CODAZZI - TENSORS IN SURFACE THEORY

by

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Known results

1. Let g be a two-dimensional Riemannian metric, b a symmetric tensor field with negative determinant and Q = det b / det g. The curves defined by

(1) $$b_{ij}du^i du^j = 0$$

form a Tschebyscheff net (T-net) $\Longleftrightarrow$ $(1/\sqrt{-Q})b_{ij}$ is a Codazzi tensor. Special case: The asymptotic lines of a surface in E^3 form a T-net $\Longleftrightarrow$ Gauss curvature K is constant negative.

2. There is no T-net without singularities, defined in the whole hyperbolic plane H^2. Corollary: There is no immersion of H^2 into E^3.

3. The curves (1) form an isothermic net $\Longleftrightarrow$ $g^{ij}b_{ij} = 0$ and there exists a function $f \neq 0$ such that $f \cdot b$ is a Codazzi tensor.

4. For a surface x in E^3 (simply connected, without umbilical points) let I be the first, II the second fundamental form and H the mean curvature. Then the following conditions are equivalent:
(a) The lines of curvature form an isothermic net.
(b) There exists a variation δx of the surface, such that
$\delta I = 0$ ("infinitesimal bending"), $\delta II \neq 0$, $\delta H = 0$.

New result:

Let x be a surface in E^3 (simply connected, with $K(H^2 - K) \neq 0$). If there exists a variation δx, such that
$\delta I \neq 0$, $\delta II = 0$ ("infinitesimal bending"), $\delta H = 0$, $\delta K = 0$, then the lines of curvature are isothermic with respect to II.