The Relation Between Indentation Hardness and Strain for Metals

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Experiments have shown[1,2] that the formula

$$S = S_t - (S_t - S_e')e^{\frac{-\eta}{\eta_c}} \qquad [1]$$

expresses very well the relation between the true stress S and the true strain η, $\left(\eta = \ln\frac{l}{l_0} = 2\ln\frac{d_0}{d}\right)$, for uniform monotonic deformation of plastic metals in single tension and compression (Fig 1). S_t (*asymptotic* or *final* stress), S_e' (*threshold* stress) and η_c (*characteristic* or *specific* strain) are constants, the significance of which immediately follows from Fig 1. Only at the initial stages of deformation the experimental curve runs below the theoretical curve. This may be due, at least partly, to the uneven stress distribution caused by the anisotropy of the crystals. It is now obvious that a similar formula

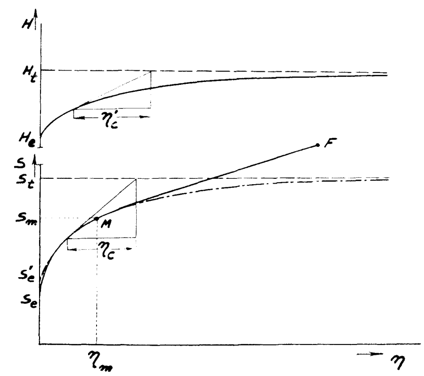

FIG 1—Schematic true stress-true strain and Vickers hardness-true strain relation.

$$H = H_t - (H_t - H_e)e^{\frac{-\eta}{\eta_c'}} \qquad [2]$$

might be valid for the relation between the hardness and the strain. The hardness can be only a function of the strain and must therefore be independent on the hydrostatic tension present in the neck during straining. Hence, if Eq 2 is correct it will also represent the relation between hardness and strain in the middle section of the neck, the strain of which is generally assumed to be almost or fully homogeneous. For aluminum, copper, and several copper alloys it is indeed observed that Eq 2 agrees very well with the experimental Vickers hardness-strain relation from zero strain on up to the strain at fracture. Moreover η_c' is, for several metals fairly equal to η_c. Hence

$$H = C_1 + C_2S \qquad [3]$$

C_1 and C_2 are constants during uniform uniaxial tension. Eq 3 was also obtained for copper by Voce.[3] According to Hencky's theory[4] the ratio of the Brinell or Vickers hardness and the yieldstress (expressed in kg per mm^2) of a non-strainhardening metal is approximately equal to 2.8. If Eq 1, 2 and 3 are correct the ratio H_t/S_t must accordingly be the same for all plastic metals and be equal to 2.8. It is now experimentally established that this ratio varies between 2.7 and 3.1 with a mean value of 2.9. Taking into account that H_t and especially S_t can be obtained only by extrapolation from the range of rather low strains, the agreement with the theory is very satisfactory.

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References

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