
§37. Inner products for protoscales

Given any protoscale E and given any $B \in [1, \circ(\mathcal{L}(E))]$
and $G \subset Q$ and $u \in Q$ we define

$$G(E(B) \geq u) = \left\{ j \in G(\mathcal{L}(E)) : \begin{array}{l} \text{inpo}(j, E(B, \tilde{B}, \hat{B}, \hat{C})) \geq u \\ \text{for all } (\tilde{B}, \hat{B}, \hat{C}) \in \text{supt}(E, B) \end{array} \right\}$$

and

$$G(E((B)) \geq u) = \left\{ j \in G(\mathcal{L}(E)) : \begin{array}{l} \text{inpo}(j, E((B, \tilde{B}, \hat{B}, \hat{C}))) \geq u \\ \text{for all } (\tilde{B}, \hat{B}, \hat{C}) \in \text{supt}((E, B)) \end{array} \right\}$$

and for any $P \in \{>, =\}$ we define

$$G(E(B)Pu) = \left\{ j \in G(E(B) \geq u) : \begin{array}{l} \text{inpo}(j, E(B, \circ(\mathcal{L}(E)), \circ(\mathcal{L}(E)), 0))Pu \end{array} \right\}$$

and

$$G(E((B))Pu) = \left\{ j \in G(E((B)) \geq u) : \begin{array}{l} \text{inpo}(j, E((B, \circ(\mathcal{L}(E)), \circ(\mathcal{L}(E)), 0)))Pu \end{array} \right\} .$$