PERTURBATION THEORY FOR SOBOLEW SPACES

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This lecture surveys a new perturbation theory for the Sobolew spaces $W^{m,p}(G)$ (cf. [3]). The functional analysis for the treatment of general perturbations of linear operators has been established in [1]. One finds a corresponding perturbation theory for elliptic sesquilinear forms on subspaces of a Hilbert space and an application to the Dirichlet problem in [2]. Within this framework, a perturbation theory for Sobolew spaces has been developed in [3] which permits the treatment of boundary value problems in partial differential equations under perturbations of coefficients and inhomogeneous terms as well as of boundary conditions and domains of definition. The theory studies the basic concepts and methods, in particular, the convergence of sequences of open sets G_ι, $\iota \in \mathbb{N}$, to G in $\mathbb{R}^n$, the strong and weak convergence of the sequence of Sobolew spaces $W^{m,p}(G_\iota)$, $\iota \in \mathbb{N}$, to $W^{m,p}(G)$ and of $W_o^{m,p}(G_\iota)$ to $W_o^{m,p}(G)$, the discrete compactness of the sequence of natural embeddings of Sobolew spaces, and the continuous convergence of continuous linear functions, boundary integrals and trace operators for the sequence of Sobolew spaces $W^{m,p}(G_\iota)$, $\iota \in \mathbb{N}$.

References

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