TERMINOLOGY AND NOTATIONS

For two sets A,B, $A \setminus B$ denotes the difference set and $A \Delta B$ the symmetric difference. If the set A is contained in a set B then χ_A is the characteristic function of A defined by

$$\chi_A(x) = \begin{cases} 1 & , x \in A \\ o & , x \in B \setminus A . \end{cases}$$

For any two numerical functions f,g on a set A let

$$[f \leqq g] = \{x \in A : f(x) \leqq g(x)\} .$$

Analogously we define $[f < g]$ and $[f = g]$.
If F is a family of numerical functions on a set A then $\sup F$ (resp. $\inf F$) is the numerical function on A defined by

$$(\sup F)(x) = \sup_{f \in F} f(x) \quad (\text{resp. } (\inf F)(x) = \inf_{f \in F} f(x)).$$

Let $(A,\leqq)$ be an ordered set and let B be a subset of A. Then we call least upper bound or supremum (resp. greatest lower bound or infimum) of B in A, if it exists, the smallest (resp. greatest) element of A which dominates any element of B (resp. which is dominated by any element of B). The ordered set A is said to be increasing (resp. decreasing) if for any $a_1, a_2 \in A$ there exists $a_3 \in A$ such that $a_3 \geqq a_1$, $a_3 \geqq a_2$ (resp. $a_3 \leqq a_1$, $a_3 \leqq a_2$).

If A is a subset of a topological space then $\bar{A}$ denotes the closure of A. If f is a numerical function on a topological space then the lower semi-continuous regularization of f is defined by

$$x \longrightarrow \liminf_{y \to x} f(y)$$

and it is the greatest lower semi-continuous function which is smaller than f.

We denote by $\mathbb{N}, \mathbb{R}, \overline{\mathbb{R}}$ the set of natural numbers (o not included), the set of real numbers and the extended real line respectively.
Further let $\mathbb{R}_+$ be the set of positive real numbers and $\overline{\mathbb{R}_+} = \mathbb{R}_+ \cup \{+\infty\}$.
If $a,b \in \overline{\mathbb{R}}$, $a < b$, then $[a,b]$ resp. $]a,b[$ denotes the closed resp. open interval with endpoints a and b.