The Incremental Rigidity Scheme for Structure from Motion : The Line-Based Formulation[†]

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Abstract: This paper presents an extension to lines of Ullman's incremental rigidity scheme, originally formulated for a set of points. The formulation is based on the angular and distance invariance of rigid configurations of lines. It is shown that the line structure can be recovered incrementally from its motion.

1. Line-based incremental rigidity scheme

The incremental rigidity scheme for the recovery of structure from motion, as proposed by Ullman [1] for a structure of points, constructs an internal estimated model of the structure which is continuously updated, as rigidly as possible, at each time a new image is available. The current estimated model is modified by the minimal "physical change" that is sufficient to account for the observed transformations in the new image. According to Ullman's results, the proposed incremental rigidity scheme converges to the correct structure. The goal of this study is to examine a line-based formulation of Ullman's scheme. Albeit with an additional weak constraint, we show that results similar to those of Ullman with point structures can be obtained with configurations of lines.

The viewing system is modelled as in Figure 1 (cartesian reference system and parallel projections, the Z-axis pointing to the observer). The line-based formulation we propose consists of two successive steps, the first step being the recovery of the orientation of the lines and the second being the complete recovery of the structure. Using the principle of angular invariance, the scheme estimates the orientations of the lines; recovery of structure is completed using distance invariance.

We will take the model $M(t)$ of the line structure at time t to be the set $\{U_i, Z_i\}$, $i = 1, ..., N$, where U_i is the unit orientation vector of line L_i, Z_i is the depth of *any*

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point on line L_i and N is the number of lines in the structure. At $t = 0$, the model is taken to be "flat" (all the lines lying in a plane parallel to the image plane).

1.1 Angular invariance: recovery of orientations

For a rigid motion of a set of lines, the principle of angular invariance states that the angles between the lines do not change as a result of this motion [2]. Let L_i and L_j be two lines in space, having projections l_i and l_j, respectively (see Figure 1). Let unit vectors on L_i and L_j be U_i, U_j at time t and U'_i, U'_j at time t'. Then, the principle of angular invariance states that (we assume that correspondence between lines, and their direction, has been established):

$$U_i \cdot U_j = U'_i \cdot U'_j \tag{1}$$

In expanded form, with unnormalized vectors V:

$$\frac{v_{1,i}v_{1,j} + v_{2,i}v_{2,j} + v_{3,i}v_{3,j}}{\|V_i\|\|V_j\|} = \frac{v'_{1,i}v'_{1,j} + v'_{2,i}v'_{2,j} + v'_{3,i}v'_{3,j}}{\|V'_i\|\|V'_j\|} \tag{2}$$

Equation 2 is written for each pair of lines (some equations may be redundant). Given the current estimate of the orientations and a new 2-D image, the problem is to determine the unknown parameters $v'_{3,i}$, $i = 1, \ldots, N$ so as to minimize the overall deviation from a rigid transformation as prescribed by the incremental rigidity scheme. The following function Ψ is considered for minimization:

$$\Psi = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \psi_{ij} \tag{3}$$

$$\psi_{ij} = \left[\frac{v_{1,i}v_{1,j} + v_{2,i}v_{2,j} + v_{3,i}v_{3,j}}{\|V_i\|\|V_j\|} - \frac{v'_{1,i}v'_{1,j} + v'_{2,i}v'_{2,j} + v'_{3,i}v'_{3,j}}{\|V'_i\|\|V'_j\|}\right]^2 \tag{4}$$

After function Ψ is minimized, resulting in a new set of $v'_{3,i}$, $i = 1, \ldots, N$, the corresponding orientations become part of the current version of model M.

1.2 Distance invariance: recovery of structure

The distances between each pair of lines remain constant during rigid motion. If we observe two lines L_i and L_j at t and t', then $d_{ij} = d'_{ij}$. The objective here is to incorporate this rigidity constraint into the incremental rigidity scheme, using the line orientations obtained at the previous step.

At time t, let $U_i = (u_{1,i}, u_{2,i}, u_{3,i})$, $U_j = (u_{1,j}, u_{2,j}, u_{3,j})$ be unit vectors on L_i and L_j respectively, and $P_i = (x_i, y_i, z_i)$ and $P_j = (x_j, y_j, z_j)$ be points on L_i and L_j. At time t', let the unit vectors be $U'_i = (u'_{1,i}, u'_{2,i}, u'_{3,i})$, $U'_j = (u'_{1,j}, u'_{2,j}, u'_{3,j})$ and points be $P'_i = (x'_i, y'_i, z'_i)$, $P'_j = (x'_j, y'_j, z'_j)$ (P_i,P_j and P'_i, P'_j are arbitrary and unrelated; no point correspondences are assumed).

According to the principle of distance invariance, the relation for lines L_i and L_j is:

$$\left|\frac{(x_j - x_i)\rho - (y_j - y_i)\sigma + (z_j - z_i)\tau}{\sqrt{\rho^2 + \sigma^2 + \tau^2}}\right| = \left|\frac{(x'_j - x'_i)\rho' - (y'_j - y'_i)\sigma' + (z'_j - z'_i)\tau'}{\sqrt{\rho'^2 + \sigma'^2 + \tau'^2}}\right| \quad (5)$$

$$\rho = (u_{2,i}u_{3,j} - u_{3,i}u_{2,j}) \qquad \sigma = (u_{1,i}u_{3,j} - u_{3,i}u_{1,j}) \qquad \tau = (u_{1,i}u_{2,j} - u_{2,i}u_{1,j}) \quad (6)$$

$$\rho' = (u'_{2,i}u'_{3,j} - u'_{3,i}u'_{2,j}) \qquad \sigma' = (u'_{1,i}u'_{3,j} - u'_{3,i}u'_{1,j}) \qquad \tau' = (u'_{1,i}u'_{2,j} - u'_{2,i}u'_{1,j}) \quad (7)$$

Equation 5 is written for each pair of lines (some of these may be redundant). Given the current estimate of distances and a new image, the problem is to find the unknown depths z'_i, $i = 1, ..., N$ in accordance with the incremental rigidity scheme. The following function Λ is minimized:

$$\Lambda = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \lambda_{ij} \quad (8)$$

$$\lambda_{ij} = \left[\left|\frac{(x_j - x_i)\rho - (y_j - y_i)\sigma + (z_j - z_i)\tau}{\sqrt{\rho^2 + \sigma^2 + \tau^2}}\right| - \left|\frac{(x'_j - x'_i)\rho' - (y'_j - y'_i)\sigma' + (z'_j - z'_i)\tau'}{\sqrt{\rho'^2 + \sigma'^2 + \tau'^2}}\right|\right]^2$$

When Λ is optimized, a new set of z'_i, $i = 1, ..., N$ is obtained and included in the current version of the model. A new image is acquired and the process is repeated: estimation of the new orientations using angular invariance, estimation of the positions using distance invariance, and update of the model.

2. Experimental results

For minimization, we used a quasi-Newton method. The initial values of $v'_{3,i}$, $i = 1, ..., N$, were set all to $+1$ or all to -1 [3]. Subsequent optimization steps used the current values of $v_{3,i}$. We have experimented with several arbitrary line structures in motion [3]. The motion reported here is a rotation about an axis parallel to the Y-axis, and located on the Z-axis. For the orientations, performance is measured with:

$$\Delta = \sum_i (1 - \cos\theta_i)/N \quad (9)$$

where $\cos\theta_i$ is the scalar product of the exact and estimated unit vectors of line i. For the positions of the lines in space, error function Γ is used:

$$\Gamma = \sum_{i,j} |d_{ij} - d'_{ij}|/N_p \tag{10}$$

where N_p is the number of pairs of lines and d_{ij}, d'_{ij} are respectively the exact and estimated distances for lines i and j.

With a rigid structure of 6 lines, typical results for the error functions Δ and Γ are shown in Figures 2 and 3 (rotations of 20°).

The scheme was experimented with *different rotation angles* (Figures 4 and 5). In general, results indicate that the convergence is accelerated with larger angular differences (also an observation in [1]). However when a sufficient level of difference is reached, greater angular variations do not necessarily increase the convergence rate.

We also examined the scheme with a *varying number of lines.* For structures containing 2 or 3 lines, the scheme exhibits an oscillatory error behavior. Results indicate that the performance gradually increases with the number of lines (Figures 6 and 7).

Experimental results also reveal that the estimation model can infer and maintain the exact structure in the presence of *small deviations from rigidity.* Performance deteriorates for larger perturbations.

Finally, better convergence results are obtained if *constraints* can be imposed on the depth parameters $v'_{3,i}$ (in practice, this amounts to knowing roughly the maximum size of the observed objects).

Summary: The incremental recovery scheme developed by Ullman [1] for point structures has been extended to line structures. The process relies on the minimization of two functions which are based on the maximum rigidity hypothesis and which involve the principles of angular and distance invariance. Albeit an additional weak constraint is met, the model eventually converges towards the exact structure which is then maintained.

References

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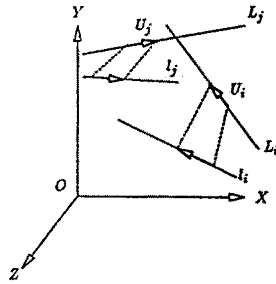

Fig. 1 Parallel projections of lines on the $X - Y$ image plane

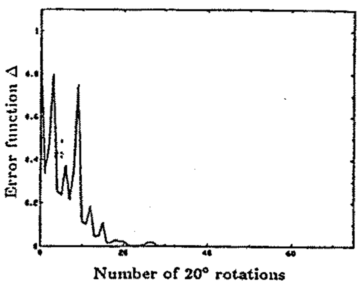

Fig. 2 Error function Δ in terms of the number of rotations of 20°

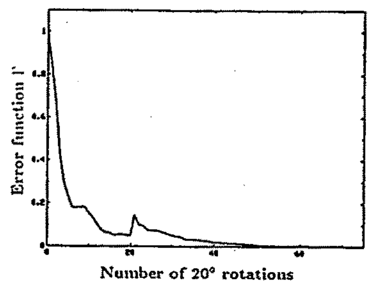

Fig. 3 Error function Γ in terms of the number of rotations of 20°

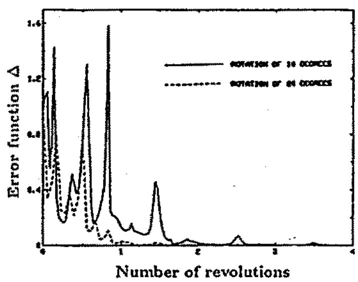

Fig. 4 Error function Δ in terms of the number or revolutions for rotation angles of 10° and 20°

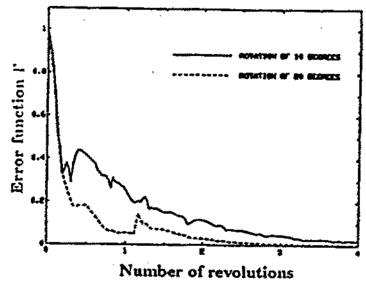

Fig. 5 Error function Γ in terms of the number or revolutions for rotation angles of 10° and 20°

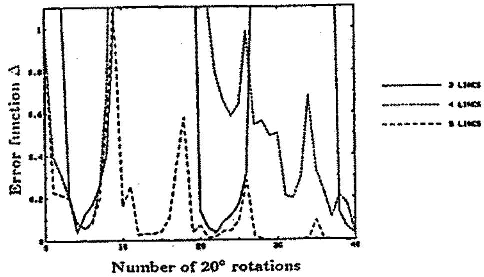

Fig. 6 Impact of the number of lines (3,4,5) on the error function Δ in terms of the number of rotations of 20°

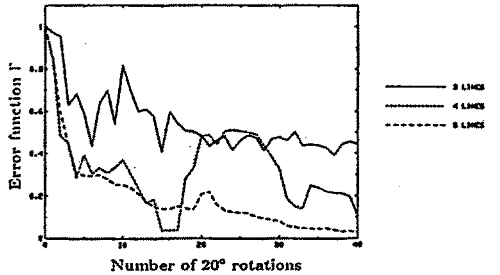

Fig. 7 Impact of the number of lines (3,4.5) on the error function Γ in terms of the number of rotations of 20°