CORRECTION

THE ROLE OF THE DRIFT IN I(2) SYSTEMS

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Summary

A correction of the proof of Theorem 5.1 is given. All the statements of the Theorem and the Tables associated with it are unaffected by the correction.

In the Appendix, bottom of page 111, the term $\beta_l' C_l W_u$ in the definition of F_u^* should be replaced by $(l, \gamma_l)' W_u$, where $l \equiv \bar{\tau}'_l C_l - \frac{1}{2} \bar{\tau}'_2 C_2$ and β_l is chosen as $\beta_l \equiv (\bar{\tau}_l, \gamma_l)$, see Paruolo (1998), lemma A3. Let $B_{2lu} \equiv c_l^{-1/2} V_{\alpha, u} = c_l^{-1/2} f' \alpha'_2 W_u$, where $f' \equiv \gamma'_2 \beta_2 (\alpha'_2 \theta \beta_2)^{-1}$, $c_l \equiv f' \alpha'_2 \Omega \alpha_2 f$. Note that the process $V_{\alpha, u}$ can be substituted with B_{2lu} in the definition of F_u^* , since $V_{\alpha, u}$ is a bijective linear function of B_{2lu} . Define also $B_{22u} \equiv c_2^{-1/2} \bar{f}'_\perp (\alpha'_2 \Omega \alpha_2)^{-1} \alpha'_2 W_u$, $c_2 \equiv \bar{f}'_\perp (\alpha'_2 \Omega \alpha_2)^{-1} \bar{f}_\perp$; B_{2l} and B_{22} are independent Brownian standard motion of dimensions $p_2 - l$ and l respectively, and B_2 can be chosen equal to $(B'_{2l}, B_{22})'$, $B_{2u} = (B'_{2lu}, B_{22u})'$, where B_l and B_2 are defined before (A.22). Next note that $(l, \gamma_l)' W_u$ is a linear function of $(B'_l, B'_2)'$, so that $(l, \gamma_l)' W_u = m_l B_{lu} + m_{2l} B_{2lu} + n B_{22u}$, for appropriate matrices m_l, m_{2l}, n ; hence $((l, \gamma_l)' W_u | l, u, B_{2lu}) = (m_l B_{lu} + n B_{22u} | l, G_u)$.

Define next $g_l \equiv (n_\perp c_3^{-1/2}, n c_4^{-1/2})$, $c_3 \equiv n'_\perp m_l m'_l n_\perp$, $c_4 \equiv c_5 + (n' n)^2$, $c_5 \equiv n' m_l m'_l n$ and let $B_{1lu} \equiv c_3^{-1/2} n'_\perp m_l B_{lu}$, $B_{12u} \equiv c_5^{-1/2} n' m_l B_{lu}$; one thus has

$$(g'_l (l, \gamma_l)' W_u | l, G_u) = ((B'_{1lu}, k_l B_{12u} + k_2 B_{22u})' | l, G_u),$$

where $k_l \equiv (c_5 / c_4)^{-1/2}$, $k_2^2 = I - k_l^2$ and $k_l B_{12u} + k_2 B_{22u}$ is a standard univariate Brownian motion independent of B_{1l} and B_{2l} .

The proof of Theorem 5.1 on page 115, between eq. (A.22) and (A.23) should be changed as follows. Let $g \equiv \text{diag}(g_1, c_1^{-1/2}, 1)$, and define $F \equiv g' F_u^* = \left\{ (B'_{11u}, k_1 B_{12u} + k_2 B_{22u}, U'_{21u}, u^2) \mid 1, u, B_{21u} \right\}$; pre-multiplying (A.22) by g' and post-multiplying it by g one obtains

$${}_2Q(p_0) \rightarrow^w \sum_{i=p_0+1}^p \psi_i = h(B, F) \quad (\text{A.23})$$

where $h(x_u, y_u) \equiv \text{tr} \left\{ \int_0^1 (dx) y \left(\int_0^1 yy' du \right)^{-1} \int_0^1 y(dx) \right\}$.

The rest of the proof is unaltered; thus ${}_2Q(p_1|p_0) \rightarrow^w h(B_2, G)$ and ${}_2Q(p_0, p_1) \rightarrow^w h(B, F) + h(B_2, G)$. We want to show that the distribution of $h(B, F) + h(B_2, G)$ does not depend on k_1 , and hence on k_2 . The statement of Theorem 5.1 then follows by choosing $k_1 = 1 - k_2 = 1$.

We start by showing that the two terms of $h(B, F) + h(B_2, G)$ are conditionally independent given $\mathcal{G} \equiv \sigma(B_{11u}, B_{21u}, u \in [0, 1])$. We then proceed to show that the conditional distribution of the first term, $h(B, F)$, and hence of $h(B, F) + h(B_2, G)$ does not depend on k_1 . Consequently also the unconditional distribution does not depend on k_1 .

In the following the information set $\mathcal{G}$ is held fixed. The only random element of the first term, $h(B, F)$, is $J \equiv \int_0^1 F d \left(B'_{12} B_{22} \right)$ whereas the only random element of the second term $h(B_2, G)$, is $K \equiv \int_0^1 G dB_{22}$. When conditioning on $\mathcal{G}$ the process G_u is fixed and, except for the component $k_1 B_{12} + k_2 B_{22}$, the process F_u is fixed as well. The two terms have the process B_{22} in common. It will be argued that J depends only on B_{22} through the process

$$B_{22u}^o \equiv B_{22u} - \int_0^u G'_s ds \left(\int_0^1 G_s G'_s ds \right)^{-1} \int_0^1 G_s dB_{22} = B_{22u} - \int_0^u G'_s ds \mathcal{K}. \quad (\text{C.1})$$

The process B_{22u}^o and the variable K are both Gaussian and arise from the same process, B_{22} . Consequently, B_{22u}^o and K , and hence also J and K , are independent since their cross variation is zero

$$\int_0^1 \left\{ I_{(t \leq u)} - \int_0^u G'_s ds \left(\int_0^1 G_s G'_s ds \right)^{-1} \right\} G'_t dt = 0.$$

The process B_{22} occurs both in the integrator and the integrand of the integral $J \equiv \int_0^l F d(B'_{12}, B_{22})$. First, it is argued that $\int_0^l F dB_{22} = \int_0^l F dB_{22}^o$. Use of (C.1) gives

$$\int_0^l F dB_{22} = \int_0^l F dB_{22}^o + \int_0^l F G' ds \mathcal{A}.$$

Since F is corrected for G the integral $\int_0^l F G' ds = 0$. Secondly, divide the process F into two components: $F_1 = (B_{11u}, k_1 B_{12u} + k_2 B_{22u})$ and $F_2 = \int_0^u G_s ds$ which is fixed given $\mathcal{G}$. The h -function has the property that $h(B, F) = h(B, MF)$ for any non-singular matrix M . Thus, F can be replaced by the vector of F_1, F_2 and F_2 . Using (C.1) once again it follows that

$$\begin{aligned} (F_1 | F_2) &= \left((B'_{11u}, k_1 B_{12u} + k_2 B_{22u}) | I, G, \int_0^u G \right) = \\ &= \left((B'_{11u}, k_1 B_{12u} + k_2 B_{22u}^o) | I, G, \int_0^u G \right). \end{aligned}$$

Consequently, $h(B, F)$ depends on B_{22} only through B_{22}^o , and hence $h(B, F)$ and $h(B, G)$ are conditionally independent for fixed $\mathcal{G}$.

Next, it has to be shown that the conditional distribution of the first term, $h(B, F)$, does not depend on k_1 . By standard properties of the trace, the h -function has the property that $h(B, F) = h(LB, F)$ for any orthogonal matrix L . Define

$$L = \begin{pmatrix} I_{p_1-1} & 0 & 0 & 0 \\ 0 & k_1 & 0 & k_2 \\ 0 & 0 & I_{p_2-1} & 0 \\ 0 & k_2 & 0 & -k_1 \end{pmatrix}, \quad L^{-1} = L'$$

and the process $B_u^* = LB_u$ which is a standard Brownian motion. Since $F' = (B'_{11}, B_{12}^*, \int_0^u G' | I, G)$, $h(B, F)$ can be expressed as $h(B^*, F)$ in terms of a Brownian motion B^* not involving k_1 .

REFERENCE

PARUOLO P. (1998), Asymptotic efficiency of the two stage estimator in I(2) systems, forthcoming in *Econometric Theory*.