2.30}$$

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It is indeed straightforward to check that the associated correction to the metric reproduces $g^{(1)}$ in (2.25). We also have

$$J = J^{(0)} + \alpha\, J^{(1)}\,, \qquad J^{(1)} = -1536 i\,(2\pi)^3 e^{-\frac{3}{2}\phi_0}\,\partial^{(0)}\bar\partial^{(0)}\,*_6^{(0)}\,(J^{(0)}\wedge F)\,, \tag{2.31}$$

and therefore

$$dJ^{(1)} = 0\ . \tag{2.32}$$

The correction to the $(3,0)$ form reads

$$\Omega = \Omega^{(0)} + \alpha\,\Omega^{(1)}\,, \qquad \Omega^{(1)} = \left[-384\,(2\pi)^3 e^{-\frac{3}{2}\phi_0}\,\Delta^{(0)}\,*_6^{(0)}\,(J^{(0)}\wedge F)\right]\Omega^{(0)}\,, \tag{2.33}$$

which implies

$$d\Omega^{(1)} = -768\,(2\pi)^3 e^{-\frac{3}{2}\phi_0}\,dQ\wedge\Omega^{(0)}\ . \tag{2.34}$$

We can summarize our conclusions in the language of SU(3) torsion classes, reviewed for instance in [41]. The most general SU(3) structure can be described by

$$\begin{aligned} dJ &= -\frac{3}{2}\mathrm{Im}(\mathcal{W}_1\,\overline{\Omega}) + \mathcal{W}_4\wedge J + \mathcal{W}_3\,,\\ d\Omega &= \mathcal{W}_1\,J\wedge J + \mathcal{W}_2\wedge J + \overline{\mathcal{W}}_5\wedge\Omega\,, \end{aligned} \tag{2.35}$$

with suitable torsion classes $\mathcal{W}_1$, $\mathcal{W}_2$, $\mathcal{W}_3$, $\mathcal{W}_4$, $\overline{\mathcal{W}}_5$. We can then see that in our case all torsion classes vanish, with the exception of

$$\overline{\mathcal{W}}_5 = 0 + \alpha\,\overline{\mathcal{W}}_5^{(1)}\,, \qquad \overline{\mathcal{W}}_5^{(1)} = -768\,(2\pi)^3 e^{-\frac{3}{2}\phi_0}\,\bar\partial^{(0)}Q\ . \tag{2.36}$$

The corrected geometry is still Kähler and the manifold is an almost Calabi-Yau threefold [23]. The situation at hand can be compared to considering, for instance, a quintic in $\mathbb{P}^4$ endowed with the metric induced by the ambient space Fubini-Study metric. It is interesting to point out, however, that if the requirement of simple connectedness is relaxed, it is possible to have Ricci-flat Kähler manifolds that are almost Calabi-Yau, but not Calabi-Yau. An example is furnished by the Enriques surfaces; recently, novel examples have been constructed in [42] in the context of solvmanifolds.

2.2.3 Fluctuations associated to Kähler moduli

In order to derive the relevant couplings in the four-dimensional effective action of type IIB compactified on Y_3 we need to activate Kähler structure deformations of the internal metric entering the 10d background solution. We thus imagine to pick a fixed, reference point in the complex structure and Kähler structure moduli space of Y_3 and to switch on small deformations in the Kähler structure moduli space directions.

It is well-known that, at zeroth-order in α, the Kähler structure deformations of the internal Ricci-flat metric take the form

$$\delta g^{(0)}_{m\bar n} = -i\,\delta v^i\,\omega_{im\bar n}\,, \tag{2.37}$$

where $i = 1,\ldots,h^{1,1}(Y_3)$, δv^i are real deformation parameters, and $\omega_{im\bar n}$ denote a basis of harmonic $(1,1)$-forms whose cohomology classes are Poincaré dual to an integral basis of the homology $H_4(Y_3,\mathbb{Z})$.

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At first order in α the structure of the internal metric deformations can be written as

$$\delta(g^{(0)}_{m\bar{n}} + \alpha\, g^{(1)}_{m\bar{n}}) = -i\,\delta v^i\,(\omega_{im\bar{m}} + \alpha\,\nabla^{(0)}_m\nabla^{(0)}_{\bar{n}}\rho_i) + \alpha\,\nabla^{(0)}_m\nabla^{(0)}_{\bar{n}}\delta\tilde{F} + \mathcal{O}(\alpha^2)\ . \tag{2.38}$$

The ρ_i functions parametrize deviations from the harmonic representative within each cohomology class in $H^{1,1}(Y_3)$, while $\delta\tilde{F}$ denotes the variation of the function $\tilde{F} = *^{(0)}_6(J^{(0)}\wedge F)$ appearing in (2.25) induced by the zeroth-order deformation (2.37). Both kinds of modifications can be combined in a single expression of the form

$$\delta(g^{(0)}_{m\bar{n}} + \alpha\, g^{(1)}_{m\bar{n}}) = -i\,\delta v^i\,(\omega_{im\bar{m}} + \alpha\,\nabla^{(0)}_m\nabla^{(0)}_{\bar{n}}\lambda_i) + \mathcal{O}(\alpha^2)\,, \tag{2.39}$$

for suitable functions λ_i. Similar results were found in [25, 26]. Crucially, these functions drop out from the dimensional reduction discussed in the next section. As a consequence, we do not need to discuss their specific form, and moreover it seems that they do not have any physical significance. Let us point out that the deformation (2.37) of the zeroth-order metric induces also a modification of the quantity Q entering the order α expressions for the Weyl rescaling function $\Phi^{(1)}$ and the dilaton correction $\phi^{(1)}$. Since we are working up to quadratic order in fluctuations, we can consider a truncated Taylor series expansion for Q of the form

$$Q[g^{(0)} + \delta g^{(0)}] = Q + Q_i\delta v^i + \frac{1}{2}Q_{ij}\delta v^i\delta v^j\ . \tag{2.40}$$

In order to compute the 4d effective action we treat the fluctuation parameters δv^i as arbitrary functions of the external coordinates. In a similar way, we promote the $\mathcal{O}(\alpha^0)$ dilaton profile ϕ_0 from a constant to an arbitrary function of external spacetime. In summary, the Ansatz utilized in the dimensional reduction takes the form

$$\begin{aligned} ds^2_{10} = \exp\left\{-192\,\alpha\, e^{-\frac{3}{2}\phi(x)}\left(Q + Q_i\delta v^i + \frac{1}{2}Q_{ij}\delta v^i\delta v^j\right)\right\}\times \\ \times\left[g_{\mu\nu}dx^\mu dx^\nu + 2\Big(g^{(0)}_{m\bar{n}} + \alpha\, g^{(1)}_{m\bar{n}} - i\delta v^i(\omega_{im\bar{n}} + \alpha\nabla^{(0)}_m\nabla^{(0)}_{\bar{n}}\lambda_i)\Big)dz^m d\bar{z}^{\bar{n}}\right] \\ + \mathcal{O}(\alpha^2) + \mathcal{O}(\delta v^3)\,, \end{aligned} \tag{2.41}$$

$$\hat{\phi}(x,y) = \phi(x) + 1152\,\tilde{c}_1\,\alpha\, e^{-\frac{3}{2}\phi(x)}\left(Q + Q_i\delta v^i + \frac{1}{2}Q_{ij}\delta v^i\delta v^j\right) + \mathcal{O}(\alpha^2) + \mathcal{O}(\delta v^3)\,. \tag{2.42}$$

Note that we have replaced the external Minkowski metric with an arbitrary metric $g_{\mu\nu}(x)$. We will drop the explicit dependence of the spacetime coordinates, writing ϕ for $\phi(x)$, for notational simplicity in the following.

2.3 Reduction results

This section is devoted to the discussion of the results of the dimensional reduction of the various terms in (2.9) according to the Ansatz (2.41). In section 2.3.1 we present the outcome of the computation and we address the problem of uplifting it from small fluctuations δv^i to a full, non-linear dependence on the Kähler structure moduli. Section 2.3.2 is then devoted to the Weyl rescaling that casts the 4d Einstein-Hilbert term into canonical form.

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2.3.1 Results and uplift

To begin with, the reduction of the Einstein-Hilbert term to four dimensions yields

$$\int_{\mathcal{M}_{10}} R *_{10} 1 \longrightarrow \int_{\mathcal{M}_4} \left(\Omega\, R + P_{ij}\, \nabla_\mu \delta v^i \nabla^\mu \delta v^j + P_i\, \nabla_\mu \delta v^i \nabla^\mu \phi\right) *_4 1 \ , \tag{2.43}$$

with

$$\Omega = \int_{Y_3} \left[1 - i\delta v^i\, \omega_{im}{}^m + \frac{1}{2}\delta v^i \delta v^j (\omega_{im\bar{n}}\omega_j{}^{\bar{n}m} - \omega_{im}{}^m \omega_{jn}{}^n) - 384 \cdot 2\alpha e^{-\frac{3}{2}\phi_0}\, Q\right] *_6^{(0)} 1, \tag{2.44}$$

$$P_{ij} = \int_{Y_3} \left[\frac{1}{2}\omega_{im\bar{n}}\omega_j{}^{\bar{n}m} - \omega_{im}{}^m \omega_{jn}{}^n - 384 \cdot 2\,\alpha\, e^{-\frac{3}{2}\phi_0}\, Q\left(\frac{1}{2}\omega_{im\bar{n}}\omega_j{}^{\bar{n}m} + \omega_{im}{}^m \omega_{jn}{}^n\right)\right] *_6^{(0)} 1\,, \tag{2.45}$$

$$P_i = \int_{Y_3} -384 \cdot 6\, i\, \alpha\, e^{-\frac{3}{2}\phi}\, \omega_{im}{}^m\, Q\ *_6^{(0)} 1\ . \tag{2.46}$$

The terms proportional to α originate from the 10d Weyl rescaling factor in the backreacted metric Ansatz (2.41). Let us point out that terms involving other quantities such as $g^{(1)}_{m\bar{n}}$ and λ_i drop out of the final result because their contributions can be organized into total derivatives in the internal space. It is also crucial to make use of the fact that the fluctuation of c_3 under a Kähler deformation is an exact six-form, in accordance with c_3 being a characteristic class.[4]

Next we consider the reduction of the dilaton kinetic terms. Note that $\nabla_\mu \phi$ can be effectively considered to be a fluctuation of the same order as δv^i, and as a result terms of the schematic form $\delta v \nabla\phi \nabla\phi$ or $\delta v \nabla \delta v \nabla \phi$ have to be neglected. To linear order in fluctuations we have

$$\nabla_\mu \phi = \nabla_\mu \phi\, \left[1 - 1728\, \alpha\, \tilde{c}_1\, e^{-\frac{3}{2}\phi} Q\right] + 1154\, \alpha\, \tilde{c}_1\, e^{-\frac{3}{2}\phi}\, Q_i\, \nabla_\mu \delta v^i\ . \tag{2.48}$$

Combining this observation with the effect of the Weyl rescaling factor we obtain

$$\int_{\mathcal{M}_{10}} -\frac{1}{2}\nabla_M \hat{\phi} \nabla^M \hat{\phi} *_{10} 1 \longrightarrow \int_{\mathcal{M}_4} \left[U\, \nabla_\mu \phi \nabla^\mu \phi + U_i\, \nabla_\mu \phi \nabla^\mu \delta v^i\right] *_4 1\,, \tag{2.49}$$

with

$$U = -\frac{1}{2}(2\pi\alpha')^3\, \mathcal{V}^{(0)} + 384\, \alpha \left(1 + \frac{9}{2}\,\tilde{c}_1\right) e^{-\frac{3}{2}\phi}\, (2\pi)^3 \chi\,, \quad U_i = -384\alpha \cdot 3i\, \tilde{c}_1\, \omega_{im}{}^m\, (2\pi)^3 \chi\,, \tag{2.50}$$

where $\mathcal{V}^{(0)}$ denotes the volume of Y_3 in units of $\sqrt{2\pi\alpha'}$ computed with the zeroth-order metric $g^{(0)}_{m\bar{n}}$ and we used (2.21) and the footnote for the integrals of Q, Q_i. We have also recalled that $\omega_{im}{}^m$ is constant on the threefold.

[4] This observation allows us to derive useful identities involving the variation of the zero-form Q. Recalling $Q = (2\pi)^3\, *_6^{(0)} c_3$ and taking into account the variation of the metric implicit in the Hodge star, one can show that

$$\int_{Y_3} \left[Q_i - i\, \omega_{im}{}^m\, Q\right] *_6^{(0)} 1 = 0\,, \qquad \int_{Y_3} \left[Q_{ij} + (\omega_{im\bar{n}}\, \omega_j{}^{\bar{n}m} + \omega_{im}{}^m\, \omega_{jn}{}^n) Q\right] *_6^{(0)} 1 = 0\,, \tag{2.47}$$

where the quantities Q_i, Q_{ij} were introduced in (2.40).

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We can now record the results of the reduction of the higher-derivative terms. First of all,

$$\int_{\mathcal{M}_{10}} e^{-\frac{3}{2}\phi}\left(t_8 t_8 + \frac{1}{8}\epsilon_{10}\epsilon_{10}\right) R^4 *_{10} 1 \longrightarrow \int_{\mathcal{M}_4}\left[-384\cdot 2(2\pi)^3 e^{-\frac{3}{2}\phi}\chi\, R + \tilde{P}_{ij}\,\nabla_\mu \delta v^i \nabla^\mu \delta v^j\right] *_4 1\,, \tag{2.51}$$

where

$$\tilde{P}_{ij} = 384(2\pi)^3 e^{-\frac{3}{2}\phi}\int_{Y_3} \omega_{im\bar{n}}\omega_j{}^{\bar{n}m}\, c_3\ . \tag{2.52}$$

Let us remind the reader that the internal six-form c_3 is the third Chern form defined in appendix A and was related to Q in (2.21). Secondly, the reduction of the higher-derivative dilaton couplings in (2.9) yields

$$\begin{aligned}&\int_{\mathcal{M}_{10}} e^{-\frac{3}{2}\phi}\left(\tilde{c}_1 \nabla^M\nabla_M\hat{\phi}\, f_1 + \tilde{c}_2\,\nabla_M\hat{\phi}\,\nabla_N\hat{\phi}\, f_2^{MN} + \tilde{c}_3\,\nabla^M\hat{\phi}\,\nabla_M\hat{\phi}\, f_3\right) \\ &\longrightarrow\ 384(2\pi)^3\int_{\mathcal{M}_4} e^{-\frac{3}{2}\phi}\left[-\frac{3}{2}(3\tilde{c}_1+2\tilde{c}_2+4\tilde{c}_3)\,\chi\,\nabla_\mu\phi\nabla^\mu\phi + 3i\,\tilde{c}_1\,\omega_{im}{}^m\,\chi\,\nabla_\mu\phi\nabla^\mu\delta v^i\right] *_4 1\,.\end{aligned} \tag{2.53}$$

As mentioned above, in the process of dimensional reduction we have implicitly chosen a reference point in the complex structure and Kähler moduli spaces of the threefold Y_3 and we have only activated Kähler fluctuations δv^i, retaining terms up to quadratic order. Our next task is to infer the form taken by the couplings written above when the fluctuations δv^i are promoted to a full dependence on the moduli space of Y_3. At two-derivative level this is a standard procedure that we briefly review in order to set up our notation.

Recall that the $(1,1)$ forms $\omega_{im\bar{n}}$, $i = 1,\ldots,h^{1,1}$ are the harmonic representatives of the cohomology classes dual to an integral basis D_i of the homology $H_4(Y_3,\mathbb{Z})$. The threefold intersection numbers are denoted

$$\mathcal{K}_{ijk} = D_i\cdot D_j\cdot D_k = (2\pi\alpha')^{-3}\int_{Y_3}\omega_i\wedge\omega_j\wedge\omega_k\ . \tag{2.54}$$

We have inserted the appropriate power of $2\pi\alpha'$ in order to make the intersection numbers $\mathcal{K}_{ijk}$ dimensionless. Indeed, the components $\omega_{im\bar{n}}$ are dimensionless, so that the $(1,1)$-forms $\omega_i = \omega_{im\bar{n}}\, dz^m\wedge d\bar{z}^{\bar{n}}$ have mass dimension -2.

The Kähler class of the zeroth-order metric at the reference point in moduli space can be expanded onto the basis of forms ω_i as

$$J^{(0)} = v^{(0)i}\omega_i\,, \tag{2.55}$$

where $v^{(0)i}$ are taken to be dimensionless. We also define the quantities

$$\mathcal{K}^{(0)}_{ij} = \mathcal{K}_{ijk}v^{(0)k}\,,\qquad \mathcal{K}^{(0)}_{i} = \frac{1}{2}\mathcal{K}_{ijk}v^{(0)j}v^{(0)k}\,,\qquad \mathcal{V}^{(0)} = \frac{1}{6}\mathcal{K}_{jik}v^{(0)i}v^{(0)j}v^{(0)k}\ . \tag{2.56}$$

The quantity $\mathcal{V}^{(0)}$ is the volume of the threefold in units of $\sqrt{2\pi\alpha'}$, so that we can write

$$\int_{Y_3} *_6^{(0)} 1 = (2\pi\alpha')^3\,\mathcal{V}^{(0)}\ . \tag{2.57}$$

Let us record the useful identities

$$\omega_{im}{}^{m} = i\frac{\mathcal{K}_i^{(0)}}{\mathcal{V}^{(0)}}\,, \qquad \omega_{im\bar{n}}\omega_j{}^{\bar{n}m}\ *_6^{(0)}\,1 = \omega_i\wedge\omega_j\wedge J - \frac{\mathcal{K}_i^{(0)}\mathcal{K}_j^{(0)}}{\mathcal{V}^{(0)2}}\ *_6^{(0)}\,1\,, \tag{2.58}$$

and stress that harmonicity of ω_i guarantees that $\omega_{im}{}^{m}$ is covariantly constant on Y_3. Using these identities we can immediately compute the $\mathcal{O}(\alpha^0)$ part of P_{ij} defined in (2.45),

$$P_{ij} = (2\pi\alpha')^3\left[\frac{1}{2}\mathcal{K}_{ij}^{(0)} + \frac{1}{2}\frac{\mathcal{K}_i^{(0)}\mathcal{K}_j^{(0)}}{\mathcal{V}^{(0)}}\right] + \mathcal{O}(\alpha)\ . \tag{2.59}$$

This expression is initially understood to be evaluated at the reference point in Kähler moduli space, but given its topological nature it uplifts naturally to the full moduli space dependence. This is achieved simply by replacing $v^{(0)i}$ with an arbitrary v^i. The quantities defined in (2.55), (2.56) in terms of $v^{(0)i}$ are promoted to v-dependent quantities denoted $\mathcal{V}$, $\mathcal{K}_i$, $\mathcal{K}_{ij}$ without the $^{(0)}$ superscript. In a similar way, the quantity Ω' naturally uplifts to the full v-dependent threefold volume form, which integrates to the full volume.

The uplift of higher-derivative couplings is considerably less under control in general, due to the fact that corrections are expected to lift some of the naïve flat directions of moduli space, making the identification of the correct massless modes a hard problem. Nonetheless, we can recast the reduction results of the higher-derivative terms under consideration in a simple form. Exploiting the fact that $\omega_{im}{}^{m}$ is covariantly constant we can immediately perform the uplift of integrals of the form

$$\int_{Y_3}\omega_{im}{}^{m}\,c_3 \longrightarrow i\,\chi\,\frac{\mathcal{K}_i}{\mathcal{V}}\,, \qquad \int_{Y_3}\omega_{im}{}^{m}\,\omega_{jn}{}^{n}\,c_3 \longrightarrow -\chi\frac{\mathcal{K}_i\mathcal{K}_j}{\mathcal{V}^2}\ . \tag{2.60}$$

A similar manipulation for the integral

$$\int_{Y_3}\omega_{im\bar{n}}\,\omega_j{}^{\bar{n}m}\,c_3 \tag{2.61}$$

is not straightforward, since the non-harmonic part F of c_3 introduced in (2.23) poses an obstruction to the factorization of this expression. In the present context, however, the overall coefficient of this term in the dimensional reduction is zero and we do not have to address this complication.

Let us introduce the field-dependent, dimensionless quantity

$$\tilde{\chi} = 384(2\pi)^3\,\frac{\alpha}{(2\pi\alpha')^3\,\mathcal{V}}\,e^{-\frac{3}{2}\phi}\,\chi = \frac{1}{16}\zeta(3)\frac{e^{-\frac{3}{2}\phi}}{\mathcal{V}}\,\chi\ . \tag{2.62}$$

In what follows it plays the role of the effective expansion parameter for the problem at hand. Combining all contributions after uplift the 4d effective action takes the form

$$\begin{aligned} S_{4\mathrm{d}} = \frac{1}{(2\pi)^4\alpha'}\int_{\mathcal{M}_4}\Big\{&\mathcal{V}\,(1+b_1\,\tilde{\chi})\,R + \left[\left(\frac{1}{2}+b_2\,\tilde{\chi}\right)\mathcal{K}_{ij} + \left(\frac{1}{2}+b_3\,\tilde{\chi}\right)\frac{\mathcal{K}_i\mathcal{K}_j}{\mathcal{V}}\right]\nabla_\mu v^i\nabla^\mu v^j \\ &+\mathcal{V}\left(-\frac{1}{2}+b_4\,\tilde{\chi}\right)\nabla_\mu\phi\nabla^\mu\phi + b_5\,\tilde{\chi}\,\mathcal{K}_i\,\nabla_\mu\phi\nabla^\mu v^i\Big\}\,*_4\,1\ . \end{aligned} \tag{2.63}$$

In this expression we have promoted the fluctuations δv^i to full 4d fields v^i. For notational convenience we have introduced the numerical coefficients

$$b_1 = -4\,, \qquad b_2 = 0\,, \qquad b_3 = 2\,, \tag{2.64}$$

$$b_4 = 1 - 3\tilde{c}_2 - 6\tilde{c}_3\,, \qquad b_5 = 6\,. \tag{2.65}$$

2.3.2 Weyl rescaling

Our next task is performing a Weyl rescaling of the external metric in (2.63) in order to bring the 4d Einstein-Hilbert term into canonical form. Let us record the identities

$$g_{\mu\nu} = e^{2\Lambda}\tilde{g}_{\mu\nu}\,, \qquad R = e^{-2\Lambda}\left[\tilde{R} - 6\tilde{\nabla}_\mu\Lambda\tilde{\nabla}^\mu\Lambda - 6\tilde{\nabla}_\mu\tilde{\nabla}^\mu\Lambda\right]\,. \tag{2.66}$$

In our case, we set

$$\Lambda = -\frac{1}{2}\log\left[\mathcal{V}\left(1 + b_1\,\tilde{\chi}\right)\right]\,. \tag{2.67}$$

Dropping the tilde on the new metric, the Weyl-rescaled 4d action can be written in the form

$$S_{4\mathrm{d}} = \frac{1}{(2\pi)^4\alpha'}\int_{\mathcal{M}_4}\left[R + \mathcal{G}_{v^iv^j}\,\nabla_\mu v^i\nabla^\mu v^j + \mathcal{G}_{\phi\phi}\,\nabla_\mu\phi\nabla^\mu\phi + 2\,\mathcal{G}_{\phi v^i}\,\nabla_\mu\phi\nabla^\mu v^i\right] *_4 1\,, \tag{2.68}$$

where

$$\mathcal{G}_{v^iv^j} = -\frac{1}{\mathcal{V}^2}\left[1 + \tilde{\chi}\left(-b_3 - \frac{5}{2}b_1\right)\right]\mathcal{K}_i\mathcal{K}_j + \frac{1}{2\mathcal{V}}\left[1 + \tilde{\chi}\left(2b_2 - b_1\right)\right]\mathcal{K}_{ij}\,, \tag{2.69}$$

$$\mathcal{G}_{\phi\phi} = -\frac{1}{2}\left[1 + \tilde{\chi}\left(-b_1 - 2b_4\right)\right]\,, \tag{2.70}$$

$$\mathcal{G}_{\phi v^i} = \left(\frac{9}{4}b_1 + \frac{1}{2}b_5\right)\tilde{\chi}\frac{\mathcal{K}_i}{\mathcal{V}}\,. \tag{2.71}$$

This form of the reduction result is a convenient starting point for the discussion of the 4d $\mathcal{N} = 2$ and $\mathcal{N} = 1$ theories in the following sections.

3 Correction to the $\mathcal{N} = 2$ prepotential

In this section we analyze the result of the dimensional reduction from the point of view of the 4d, $\mathcal{N} = 2$ effective theory obtained without including branes or performing any orientifold projection. We show that our findings are compatible with the known results about the correction to the geometry of the $\mathcal{N} = 2$ hypermultiplet scalar manifold induced by a perturbative α' correction to the Kähler moduli space prepotential [21, 40, 43].

3.1 Translation into $\mathcal{N} = 2$ field variables

Let us recall some well-known facts about the 4d, $\mathcal{N} = 2$ effective theory arising from compactification of type IIB superstring on a Calabi-Yau threefold [20, 22, 44, 45]. The complex structure moduli of the threefold fit into $n_V = h^{1,2}$ vector multiplets. Their scalar manifold is special Kähler and its geometry is tree-level exact both in g_s and α'. The Kähler structure moduli, the dilaton, the axion, as well as the scalars coming from the expansion

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of B_2, C_2, C_4 onto internal even harmonic forms, all fit into $n_H = h^{1,1}+1$ hypermultiplets. Their scalar manifold $\mathcal{M}_{\rm Q}$ is quaternionic of real dimension $4(h^{1,1}+1)$.

The structure of the metric on $\mathcal{M}_{\rm Q}$ is most conveniently analyzed using the string frame Kähler moduli $v^i_{\rm s}$ and the 4d dilaton ϕ_4. In terms of these variables the quaternionic metric on $\mathcal{M}_{\rm Q}$ can be written schematically as [20, 44]

$$ds^2(\mathcal{M}_{\rm Q}) = (d\phi_4)^2 + G_{i\bar{\jmath}}(w_{\rm s},\bar{w}_{\rm s})\, dw^i\, d\bar{w}^{\bar{\jmath}} + \mathcal{G}_{xy}(w_{\rm s},\bar{w}_{\rm s},\phi_4,q) dq^x\, dq^y\ . \tag{3.1}$$

In this expression $w^i_{\rm s} = u^i_{\rm s} + i\, v^i_{\rm s}$ are the complexified Kähler moduli arising from expansion of $B_2 + iJ_{\rm s}$ onto the basis ω_i of harmonic $(1,1)$-forms. The metric $G_{i\bar{\jmath}}$ is the special Kähler metric on the Kähler moduli space $\mathcal{M}_{\rm K}$ of the threefold. The fields q^x denote collectively all real scalars in $\mathcal{M}_{\rm Q}$ different from ϕ_4, $u^i_{\rm s}$, $v^i_{\rm s}$. The metric components $\mathcal{G}_{xy}$ are entirely determined by the special Kähler metric on $\mathcal{M}_{\rm K}$. For this reason, the quaternionic metric on $\mathcal{M}_{\rm Q}$ is sometimes referred to as special quaternionic [20, 44]. Let us stress that the structure (3.1) of the metric on $\mathcal{M}_{\rm Q}$ is expected to hold not only at tree level, but also including α' corrections, thanks to mirror symmetry considerations, reviewed for instance in [46, 47].

In our analysis all the fields q^x as well as the axions $u^i_{\rm s}$ are effectively frozen to zero. Our next task is therefore to connect the string frame Kähler moduli $v^i_{\rm s}$ and the 4d dilaton ϕ_4 to the Einstein frame Kähler moduli v^i and the 10d dilaton ϕ. At two-derivative level we have the relations

$$e^{-2\phi_4} = e^{-\frac{1}{2}\phi}\,\mathcal{V}\ , \qquad v^i_{\rm s} = e^{\frac{1}{2}\phi}\, v^i\ . \tag{3.2}$$

Let us remind the reader that in our conventions $\mathcal{V}$ is the volume of the threefold in units of $\sqrt{2\pi\alpha'}$. In order to take into account the effect of higher-derivative corrections we deform these relations into

$$e^{-2\phi_4} = e^{-\frac{1}{2}\phi}\,\mathcal{V}\,(1+\Upsilon_1\,\tilde{\chi})\ , \qquad v^i_{\rm s} = e^{\frac{1}{2}\phi}\, v^i\ \ (1+\Upsilon_2\,\tilde{\chi})\ , \tag{3.3}$$

where Υ_1, Υ_2 are constants that will be fixed momentarily.

We can now invert (3.3) to leading order in α'^3 to obtain v^i, ϕ in terms of $v^i_{\rm s}$, ϕ_4, and plug the resulting expressions into the 4d effective action (2.68). The result is conveniently expressed in terms of the quantities

$$\mathcal{V}_{\rm s} = \frac{1}{6}\mathcal{K}_{ijk}\, v^i_{\rm s}\, v^j_{\rm s}\, v^k_{\rm s}\,, \qquad \mathcal{K}^{\rm s}_i = \frac{1}{2}\mathcal{K}_{ijk}\, v^j_{\rm s}\, v^k_{\rm s}\,, \qquad \mathcal{K}^{\rm s}_{ij} = \mathcal{K}_{ijk}\, v^k_{\rm s}\ . \tag{3.4}$$

Furthermore, it is convenient to introduce a new field-dependent dimensionless quantity $\tilde{\chi}_{\rm s}$, which is the string-frame analog of $\tilde{\chi}$ and is given by

$$\tilde{\chi}_{\rm s} = \frac{1}{16}\zeta(3)\frac{1}{\mathcal{V}_{\rm s}}\,\chi\ . \tag{3.5}$$

With this notation, the 4d effective action can be written as

$$\begin{aligned} S_{\rm 4d} = \frac{1}{(2\pi)^4\alpha'}\int_{\mathcal{M}_4}\Bigg\{ R + \left[\frac{1}{2\mathcal{V}_{\rm s}}\,(1+a_1\,\tilde{\chi}_{\rm s})\,\mathcal{K}^{\rm s}_{ij} - \frac{1}{2\mathcal{V}_{\rm s}}\,(1+a_2\,\tilde{\chi}_{\rm s})\,\frac{\mathcal{K}^{\rm s}_i\mathcal{K}^{\rm s}_j}{\mathcal{V}_{\rm s}}\right]\nabla_\mu v^i_{\rm s}\nabla^\mu v^j_{\rm s} \\ - 2\,(1+a_3\,\tilde{\chi}_{\rm s})\,\nabla_\mu\phi_4\nabla^\mu\phi_4 + a_4\,\tilde{\chi}_{\rm s}\,\frac{\mathcal{K}^{\rm s}_i}{\mathcal{V}_{\rm s}}\,\nabla_\mu\phi_4\nabla^\mu v^i_{\rm s}\Bigg\}\,*_4\,1\,, \end{aligned} \tag{3.6}$$

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where the numerical coefficients a are given in terms of the b coefficients introduced in (2.63) by

$$a_1 = 2b_2 - b_1\,, \qquad a_2 = -\frac{37}{16}b_1 + \frac{5}{4}b_2 - \frac{1}{8}b_3 - \frac{1}{2}b_4 - \frac{1}{4}b_5 + 2\Upsilon_2\,,$$
$$a_3 = \frac{11}{16}b_1 - \frac{3}{4}b_2 - \frac{9}{8}b_3 - \frac{1}{2}b_4 + \frac{3}{4}b_5\,, \quad a_4 = -\frac{27}{8}b_1 - \frac{1}{2}b_2 - \frac{3}{4}b_3 + b_4 - \frac{1}{2}b_5 + 2\Upsilon_1\,. \tag{3.7}$$

If we plug in the values of the b coefficients given in (2.64), we find

$$a_1 = 4\,, \qquad a_2 = 7 + \frac{3}{2}\tilde{c}_2 + 3\tilde{c}_3 + 2\Upsilon_2\,,$$
$$a_3 = -1 + \frac{3}{2}\tilde{c}_2 + 3\tilde{c}_3\,, \qquad a_4 = 13 - 3\tilde{c}_2 - 6\tilde{c}_3 + 2\Upsilon_1\,. \tag{3.8}$$

We are now in a position to discuss the correction to the $\mathcal{N}=2$ prepotential.

3.2 Implications for $\mathcal{N}=2$ prepotential

As noted above, the form (3.1) of the quaternionic metric on $\mathcal{M}_{\rm Q}$ should be preserved by α' corrections. In particular, cross terms between ϕ_4 and $v_{\rm s}^i$ are not allowed, and also $v_{\rm s}^i$-dependent corrections to the $(d\phi_4)^2$ terms are forbidden. These considerations lead us to impose $a_3 = 0$, $a_4 = 0$. As a result we can fix the value of Υ_1 and derive a linear constraint on the dilaton coupling coefficients $\tilde{c}_2$, $\tilde{c}_3$,

$$\Upsilon_1 = -4\,, \qquad 3\tilde{c}_2 + 6\tilde{c}_3 - 2 = 0\,. \tag{3.9}$$

We now aim at demonstrating that the effective action (3.6) can be written as

$$S = \frac{1}{(2\pi)^4\alpha'}\int_{\mathcal{M}_4}\Big\{R - 2\nabla_\mu\phi_4\nabla^\mu\phi_4 - 2G_{i\bar{\jmath}}\nabla_\mu v_{\rm s}^i\nabla^\mu v_{\rm s}^j\Big\}*_4 1\,, \quad G_{i\bar{\jmath}} = \partial_{w_{\rm s}^i}\partial_{\bar{w}_{\rm s}^{\bar{\jmath}}}K^{\mathcal{N}=2}(w_{\rm s},\bar{w}_{\rm s})\,, \tag{3.10}$$

where we remind the reader that $w_{\rm s}^i = u_{\rm s}^i + i\,v_{\rm s}^i$ are the complexified Kähler moduli, and where the Kähler potential $K^{\mathcal{N}=2}(w_{\rm s},\bar{w}_{\rm s})$ can be derived from a holomorphic prepotential.

In order to set our notation, it is useful to review the relation between the Käher potential and the prepotential. The geometry of the $h^{1,1}$ complex-dimensional Kähler moduli space is conveniently described by $h^{1,1}+1$ complex projective coordinates $X^I = (X^0, X^i)$ related to the complex coordinates $w_{\rm s}^i$ by

$$w_{\rm s}^i = \frac{X^i}{X^0}\,. \tag{3.11}$$

The geometry of $\mathcal{M}_{\rm K}$ is encoded in a holomorphic prepotential $F(X)$, which is a homogeneous function of X^I of degree 2. The Kähler potential $K^{\mathcal{N}=2}(w_{\rm s},\bar{w}_{\rm s})$ for the metric on $\mathcal{M}_{\rm K}$ is then extracted from F via

$$e^{-K^{\mathcal{N}=2}(w_{\rm s},\bar{w}_{\rm s})} = -i(X^I\overline{F}_I - F_I\overline{X}^I)\,, \qquad F_I = \partial_{X^I}F(X)\,. \tag{3.12}$$

Mirror symmetry considerations [21, 46, 47] ensure that the prepotential $F(X)$ takes the form

$$F(X) = \frac{1}{X^0}\frac{1}{6}\mathcal{K}_{ijk}X^iX^jX^k + i\,\lambda\,(X^0)^2 + \dots\,, \tag{3.13}$$

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where the first term is the classical prepotential, the second is a perturbative α' effect, and the ellipsis stands for terms that are exponentially suppressed in the large volume limit and originate from worldsheet instantons. The $\mathcal{N}=2$ Kähler potential derived from (3.13) using (3.12) is, up to instanton corrections and an immaterial constant,

$$K^{\mathcal{N}=2} = -\log\left[\mathcal{V}_{\rm s} + \frac{1}{2}\lambda\right] \ . \tag{3.14}$$

We can finally relate this to our dimensional reduction by comparing the metric computed from (3.14) with the metric in (3.6). We find that, in order for the match to be possible, we have to impose

$$\Upsilon_2 = 0 \ . \tag{3.15}$$

The constant λ is then fixed to be

$$\lambda = -\frac{1}{2}\,\zeta(3)\,\chi\,, \tag{3.16}$$

so that the $\mathcal{N}=2$ Kähler potential reads

$$K^{\mathcal{N}=2} = -\log\left[\frac{\mathcal{V}_{\rm s}}{(2\pi\alpha')^3} - \frac{1}{4}\zeta(3)\,\chi\right] \ . \tag{3.17}$$

In this last expression we have reinstated all factors of α' and $\mathcal{V}_{\rm s}$ denotes the dimensionful volume of the threefold. Let us close this section with a comparison between our findings and the analogous quantities in BBHL [6]. Using equations (3.11), (3.12), (3.13) and the comment before (3.15) in that paper, we infer that their $\mathcal{N}=2$ Kähler potential in our notation coincides exactly with (3.17).

4 $\mathcal{N}=1$ Kähler coordinates and Kähler potential

In this section we analyze the results of the dimensional reduction and identify the Kähler coordinates and Kähler potential in the 4d, $\mathcal{N}=1$ effective action. We briefly comment on our findings.

4.1 Correction to the Kähler potential and coordinates

Upon dimensional reduction on a Calabi-Yau threefold, the two-derivative action of type IIB supergravity yields a 4d effective action with $\mathcal{N}=2$ supersymmetry. If orientifold planes are included in the setup, the 4d spectrum is suitably projected and one obtains an effective action with 4d, $\mathcal{N}=1$ supersymmetry [30–32]. For definiteness, we consider here the projection relevant to the case with $O3/O7$-planes. The dilaton ϕ and the Kähler moduli v^i fit into 4d, $\mathcal{N}=1$ chiral multiplets whose scalar components are of the form

$$\tau_0 = C_0 + ie^{-\phi}\,, \tag{4.1}$$

$$T_i = \rho_i + i\,\mathcal{K}_i + \zeta_i \ . \tag{4.2}$$

Several remarks are in order. We used the symbol τ_0 to denote the 4d axio-dilaton. Its real part C_0 is the straightforward dimensional reduction of the fluctuations of the 10d axion

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around its zero background. Its imaginary part is built with ϕ which is not directly identified with the 10d dilaton because of the backreaction term $\phi^{(1)}$ discussed in section 2.2.1. This discrepancy has consequences for the transformation property of 4d fields under the 10d $\mathrm{SL}(2,\mathbb{Z})$ symmetry, as discussed in [6]. In the expression for T_i, the scalars ρ_i come from the expansion of the RR four-form, and ζ_i denotes a complex quantity built with the scalars coming from the reduction of the NSNS and RR two-forms. Since we are freezing all such scalars to zero, the quantity ζ_i vanishes. Let us point out that the orientifold projection to $\mathcal{N}=1$ supersymmetry is implicit in the range of the index i. Indeed, by an abuse of notation, the index i in (4.1) runs only over the elements of $H^{1,1}_{+}(Y_3)$, i.e. the $(1,1)$ forms that are even under the isometric involution of Y_3 considered in the implementation of the orientifold projection. Let us stress that equations (2.56) still hold after restricting the range of i, by virtue of the restrictions imposed on intersection numbers by the orientifold projection, as explained in detail in [30–32].

The kinetic terms of τ_0 and T_i are governed by a Kähler potential K. If we write $T_0 \equiv \tau_0$ and introduce the collective index $I=(0,i)$, the effective action contains the terms

$$S_{4\mathrm{d}} \supset \frac{1}{(2\pi)^4\alpha'}\int_{\mathcal{M}_4}\Big[R + G_{T_I\overline{T}_J}\,\nabla_\mu T_I \nabla^\mu \overline{T}_J\Big]\,*_4\,1\,, \tag{4.3}$$

where

$$G_{T_I\overline{T}_J} = -2\,\partial_{T_I}\partial_{\overline{T}_J}K\,, \qquad K=\phi-2\log\mathcal{V}\,. \tag{4.4}$$

The Kähler potential is understood as a function of T_I, $\overline{T}_J$ determined implicitly via (4.1).

Upon inclusion of higher-derivative terms in the 10d action, the coefficients of the terms in the two-derivative 4d effective action are modified, but the form of the action is still expected to obey the constraints deriving from 4d, $\mathcal{N}=1$ supersymmetry. In this section we show that our results are consistent with this expectation.

Let us introduce the notation $v^0 \equiv \phi$, in such a way that $v^I=(\phi, v^i)$. The kinetic terms specified by (2.69) can be written compactly as

$$\mathcal{G}_{v^Iv^J}\,\nabla_\mu v^I\nabla^\mu v^J = \mathcal{G}_{\phi\phi}\,\nabla_\mu\phi\nabla^\mu\phi + \mathcal{G}_{v^iv^j}\,\nabla_\mu v^i\nabla^\mu v^j + 2\mathcal{G}_{v^i\phi}\,\nabla_\mu v^i\nabla^\mu\phi\,. \tag{4.5}$$

Recall that C_0, ρ_i and ζ_i are frozen to zero in our discussion. The problem at hand is the determination of the $\mathcal{O}(\tilde{\chi})$ corrections to T_I and K in such a way that the relation

$$\mathcal{G}_{v^Iv^J}\,\nabla_\mu v^I\nabla^\mu v^J = -2\,(\partial_{T_I}\partial_{\overline{T}_J}K)\,\nabla_\mu T_I\nabla^\mu\overline{T}_J \tag{4.6}$$

holds including terms up to order $\tilde{\chi}$.

The first outcome of our analysis is the observation that, regarding the b coefficients in (2.63) as input data, a solution exists only if the following linear constraint is satisfied,

$$11b_1 - 12b_2 - 18b_3 - 8b_4 + 12b_5 = 0\,. \tag{4.7}$$

Plugging in the values of the b coefficients given in (2.64), we obtain

$$3\tilde{c}_2 + 6\tilde{c}_3 - 2 = 0\,, \tag{4.8}$$

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which is the same constraint on $\tilde{c}_2$, $\tilde{c}_3$ found in (3.9) in the analysis of the $\mathcal{N}=2$ case. If this requirement is met, the Kähler coordinates and Kähler potential are given in terms of the b coefficients by

$$\tau_0 = ie^{-\phi}\left[1+\tilde{\chi}\left(-\frac{3}{4}b_1+\frac{3}{2}b_3-b_5\right)\right], \tag{4.9}$$

$$T_i = i\,\mathcal{K}_i\left[1+\tilde{\chi}\left(\frac{1}{4}b_1+2b_2+\frac{1}{2}b_3\right)\right], \tag{4.10}$$

$$K = \phi - 2\log\left[\mathcal{V}\left(1+\tilde{\chi}\left(b_1+b_2+\frac{3}{2}b_3-\frac{1}{2}b_5\right)\right)\right]. \tag{4.11}$$

The actual values taken by these quantities upon substituting the numerical values of the b coefficients are summarized at the end of this section.

An important feature of the zeroth-order Kähler potential is the property of extended no-scale structure [30–32]. This property is tested by computing the following quantity,

$$\frac{\partial K}{\partial T_I}\,(K^{-1})_{I\bar{J}}\,\frac{\partial K}{\partial \overline{T}_{\bar{J}}} = 4\left[1+0\cdot\tilde{\chi}+\mathcal{O}(\tilde{\chi}^2)\right], \tag{4.12}$$

where $(K^{-1})_{I\bar{J}}$ denotes the inverse of the matrix $K^{I\bar{J}}=\partial_{T_I}\partial_{\overline{T}_J}K$. The zeroth order value 4 signals extended no-scale structure [48], and interestingly the leading correction has a vanishing coefficient for any value of the b coefficients. This does not mean that the correction under examination has no physical effect. For instance, in a scenario like KKLT the axio-dilaton and the complex structure moduli are stabilized first at a supersymmetric value by means of the Gukov-Vafa-Witten superpotential [49]. The relevant quantity in the computation of the scalar potential for the Kähler moduli is no longer the quantity in (4.12), but rather the same object with summation restricted to i, j indices only, implicitly evaluated at the fixed value of the axio-dilaton. One finds

$$\frac{\partial K}{\partial T_i}\,(K^{-1})_{i\bar{j}}\,\frac{\partial K}{\partial \overline{T}_{\bar{j}}} = 3\left[1+\left(\frac{1}{2}b_1-b_2\right)\tilde{\chi}+\mathcal{O}(\tilde{\chi}^2)\right]. \tag{4.13}$$

As we can see, the zeroth-order value 3 associated to no-scale structure receives a non-zero correction at order $\tilde{\chi}$ ($\frac{1}{2}b_1-b_2=-2$).

In order to elucidate further the physical effects of the correction to the Kähler potential an alternative formulation can be used, in which the chiral multiplets T_i are dualized into linear multiplets L_i [30–32]. The dynamics of the system is encoded in a kinetic potential $\tilde{K}$, which is the Legendre transform of the original Kähler potential K. Let us adopt the following conventions for the Legendre transform,

$$L^i = -\frac{\partial K}{\partial \mathrm{Im}T_i}\,, \quad \tilde{K} = K + L^i\,\mathrm{Im}T_i\,, \quad \mathrm{Im}T_i = \frac{\partial \tilde{K}}{\partial L^i}\,. \tag{4.14}$$

A straightforward computation gives, up to terms of order $\tilde{\chi}^2$ and higher,

$$L_i = \frac{v^i}{\mathcal{K}}\left[1+\tilde{\chi}\left(-\frac{5}{4}b_1-\frac{1}{2}b_3\right)\right], \tag{4.15}$$

$$L^i\,\mathrm{Im}T_i = 3\left[1+\tilde{\chi}\,(-b_1+2b_2)\right]. \tag{4.16}$$

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In this formulation the hallmark of zeroth-order no-scale structure is the value 3, which again receives corrections at order $\tilde{\chi}$. As a further check that the 4d dynamics is really affected by the corrections under examination we can examine the expression for the kinetic potential $\tilde{K}$ in terms of the fields L^i,

$$\tilde{K} = 3 - \log \mathrm{Im}\tau + \log\left(\frac{1}{6}\mathcal{K}_{ijk}L^iL^jL^k\right) + \frac{1}{16}\zeta(3)\,\chi\,(-2b_1+4b_2)\,(\mathrm{Im}\tau)^{3/2}\sqrt{\frac{1}{6}\mathcal{K}_{ijk}L^iL^jL^k}\,, \tag{4.17}$$

which has manifestly a different functional form as compared to the zeroth-order result. Note indeed that $-2b_1+4b_2 = 8$.

Let us now summarize the expressions for the Kähler coordinates and potential after plugging in the values of the b coefficients given in (2.64). We have

$$\tau_0 = ie^{-\phi}\,, \tag{4.18}$$

$$T_i = i\,\mathcal{K}_i\,, \tag{4.19}$$

$$K = \phi - 2\log\left[\frac{\mathcal{V}}{(2\pi\alpha')^3} - \frac{1}{4}\,\zeta(3)\,e^{-\frac{3}{2}\phi}\,\chi\right]\,, \tag{4.20}$$

$$L_i = \frac{v^i}{\mathcal{V}}\left[1 + \frac{1}{4}\,\frac{(2\pi\alpha')^3}{\mathcal{V}}\,\zeta(3)\,e^{-\frac{3}{2}\phi}\,\chi\right]\,, \tag{4.21}$$

$$L^i\,\mathrm{Im}T_i = 3\left[1 + \frac{1}{4}\,\frac{(2\pi\alpha')^3}{\mathcal{V}}\,\zeta(3)\,e^{-\frac{3}{2}\phi}\,\chi\right]\,. \tag{4.22}$$

Clearly these expressions are valid up to α' corrections of higher order than the α'^3 corrections under examination. We have utilized the explicit expression (2.62) for $\tilde{\chi}$ and we have used $\mathcal{V}$ for the dimensionful volume of the threefold, thus reinstating explicitly all factors of α' for the convenience of the reader. As we can see, the leading correction to τ_0, T_i is vanishing. Let us also remark that our finding for the correction to the $\mathcal{N}=1$ Kähler potential is in perfect agreement with the corresponding result in BBHL [6].

4.2 Some comments

Some comments about the results of the previous section are in order. We consider the 4d, $\mathcal{N}=1$ theory arising from a Calabi-Yau orientifold with $O7$-planes and $D7$-branes, but we do not take into account explicitly the backreaction of these extended objects. For instance, the Chern-Simons part of the effective action for a $D7$-brane wrapping a divisor in the Calabi-Yau threefold contains higher-curvature terms which are essential in the derivation of the contribution of $D7$-branes to the $D3$-brane tadpole cancellation condition [35]. From the point of view of the uplift to F-theory, $D7$-branes and $O7$-branes are encoded in the Calabi-Yau fourfold geometry, and the metric Ansatz for dimensional reduction to three dimensions is modified by higher-derivative corrections [24–28]. These considerations suggest that $D7$-branes and $O7$-branes could play an important role in the context of higher-derivative corrections to the bulk $\mathcal{N}=1$ Kähler potential, which is however beyond the scope of this work and left for future investigation.

It is interesting to observe that our computation shows that the leading correction to the $\mathcal{N}=1$ Kähler coordinates τ_0, T_i is vanishing. As far as the 4d axio-dilaton τ_0

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is concerned, this result is consistent with the expectations from F-theory, since in the context of F-theory compactification the axio-dilaton is identified with one of the complex structure moduli of the elliptically fibered Calabi-Yau fourfold, and is therefore expected not to receive corrections from the Kähler moduli sector proportional to the Euler number χ of the base threefold.

5 Conclusions

In this paper we revisited the leading α'^3 perturbative correction to the Kähler potential of a 4d, $\mathcal{N}=1$ orientifold compactification of type IIB superstring theory on a Calabi-Yau threefold. We reproduced the result of Becker, Becker, Haack, and Louis (BBHL) [6] by showing that a correction to the Kähler potential emerges, which is proportional to the Euler characteristic of the leading order Calabi-Yau threefold background.

Our derivation is based on an explicit Kaluza-Klein reduction of the type IIB bulk action from ten to four dimensions. The 10d two-derivative action is supplemented by the well-known gravitational correction of the schematic form $(t_8 t_8 + \frac{1}{8}\epsilon_{10}\epsilon_{10})R^4$ [10–19], as well as by related dilaton couplings. Since the latter are not completely known, we parametrized them in terms of three constants $\tilde{c}_1$, $\tilde{c}_2$, $\tilde{c}_3$, see (2.8). Supersymmetry considerations in four dimensions allowed us to derive the linear relation (4.8) among these parameters in the 10d action, which could be checked against proposals in the literature for the complete axio-dilaton α'^3 sector.

The background geometry used in the dimensional reduction at two-derivative level must be modified in order to solve the higher-dimensional equations of motion in the presence of the α'^3 corrections under examination. We presented an explicit solution for this backreacted background in terms of the non-harmonic part of the third Chern form c_3 of the zeroth order Calabi-Yau threefold. The α'-corrected internal metric is still Kähler but no longer Ricci-flat. As a result, the manifold is no longer a Calabi-Yau threefold with SU(3) holonomy, but rather an almost Calabi-Yau threefold [23] endowed with an SU(3) structure for which the only non-zero torsion class is $\mathcal{W}_5$ in the notation of [41]. We also found that the entire 10d background metric is corrected at order α'^3 by an overall Weyl factor proportional to the Euler density of the zeroth order Calabi-Yau threefold. This is in analogy with the results of [24–26] in the context of three-dimensional M-theory vacua in the presence of higher-derivative corrections.

The dimensional reduction is performed by allowing for small spacetime dependent fluctuations δv^i of the Kähler moduli of the threefold, for arbitrary $h^{1,1}$. We have showed that the outcome of the dimensional reduction can be uplifted from infinitesimal variations to a full non-linear dependence on the moduli space. The latter is captured by simple topological expressions proportional to the Euler characteristic of the threefold.

The resulting kinetic terms for the dilaton and the Kähler moduli are consistent with 4d, $\mathcal{N}=2$ supersymmetry, once the appropriate $\mathcal{N}=2$ field variables are identified. The prepotential for the special Kähler geometry of the Kähler moduli space is found to be corrected by the expected term proportional to the Euler characteristic of the threefold [43].

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The kinetic terms in the 4d effective action can also be truncated in accordance with the standard O3/O7 orientifold projection, yielding a theory with $\mathcal{N}=1$ supersymmetry. We reproduce the expected BBHL correction to the $\mathcal{N}=1$ Kähler potential proportional to the Euler characteristic of the classical Calabi-Yau threefold. We also computed the leading α' corrections to the Kähler coordinates as a function of the threefold Kähler moduli, and found that they are vanishing.

Our investigation can be extended in several directions. It would be desirable to switch on additional type IIB fields, for instance the axion C_0, and examine their kinetic terms to confirm the structure of the Kähler potential. It would be also interesting to repeat a similar analysis for the complex structure moduli sector, even though no analogous correction to the Kähler potential is expected. Finally, it would be interesting to consider explicitly the effect of both local sources and bulk higher-derivative corrections on the background solution. To pursue these directions further, however, a preliminary investigation of the complete α'^3-corrected type IIB bulk action would be probably necessary. By a similar token, the detailed knowledge of the α'-corrected 10d gravitino and dilatino supersymmetry variations would allow a direct check of supersymmetry of the background in the presence of higher-derivative corrections.

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A Conventions, definitions, and identities

In this work we denote 10d spacetime indices by capital Latin letters $M,N=0,\ldots,9$, 4d spacetime indices by $\mu,\nu=0,1,2,3$, and the internal complex indices by $m,n,p=1,\ldots,3$ and $\bar{m},\bar{n},\bar{p}=1,\ldots,3$. We occasionally also make use of real internal indices $a,b=1,\ldots,6$. The metric signature of the ten-dimensional space is $(-,+,\ldots,+)$. Our conventions for the totally anti-symmetric tensor in Lorentzian signature in an orthonormal frame are $\epsilon_{012\ldots9}=\epsilon_{0123}=+1$. The epsilon tensor in d dimensions then satisfies

$$\epsilon^{R_1\cdots R_p N_1\ldots N_{d-p}}\epsilon_{R_1\ldots R_p M_1\ldots M_{d-p}}=(-1)^s(d-p)!p!\delta^{N_1}{}_{[M_1}\ldots\delta^{N_{d-p}}{}_{M_{d-p}]}\,, \tag{A.1}$$

where $s=0$ if the metric has Euclidean signature and $s=1$ for a Lorentzian metric.

We adopt the following conventions for the Christoffel symbols and Riemann tensor

$$\begin{aligned}\Gamma^R{}_{MN}&=\frac{1}{2}g^{RS}(\partial_M g_{NS}+\partial_N g_{MS}-\partial_S g_{MN})\,, & R_{MN}&=R^R{}_{MRN}\,,\\ R^M{}_{NRS}&=\partial_R\Gamma^M{}_{SN}-\partial_S\Gamma^M{}_{RN}+\Gamma^M{}_{RT}\Gamma^T{}_{SN}-\Gamma^M{}_{ST}\Gamma^T{}_{RN}\,, & R&=R_{MN}g^{MN}\,,\end{aligned} \tag{A.2}$$

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with equivalent definitions on the internal and external spaces. Differential p-forms are expanded in a basis of differential one-forms as

$$\Lambda = \frac{1}{p!}\Lambda_{M_1\ldots M_p} dx^{M_1} \wedge \ldots \wedge dx^{M_p} \ . \tag{A.3}$$

The wedge product between a p-form $\Lambda^{(p)}$ and a q-form $\Lambda^{(q)}$ is given by

$$(\Lambda^{(p)} \wedge \Lambda^{(q)})_{M_1\ldots M_{p+q}} = \frac{(p+q)!}{p!q!}\Lambda^{(p)}_{[M_1\ldots M_p}\Lambda^{(q)}_{M_1\ldots M_q]} \ . \tag{A.4}$$

Furthermore, the exterior derivative on a p-form Λ reads

$$(d\Lambda)_{NM_1\ldots M_p} = (p+1)\partial_{[N}\Lambda_{M_1\ldots M_p]} \ , \tag{A.5}$$

while the Hodge star of p-form Λ in d real coordinates is given by

$$(*_d\Lambda)_{N_1\ldots N_{d-p}} = \frac{1}{p!}\Lambda^{M_1\ldots M_p}\epsilon_{M_1\ldots M_p N_1\ldots N_{d-p}} \ . \tag{A.6}$$

Moreover, the identity

$$\Lambda^{(1)} \wedge *\Lambda^{(2)} = \frac{1}{p!}\Lambda^{(1)}_{M_1\ldots M_p}\Lambda^{(2)\,M_1\ldots M_p} * 1 \tag{A.7}$$

holds for two arbitrary p-forms $\Lambda^{(1)}$ and $\Lambda^{(2)}$.

Let us specify in more detail our conventions regarding complex coordinates in the internal space. For a complex manifold M with complex dimension n the complex coordinates $z^1, \ldots, z^n$ and the underlying real coordinates $y^1, \ldots, y^{2n}$ are related by

$$(z^1, \ldots, z^n) = \left(\frac{1}{\sqrt{2}}(y^1 + iy^2), \ldots, \frac{1}{\sqrt{2}}(y^{2n-1} + iy^{2n})\right) . \tag{A.8}$$

Using these conventions one finds

$$\sqrt{g}\, dy^1 \wedge \ldots \wedge dy^{2n} = \sqrt{g}(-1)^{\frac{(n-1)n}{2}} i^n dz^1 \wedge \ldots \wedge dz^n \wedge d\bar{z}^1 \wedge \ldots \wedge d\bar{z}^n = \frac{1}{n!}J^n \,, \tag{A.9}$$

with g the determinant of the metric g_{ab} in real coordinates and $\sqrt{\det g_{ab}} = \det g_{m\bar{n}}$. The Kähler form is given by

$$J = i g_{m\bar{n}} dz^m \wedge d\bar{z}^{\bar{n}} \,. \tag{A.10}$$

Let $\omega_{p,q}$ be a (p,q)-form, then its Hodge dual is the $(n-q, n-p)$ form

$$\begin{aligned} *\omega_{p,q} = {} & \frac{(-1)^{\frac{n(n-1)}{2}}\, i^n\, (-1)^{pn}}{p!q!(n-p)!(n-q)!}\, \omega_{m_1\ldots m_p\bar{n}_1\ldots\bar{n}_q}\epsilon^{m_1\ldots m_p}{}_{\bar{r}_1\ldots\bar{r}_{n-p}}\epsilon^{\bar{n}_1\ldots\bar{n}_q}{}_{s_1\ldots s_{n-q}} \times \\ & \times dz^{s_1} \wedge \cdots \wedge dz^{s_{n-q}} \wedge d\bar{z}^{\bar{r}_1} \wedge \cdots \wedge d\bar{z}^{\bar{r}^{n-p}} . \end{aligned}$$

Finally, let us record our conventions regarding Chern forms. To begin with, we define the curvature two-form for Hermitian manifolds to be

$$\mathcal{R}^m{}_n = R^m{}_{nr\bar{s}}\, dz^r \wedge d\bar{z}^{\bar{s}} \ , \tag{A.11}$$

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and we set

$$\begin{aligned}
\mathrm{Tr}\mathcal{R} &= R^m{}_{mr_1\bar{s}_2}\, dz^{r_1}\wedge d\bar{z}^{\bar{s}_1}\,,\\
\mathrm{Tr}\mathcal{R}^2 &= R^m{}_{nr_1\bar{s}_1}\, R^n{}_{mr_2\bar{s}_2}\, dz^{r_1}\wedge d\bar{z}^{\bar{s}_1}\wedge dz^{r_2}\wedge d\bar{z}^{\bar{s}_2}\,,\\
\mathrm{Tr}\mathcal{R}^3 &= R^m{}_{nr_1\bar{s}_1}\, R^n{}_{pr_2\bar{s}_2}\, R^p{}_{mr_3\bar{s}_3}\wedge dz^{r_1}\wedge d\bar{z}^{\bar{s}_1}\wedge dz^{r_2}\wedge d\bar{z}^{\bar{s}_2}\wedge dz^{r_3}\wedge d\bar{z}^{\bar{s}_3}\,.
\end{aligned}\tag{A.12}$$

The Chern forms can then be expressed in terms of the curvature two-form as

$$\begin{aligned}
c_0 &= 1\,,\\
c_1 &= \frac{1}{2\pi} i\mathrm{Tr}\mathcal{R}\,,\\
c_2 &= \frac{1}{(2\pi)^2}\frac{1}{2}\left(\mathrm{Tr}\mathcal{R}^2-(\mathrm{Tr}\mathcal{R})^2\right)\,,\\
c_3 &= \frac{1}{3}c_1c_2+\frac{1}{(2\pi)^2}\frac{1}{3}c_1\wedge\mathrm{Tr}\mathcal{R}^2-\frac{1}{(2\pi)^3}\frac{i}{3}\mathrm{Tr}\mathcal{R}^3\,,\\
c_4 &= \frac{1}{24}\left(c_1^4-\frac{1}{(2\pi)^2}6c_1^2\mathrm{Tr}\mathcal{R}^2-\frac{1}{(2\pi)^3}8ic_1\mathrm{Tr}\mathcal{R}^3\right)+\frac{1}{(2\pi)^4}\frac{1}{8}((\mathrm{Tr}\mathcal{R}^2)^2-2\mathrm{Tr}\mathcal{R}^4)\,.
\end{aligned}\tag{A.13}$$

The Chern forms of an n-dimensional Calabi-Yau manifold Y_n reduce to

$$c_3(Y_{n\geq 3})=-\frac{1}{(2\pi)^3}\frac{i}{3}\mathrm{Tr}\mathcal{R}^3\ \text{ and }\ c_4(Y_{n\geq 4})=\frac{1}{(2\pi)^4}\frac{1}{8}((\mathrm{Tr}\mathcal{R}^2)^2-2\mathrm{Tr}\mathcal{R}^4)\,.\tag{A.14}$$

B Higher-derivative dilaton terms

In this appendix we record the expression of the quantities f_1, f_2^{MN}, f_3 introduced in (2.9). We have

$$\begin{aligned}
f_1 = {}& 192R_M{}^R{}_O{}^S R^{MNOP}R_{NSPR}-48R_{MN}{}^{RS}R^{MNOP}R_{OPRS}\\
&+576R^{MN}{}_M{}^O R_N{}^{PRS}R_{OPRS}+384R^{MN}{}_M{}^O R_N{}^P{}_P{}^R R_O{}^S{}_{RS}\\
&-72R^{MN}{}_{MN}R_{OPRS}R^{OPRS}-576R^{MN}{}_M{}^O R_N{}^P{}_O{}^R R_P{}^S{}_{RS}\\
&+288R^{MN}{}_{MN}R^{OP}{}_O{}^R R_P{}^S{}_{RS}-24R^{MN}{}_{MN}R^{OP}{}_{OP}R^{RS}{}_{RS}\,,
\end{aligned}\tag{B.1}$$

$$\begin{aligned}
f_2^{MN} = {}& g^{MN}\big(192R_R{}^Q{}_O{}^V R^{RSOP}R_{SVPQ}-48R_{RS}{}^{QV}R^{RSOP}R_{OPQV}\\
&+576R^{RS}{}_R{}^O R_S{}^{PQV}R_{OPQV}+384R^{RS}{}_R{}^O R_S{}^P{}_P{}^Q R_O{}^V{}_{QV}\\
&-72R^{RS}{}_{RS}R_{OPQV}R^{OPQV}-576R^{RS}{}_R{}^O R_S{}^P{}_O{}^Q R_P{}^V{}_{QV}\\
&+288R^{RS}{}_{RS}R^{OP}{}_O{}^Q R_P{}^V{}_{QV}-24R^{RS}{}_{RS}R^{OP}{}_{OP}R^{QV}{}_{QV}\big)+\cdots = g^{MN}\,f_1+\ldots
\end{aligned}\tag{B.2}$$

where the ellipsis denote terms where the free indices M,N are on the Riemann tensors and thus will not contribute to our discussion. Furthermore,

$$\begin{aligned}
f_3 = {}& 768R_{MO}{}^{RS}R^{MNOP}R_{NRPS}+384R_M{}^R{}_O{}^S R^{MNOP}R_{NSPR}\\
&-96R_{MN}{}^{RS}R^{MNOP}R_{OPRS}+1536R^{MN}{}_M{}^O R_N{}^{PRS}R_{OPRS}\\
&-384R_{MN}{}^{RS}R^{MNOP}R_{ORPS}+768R^{MN}{}_M{}^O R_N{}^{PRS}R_{ORPS}\\
&+1536R^{MN}{}_M{}^O R_N{}^P{}_P{}^R R_O{}^S{}_{RS}-240R^{MN}{}_{MN}R_{OPRS}R^{OPRS}\\
&-1920R^{MN}{}_M{}^O R_N{}^P{}_O{}^R R_P{}^S{}_{RS}+1152R^{MN}{}_{MN}R^{OP}{}_O{}^R R_P{}^S{}_{RS}\\
&-96R^{MN}{}_{MN}R^{OP}{}_{OP}R^{RS}{}_{RS}\,.
\end{aligned}\tag{B.3}$$

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