Parameterfree Information-Preserving Surface Restoration

Uwe Weidner*

Institut für Photogrammetrie
Nußallee 15 53115 Bonn
email: weidner@ipb.uni-bonn.de

Abstract. In this paper we present an algorithm for parameterfree information-preserving surface restoration. The algorithm is designed for 2.5D and 3D surfaces. The basic idea is to extract noise and signal properties of the data simultaneously by variance-component estimation and use this information for filtering. The variance-component estimation delivers information on how to weigh the influence of the data dependent term and the stabilizing term in regularization techniques, and therefore no parameter which controls this relation has to be set by the user.

1 Introduction

The first step for gaining a description of a 3D scene is the measurement of 3D point coordinates. During the data aqcuisition errors occur. These errors include systematic and random errors. The systematic errors can often be eliminated by calibrating the measurement equipment. Then the result of the measurement is a noisy discrete data set.

This data is the basis for the computation of the surface, on which the following first step of object recognition, i. e. feature extraction, is performed. Often features are related to some discontinuities in the data. Derivatives of the initial surface are commonly used for their extraction. The computation of these derivatives as well as the computation of the surface are ill-posed problems (c. f. [14]). These ill-posed problems have to be reformulated as well-posed problems.

Filters, surface approximations, or general regularization techniques are used to achieve this goal. Global techniques like linear filters or standard regularization (e.g. Tikhonov regularization) lead to a reduction of noise, but also affect the features and discontinuities, i. e. the information, by blurring the data. Therefore, the goal of surface restoration should be the regularization of the data including the suppression of noise with a minimal loss of information.

In this contribution we formulate filtering for regularization and noise suppression as a minimization problem which depends on local features based on an explicit physical and/or geometric model and the noise, and includes a procedure for estimating noise. Thus all parameters are estimated within a statistical framework. In this respect our approach differs significantly from other regularization techniques (cf. [13], [3], [7], [12]).

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The basic idea is to extract signal and noise properties from the data and use this information for the filtering of the data. The extraction of these properties is based on generic prior knowledge about the surface. This a priori knowledge also puts constraints on the data and is used for the regularization via the stabilizing function. If the data does not correspond to the a priori knowledge or the model respectively, the influence of regularization is weakened.

In our approach the function

$$F = \sum \frac{(z_i - \hat{z}_i)^2}{\sigma_z^2} + \sum \left(\frac{k_{1i}}{\sigma_{k_{1i}}} \right)^2 + \sum \left(\frac{k_{2i}}{\sigma_{k_{2i}}} \right)^2 \tag{1}$$

is to be minimized. The principal curvatures k_1 and k_2 are used because they lead to a unique model surface and include directional information. Obviously the degree of regularization is made dependent on the variances of the data and of the true surface's smoothness. As both variances are derived from the data, this filter is a parameterfree information-preserving approach to surface restoration. The filter can be used for all applications in which surfaces are given by measured discrete points. The coordinates of these discrete points can be given either in a 3D coordinate sytem using arbitrary surface coordinates or in a 2.5D sensor coordinate system using a graph surface representation.

The kind of the input data is not really fixed and not only includes geometric surfaces, but may also represent the density distribution of a material. The only requirements of the data are that an interpretation of the data as a surface is possible, that the data is dense in order to have a sufficient redundancy, and, for ease of representation, that the data is regularly located on a grid given by arbitrary surface coordinates (u, v). Additionally, the data must have properties suited for regularization. Though our approach is more general, we restrict to using curvatures in our implementation.

In this paper we assume uncorrelated signal independent white Gaussian noise. This does not affect the presented approach as other knowledge about noise can be integrated easily in the estimation process via weights. Similarily, correlated noise could be taken into consideration.

The basic idea is to simultaneously estimate the variance of noise and of appropriate smoothness. Estimation techniques for the noise only (c. f. [9]) do not solve the problem we deal with, as they do not take the variance of smoothness into account. In [4] it is shown that noise and signal in observed autoregressive processes are seperable, using both Fourier analysis and variance-component estimation. Here, variance-components estimation is used for the estimation of signal and noise properties.

2 Algorithm

The algorithm is based on a geometric model. It is assumed that the expectations of the principal curvatures are zero, i. e. the surface can be locally approximated using planes. If the principal directions and the surface normals are known for the 3D representation, the principal curvatures can be computed by convolution. Those convolution kernels are gathered in the matrix of coefficients $\mathbf{X}$.

The information about a surface's curvature properties is fully contained in the Weingarten map or shape operator $\mathbf{W}$ (c. f. [2]). The eigenvalues of $\mathbf{W}^2 = \mathbf{W}\,\mathbf{W}$ are the squared eigenvalues of $\mathbf{W}$. The eigenvectors of $\mathbf{W}^2$ are equal to those of $\mathbf{W}$. We use the eigenvalues for estimating the local variance of the curvature of the surface.

The algorithm is based on the assumptions that

$$\mathbf{d} = \left(\mathbf{d}_1^T\ \mathbf{d}_2^T\ \mathbf{d}_3^T\right)^T = \mathbf{u} + \mathbf{n}, \quad E(\mathbf{d}) = \mathbf{u} \quad \text{and} \quad D(\mathbf{d}) = \sigma_d^2\,\mathbf{I} \tag{2}$$
$$\text{with} \quad \mathbf{d}_1 = \mathbf{x}(u,v) \quad \mathbf{d}_2 = \mathbf{y}(u,v) \quad \mathbf{d}_3 = \mathbf{z}(u,v)$$
$$E(\mathbf{k}_1) = \mathbf{0},\ D(\mathbf{k}_1) = Diag(\sigma_{k1i}^2), \quad E(\mathbf{k}_2) = \mathbf{0},\ D(\mathbf{k}_2) = Diag(\sigma_{k2i}^2)$$

where $Diag(p_i)$ denotes a diagonal matrix with entries p_i. If the surface normal and the principal directions are given, the following linear model with $m = 5$ groups of observations results (c. f. [6]):

$$E(\mathbf{y}) = \mathbf{X}\,\mathbf{u} \quad D(\mathbf{y}) = \sum_{i=1}^{5} \mathbf{V}_i\,\sigma_i^2 \tag{3}$$
$$\text{with} \quad \mathbf{X} = \left(\mathbf{X}_x^T\ \mathbf{X}_y^T\ \mathbf{X}_z^T\ \mathbf{X}_{k1}^T\ \mathbf{X}_{k2}^T\right)^T,\ \mathbf{y} = \left(\mathbf{d}_1^T \dots \mathbf{d}_5^T\right)^T,\ \mathbf{d}_4 = \mathbf{k}_1,\ \mathbf{d}_5 = \mathbf{k}_2$$

The matrix of coefficients, which describes the linear relation between the observations and the unknown parameters $\mathbf{u}$, splits into five submatrices, where the matrices $\mathbf{X}_x$, $\mathbf{X}_y$ and $\mathbf{X}_z$ are identity matrices and the rows of the matrices $\mathbf{X}_{k1}$ and $\mathbf{X}_{k2}$ contain the convolution kernels for the principal curvatures $\mathbf{k}_1$ and $\mathbf{k}_2$. The structure of $\mathbf{V}_i$ must be known in advance (c. f. [6]). Assuming independence of the obsvervations and equal variances for the coordinates simplifies (3) to

$$E(\mathbf{y}) = \mathbf{X}\,\mathbf{u} \quad D(\mathbf{y}) = \sigma_d^2\mathbf{V}_d + \sigma_{k1}^2\mathbf{V}_{k1} + \sigma_{k2}^2\mathbf{V}_{k2} \tag{4}$$
$$\text{with} \quad \mathbf{V}_d = \sigma_d^2\,\mathbf{I}, \quad \mathbf{V}_{k1} = Diag(\overline{\sigma}_{k1i}^2) \quad \text{and} \quad \mathbf{V}_{k2} = Diag(\overline{\sigma}_{k2i}^2)$$

where $\overline{\sigma}_k^2$ denote a local estimate of the curvatures' variances.

Based on this, the unknown parameters $\mathbf{u}$, i. e. the coordinates, and the variances can be estimated using iterative estimation. Details are given in [16].

3 Results

In this section we want to show the results of the parameterfree information-preserving filter and compare these results with the results of other restoration techniques. For this purpose we use synthetic test data sets. The advantage of synthetic data is that the values of the true signal $\mathbf{z}_0$ are known and can be used as reference. A reference is of importance because we do not only want to compare the results qualitatively by visual inspection, but also give a quantitative comparison.

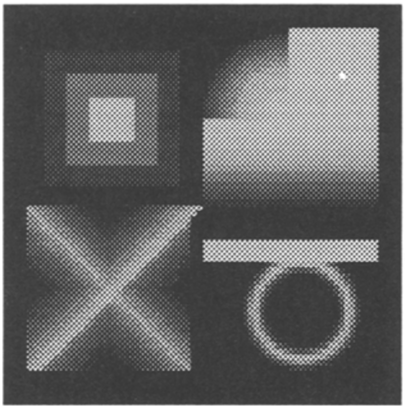

Fig. 1. Test image

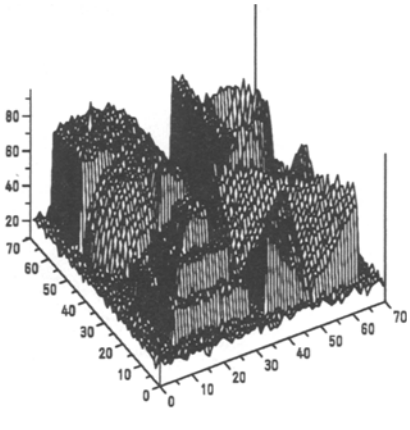

Fig. 2. Test image: $\sigma = 2$ [gr], range: 20-90 [gr]

In order to derive such quantitative measures, the area of the surface $\mathcal{S}$ is devided into two components $\mathcal{R}$ and $\mathcal{B}$, where $\mathcal{B} = \{\mathcal{B}_i\}$ is the set of boundary regions, i. e. regions of discontinuities, and $\mathcal{R} = \{\mathcal{R}_i\}$ is the set of mutally exclusive segments of continous regions. Based on this division, two quantities can be computed.

The first quantity is the *property of smoothing*

$$PS = \frac{\hat{\sigma}_n(\hat{\mathbf{z}})}{\sigma_n(\mathbf{z})} \quad \text{with} \quad \hat{\sigma}_n = 1.4826 * med_{i \in \mathcal{R}}(|\hat{z}_i - z_{0i}|) \tag{5}$$

$\hat{\sigma}_n$ is the robust estimate of the noise standard deviation in homogeneous regions $\mathcal{R} = \{\mathcal{R}_j\}$ (cf. [11]) based on the restored surface. $\sigma_n(\mathbf{z})$, which in case of synthetic data is known, is the standard deviation of the observed signal $\mathbf{z}$. If all noise is removed, i. e. $\hat{\sigma}_n(\hat{\mathbf{z}}) = 0$, PS is equal to zero.

The second quantity is the *property of preserving discontinuities*

$$PP = \frac{\sigma_{\Delta_S}}{\sigma_s(\mathbf{z})} \quad \text{with} \quad \sigma_{\Delta_S} = \frac{\sum_{i \in \mathcal{B}} |\hat{S}_i - S_i|}{|\mathcal{B}|} \quad \text{and} \quad S_i = (\sqrt{g_r^2 + g_c^2})_i \tag{6}$$

where $\hat{S}_i$ and S_i, the local edge strength, are computed based on the restored image and the noiseless test image respectively using the Sobel-operator and $|\mathcal{B}|$ is the number of points in $\mathcal{B}$. This quantity is equal to zero, if the information is maintained.

The quantity PP is related to the Sobel's standard deviation σ_s of the observed signal and is computed based only using pixels within the boundary regions $\mathcal{B}$. Therefore the two quantities for the quantitative evaluation are independent of each other.

The test data set (Fig. 1) is similar to the image [1] used. For further examination white Gaussian noise with standard deviation $\sigma = 2$ [gr] has been added to the original data set (Fig. 2).

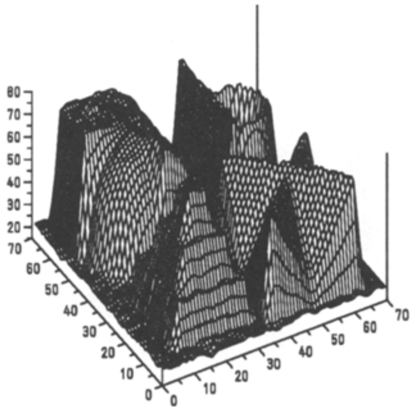

Fig. 3. Linearily filtered test image: 3D-plot

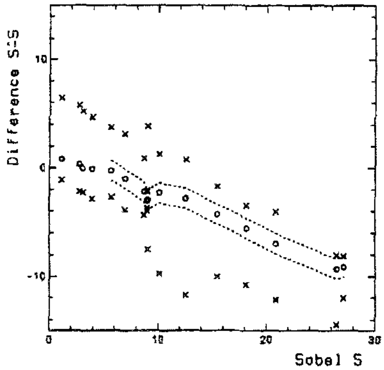

Fig. 4. Linearily filtered test image: plot of differences $\Delta\hat{S}_i(S)$

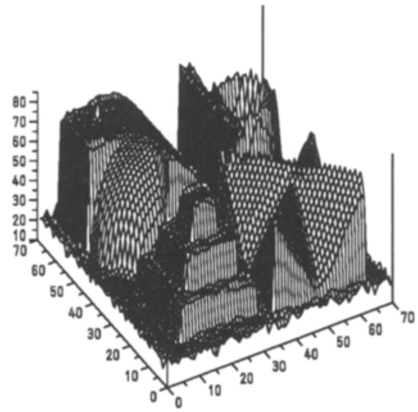

Fig. 5. Restored test image: 3D-plot

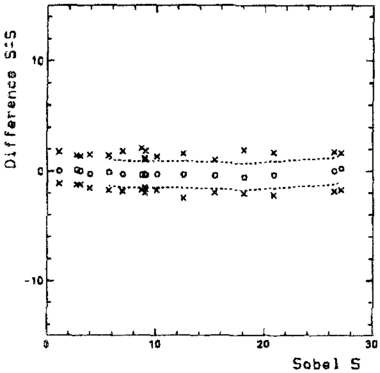

Fig. 6. Restored test image: plot of differences $\Delta\hat{S}_i(S)$

Qualitative effects of the restoration techniques can be easily seen in the 3D-plots and the additional plots (Fig. 4, Fig. 6), where the mean differences of $\hat{S}_i - S_i$ are plotted against S. $\times$ signifies the extrema and $\circ$ the mean difference. The tolerance of $3\sigma_S$ is represented by the lines, where σ_S is computed based on the estimated standard deviation $\hat{\sigma}_n(\hat{\mathbf{z}})$.

Figure 3 shows the result of restoration using a linear filter, the mean filter (MEAN(std),c. f. [17]). The changes of the signal are evident. The peak almost diminishes and the edges, corners and the tops of the roofs are smeared. The effect on the edges can also easily be seen in figure 4. The values of the quantities $\hat{S}_i$ are reduced, the differences are negative. This filter has the best smoothing properties in homogeneous regions, but also the worst information preserving properties ($PS = 0.328$; $PP = 3.976$).

Figure 5 shows the result of the information-preserving filter. The smoothing properties of this filter applied to the noisy image are almost equal in the entire

Algorithm/Approach	PS	PP	Algorithm/Approach	PS	PP
Gaussian Filter	0.367	3.040	MEAN(std)	0.328	3.976
MEAN(knn)	0.823	0.762	MEAN(sig)	0.655	0.822
MEAN(snn)	0.835	1.282	MEDIAN(std)	0.476	1.366
MEDIAN(knn)	0.869	0.841	MEDIAN(sig)	0.714	0.755
MEDIAN(snn)	0.935	1.421	Nagao/Matsuyama(1979)	0.978	1.610
GRIN	0.413	0.913	Perona/Malik(1990)	0.391	1.002
Inform.Pres.Filter(IPF)	0.589	0.660			

Fig. 7. Evaluation of various filters on an artificial test image

image. The peak and the tops of the roofs are maintained. Figure 6 indicates that the information is maintained because the mean differences $\hat{S}_i - S_i$ are close to zero. These properties are also evident in the quantities $PS = 0.589$ and $PP = 0.660$.

The results for other filters can only be given in Fig. 7 using the enchantments for the filters like [17]. Some of the results are also given in [15]. The trade-off between smoothing and preserving the information for non-adaptive techniques (Gaussian, mean and median using the entire neighbourhood of a point) is evident. Adaptive techniques which are based on selecting points from the neighbourhood using a criterion like the k-Nearest-Neighbourhood (knn), the Sigma-Neighbourhood (sig), and the Symmetric-Nearest-Neighbourhood (snn) have knobs to be tuned with. The tuning of these knobs depends either on the user or the information the user has in advance, e. g. the standard deviation for the Sigma-Neighbourhood. All techniques except the information-preserving filter have no criteria, when iterations should be stopped. For the results of these techniques the knobs have been tuned in order to gain the optimal result for each technique.

The disadvantage of the information-preserving filter is the high computational effort due to solving a linear equation system of $U \times V$ unknowns, where U and V are the number of nodes in each surface coordinate direction. Furthermore the convergence of the SOR-iteration is dependent on the degree of noise. The rigorous solution via least squares adjustment can be approximated for not too high noise efficiently yielding comparable results. (c. f. [5]).

4 Conclusion

We presented an algorithm for parameterfree information-preserving surface restoration. The basic idea of this algorithm is to extract noise and signal properties of the data, which are 2.5D or 3D surfaces, and to use these properties for filtering. The properties which are estimated within a statistical framework are the variances of the noise and the smoothness of the data. The ratio of these quantities determines the parameter λ of standard regularization techniques for each point. Therefore this knob has been eliminated and no parameters have to be tuned by the user in order to obtain optimal results with regard to the smoothness and the preservation of information. The minimization of the resulting functional is done by least squares adjustment.

The results of our algorithm for 2.5D surfaces have been quantitatively compared to those of wellknown filter techniques. The comparison outlines our approach's property of preserving information. It also has been tested on images with signal-dependent noise and aerial images with similar results.

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