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Renormalization of vacuum expectation values in spontaneously broken gauge theories: two-loop results

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Abstract: We complete the two-loop calculation of β-functions for vacuum expectation values (VEVs) in gauge theories by the missing $\mathcal{O}(g^4)$-terms. The full two-loop results are presented for generic and supersymmetric theories up to two-loop level in arbitrary R_ξ-gauge. The results are obtained by means of a scalar background field, identical to our previous analysis. As a by-product, the two-loop scalar anomalous dimension for generic supersymmetric theories is presented. As an application we compute the β-functions for VEVs and $\tan\beta$ in the MSSM, NMSSM, and E_6SSM.

Keywords: Spontaneous Symmetry Breaking, Renormalization Group, Supersymmetric gauge theory

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Contents

1 Introduction

The renormalization of vacuum expectation values (VEVs) in general gauge theories with R_ξ-gauge has been studied in our earlier work [1]. We showed that in R_ξ-gauge the VEVs renormalize differently from the respective scalar fields and explained the origin and behaviour of this difference. We computed VEV-counterterms and β-functions at one-loop and leading two-loop level. The purpose of this subsequent paper is to complete the two-loop renormalization of VEVs in general gauge theories and generic supersymmetric theories.

The renormalization of a VEV v can generically be written in the two equivalent forms

$$v \to v + \delta v = \sqrt{Z}\,(v + \delta \bar{v})\,, \tag{1.1}$$

with $\sqrt{Z}$ being the field renormalization constant of the corresponding scalar field. The main insight of ref. [1] has been that $\delta\bar{v}$ can be interpreted by the field renormalization $\sqrt{\hat{Z}}$ of a suitable chosen scalar background field. Thus, a simple computation becomes possible in terms of a single two-point function.

In the present paper we address the following points:

1. The missing two-loop terms of the order g^4 in $\sqrt{\hat{Z}}$ are computed and the complete two-loop VEV β-function for general gauge theories with R_ξ gauge fixing can be provided.

2. Gauge kinetic mixing in case of several U(1) gauge factors is taken into account in the computation of the g^4 terms.

3. The complete results are specialised to general supersymmetric theories in the $\overline{\text{DR}}$ scheme.

4. As a by-product the anomalous dimension $\gamma^{(2)}$ for generic $N = 1$ supersymmetric theories is derived in $\overline{\text{DR}}$ for arbitrary values of ξ.

5. As application, the concrete results for anomalous dimensions and β-functions of VEVs and $\tan\beta$ are provided in the well-known supersymmetric models MSSM, NMSSM, and E_6SSM. These results can be readily applied in practical applications. Moreover, they highlight various characteristic features of the general results.

This paper is organized as follows: section 2 provides a brief summary of the formalism and notation. Section 3 is centred on the computation of the full two-loop results for general gauge theories and supersymmetric theories. The application to the MSSM, NMSSM, and E_6SSM is carried out in section 4. Generally this paper provides a complete picture up to two-loop level and summarizes all relevant expressions, but the one-loop and Yukawa-enhanced two-loop results have already been published in [1].

2 General gauge theory and scalar background fields

The renormalization of vacuum expectations can be cast in an elegant scheme by employing a scalar background field. As elaborated in our previous publication [1], we use the general setting of real scalar fields φ_a, Weyl 2-spinors $\psi_{p\alpha}$, and real (non-abelian) gauge fields V^A_μ in the notation of [2–5]. The Lagrangian is given as

$$\begin{aligned}\mathcal{L}_{\text{inv}} = &-\frac{1}{4}F^A_{\mu\nu}F^{A\mu\nu} + \frac{1}{2}\left(D_\mu\varphi\right)_a\left(D^\mu\varphi\right)_a + i\psi^\alpha_p\sigma^\mu_{\alpha\dot\alpha}\left(D^\dagger_\mu\bar\psi^{\dot\alpha}\right)_p\\ &-\frac{1}{2!}m^2_{ab}\varphi_a\varphi_b - \frac{1}{3!}h_{abc}\varphi_a\varphi_b\varphi_c - \frac{1}{4!}\lambda_{abcd}\varphi_a\varphi_b\varphi_c\varphi_d\\ &-\frac{1}{2}\left[\left(m_f\right)_{pq}\psi^\alpha_p\psi_{q\alpha} + \text{h.c.}\right] - \frac{1}{2}\left[Y^a_{pq}\psi^\alpha_p\psi_{q\alpha}\varphi_a + \text{h.c.}\right].\end{aligned} \tag{2.1}$$

The VEVs v_a are replaced in this formalism by scalar background fields $(\hat\varphi_a + \hat v_a)$. These auxiliary fields allow to formulate a rigid (global) gauge invariant gauge fixing; analogous to ref. [6] the gauge-fixing functional reads

$$F^A = \partial^\mu V^A_\mu + ig\xi\xi'\left(\hat\varphi + \hat v\right)_a T^A_{ab}\varphi_b\,. \tag{2.2}$$

By setting $\hat\varphi_a$ to zero, one recovers the gauge theory in standard R_ξ-gauge. But the inclusion of $\hat\varphi_a$ and the rigid (global) gauge invariant gauge fixing imply that the following renormalization transformations are sufficient

$$\varphi_a \to \sqrt{Z}_{ab}\,\varphi_b\,, \tag{2.3a}$$

$$(\hat\varphi + \hat v)_a \to \sqrt{Z}_{ab}\,\sqrt{\hat Z}_{bc}\,(\hat\varphi + \hat v)_c\,. \tag{2.3b}$$

An additional VEV counterterm is then prohibited. In the standard approach, without background fields, the most generic renormalization transformation of the scalar fields with shifts reads

$$\varphi_a + v_a \to \sqrt{Z}_{ab}\left(\varphi_b + v_b + \delta\bar{v}_b\right) = \sqrt{Z}_{ab}\left(\varphi_b + v_b\right) + \delta v_a \,. \tag{2.4}$$

The two formalisms are equivalent, with the following identifications

$$\delta v_a = \left(\sqrt{Z}\sqrt{\hat{Z}} - 1\right)_{ab}\hat{v}_b = \frac{1}{2}\left(\delta Z + \delta\hat{Z}\right)_{ab}\hat{v}_b + \mathcal{O}\left(\hbar^2\right)\ , \tag{2.5a}$$

$$\delta\bar{v}_a = \left(\sqrt{\hat{Z}} - 1\right)_{ab}\hat{v}_b = \frac{1}{2}\delta\hat{Z}_{ab}\hat{v}_b + \mathcal{O}\left(\hbar^2\right). \tag{2.5b}$$

As a result, the β function of the VEV can be obtained as

$$\beta(v_a) = \left(\gamma_{ab} + \hat{\gamma}_{ab}\right) v_b \,, \tag{2.6}$$

with the anomalous dimensions γ and $\hat{\gamma}$ corresponding to the field renormalizations $\sqrt{Z}$ and $\sqrt{\hat{Z}}$, respectively.

One of the main results of ref. [1] was that the computation of $\delta\hat{Z}$ can be reduced to the very simple, unphysical two-point function

$$\Gamma^{\mathrm{CT},(n)}_{\hat{q}_a, K_{\varphi_b}} = -\frac{i}{2}\delta\hat{Z}^{(n)}_{ba}\ . \tag{2.7}$$

Here K_{φ_b} are the sources of the BRS transformation of the scalar field, and $\hat{q}_a$ is the BRS transformation of $\hat{\varphi}_a$. Both of these unphysical fields appear in a very simple and well prescribed way in the Lagrangian.

Our formalism is independent of the actual value assigned to $\hat{v}_a$. We can therefore choose $\hat{v}_a$ as the minimum of the full loop-corrected scalar potential. Hence, our β-functions describe the running of the full VEV, which is required, for example, in many supersymmetry applications such as spectrum generators [7, 8]. Note that this running VEV has to be distinguished from other definitions used for example in the Standard Model [9, 10], which corresponds to the VEV defined explicitly in terms of the running tree-level potential parameters

$$v(\mu) = \sqrt{\frac{m^2(\mu)}{\lambda(\mu)}}. \tag{2.8}$$

Ref. [9] contains a diagram exposing the difference in the running between the different definitions.

JHEP01(2014)068

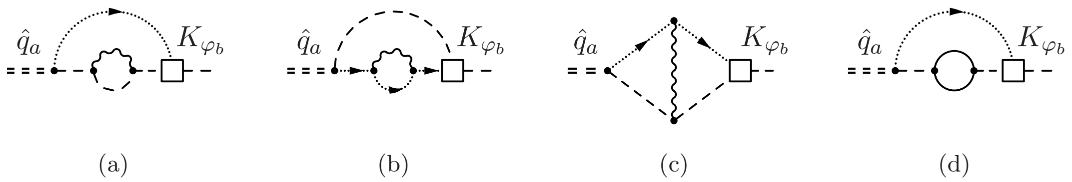

Figure 1. All relevant graphs for determination of two-loop corrections to $\Gamma_{\hat{q}_a,K_{\varphi_b}}$: graphs 1(a), 1(b), and 1(c) are $\mathcal{O}(g^4)$-contributions; graph 1(d) corresponds to $\mathcal{O}(g^2YY^\dagger)$.

Diagram	$\hat{S}_{ab}$	A	B
1(a)	$g^4\xi\xi' C^2_{ac}(\mathrm{S}) C^2_{cb}(\mathrm{S})$	$-3+\xi$	$1+\xi$
1(b)	$g^4\xi\xi' C_2(\mathrm{G}) C^2_{ab}(\mathrm{S})$	$\frac{-3+\xi}{4}$	$\frac{1+\xi}{4}$
1(c)	$g^4\xi\xi' C_2(\mathrm{G}) C^2_{ab}(\mathrm{S})$	$-\frac{\xi}{2}$	$\frac{3-\xi}{2}$
1(d)	$g^2\xi\xi' C^2_{ac}(\mathrm{S}) Y^2_{cb}(\mathrm{S})$	1	-1

Table 1. Singular parts of the two-loop diagrams for $\Gamma_{\hat{q}_a,K_{\varphi_b}}$. All relevant one-loop subdiagrams have been renormalized such that the above expressions correspond to the two-loop diagrams depicted plus the necessary diagrams with one-loop counterterm insertions.

3 Results

3.1 General gauge theory

The one-loop results for the anomalous dimensions $\gamma_{ab}(\mathrm{S})$, $\hat{\gamma}_{ab}(\mathrm{S})$ and β-functions $\beta(v_a)$ in a general gauge theory have been presented in [1] and read

$$\gamma^{(1)}_{ab}(\mathrm{S}) = \frac{1}{(4\pi)^2}\left[g^2\left(3-\xi\right)C^2_{ab}(\mathrm{S}) - Y^2_{ab}(\mathrm{S})\right] \ , \tag{3.1a}$$

$$\hat{\gamma}^{(1)}_{ab}(\mathrm{S}) = \frac{1}{(4\pi)^2} 2g^2\xi\xi' C^2_{ab}(\mathrm{S}) \ , \tag{3.1b}$$

$$\beta^{(1)}(v_a) = \frac{1}{(4\pi)^2}\left[g^2\left(3-\xi+2\xi\xi'\right)C^2_{ab}(\mathrm{S}) - Y^2_{ab}(\mathrm{S})\right] v_b \ . \tag{3.1c}$$

At the two-loop level, the terms of $\mathcal{O}(g^2YY^\dagger)$ of $\hat{\gamma}^{(2)}$ [1] and the full $\gamma^{(2)}$ [2, 5] have already been published. Therefore, the computation of $\mathcal{O}(g^4)$-terms in $\hat{\gamma}^{(2)}$ remains at two-loop. Figure 1 contains the four relevant graphs that generate the divergencies in the loop corrections of $\Gamma_{\hat{q}_a,K_{\varphi_b}}$, wherein we implicitly understand one-loop subdivergencies to be subtracted. As before, all calculations are carried out in $\overline{\mathrm{MS}}$ or equivalently MS scheme.

In analogy to the presentation of Machacek & Vaughn [2–4], we provide the contributions of each diagram of figure 1 in table 1 with the notation

$$\delta\hat{Z}^{(2)}_{ab} = \frac{1}{(4\pi)^4}\hat{S}_{ab}\left(\frac{A}{\eta^2}+\frac{B}{\eta}\right), \tag{3.2}$$

wherein $1/\eta = 1/\epsilon + \ln(4\pi) - \gamma_E$.

The completed two-loop results in the $\overline{\text{MS}}$ scheme read as follows

$$\gamma_{ab}^{(2)}(\text{S}) = \frac{1}{(4\pi)^4}\Bigg\{g^4 C_{ab}^2(\text{S})\left[\left(\frac{35}{3} - 2\xi - \frac{1}{4}\xi^2\right) C_2(\text{G}) - \frac{10}{6}S_2(\text{F}) - \frac{11}{12}S_2(\text{S})\right] \tag{3.3a}$$
$$- \frac{3}{2}g^4 C_{ac}^2(\text{S})C_{cb}^2(\text{S}) + \frac{3}{2}H_{ab}^2(\text{S}) + \bar{H}_{ab}^2(\text{S}) - \frac{10}{2}g^2 Y_{ab}^{2F}(\text{S}) - \frac{1}{2}\Lambda_{ab}^2(\text{S})\Bigg\},$$

$$\hat{\gamma}_{ab}^{(2)}(\text{S}) = \frac{\xi\xi'}{(4\pi)^4}\Bigg\{g^4\left[2\left(1+\xi\right) C_{ac}^2(\text{S})C_{cb}^2(\text{S}) + \frac{7-\xi}{2}C_2(\text{G})C_{ab}^2(\text{S})\right] \tag{3.3b}$$
$$- 2g^2 C_{ac}^2(\text{S})Y_{cb}^2(\text{S})\Bigg\},$$

$$\beta^{(2)}(v_a) = \frac{1}{(4\pi)^4}\Bigg\{g^4 C_{ab}^2(\text{S})\left[\left(\frac{35}{3} - 2\xi - \frac{1}{4}\xi^2 + \frac{7-\xi}{2}\xi\xi'\right) C_2(\text{G}) - \frac{10}{6}S_2(\text{F}) - \frac{11}{12}S_2(\text{S})\right]$$
$$+ g^4\left[2\xi\xi'\left(1+\xi\right) - \frac{3}{2}\right] C_{ac}^2(\text{S})C_{cb}^2(\text{S}) - \frac{1}{2}\Lambda_{ab}^2(\text{S}) \tag{3.3c}$$
$$+ \frac{3}{2}H_{ab}^2(\text{S}) + \bar{H}_{ab}^2(\text{S}) - \frac{10}{2}g^2 Y_{ab}^{2F}(\text{S}) - 2\xi\xi' g^2 C_{ac}^2(\text{S})Y_{cb}^2(\text{S})\Bigg\} v_b\,.$$

3.2 Kinetic mixing

The results of section 3.1 hold for simple gauge groups. The generalization to product groups is obvious, except for gauge kinetic mixing of U(1) field strength tensors. In the recent literature, the impact of gauge kinetic mixing on RGEs has been studied quite extensively up to two-loop level [11–13]. Following the approach of refs. [12, 13], we need to provide substitution rules for $\hat{\gamma}$ to take kinetic mixing into account.

A generic gauge group G can be decomposed into

$$G = \left(\bigotimes_{k\in I} G_k\right) \otimes \left(\bigotimes_{a\in J} \text{U}(1)_a\right), \tag{3.4}$$

with the simple groups G_k and the two (finite) sets $I, J \subset \mathbb{N}$. The part of the Lagrangian describing kinetic mixing reads

$$\mathcal{L} = -\frac{1}{4}\sum_{k\in I} F_{k,\mu\nu}^{A_k} F_k^{A_k,\mu\nu} - \frac{1}{4}\sum_{a,b\in J} F_{a,\mu\nu}\Xi_{ab}F_b^{\mu\nu} + \cdots\,. \tag{3.5}$$

Analogously to refs. [12, 13], we define

$$\hat{g}_{ab} := \sum_{c\in J}\delta_{ac}g'_c\sqrt{\Xi}_{cb}^{-1} \quad \text{and} \quad W_a := \sum_{b\in J} Q_b\hat{g}_{ba}\,, \tag{3.6}$$

with the root defined by $\sqrt{\Xi}\sqrt{\Xi} = \Xi$.

The inspection of the graphs in figure 1 implies that there do not exist any gauge kinetic mixing contributions to $\hat{\gamma}^{(1)}$ and the $\mathcal{O}(g^2 YY^\dagger)$-part of $\hat{\gamma}^{(2)}$, because BRS-ghost and -antighost are not affected by kinetic mixing. Graphs 1(b) and 1(c) are not affected either, as U(1)-gauge fields do not interact with the corresponding Faddeev-Popov-ghosts. Hence, the only change for kinetic mixing stems from graph 1(a), in particular from the one-loop insertion of the scalar self-energy. The relevant substitution rule is given by

$$g^4 C^2(\mathrm{S}) C^2(\mathrm{S}) \xrightarrow[\text{kin. mix}]{\hat{\gamma}} \left[\sum_{k\in I} g_k^2 C^2_{G_k}(X) + \sum_{d\in J} W_d(X) W_d(X)\right] \times \left[\sum_{k\in I} g_k^2 C^2_{G_k}(X) + \sum_{d\in J} g'^2_d Q_d^2(X)\right] . \tag{3.7}$$

Here g_k denote the non-abelian gauge couplings and g'_d the abelian ones, with the corresponding quantum numbers Q_d. Further, X denotes the field under consideration, e.g. up- or down-type Higgs. The substitution rules for γ can be found in [12, 13].

3.3 Supersymmetric gauge theory

The treatment of supersymmetric theories requires to take three subtleties into account: (i) supersymmetric theories are formulated in terms of complex scalar fields, (ii) the coupling structure is severely restricted by supersymmetry, and (iii) the use of the supersymmetry-preserving renormalization scheme $\overline{\mathrm{DR}}$.

The first two points are merely computational issues, in the sense that one needs to take care of the changed coupling structure and the scalar field representation. Hence, these aspects will not be spelled out in detail and we directly present the results for complex scalar fields in a notation based on ref. [14]. We will, however, give some details on the conversion to $\overline{\mathrm{DR}}$, which requires transition counterterms for parameters [15] and fields [16]. The existence of such transition counterterms is due to the equivalence of dimensional reduction and dimensional regularisation as shown in ref. [17].

At one-loop level the results have been provided earlier [1] and read

$$\gamma^{(1)}_{ab}(\mathrm{S})\Big|^{\overline{\mathrm{DR}}}_{\mathrm{SUSY}} = \frac{1}{(4\pi)^2}\left[g^2\left(1-\xi\right) C^2_{ab}(\mathrm{S}) - \frac{1}{2} Y^*_{apq} Y_{bpq}\right] , \tag{3.8a}$$

$$\hat{\gamma}^{(1)}_{ab}(\mathrm{S})\Big|^{\overline{\mathrm{DR}}}_{\mathrm{SUSY}} = \frac{1}{(4\pi)^2} 2 g^2 \xi\xi' C^2_{ab}(\mathrm{S}) , \tag{3.8b}$$

$$\beta^{(1)}(v_a)\Big|^{\overline{\mathrm{DR}}}_{\mathrm{SUSY}} = \frac{1}{(4\pi)^2}\left[g^2\left(1-\xi+2\xi\xi'\right) C^2_{ab}(\mathrm{S}) - \frac{1}{2} Y^*_{apq} Y_{bpq}\right] v_b . \tag{3.8c}$$

The first two-loop renormalization studies of softly broken $N = 1$ SUSY theories in $\overline{\mathrm{DR}}$ have been performed in [18–20], though not always in component fields as used here. To our knowledge, the full result for $\gamma^{(2)}$ in a general supersymmetric theory is not available in the literature, except for Landau gauge ($\xi = 0$) [14]. In order to obtain the result for arbitrary ξ we proceed in the following steps. We first reevaluate the Feynman graphs in ref. [2] with a generic $N = 1$ supersymmetric Lagrangian.[1] Then we apply transition counterterms for the conversion from $\overline{\mathrm{MS}}$ to $\overline{\mathrm{DR}}$. This step differs from the case of the $\overline{\mathrm{DR}}$ β-functions computed in ref. [18]. Since the β-functions in that reference are gauge invariant, physical

[1] Note the remarks by ref. [5] on the implicitly real spinors of Machacek & Vaughn.

quantities, only transition counterterms for physical parameters were required, and those were provided in ref. [15]. In the present case of γ-functions, also transition counterterms for field renormalization and gauge parameters are necessary. These were presented in ref. [16]. Fortunately, however, the needed additional transition counterterms for the scalar field renormalization and for the gauge parameter are zero,

$$\delta Z_{\varphi}^{(1),\mathrm{trans}} = 0, \tag{3.9}$$

$$\delta Z_{\xi}^{(1),\mathrm{trans}} = 0. \tag{3.10}$$

The transition for $\hat{\gamma}$ to supersymmetry and $\overline{\mathrm{DR}}$ could be carried out in an analogous way, by employing transition counterterms. However, it is also possible and simpler to use the fact that there is no difference between $\overline{\mathrm{MS}}$ and $\overline{\mathrm{DR}}$ for any diagram contributing to $\delta\hat{Z}$ at the two-loop level. Hence, $\hat{\gamma}$ is equal in the $\overline{\mathrm{MS}}$ and $\overline{\mathrm{DR}}$ schemes. From this knowledge, one can then derive additional transition counterterms as a by-product: $\delta\hat{Z}^{(1),\mathrm{trans}} = 0$, and owing to the non-renormalization of the gauge fixing,

$$\delta Z_{\xi'}^{(1),\mathrm{trans}} = -\delta Z_{g}^{(1),\mathrm{trans}} + \frac{1}{2}\delta Z_{V}^{(1),\mathrm{trans}} = \frac{1}{(4\pi)^2}\frac{g^2}{3}C_2(\mathrm{G})\;, \tag{3.11}$$

where $\delta Z_{V}^{(1),\mathrm{trans}}$ denotes the transition counterterm for the gauge field, as obtained in ref. [16]. With these ingredients, the full gauge-dependent two-loop results for the anomalous dimensions γ and $\hat{\gamma}$ as well as for the VEV β-function can be obtained. In $\overline{\mathrm{DR}}$ they read

$$\begin{aligned}\gamma_{ab}^{(2)}(\mathrm{S})\Big|_{\mathrm{SUSY}}^{\overline{\mathrm{DR}}} = \frac{1}{(4\pi)^4}\Bigg\{ & g^4\left[\left(\frac{9}{4}-\frac{5}{3}\xi-\frac{1}{4}\xi^2\right)C_2(\mathrm{G})-S_2(\mathrm{S})\right]C_{ab}^2(\mathrm{S}) \\ & - 2g^4 C_{ac}^2(\mathrm{S})C_{cb}^2(\mathrm{S}) + \frac{1}{2}Y_{arc}^{*}Y_{rpq}Y_{pqd}^{*}Y_{bcd} \\ & + g^2\left[C_{ac}^2(\mathrm{S})Y_{cpq}^{*}Y_{bpq} - 2Y_{apq}^{*}C_{pr}^2(\mathrm{S})Y_{brq}\right]\Bigg\}\;,\end{aligned} \tag{3.12a}$$

$$\begin{aligned}\hat{\gamma}_{ab}^{(2)}(\mathrm{S})\Big|_{\mathrm{SUSY}}^{\overline{\mathrm{DR}}/\overline{\mathrm{MS}}} = \frac{\xi\xi'}{(4\pi)^4}\Bigg\{ & g^4\left[\frac{7-\xi}{2}C_2(\mathrm{G})C_{ab}^2(\mathrm{S}) - 2\left(1-\xi\right)C_{ac}^2(\mathrm{S})C_{cb}^2(\mathrm{S})\right] \\ & - g^2 C_{ac}^2(\mathrm{S})Y_{cpq}^{*}Y_{bpq}\Bigg\}\;,\end{aligned} \tag{3.12b}$$

$$\begin{aligned}\beta^{(2)}(v_a)\Big|_{\mathrm{SUSY}}^{\overline{\mathrm{DR}}} = \frac{1}{(4\pi)^4}\Bigg\{ & g^4\left[\left(\frac{9}{4}-\frac{5}{3}\xi-\frac{1}{4}\xi^2+\frac{7-\xi}{2}\xi\xi'\right)C_2(\mathrm{G})-S_2(\mathrm{S})\right]C_{ab}^2(\mathrm{S}) \\ & - g^4\left[2\xi\xi'\left(1-\xi\right)+2\right]C_{ac}^2(\mathrm{S})C_{cb}^2(\mathrm{S}) + \frac{1}{2}Y_{arc}^{*}Y_{rpq}Y_{pqd}^{*}Y_{bcd} \\ & + g^2\left[1-\xi\xi'\right]C_{ac}^2(\mathrm{S})Y_{cpq}^{*}Y_{bpq} - 2g^2 Y_{apq}^{*}C_{pr}^2(\mathrm{S})Y_{brq}\Bigg\}\,v_b\;.\end{aligned} \tag{3.12c}$$

4 Application to concrete supersymmetric models

This section provides the explicit two-loop results for the renormalization of all VEVs in the MSSM, NMSSM, and E_6SSM, using the notation of ref. [1]. For completeness and convenience, we provide the full results including previously known ones.

4.1 MSSM

One-loop. The one-loop results for the anomalous dimensions of the MSSM Higgs doublets read

$$(4\pi)^2\gamma^{(1),\overline{\mathrm{DR}}}_{\mathrm{MSSM}}(H_u) = (1-\xi)\left(\frac{3}{20}g_1^2+\frac{3}{4}g_2^2\right) - N_c \operatorname{Tr}\left(y^u y^{u\dagger}\right) , \tag{4.1a}$$

$$(4\pi)^2\hat{\gamma}^{(1),\overline{\mathrm{DR}}}_{\mathrm{MSSM}}(H_u) = 2\xi\xi'\left(\frac{3}{20}g_1^2+\frac{3}{4}g_2^2\right) . \tag{4.1b}$$

$$(4\pi)^2\gamma^{(1),\overline{\mathrm{DR}}}_{\mathrm{MSSM}}(H_d) = (1-\xi)\left(\frac{3}{20}g_1^2+\frac{3}{4}g_2^2\right) - N_c \operatorname{Tr}\left(y^d y^{d\dagger}\right) - \operatorname{Tr}\left(y^e y^{e\dagger}\right) , \tag{4.2a}$$

$$(4\pi)^2\hat{\gamma}^{(1),\overline{\mathrm{DR}}}_{\mathrm{MSSM}}(H_d) = 2\xi\xi'\left(\frac{3}{20}g_1^2+\frac{3}{4}g_2^2\right) . \tag{4.2b}$$

The β-function of $\tan\beta$ follows then as

$$\frac{\beta^{(1),\overline{\mathrm{DR}}}_{\mathrm{MSSM}}(\tan\beta)}{\tan\beta} = -\frac{1}{(4\pi)^2}\left[N_c \operatorname{Tr}\left(y^u y^{u\dagger}\right) - N_c \operatorname{Tr}\left(y^d y^{d\dagger}\right) - \operatorname{Tr}\left(y^e y^{e\dagger}\right)\right] . \tag{4.3}$$

Two-loop. The application of the general two-loop results yields for the MSSM

$$\begin{aligned}(4\pi)^4\gamma^{(2),\overline{\mathrm{DR}}}_{\mathrm{MSSM}}(H_u) = {}& -\frac{207}{200}g_1^4 - \frac{9}{20}g_1^2g_2^2 - \left(3+\frac{5}{2}\xi+\frac{3}{8}\xi^2\right)g_2^4 \\ & - \left(\frac{4}{15}g_1^2+\frac{16}{3}g_3^2\right)N_c \operatorname{Tr}\left(y^u y^{u\dagger}\right) \\ & + N_c \operatorname{Tr}\left(y^u y^{d\dagger} y^d y^{u\dagger}\right) + 3N_c \operatorname{Tr}\left(y^u y^{u\dagger} y^u y^{u\dagger}\right) ,\end{aligned} \tag{4.4a}$$

$$(4\pi)^4\hat{\gamma}^{(2),\overline{\mathrm{DR}}}_{\mathrm{MSSM}}(H_u) = -\xi\xi'\left\{\left(\frac{3}{10}g_1^2+\frac{3}{2}g_2^2\right)\left[N_c \operatorname{Tr}\left(y^u y^{u\dagger}\right)\right] + \mathcal{R}_{\mathrm{MSSM}}\right\} , \tag{4.4b}$$

$$\begin{aligned}(4\pi)^4\gamma^{(2),\overline{\mathrm{DR}}}_{\mathrm{MSSM}}(H_d) = {}& -\frac{207}{200}g_1^4 - \frac{9}{20}g_1^2g_2^2 - \left(3+\frac{5}{2}\xi+\frac{3}{8}\xi^2\right)g_2^4 \\ & - \left(-\frac{2}{15}g_1^2+\frac{16}{3}g_3^2\right)N_c \operatorname{Tr}\left(y^d y^{d\dagger}\right) - \frac{6}{5}g_1^2 \operatorname{Tr}\left(y^e y^{e\dagger}\right) \\ & + 3N_c \operatorname{Tr}\left(y^d y^{d\dagger} y^d y^{d\dagger}\right) + N_c \operatorname{Tr}\left(y^d y^{u\dagger} y^u y^{d\dagger}\right) + 3\operatorname{Tr}\left(y^e y^{e\dagger} y^e y^{e\dagger}\right) ,\end{aligned} \tag{4.5a}$$

$$(4\pi)^4\hat{\gamma}^{(2),\overline{\mathrm{DR}}}_{\mathrm{MSSM}}(H_d) = -\xi\xi'\left\{\left(\frac{3}{10}g_1^2+\frac{3}{2}g_2^2\right)\left[N_c \operatorname{Tr}\left(y^d y^{d\dagger}\right)+\operatorname{Tr}\left(y^e y^{e\dagger}\right)\right]+\mathcal{R}_{\mathrm{MSSM}}\right\} , \tag{4.5b}$$

with

$$\mathcal{R}_{\mathrm{MSSM}} = (1-\xi)\frac{9}{2}\left(\frac{1}{100}g_1^4+\frac{1}{10}g_1^2g_2^2+\frac{1}{4}g_2^4\right) - 3\frac{7-\xi}{4}g_2^4 . \tag{4.6}$$

JHEP01(2014)068

The explicit calculations confirm our earlier statement [1] that the same $\mathcal{R}_{\rm MSSM}$ terms in $\hat\gamma^{(2)}$ appear for up- and down-Higgs. Thus, we obtain the two-loop β-function for $\tan\beta$ as

$$\frac{\beta^{(2),\overline{\rm DR}}_{\rm MSSM}(\tan\beta)}{\tan\beta} = \frac{1}{(4\pi)^4}\Bigg\{-\left(\frac{4}{15}g_1^2+\frac{16}{3}g_3^2\right)N_c\,{\rm Tr}\left(y^u y^{u\dagger}\right) \\ +\left(-\frac{2}{15}g_1^2+\frac{16}{3}g_3^2\right)N_c\,{\rm Tr}\left(y^d y^{d\dagger}\right)+\frac{6}{5}g_1^2\,{\rm Tr}\left(y^e y^{e\dagger}\right) \\ +3N_c\,{\rm Tr}\left(y^u y^{u\dagger}y^u y^{u\dagger}\right)-3N_c\,{\rm Tr}\left(y^d y^{d\dagger}y^d y^{d\dagger}\right)-3\,{\rm Tr}\left(y^e y^{e\dagger}y^e y^{e\dagger}\right)\Bigg\} \\ +\frac{1}{(4\pi)^2}\xi\xi'\left(\frac{3}{10}g_1^2+\frac{3}{2}g_2^2\right)\frac{\beta^{(1),\overline{\rm DR}}_{\rm MSSM}(\tan\beta)}{\tan\beta}\,. \tag{4.7}$$

The gauge-dependence of $\tan\beta$ at two-loop stems solely from the $\hat\gamma$ terms.

4.2 NMSSM

One-loop. The one-loop anomalous dimensions for the Higgs doublets $H_{u,d}$ in the NMSSM resemble the corresponding MSSM results:

$$\gamma^{(1),\overline{\rm DR}}_{\rm NMSSM}(H_{u,d}) = \gamma^{(1),\overline{\rm DR}}_{\rm MSSM}(H_{u,d}) - \frac{1}{(4\pi)^2}|\lambda|^2\,, \tag{4.8a}$$

$$\hat\gamma^{(1),\overline{\rm DR}}_{\rm NMSSM}(H_{u,d}) = \hat\gamma^{(1),\overline{\rm DR}}_{\rm MSSM}(H_{u,d})\,. \tag{4.8b}$$

The NMSSM Higgs singlet S has the following RGE coefficients:

$$\gamma^{(1),\overline{\rm DR}}_{\rm NMSSM}(S) = -\frac{1}{(4\pi)^2}2\left(|\lambda|^2+|\kappa|^2\right)\,, \tag{4.9a}$$

$$\hat\gamma^{(1),\overline{\rm DR}}_{\rm NMSSM}(S) = 0\,. \tag{4.9b}$$

Due to the unchanged gauge group the one-loop result for $\tan\beta$ is identical to the MSSM

$$\beta^{(1),\overline{\rm DR}}_{\rm NMSSM}(\tan\beta) = \beta^{(1),\overline{\rm DR}}_{\rm MSSM}(\tan\beta)\,. \tag{4.10}$$

Two-loop. The two-loop results for the Higgs-doublets are given by

$$\gamma^{(2),\overline{\rm DR}}_{\rm NMSSM}(H_u) = \gamma^{(2),\overline{\rm DR}}_{\rm MSSM}(H_u)+\frac{|\lambda|^2}{(4\pi)^4}\left[2|\kappa|^2+3|\lambda|^2+N_c\,{\rm Tr}\left(y^d y^{d\dagger}\right)+{\rm Tr}\left(y^e y^{e\dagger}\right)\right]\,, \tag{4.11a}$$

$$\hat\gamma^{(2),\overline{\rm DR}}_{\rm NMSSM}(H_u) = -\frac{\xi\xi'}{(4\pi)^4}\left\{\left(\frac{3}{10}g_1^2+\frac{3}{2}g_2^2\right)\left[N_c\,{\rm Tr}\left(y^u y^{u\dagger}\right)+|\lambda|^2\right]+\mathcal{R}_{\rm NMSSM}\right\}, \tag{4.11b}$$

$$\gamma^{(2),\overline{\rm DR}}_{\rm NMSSM}(H_d) = \gamma^{(2),\overline{\rm DR}}_{\rm MSSM}(H_d)+\frac{|\lambda|^2}{(4\pi)^4}\left[2|\kappa|^2+3|\lambda|^2+N_c\,{\rm Tr}\left(y^u y^{u\dagger}\right)\right]\,, \tag{4.12a}$$

$$\hat\gamma^{(2),\overline{\rm DR}}_{\rm NMSSM}(H_d) = -\frac{\xi\xi'}{(4\pi)^4}\Bigg\{\left(\frac{3}{10}g_1^2+\frac{3}{2}g_2^2\right)\left[N_c\,{\rm Tr}\left(y^d y^{d\dagger}\right)+{\rm Tr}\left(y^e y^{e\dagger}\right)+|\lambda|^2\right] \\ +\mathcal{R}_{\rm NMSSM}\Bigg\}, \tag{4.12b}$$

with $\mathcal{R}_{\text{NMSSM}} = \mathcal{R}_{\text{MSSM}}$. Again, the $\mathcal{R}_{\text{NMSSM}}$ terms in $\hat{\gamma}^{(2)}$ are equal for up- and down-Higgs. Next, we can provide the results for the two-loop gauge singlet:

$$\begin{aligned}(4\pi)^4\gamma^{(2),\overline{\text{DR}}}_{\text{NMSSM}}(S) &= 8|\kappa|^4 + 8|\kappa|^2|\lambda|^2 + 4|\lambda|^4 - \left(\frac{6}{5}g_1^2 + 6g_2^2\right)|\lambda|^2 \qquad (4.13\text{a})\\ &\quad + 2|\lambda|^2\left[N_c \operatorname{Tr}\left(y^d y^{d\dagger}\right) + \operatorname{Tr}\left(y^e y^{e\dagger}\right) + N_c \operatorname{Tr}\left(y^u y^{u\dagger}\right)\right] ,\\ \hat{\gamma}^{(2),\overline{\text{DR}}}_{\text{NMSSM}}(S) &= 0 . \qquad (4.13\text{b})\end{aligned}$$

Finally, the two-loop β-function for $\tan\beta$ turns out to be modified by the additional Yukawa-coupling λ in comparison to the MSSM

$$\begin{aligned}\frac{\beta^{(2),\overline{\text{DR}}}_{\text{NMSSM}}(\tan\beta)}{\tan\beta} &= \gamma^{(2),\overline{\text{DR}}}_{\text{MSSM}}(H_u) - \gamma^{(2),\overline{\text{DR}}}_{\text{MSSM}}(H_d) + \frac{|\lambda|^2}{(4\pi)^2}\frac{\beta^{(1),\overline{\text{DR}}}_{\text{MSSM}}(\tan\beta)}{\tan\beta} \qquad (4.14\text{a})\\ &\quad + \frac{1}{(4\pi)^2}\xi\xi'\left(\frac{3}{10}g_1^2 + \frac{3}{2}g_2^2\right)\frac{\beta^{(1),\overline{\text{DR}}}_{\text{MSSM}}(\tan\beta)}{\tan\beta}\\ &= \frac{\beta^{(2),\overline{\text{DR}}}_{\text{MSSM}}(\tan\beta)}{\tan\beta} + \frac{|\lambda|^2}{(4\pi)^2}\frac{\beta^{(1),\overline{\text{DR}}}_{\text{MSSM}}(\tan\beta)}{\tan\beta} . \qquad (4.14\text{b})\end{aligned}$$

4.3 E$_6$SSM

The E$_6$SSM introduces a new feature: the U$(1)_N$-extension of the SM-gauge group leads inevitably to gauge kinetic mixing. The notations for kinetic mixing of section 3.2 can be specialized to the E$_6$SSM as

$$\hat{g} = \begin{pmatrix} g_1 & g_{11'} \\ g_{1'1} & g_1' \end{pmatrix} \quad \text{and} \quad Q(X) := \begin{pmatrix} \sqrt{\frac{3}{5}}Q_Y(X) \\ \sqrt{\frac{1}{40}}Q_N(X) \end{pmatrix} . \qquad (4.15)$$

Note that eq. (4.15) contains the GUT-normalized U$(1)_Y$- and U$(1)_N$-charges for any field X. The quantum-numbers $Q_Y(X)$ and $Q_N(X)$ are those of ref. [21].

One-loop. In comparison to our earlier results [1] the one-loop anomalous dimensions γ and $\hat{\gamma}$ are now extended for the general case of gauge kinetic mixing already present at tree-level. For the Higgs-doublets $H_{u/d,3}$ and the SM-singlet S_3 our computations yield

$$\begin{aligned}\gamma^{(1),\overline{\text{DR}}}_{\text{E}_6\text{SSM}}(H_{u,3}) &= \gamma^{(1),\overline{\text{DR}}}_{\text{MSSM}}(H_u) + \frac{1}{(4\pi)^2}\left[\frac{1}{10}(1-\xi){g_1'}^2 - |\lambda_3|^2\right] \qquad (4.16\text{a})\\ &\quad + \frac{1-\xi}{(4\pi)^2}\left(\frac{3}{20}g_{11'}^2 + \frac{1}{10}g_{1'1}^2 - \frac{1}{5}\sqrt{\frac{3}{2}}g_{11'}g_1' - \frac{1}{5}\sqrt{\frac{3}{2}}g_{1'1}g_1\right) ,\\ \hat{\gamma}^{(1),\overline{\text{DR}}}_{\text{E}_6\text{SSM}}(H_{u,3}) &= \hat{\gamma}^{(1),\overline{\text{DR}}}_{\text{MSSM}}(H_u) + \frac{1}{(4\pi)^2}\frac{1}{5}\xi\xi'{g_1'}^2 . \qquad (4.16\text{b})\end{aligned}$$

$$\gamma^{(1),\overline{\rm DR}}_{\rm E_6SSM}(H_{d,3}) = \gamma^{(1),\overline{\rm DR}}_{\rm MSSM}(H_d) + \frac{1}{(4\pi)^2}\left[\frac{9}{40}(1-\xi)\,{g_1'}^2 - |\lambda_3|^2\right] + \frac{1-\xi}{(4\pi)^2}\left(\frac{3}{20}g_{11'}^2 + \frac{9}{40}g_{1'1}^2 + \frac{3}{10}\sqrt{\frac{3}{2}}g_{11'}g_1' + \frac{3}{10}\sqrt{\frac{3}{2}}g_{1'1}g_1\right), \tag{4.17a}$$

$$\hat\gamma^{(1),\overline{\rm DR}}_{\rm E_6SSM}(H_{d,3}) = \hat\gamma^{(1),\overline{\rm DR}}_{\rm MSSM}(H_d) + \frac{1}{(4\pi)^2}\frac{9}{20}\xi\xi'{g_1'}^2 . \tag{4.17b}$$

$$(4\pi)^2\gamma^{(1),\overline{\rm DR}}_{\rm E_6SSM}(S_3) = \frac{5}{8}(1-\xi)\left({g_1'}^2 + g_{1'1}^2\right) - 2\,{\rm Tr}\left(\lambda\lambda^\dagger\right) - N_c\,{\rm Tr}\left(\kappa\kappa^\dagger\right), \tag{4.18a}$$

$$(4\pi)^2\hat\gamma^{(1),\overline{\rm DR}}_{\rm E_6SSM}(S_3) = \frac{5}{4}\xi\xi'{g_1'}^2 . \tag{4.18b}$$

Thus, the one-loop β-function for $\tan\beta$ is given by

$$\frac{\beta^{(1),\overline{\rm DR}}_{\rm E_6SSM}(\tan\beta)}{\tan\beta} = \frac{\beta^{(1),\overline{\rm DR}}_{\rm MSSM}(\tan\beta)}{\tan\beta} - \frac{1}{(4\pi)^2}\frac{1}{8}\left(1-\xi+2\xi\xi'\right){g_1'}^2 - \frac{1-\xi}{(4\pi)^2}\left[\frac{1}{8}g_{1'1}^2 + \frac{1}{2}\sqrt{\frac{3}{2}}\left(g_{11'}g_1' + g_{1'1}g_1\right)\right]. \tag{4.19}$$

Eq. (4.19) illustrates once more the gauge dependence of $\tan\beta$ at one-loop level due to the different $\mathrm{U}(1)_N$-quantum numbers of the Higgs doublets, see [1].

Two-loop. We restrict the list of two-loop results to the $\hat\gamma$ and the β-function for $\tan\beta$. The two-loop results for the $\mathrm{E_6SSM}$ Higgs doublets are

$$(4\pi)^4\hat\gamma^{(2),\overline{\rm DR}}_{\rm E_6SSM}(H_{u,3}) = -\xi\xi'\left\{\left(\frac{3}{10}g_1^2+\frac{3}{2}g_2^2+\frac{1}{5}{g_1'}^2\right)\left[N_c\,{\rm Tr}\left(y^u y^{u\dagger}\right)+|\lambda_3|^2\right]+\mathcal{R}_u\right\}, \tag{4.20a}$$

$$(4\pi)^4\hat\gamma^{(2),\overline{\rm DR}}_{\rm E_6SSM}(H_{d,3}) = -\xi\xi'\left\{\left(\frac{3}{10}g_1^2+\frac{3}{2}g_2^2+\frac{9}{20}{g_1'}^2\right)\times\left[N_c\,{\rm Tr}\left(y^d y^{d\dagger}\right)+{\rm Tr}\left(y^e y^{e\dagger}\right)+|\lambda_3|^2\right]+\mathcal{R}_d\right\}, \tag{4.20b}$$

with

$$\mathcal{R}_u = \mathcal{R}_{\rm MSSM} + (1-\xi)\frac{1}{10}{g_1'}^2\left[\frac{3}{5}g_1^2+3g_2^2+\frac{1}{5}{g_1'}^2\right] + (1-\xi)\frac{1}{200}\left[3g_{11'}^2+2g_{1'1}^2-2\sqrt{6}\left(g_{11'}g_1'+g_{1'1}g_1\right)\right]\left(2{g_1'}^2+3g_1^2+15g_2^2\right), \tag{4.21a}$$

$$\mathcal{R}_d = \mathcal{R}_{\rm MSSM} + (1-\xi)\frac{9}{40}{g_1'}^2\left[\frac{3}{5}g_1^2+3g_2^2+\frac{9}{20}{g_1'}^2\right] + (1-\xi)\frac{9}{800}\left[2g_{11'}^2+3g_{1'1}^2+2\sqrt{6}\left(g_{11'}g_1'+g_{1'1}g_1\right)\right]\left(3{g_1'}^2+2g_1^2+10g_2^2\right). \tag{4.21b}$$

The new result of eqs. (4.20) and (4.21) are the $\mathcal{R}$-terms for up- and down-type Higgs. They differ non-trivially because of the $\mathrm{U}(1)_N$-quantum numbers, and $\mathcal{R}_u - \mathcal{R}_d$ does not vanish in the E_6SSM in contrast to the MSSM and NMSSM cases. The two-loop $\hat{\gamma}$ for the singlet field reads

$$(4\pi)^4\hat{\gamma}^{(2),\overline{\mathrm{DR}}}_{\mathrm{E_6SSM}}(S_3) = -\xi\xi'\left\{\frac{5}{4}{g'_1}^2\left[2\,\mathrm{Tr}\left(\lambda\lambda^\dagger\right) + N_c\,\mathrm{Tr}\left(\kappa\kappa^\dagger\right)\right] + \mathcal{R}_s\right\}\,, \tag{4.22a}$$

$$\mathcal{R}_s = \frac{25}{32}(1-\xi){g'_1}^2\left({g'_1}^2 + g^2_{1'1}\right)\,. \tag{4.22b}$$

The complete two-loop β-function of $\tan\beta$ requires additionally the two-loop γ's, which can be computed but will not be spelled out here. The RGE coefficients then reads

$$\begin{aligned}
(4\pi)^4\frac{\beta^{(2),\overline{\mathrm{DR}}}_{\mathrm{E_6SSM}}(\tan\beta)}{\tan\beta} &= (4\pi)^4\frac{\beta^{(2),\overline{\mathrm{DR}}}_{\mathrm{MSSM}}(\tan\beta)}{\tan\beta} + (4\pi)^2|\lambda_3|^2\frac{\beta^{(1),\overline{\mathrm{DR}}}_{\mathrm{MSSM}}(\tan\beta)}{\tan\beta} \\
&+ \frac{3}{40}\left[-3+\xi\xi'\left(1-\xi\right)\right]{g'_1}^2g_1^2 + \frac{3}{8}\left[1+\xi\xi'\left(1-\xi\right)\right]{g'_1}^2g_2^2 \\
&+ \frac{1}{160}\left[201+13\xi\xi'\left(1-\xi\right)\right]{g'_1}^4 \\
&- \frac{1}{5}\left(1-\frac{9}{4}\xi\xi'\right){g'_1}^2\left[3\,\mathrm{Tr}\left(y^dy^{d\dagger}\right) + \mathrm{Tr}\left(y^ey^{e\dagger}\right)\right] \\
&+ \frac{3}{10}\left(1-2\xi\xi'\right){g'_1}^2\,\mathrm{Tr}\left(y^uy^{u\dagger}\right) - \frac{1}{2}\left(1-\frac{1}{2}\xi\xi'\right){g'_1}^2|\lambda_3|^2 \\
&+ \frac{3}{40}\left[11+\frac{1}{2}\xi\xi'\left(1-\xi\right)\right]\left(g^2_{11'}{g'_1}^2 + g^2_{1'1}g_1^2\right) \\
&+ \frac{1}{80}\left[201+\frac{13}{2}\xi\xi'\left(1-\xi\right)\right]g^2_{1'1}{g'_1}^2 + \frac{3}{8}\left[1+\frac{1}{2}\xi'\xi\left(1-\xi\right)\right]g^2_{1'1}g_2^2 \\
&+ \frac{1}{20}\sqrt{\frac{3}{2}}\left[99+\frac{7}{2}\xi\xi'\left(1-\xi\right)\right]\left(g_{11'}g'_1 + g_{1'1}g_1\right){g'_1}^2 \\
&+ \frac{1}{10}\sqrt{\frac{3}{2}}\left[51+\frac{3}{2}\xi\xi'\left(1-\xi\right)\right]\left(g_{11'}g'_1 + g_{1'1}g_1\right)g_1^2 \\
&+ \frac{3}{2}\sqrt{\frac{3}{2}}\left[1+\frac{1}{2}\xi\xi'\left(1-\xi\right)\right]\left(g_{11'}g'_1 + g_{1'1}g_1\right)g_2^2 \\
&+ \frac{51}{10}\sqrt{\frac{3}{2}}\left(g_{11'}g'_1 + g_{1'1}g_1\right)g^2_{11'} + \frac{99}{20}\sqrt{\frac{3}{2}}\left(g_{11'}g'_1 + g_{1'1}g_1\right)g^2_{1'1} \\
&+ \frac{21}{10}g_{11'}g_{1'1}g'_1g_1 + \frac{201}{160}g^4_{1'1} - \frac{9}{40}g^2_{11'}g^2_{1'1} \\
&- \left[\frac{1}{2}g^2_{1'1} + \sqrt{6}\left(g_{11'}g'_1 + g_{1'1}g_1\right)\right]|\lambda_3|^2 \\
&- \left[\frac{2}{5}g^2_{11'} + \frac{3}{5}g^2_{1'1} + \frac{2}{5}\sqrt{6}\left(g_{11'}g'_1 + g_{1'1}g_1\right)\right]\mathrm{Tr}\left(y^dy^{d\dagger}\right) \\
&- \left[-\frac{6}{5}g^2_{11'} + \frac{1}{5}g^2_{1'1} + \frac{3}{5}\sqrt{\frac{3}{2}}\left(g_{11'}g'_1 + g_{1'1}g_1\right)\right]\mathrm{Tr}\left(y^ey^{e\dagger}\right) \\
&- \left[+\frac{4}{5}g^2_{11'} - \frac{3}{10}g^2_{1'1} + \frac{3}{5}\sqrt{\frac{3}{2}}\left(g_{11'}g'_1 + g_{1'1}g_1\right)\right]\mathrm{Tr}\left(y^uy^{u\dagger}\right)\,.
\end{aligned} \tag{4.23}$$

JHEP01(2014)068

The connection with the more conventional treatment [21, 22] of the kinetic mixing in the E_6SSM

$$\mathcal{L} = -\frac{1}{4}F_Y^{\mu\nu}F_{Y,\mu\nu} - \frac{1}{4}F_N^{\mu\nu}F_{N,\mu\nu} - \frac{\sin\chi}{2}F_Y^{\mu\nu}F_{N,\mu\nu} + \cdots \tag{4.24a}$$

is established by the coupling matrix (cf. eq. (4.15))

$$\hat{g} = \begin{pmatrix} g_1 & -g_1 \tan\chi \\ 0 & \frac{g_1'}{\cos\chi} \end{pmatrix} . \tag{4.24b}$$

5 Conclusions

We completed the calculation of the two-loop VEV β-functions for general gauge theories and generic supersymmetric theories. The result complements the well-known set of RGE coefficients of refs. [2–5] for general gauge theories as well as the supersymmetric gauge theories of refs. [14, 18]. In particular, we achieved the following

- Completion of $\hat{\gamma}^{(2)}$ by the missing $\mathcal{O}(g^4)$-contributions of our earlier results [1].
- Extension of $\gamma^{(2)}|^{\overline{\text{DR}}}_{\text{SUSY}}$ to arbitrary values of the gauge fixing parameter ξ.

As a consequence, we were able to provide the full VEV β-function for general and supersymmetric gauge theories in the $\overline{\text{MS}}$ and $\overline{\text{DR}}$ scheme up to the two-loop level. The result was applied to the MSSM, NMSSM, and E_6SSM and we proved the statements made in [1] on the $\mathcal{O}(g^4)$-terms:

1. $\mathcal{R}_u - \mathcal{R}_d = 0$ in the MSSM and NMSSM,
2. $\mathcal{R}_u - \mathcal{R}_d \neq 0$ for the E_6SSM.

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JHEP01(2014)068