Correction to “Determining the Relaxation Tensor in Linear Viscoelasticity of Integral Type”

(Vol. 11, No. 1, pp. 131–153, February 1994)

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Received June 5, 1995

In [2, proof of Lemma 6.1] identities (6.12–13) are used. Unfortunately, these identities do not allow to treat the convolution terms $\alpha * \Delta \mathbf{v}$ and $(\alpha + \beta) * \nabla \nabla \cdot \mathbf{v}$ (see [2, eq. (6.10)]). Therefore, identity (6.15) in [2] is wrong.

Here we give a correct proof of Lemma 6.1.

Proof of Lemma 6.1. The argument is similar to the one used in [1, Appendix, proof of Lemma 2.1]). Consider the weak solution $\mathbf{v} \in C^0([0, T]; V) \cap C^1([0, T]; H)$ to the Cauchy problem [2, eqs. (3.13–15)] given by [2, Thm. 6.1]. On account of [2, eq. (1.1)], [3, Chap. I, Théo. 4.1], and [3, Chap. IV, Sec. 1.2]), one (formally) obtains

$$(0.1) \quad \|\mathbf{v}_n\|_{L^2(\Gamma \times (0, t); \mathbf{R}^3)} \leq \tilde{A}_1(T) \{ \|\nabla \cdot \tilde{\sigma}(\mathbf{v})\|_{L^1(0, t; H)} + \|\mathbf{F}\|_{L^1(0, t; H)} \\ + \|\mathbf{v}_0\|_V + \|\mathbf{v}_1\|_H \} \quad \forall t \in [0, T].$$

Here and in the sequel $\tilde{A}_j$, $j \in \mathbf{N}$, stands for a non-negative, continuous function of its arguments. In addition, $\tilde{A}_j$ is non-decreasing in each of its variables and depends on Ω , μ , κ , $\alpha(0)$, and $\beta(0)$, at most.

On the other hand, recalling [2, eqs. (1.1), (1.6–7), (3.9), and (6.4)], an integration by parts in time gives

$$(0.2) \quad \nabla \cdot \tilde{\sigma}(\mathbf{v}) = \alpha(0) \Delta(1 * \mathbf{v}) + (\alpha + \beta)(0) \nabla \nabla \cdot (1 * \mathbf{v}) \\ + \alpha' * \Delta(1 * \mathbf{v}) + (\alpha + \beta)' * \nabla \nabla \cdot (1 * \mathbf{v}) \quad \text{in } V^*.$$

Observe that $\mathbf{w} := 1 * \mathbf{v} \in C^1([0, T]; V) \cap C^2([0, T]; H)$ solves the Cauchy problem

$$(0.3) \quad \mathbf{w}_{tt} = \nabla \cdot \sigma(\mathbf{w}) + 1 * \mathbf{F} + \mathbf{v}_1 \quad \text{a.e. in } \Omega \times (0, T),$$

$$(0.4) \quad \mathbf{w}(0) = \mathbf{0} \quad \text{a.e. in } \Omega,$$

$$(0.5) \quad \mathbf{w}_t(0) = \mathbf{v}_0 \quad \text{a.e. in } \Omega,$$

which is derived from [2, eqs. (3.13–15)] via integration in time over $(0, t)$, $t \in (0, T]$. Equation (0.3) implies $\nabla \cdot \sigma(\mathbf{w}) \in C^0([0, T]; H)$, then $\mathbf{w} \in C^0([0, T]; W)$ and

equation (0.2) holds a.e. in $\Omega \times (0, T)$ as well. Moreover, owing to Young inequality for convolution products, from (0.2) we easily deduce, for any $t \in [0, T]$,

$$(0.6) \quad \|\nabla \cdot \tilde{\sigma}(\mathbf{v})\|_{L^1(0,t;H)} \leq \tilde{A}_2(\|\alpha\|_{W^{1,1}(0,T)} + \|\beta\|_{W^{1,1}(0,T)}, T) \|\mathbf{w}\|_{L^1(0,t;W)}.$$

Using now [2, ineq. (6.5)], we obtain, for any $t \in [0, T]$,

$$(0.7) \quad \begin{aligned} & \|\mathbf{w}\|_{C^1([0,t];V)} + \|\mathbf{w}\|_{C^2([0,t];H)} \\ & \leq \tilde{A}_3(\|\alpha\|_{W^{1,1}(0,T)} + \|\beta\|_{W^{1,1}(0,T)}, T) \{ \|\mathbf{F}\|_{L^1(0,t;H)} + \|\mathbf{v}_0\|_V + \|\mathbf{v}_1\|_H \}. \end{aligned}$$

Besides, taking [2, eq. (1.1)] into account, from equation (0.3) one infers

$$(0.8) \quad \begin{aligned} \|\nabla \cdot \sigma^0(\mathbf{w})(\cdot, t)\|_H & \leq \|\nabla \cdot \tilde{\sigma}(\mathbf{w})(\cdot, t)\|_H + \|\mathbf{w}_{tt}(\cdot, t)\|_H \\ & \quad + \|(1 * \mathbf{F})(\cdot, t)\|_H + \|\mathbf{v}_1\|_H \quad \forall t \in [0, T]. \end{aligned}$$

Then, combining (0.7) and (0.8), an application of Gronwall lemma yields (cf. also [2, ineq. (2.2)])

$$(0.9) \quad \begin{aligned} & \|\mathbf{w}\|_{C^0([0,t];W)} \\ & \leq \tilde{A}_4(\|\alpha\|_{W^{1,1}(0,T)} + \|\beta\|_{W^{1,1}(0,T)}, T) \{ \|\mathbf{F}\|_{L^1(0,t;H)} + \|\mathbf{v}_0\|_V + \|\mathbf{v}_1\|_H \}, \end{aligned}$$

for any $t \in [0, T]$.

Finally, inequalities (0.1), (0.6), and (0.9) entail [2, ineq. (6.7)] and, in particular, $\mathbf{v}_n \in L^2(\Gamma \times (0, T); \mathbf{R}^3)$. This completes the proof.

References

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