

For the historical record

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In the interest of historical accuracy, it should be noted that a complete, self-contained, probabilistic proof of the Martingale convergence theorem in Banach spaces with RNP (Every L^1 - bounded Martingale taking values in a Banach space E with the Radon-Nikodym Property converges strongly to a limit almost surely) was already available in 1963, in the paper with the unlikely title "Abstract ergodic theorems" by A. Ionescu Tulcea and C. Ionescu Tulcea, Trans. A.M.S., vol.107, No.1, p. 107-124 : see Part II, sections 2 and 3, pp.118-122.

The proof is based on a Kolmogorov-type extension for vector-valued measures in infinite product spaces (Proposition 4). It obviously suffices to assume that the Banach space E is separable. The separability of E is used in applying Proposition 4. The additional assumption that E is a dual space, or a reflexive space is not needed in Proposition 5 and Theorem 4: as the Remark on p.121 at the end of Proposition 5 clearly spells out, the only additional property of E used in the proof is the one that makes it possible to assert that "Every countably additive mapping of \mathcal{J} into E with finite total variation has a Bochner density with respect to its total variation". Note also that Theorem 4 holds whenever the Banach space E is such that Proposition 5 holds: Remark 3 at the end of Theorem 4.