On the differential equation $\sum_{r=0}^{n} c_r (pD_1 + qD_2)^r u = 0$

I. Fenyő

Let u be a distribution of $D'(R^2)$ which is a solution of

$$\sum_{r=0}^{n} c_r (pD_1 + qD_2)^r u = 0 \tag{1}$$

(D_1 and D_2 are the partial differential operators) and ψ an arbitrary Schwartz-test function of one variable. We denote by $\odot$ a mapping from $D'(R^2) \times D(R)$ into $D'(r)$ such that $(u \odot \psi, \varphi) = (u, \varphi \otimes \psi)$ (φ is an abritrary testfunction of $D(R)$). It is proved, that the distribution $u \odot \psi$ possess in every point x_0 a local value (in the sense of Lojasiewicz) and the value of $u \odot \psi$ in x_0 is the scalar product of a distribution $u(x_0) \in D'(R)$ with ψ (i.e. $u \odot \psi|_{x_0} = (u(x_0), \psi)$). Now the following problem can be posed. There are given n points of the real axis: $x_1, x_2, \ldots, x_n$ ($x_i \neq x_k$, $i \neq k$) and n distributions $f_1, \ldots, f_n$ (of $D'(R)$). We are looking for a distribution $u \in D'(R^2)$ which is a solution of (1) and for which $u(x_i) = f_i$ ($i = 1, 2, \ldots, n$) holds. It is proved that this problem has always a unique solution.

Reference

1. Fenyő, I. : On a partial differential equation in two variables, Demonstration Math. (Poland) Vol 1973.