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GENERAL RELATIVISTIC THEORY OF ONE-DIMENSIONAL LAGRANGIAN FUNCTIONS

PART III.

ONE-COMPONENT MASSOR FIELDS

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Massors are objects which describe the internal energy (mass) of the fields (particles) in a way which is similar to the description of the spin by spinors, vectors, etc. After a brief survey of the mathematical theory of the massors the general relativistic and quantized free unitary massor field model is treated. The rest mass of the field appears spontaneously in the Lagrangian as well as in the equations of motion of the system. The field has charge, which is a scalar, and energy, which is an invariant implet. The energy is copletisable by the charge. The quantized values of the charge are $Q = n$, and those of the energy $T = \varkappa n$, where $\varkappa$ is the rest mass of a quantum, and n equals 0, 1, 2, ... in the case of the Bose—Einstein statistics, or n equals 0,1 in the case of the Fermi—Dirac one.

26. Introduction

In preceding parts of this series of papers [1] we have dealt with the fundamental one-component coplet fields in the space X_1, the time. In the present publication we consider the remaining one-component coplets, the massors.

(D.28) Massors are those objects of the Brandt semigrupoids of the coordinate transformations whose transformators contain explicitly the coordinates, i.e.,

$$\overline{\psi}_A(\bar{x}) = T_A\left[\psi_B(x(\bar{x})), \bar{x}, x(\bar{x}), \frac{\partial x}{\partial \bar{x}}, \ldots\right]. \tag{165}$$

In Chapter VI the physical applicability of these objects is treated, in Chapter VII their mathematical theory is surveyed and in Chapter VIII a unitary massor model is presented.

Chapter VI is treated on the special relativistic level, but Chapters VII and VIII on the general relativistic one.

VI. The physical applicability of massors

27. The discontinuous spaces

The solution of the problem of the structure of the subnuclear particles, and that of the collapsing objects of the Universe, is far from being solved. Therefore one cannot be sure that physical space-time is continuous in the interior of these objects. We think of the possibility that in the interior of the physical objects with an extraordinary high density of mass space-time may have bubbles, or isolated points, or isolated regions, or any other kind of discontinuity.

In the discontinuous spaces the derivatives of the coordinates, i.e.,

$$\frac{\partial x^i}{\partial \bar{x}^k}, \frac{\partial^2 x^i}{\partial \bar{x}^k \partial \bar{x}^l}, \ldots \tag{166}$$

are meaningless. The transformators

$$\bar{\psi}_A(\bar{x}) = T_A \left[\psi_B(x), \frac{\partial x}{\partial \bar{x}}, \ldots\right] \tag{167}$$

which contain derivatives, and the objects belonging to them, are also meaningless. Therefore the concepts of vector, tensor (and that of the spinor) are meaningless in the exotic regions of these general spaces.

If so, the differential geometric objects cannot describe all properties of Nature and they cannot express all its laws. The only objects, which remain meaningful in these (at least partly) discontinuous spaces are the massors of class zero, whose transformator reads

$$\bar{\psi}_A(\bar{x}) = T_A \left[\psi(x'\bar{x})), \bar{x}, x(\bar{x})\right]. \tag{168}$$

The circumstance that only the massors remain meaningful at all points of any space-time shows that the massors have a much greater domain of applicability than the nonmassors (pure differential geometric objects), which are mostly applied in theoretical physics and in differential geometry.

28. The charges

It is a known fact that the subnuclear particles possess charges (baryonic, electric, leptonic, etc.), and that these charges are connected with the gauge groups. The descriptors $\psi_A(x)$ of the particles which possess charges *are massors of class zero* with respect to these gauge groups. E.g., the electric gauge transformation of the proton field reads

$$\bar{\psi}_\mu(x) = e^{ie\varphi(x)} \psi_\mu(x), \tag{169}$$

where $\varphi(x)$ is the local parameter of the electric group. Since $\varphi(x)$ is explicitly present in (169), ψ_μ is a gauge nullor, i.e., a gauge massor of class zero. Thus the massors are already applied in physics.

In the present paper we show that the massors of the coordinate Brandt semigroupoid, which contains the Poincaré group, are very suitable to describe the fields (particles) whose rest mass is different from zero.

The massors are those objects which describe fields which have internal energy. As will be seen in subsequent paragraphs this quantity is similar to the different charges and to the spin of the particles.

29. The internal properties of the particles

In order to explain the above statements, we will consider more precisely the internal properties of the fields.

In the special relativistic Lagrangian theory of fields different properties of the systems are expressed as different concomitants of descriptors $\psi(x)$, of its transformators T_A under some symmetry bioperatives and of the Lagrangian L of the system under consideration. E.g., the expression of the charge-current density j^k is

$$j^k = \frac{\partial L}{\partial \psi_{A,k}} \frac{\delta \psi_A}{\delta \varphi}\,, \tag{170}$$

where

$$\frac{\delta \psi_A}{\delta \varphi} \equiv ie\psi_A \tag{171}$$

where the Lagrangian L is invariant under the electric gauge group

$$\overline{\psi}_A(x) = e^{ie\varphi(x)}\,\psi_A(x) \equiv T_A\left[\psi_B(x), \varphi(x), \varphi_{,k}(x), \ldots\right]. \tag{172}$$

Since, according to (170), the variation of ψ_A is

$$\delta\psi_A = ie\,\psi_A \delta\varphi \equiv \left.\frac{\partial T_A}{\partial \varphi}\right|_{\varphi=0} \delta\varphi + \left.\frac{\partial T_A}{\partial \varphi_k}\right|_{\varphi=0} \delta\varphi_{,k} + \ldots, \tag{173}$$

it is seen from the comparison of (172) and (174) that expression (170) contains the derivatives of the transformator (172) too. Taking this into account one sees from (170) that j_k depends on ψ_A, T_A and L, i.e., it is a concomitant of them.

Similar rule is valid for all properties of the physical systems.

According to this general rule the internal mass (internal energy) of fields should also be concomitant of ψ_A, T_A and L. The form of this concomitant is given in the subsequent paragraph.

30. The concomitant of the internal energy

In the recent special relativistic field theory of subnuclear particles the canonical energy-momentum concomitant has the form [3] [4]

$$T_l^k = \frac{\partial L}{\partial \psi_{A,k}} \cdot \frac{\delta \psi_A}{\delta x^l} - \frac{\partial L}{\partial \psi_{A,k}} \psi_{A,l} + \delta_l^k L, \tag{174}$$

where $\delta \psi_A$ is the isolocal variation of ψ_A under the infinitesimal Poincaré transformation δx^l of coordinates. The third term of (174), $\delta_l^k L$, is a pressure. The pressure is a result of the interaction (or selfinteraction), therefore it is not part of the internal energy. The second term, $-\frac{\partial L}{\partial \psi_{A,k}} \psi_{A,l}$, is the four dimensional path energy. It does not belong to the internal mass either. Thus the density of the internal energy is either identical with, or contained in the first term

$$T_0^0(1) = \frac{\partial L}{\partial \psi_{A,0}} \frac{\delta \psi_A}{\delta x^0}. \tag{175}$$

If we compare this expression with the expression of the electric charge (170), and with the expression of the spin

$$S_{kl}^0 = \frac{\partial L}{\partial \psi_{A,0}} \frac{\delta \psi_A}{\delta \pi^{kl}}, \tag{176}$$

where π^{kl} are the parameters of the rotation group, we see that all internal properties of the matter have similar concomitants

$$I_{(p)}^{\circ} = \frac{\partial L}{\partial \psi_{A,0}} \frac{\delta \psi_A}{\delta p}. \tag{177}$$

Now the question arises: is it natural that the subnuclear particles possess a number of charges, that they possess internal angular momentum, and that they do not possess even internal mass?

31. The role of the transformators

It is a known fact that the descriptor fields of the particles with different spin have different transformation character under the rotation subgroup of the Poincaré group. In the present nomenclature: they have different transformators. E.g., the scalar fields describe 0-spin particles, the spinor fields

$\frac{\hbar}{2}$-spin ones, etc. The spin is thus "determined" by the transformator T of ψ

$$\overline{\psi}_A(\overline{x}) = T_A\left[\psi_B(x(\overline{x})), \overline{x}, x, \frac{\partial x}{\partial \overline{x}}, \ldots\right] \tag{178}$$

under the rotation (or Lorentz) group of the coordinate transformations.

Similarly the electric (baryonic, etc.) charge of a field ψ_A is determined by the transformator T of ψ

$$\overline{\psi}_A(x) = T_A\left[\psi_B(x), \varphi(x), \frac{\partial \varphi}{\partial x}, \ldots\right] \tag{179}$$

under the electric (baryonic, etc.) gauge group. For example, the electromagnetic field A_i has no charge, because its transformator under the electric gauge group

$$\bar{A}_k(x) = A_k(x) + \varphi_{,k}(x) \tag{180}$$

does not contain φ explicitly. A_i is a gauge connector.

These facts mean that the dependence of the internal quantities $I^0{}_{(p)}$ (177) on $\frac{\partial L}{\partial \psi_{A,k}}$ is either universal, or apparent one. In any case the values of the internal quatities $I^0_{(p)}$ are determined by the transformators T via the variations $\delta\psi_A$.

According to this rule the internal energy should be also determined by the transformator T of ψ_A. Particles of different internal mass should have different transformation properties in the case of coordinate transformations.

In this context we mention a theorem:

(T.35) Those objects of the Poincaré group whose transformators do not contain the coordinates explicitly, describe particles of zero internal mass (energy).

(Proof. The transformators in question are of the form

$$\overline{\psi}_A(\overline{x}) = T_A\left[\psi_B(x(\overline{x})), \frac{\partial x^i}{\partial \overline{x}^k}\right]. \tag{181}$$

The infinitesimal isolocal variation of ψ_A is therefore

$$\delta\psi_A = - \left.\frac{\partial T_A}{\partial \dfrac{\partial x^i}{\partial \overline{x}^k}}\right|_{x=\overline{x}} \delta \frac{\partial x^i}{\partial \overline{x}^k}. \tag{182}$$

Since in the case of the Poincaré group the variations $\delta \frac{\partial x^i}{\partial \overline{x}^k}$ are infinitesimal constants, which do not depend on δx_0 at all, $\delta \frac{\partial x^i}{\partial \bar{x}^k} / \delta x_0$ is zero. If so, $\delta \psi$ is also zero. Thus

$$T_0^0(1) = \frac{\partial L}{\partial \psi_{A,0}} \frac{\delta \psi_A}{\delta x^0} = 0 \tag{183}$$

and the theorem is proved).

This theorem means that scalars, spinors, vectors, tensors, etc. can describe particles with zero internal mass only. They are not able to describe particles whose internal mass is different from zero.

Thus in the description of the particles of nonzero internal mass we should take into consideration the massors, whose transformator under the Poincaré group, as well as under the Brandt semigroupoid of coordinate transformations, contains explicitly the coordinates, i.e.,

$$\bar{\psi}_A(\bar{x}) = T_A\left[\psi_B(x(\bar{x})), \bar{x}, x(\bar{x}), \frac{\partial x}{\partial \bar{x}}, \ldots\right]. \tag{184}$$

32. The massors and the restmass

The theory of fields with internal energy does not lead to difficulties as, for example, the appearance of the internal linear momentum, or the possibility of

$$E^2 < c^2 p^2 \tag{185}$$

as in the case of the "tachions" [2], because in the general relativistic theory of fields the value of the canonical energy-momentum density is identically zero.

On the contrary the massor fields have an advantageous property. In the cases we have studied *the rest mass of the massor fields appears spontaneously in the Lagrangian and in the equations of motion of the fields.* In Section VIII we present a model of this kind.

Since this circumstance is a remarkable property of the massors, the study of the massors has physical interest.

VII. The one-component massors of Bd (1)

33. The spectrum of the coplets

The one-component coplets of the Brandt semigroupoid Bd (1) of coordinate transformations in the space X_1 constitute a "bundle", which consists of eight "rays". Every ray contains one of the fundamental coplets and infinitely many other coplets. The fundamental object characterises the ray and conversely the later belongs to it. The coplets of the ray, with the exception of the basic one, are the massors which belong to the basic coplet.

Since the fundamental one-component coplets constitute a discrete spectrum of objects, the different rays are also disconnected. But the massors belonging to a single ray constitute a nondiscrete spectrum, which consists of functionally infinite coplets.

These assumptions follow from the following fundamental spectral theorem of the one-component massors.

34. The spectral theorem

(T.36) The transformators of the one-component massors $\psi(\tau)$ of the Bd (1) have the form

$$\bar{\psi}(\bar{\tau}) = f\left\{T\left[f^{-1}(\psi(\tau(\bar{\tau})), \tau(\bar{\tau})), \frac{d\tau}{d\bar{\tau}}, \frac{d^2\tau}{d\bar{\tau}^2}, \ldots\right], \bar{\tau}\right\}, \tag{186}$$

where T is any of the eight transformators of the fundamental one-component coplets of Bd (1), $f(\omega, \tau)$ is an arbitrary invertible function with respect to its first argument $\omega(\tau)$, and f^{-1} is the inverse of f.

(Proof: [5] p. 22, [6] theor. 13.)

The fundamental spectral theorem of the massors is equivalent to the following assumption.

(T.37) Every massor may be expressed as a bicomitant of one of the eight fundamental coplets $\omega(\tau)$ and of the paratime τ, which is the primary object of Bd (1), in the form

$$\psi(\tau) = f(\omega(\tau), \tau) \tag{187}$$

where $f(\omega, \tau)$ is the function given above.

(Proof: [7] p. 20)

Theorem (T.37) does not mean that $\psi(\tau)$ and $\omega(\tau)$ are equivalent. Namely the densities $D(\tau)$, which belong to the fundamental coplets, may also be expressed as bicomitants of a scalar $S(\tau)$, and of the derivative of time in the form

$$D(\tau) = f\left(S(\tau), \frac{d\tau}{dp}\right) \tag{188}$$

where p is a scalar parameter. In this case the transformator reads

$$\bar{D}\,\bar{\tau}) = f\left\{f^{-1}\left[D(\tau(\bar{\tau})), \frac{d\bar{\tau}}{dp}\right], \frac{d\bar{\tau}}{dp}\right\}. \tag{189}$$

If we take

$$f\left(s, \frac{d\tau}{dp}\right) \equiv s\,\frac{d\tau}{dp} \tag{190}$$

then (189) reads

$$\bar{D}(\bar{\tau}) = D(\tau)\,\frac{dv}{dp}\left(\frac{d\bar{\tau}}{dp}\right)^{-1}, \tag{191}$$

i.e., we get the transformation formula of the densities

$$\bar{D} = \frac{d\tau}{d\bar{\tau}}\,D\,. \tag{192}$$

The massors are to the same degree non-equivalent with the basic coplets as the latest are non-equivalent among themselves.

In Tables 3 and 4 we give some massors and some objects which belong to them. Since the number of the massors is functionally infinite there is no possibility to give any exhaustive table of them.

Table 3

Some massors of class zero

Number	Name	Transformator
19	multiplicative nullor	$\bar{N}(\bar{\tau}) = e^{f(\tau)-(\bar{\tau})}\,N(\tau)$
20	unitary nullor	$\bar{\psi}(\bar{\tau}) = e^{ik(\tau-\bar{\tau})}\,\psi(\tau)$
21	additive nullor	$N(\bar{\tau}) = f(\tau) - f(\bar{\tau}) + N(\tau)$
22	special additive nullor	$N(\bar{\tau}) = K(\tau - \bar{\tau}) + N(\tau)$

Table 4

Some objects belonging to the massors

Number	Name	Class	Transformator	Note
23	connector of No. 20, or gradient of No. 22.	1	$\bar{U}(\bar{\tau}) = \frac{d\tau}{d\bar{\tau}}\,U(\tau) + k\left(\frac{d\tau}{d\bar{\tau}} - 1\right)$	
24	connector of $\omega = ND_w$	2	$\bar{\Gamma}(\bar{\tau}) = \frac{d\tau}{d\bar{\tau}}\,\Gamma(\tau) - w\,\frac{d^2\tau}{d\bar{\tau}^2}\Big/\frac{d\tau}{d\bar{\tau}} + k\left(\frac{d\tau}{d\bar{\tau}} - 1\right)$	D is a density of weight w, N is a multiplicative nullor
25	pronector	3	$\bar{\Pi}(\bar{\tau}) = \left(\frac{d\tau}{d\bar{\tau}}\right)^2 \Pi(\tau) - \frac{3w}{2}\left(\frac{d^2\tau}{d\bar{\tau}^2}\Big/\frac{d\tau}{d\bar{\tau}}\right)^2 + w\,\frac{d^3\tau}{d\bar{\tau}^3}\Big/\frac{d\tau}{d\bar{\tau}} + k\left[\left(\frac{d\tau}{d\bar{\tau}}\right)^2 - 1\right]$	

VIII. The free unitary massor field model

Now, turning back to physics, we consider in full the free unitary massor field model in the space X_1. Despite its simplicity, produced by the dimensionality 1, many features of general relativistic field theories clearly appear, among others the role of the energy within general relativity theory.

35. The unitary massor

(D.29) The transformator of the unitary massor $\psi(\tau)$ reads as follows

$$\bar{\psi}(\bar{\tau}) = e^{i\varkappa(\tau-\bar{\tau})}\,\psi(\tau)\,, \tag{193}$$

where $\varkappa$ is an arbitrary constant, which is equal to the restmass of the system to be considered.

(T.38) The transformator of the "velocity" $d\psi/d\tau$ is

$$\bar{\psi}_{\bar{\tau}}(\bar{\tau}) = \frac{d\tau}{d\bar{\tau}}\,e^{i\varkappa(\tau-\bar{\tau})}\,\psi_\tau(\tau) + i\varkappa\left(\frac{d\tau}{d\bar{\tau}} - 1\right)e^{i\varkappa(\tau-\bar{\tau})}\,\psi(\tau)\,. \tag{194}$$

As is seen from (194) the velocity ψ_τ is an implet only. But it is not the only implet of the system under consideration. The transformators of the conjugate complex fields ψ^* and ψ_τ^* are

$$\bar{\psi}^*(\bar{\tau}) = e^{-i\varkappa(\tau-\bar{\tau})}\,\psi^*(\tau)\,, \tag{195}$$

and

$$\bar{\psi}^*(\bar{\tau}) = e^{-i\varkappa(\tau-\bar{\tau})}\left[\frac{d\tau}{d\bar{\tau}}\,\psi_\tau^*(\tau) - i\varkappa\left(\frac{d\tau}{d\bar{\tau}} - 1\right)\psi^*(\tau)\right]\,, \tag{196}$$

respectively.

36. The Lagrangian

(T.39) The most general Lagrangian $L(\psi, \psi^*, \psi_\tau, \psi_\tau^*)$ depending on the field ψ, its velocity ψ_τ and their complex conj. fields is

$$L = (\varkappa\psi^*\psi - i\psi^*\psi_\tau)\cdot I\left(\psi^*\psi, \frac{\varkappa\psi\psi^* + i\psi\psi_\tau^*}{\varkappa\psi^*\psi - i\psi^*\psi_\tau}\right), \tag{197}$$

where I is a scalar concomitant of its scalar arguments.

(Proof. The Lagrangian (197) is the general solution of the functional equation

$$L(\bar{\psi}, \bar{\psi}^*, \bar{\psi}_{\bar{\tau}}, \bar{\psi}_{\bar{\tau}}^*) = \frac{d\tau}{d\bar{\tau}}\,L(\psi, \psi^*, \psi_\tau, \psi_\tau^*)\,, \tag{198}$$

i.e., of

$$L\left\{e^{\varkappa i(\tau-\bar\tau)}\,\psi, e^{-i\varkappa(\tau-\bar\tau)}\,\psi^*, e^{i\varkappa(\tau-\bar\tau)}\left[\frac{d\tau}{d\bar\tau}\psi_\tau + i\varkappa\left(\frac{d\tau}{d\bar\tau}-1\right)\psi\right],\right.$$
$$\left. e^{-i\varkappa(\tau-\bar\tau)}\left[\frac{d\tau}{d\bar\tau}\psi_\tau^* - i\varkappa\left(\frac{dv}{d\bar\tau}-1\right)\psi^*\right]\right\} = \frac{d\tau}{d\bar\tau}L(\psi,\psi^*,\psi_\tau,\psi_\tau^*), \tag{199}$$

where τ, $\bar\tau$ and $d\tau/d\bar\tau$, as well as ψ, ψ^*, ψ_τ and ψ_τ^* are the free variables. Denoting

$$\frac{d\tau}{d\bar\tau} \equiv b \tag{200}$$

and

$$e^{i\varkappa(\tau-\bar\tau)} \equiv t \tag{201}$$

eq. (199) reads

$$L\left\{t\psi, \frac{\psi^*}{t}, t\left[b\psi_\tau + i\varkappa(b-1)\psi\right], \frac{1}{t}\left[b\psi_\tau^* - i\varkappa(b-1)\psi^*\right]\right\} =$$
$$= bL(\psi,\psi^*,\psi_\tau,\psi_\tau^*). \tag{202}$$

This homogeneous functional equation should be satisfied for every value of the arguments t, b, ψ, ψ^*, ψ_τ and ψ_τ^*. If so, (202) should be valid also in the case when

$$t\psi = 1. \tag{203}$$

Then

$$t = 1/\psi, \tag{204}$$

and (202) reduces to

$$L\left\{1, \psi^*\psi, \frac{b\psi_\tau + i\varkappa(b-1)\psi}{\psi}, \psi\left[b\psi_\tau^* - i\varkappa(b-1)\psi^*\right]\right\} = bL(\psi,\psi^*,\psi_\tau,\psi_\tau^*). \tag{205}$$

This equation should be satisfied also if

$$b\psi_\tau + i\varkappa(b-1)\psi = 0. \tag{206}$$

Inserting b from equ. (206), (208) into (205) we get

$$L(\psi,\psi^*,\psi_\tau,\psi_\tau^*) = \frac{\psi_\tau + i\varkappa\psi}{i\varkappa\psi} H\left[\psi^*\psi, i\varkappa\psi\psi^* + \frac{i\varkappa\psi}{\psi_\tau + i\varkappa\psi}(\psi\psi_\tau^* - i\varkappa\psi\psi^*)\right], \tag{207}$$

thereafter

$$L(\psi,\psi^*,\psi_\tau,\psi_\tau^*) = \frac{\psi^*\psi_\tau + i\varkappa\psi^*\psi}{i\varkappa\psi^*\psi} J\left[\psi^*\psi, \frac{\psi\psi_\tau^* - i\varkappa\psi^*\psi}{\psi^*\psi_\tau + i\varkappa\psi^*\psi}\right], \tag{208}$$

and at last

$$L = (\psi^* \psi_\tau + i\varkappa\psi^*\psi)\, I \left(\psi^*\psi, \frac{\psi\psi_\tau^* - i\varkappa\psi\psi^*}{\psi^*\psi_\tau + i\varkappa\psi^*\psi}\right), \tag{209}$$

where H, J and I are invariant functions of their arguments. QED.

It is to be noticed that the density

$$L_0 = \varkappa\psi^*\psi - i\psi^*\psi_\tau \tag{210}$$

is not a sum of two densities, i.e. it is in this sense irreducible. This is an advantageous property of L, because in general relativity theory every densitant leads to strong conservation rules. If the Lagrangian consists of many densities, then the number of the strongly conserved quantities becomes enormous and this circumstance leads to more difficulties [8].

37. The equations of motion

For the sake of simplicity we consider here the free field (212). We take

$$L = \varkappa\psi^*\psi - i\psi^*\psi_{,\tau}\,. \tag{211}$$

This model is the one-dimensional version of the free Dirac field.

The Euler-equations of this Lagrangian (211) are

$$L_{\psi^*} \equiv \varkappa\psi - i\psi_{,\tau} = 0 \tag{212}$$

and

$$L_\psi \equiv \varkappa\psi^* + i\psi^*{}_{,\tau} = 0\,. \tag{213}$$

It is clearly seen from (211), (212) and (213) that the *rest mass $\varkappa$ appeared spontaneously*, because it was built into the transformator of ψ. This method of the introduction of the rest mass into the theory is more advantageous than the old one, where the rest mass is built into the Lagrangian. The method of introducing experimental data into the transformators means therefore a step forward.

The solutions of (212) and (213) are

$$\psi(\tau) = Ae^{-i\varkappa\tau} \tag{214}$$

and

$$\psi^*(\tau) = A^*e^{i\varkappa\tau}, \tag{215}$$

where the amplitude A is a scalar constant.

38. The conserved comitants

The Euler equations yield two "old" conserved quantities and a "new" one.

1. *The charge*

The first old concomitant is the charge

$$Q\,[\psi] = \psi^* \psi, \tag{216}$$

whose value is

$$Q = A^* A. \tag{217}$$

The charge is conserved because

$$\frac{dQ}{d\tau} = 0. \tag{218}$$

The proof of this statement is known from the theory of the Dirac equation.

The expression of the charge $Q[\psi]$ is an invariant. Its value Q is a scalar.

2. *The energy*

The second old conserved concomitant is the energy

$$T[\psi] = \frac{i}{2}\,(\psi^* \psi_\tau - \psi \psi_\tau^*), \tag{219}$$

whose value is

$$T = \varkappa A^* A. \tag{220}$$

The energy is also conserved

$$\frac{dT}{d\tau} = 0. \tag{221}$$

The proof of this statement is also known from the Dirac equation.

Now we derive the transformator of $T[\psi]$. As a first step we have

$$\begin{aligned}\frac{i}{2}\,(\overline{\psi}^*\,\overline{\psi}_{\bar\tau} - \overline{\psi}\,\overline{\psi}_{\bar\tau}^*) = \frac{i}{2}\Big\{& e^{-i\varkappa(\tau-\bar\tau)}\,\psi^*\, e^{i\varkappa(\tau-\bar\tau)}\left[t\varkappa\left(\frac{d\tau}{d\bar\tau} - 1\right)\psi + \frac{d\tau}{d\bar\tau}\,\psi_\tau\right] - \\ & - e^{i\varkappa(\sigma-\bar\tau)}\,\psi\, e^{-i\varkappa(\tau-\bar\tau)}\left[-i\varkappa\left(\frac{d\tau}{d\bar\tau} - 1\right)\psi^* + \frac{d\tau}{d\bar\tau}\,\psi_{\bar\tau}^*\right]\Big\},\end{aligned} \tag{222}$$

i.e.

$$\frac{i}{2}\,(\overline{\psi}^*\,\overline{\psi}_{\bar\tau} - \overline{\psi}\,\overline{\psi}_{\bar\tau}^*) = \frac{d\tau}{d\bar\tau}\,\frac{i}{2}\,(\psi^*\,\psi_\tau - \psi\psi_\tau^*) - \left(\frac{d\tau}{d\bar\tau} - 1\right)\varkappa\psi^*\psi. \tag{223}$$

Inserting (219) and (216) into (223) we get that

(T.40) the transformator of $T[\psi]$ under general time transformations reads

$$\bar{T}[\bar{\psi}] = \frac{d\tau}{d\bar{\tau}} T[\psi] + \left(1 - \frac{d\tau}{d\bar{\tau}}\right) \varkappa Q[\psi]. \tag{224}$$

(D.30) If some concomitant transforms as a coplet, it is a copletant.

(D.31) If some concomitant transforms as an implet, it is an impletant.

It is seen from (224) that the expression of the energy $T[\psi]$ is an impletant. Briefly, energy is an implet (and not a coplet as in the case of special relativistic theories !). Its *catalisator* [for its definition see (Def. 18)] is the charge $Q[\psi]$. Therefore

(T.41) the unified object ($Q[\psi]$, $T[\psi]$) is a coplet, whose transformator reads

$$\bar{Q}[\bar{\psi}] = Q[\psi], \tag{225}$$

$$\bar{T}[\bar{\psi}] = \frac{d\tau}{d\bar{\tau}} T[\psi] + \left(1 - \frac{d\tau}{d\bar{\tau}}\right) \varkappa Q[\psi]. \tag{226}$$

The implet character of the (four dimensional) general relativistic energy expressions was found by MØLLER [9], MICKEVICH [10] and PELLEGRINI—PLEBANSKI [11].

The transformators (225) and (226) mean that the energy is subordinate to the charge. It seems that in general relativistic theories the energy-moment is a quantity of second order only.

Inserting the value (220) into (226) one gets

$$\bar{T} = T. \tag{227}$$

This last equation means that the energy *value* of the system is an invariant implet (but not a scalar). The concept of the invariant implet is similar to that of the invariant Kronecker tensor δ^i_k, the totally antisymmetric invariant Levi—Civita tensor density ε^{iklm}, etc. The facts that T does not vary, do not reduce it to a scalar ! For example, ε^{iklm} does not vary nevertheless it is not a scalar.

The implet character explains all "difficulties" connected with the energy in general relativistic theories.

Lastly it should be noticed that the canonical energy identically vanishes, i.e.

$$\frac{\partial L}{\partial \psi_\tau} \frac{\delta \psi}{\delta \tau} - \frac{\partial L}{\partial \psi_\tau} \psi_\tau + \frac{\partial L}{\partial \psi^*_\tau} \frac{\delta \psi^*}{\delta \tau} - \frac{\partial L}{\partial \psi^*_\tau} \psi^*_\tau + L \equiv 0. \tag{228}$$

3. *The specific energy*

One of the interesting problems of the theory of implets is their copletisation. This task may be achieved in different ways. One of them is to join the implet (or implets) with some coplet (or coplets). Examples of this kind are represented by equations (97), as well as (225) and (226). Another way of the completisation is the following.

Dividing eq. (226) by eq. (225) one gets

$$\frac{\bar{T}[\bar{\psi}]}{\bar{Q}[\bar{\psi}]} = \frac{d\tau}{d\bar{\tau}} \frac{T[\psi]}{Q[\psi]} + \left(1 - \frac{d\tau}{d\bar{\tau}}\right)\varkappa. \tag{229}$$

This equation is the transformator of an inhomogeneous coplet. The copletisation of $T[\psi]$ is achieved.

(D.32) The new coplet

$$t[\psi] \equiv \frac{T[\psi]}{Q[\psi]} \tag{230}$$

will be called specific energy (energy/charge).

The specific energy is also conserved

$$\frac{dt}{d\tau} = 0, \tag{231}$$

because both T and Q are conserved. t is identical with the rest mass of the model.

It seems that the specific energy is a useful quantity.

4. *Some other quantities*

We note that

$$\bar{T}[\bar{\psi}] - \varkappa\bar{Q}[\bar{\psi}] = \frac{d\tau}{d\bar{\tau}} (T[\psi] - \varkappa Q[\psi]) \tag{232}$$

is a density of weight one. Similarly

$$\bar{t}[\bar{\psi}] - \varkappa = \frac{d\tau}{d\bar{\tau}} (t[\psi] - \varkappa) \tag{233}$$

is also a density.

The real part of $i\psi^*\psi_\tau$ is the energy implet, the imaginary one is the derivative of the charge.

39. Quantization

The procedure of the quantization consists of supplementing the equations of motion by some algebraic constraints, which also should be obeyed by the descriptors of the physical system in question. Of course, the constraint cannot be in contradiction with the equations of motion.

Two kinds of quantization methods have been successful in the quantum theory of fields. The first one is based on Heisenberg's relation, the second one on Pauli's exclusion principle.

The model under consideration may be quantized in both forms.

1. *Heisenberg quantization*

The basic constraint of this quantization method is given by the commutator. It reads

$$[\psi^*(\tau), \psi(\tau)]_- \equiv \psi^* \psi - \psi\psi^* = 1 , \tag{234}$$

i.e.

$$A^* A - AA^* = 1 . \tag{235}$$

Since A is constant, (235) does not contradict the equations of motion (212) and (213).

The solution of (235) is the matrix

$$A = \begin{bmatrix} 0 & 0 & 0 & - - - \\ 1 & 0 & 0 & \\ 0 & \sqrt{2} & 0 & \\ 0 & 0 & \sqrt{3} & \\ - & - & - & \end{bmatrix}, \quad A^* = \begin{bmatrix} 0 & 1 & 0 & 0 & - - - \\ 0 & 0 & \sqrt{2} & 0 & - - - \\ 0 & 0 & 0 & \sqrt{3} & \\ 0 & 0 & 0 & 0 & \\ - & - & - & & \end{bmatrix} . \tag{236}$$

According to this solution the values of the charge Q are

$$Q \equiv A^* A = n , \quad n = 1, 2, 3, \ldots , \tag{237}$$

but there is another charge Q' too

$$Q' = AA^* = n - 1 . \tag{238}$$

The values of the energy are

$$T = \varkappa A^* A = \varkappa n , \tag{239}$$

but there are other energies too. The value of the specific energy is

$$t = \frac{\varkappa n}{n} = \varkappa . \tag{240}$$

2. *Pauli quantization*

The constraint in this case is the anticommutator of Jordan. It reads

$$[\psi^*, \psi]_+ \equiv \psi^* \psi + \psi\psi^* = 1 , \quad \psi\psi = 0, \quad \psi^* \psi^* = 0, \tag{241}$$

i.e.

$$A^* A + AA^* = 1, \quad AA = 0, \quad A^* A^* = 0 . \tag{242}$$

The solution of this constraint is

$$A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad A^* = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}. \tag{243}$$

Accordingly the charge of the system is

$$Q = A^* A = n\,, \quad n = 0\,, \quad \text{or} \quad 1\,. \tag{244}$$

The energy is

$$T = \varkappa^* A = \varkappa n\,. \tag{245}$$

The specific energy is

$$t = \begin{Bmatrix} \text{indefinite}\,, & \text{if} \quad n = 0 \\ \varkappa & n = 1 \end{Bmatrix}. \tag{246}$$

40. Conclusion

The free unitary model discussed is both generally relativistic and quantized. This example shows that quantum theory and general relativity theory are not in contradiction.

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ОБЩЕ РЕЛЯТИВИСТСКАЯ ТЕОРИЯ ОДНОМЕРНЫХ ЛАГРАНЖИАНОВ

Часть III.

Однокомпонентные масорные поля

Г. КНАПЕЦ

Массоры являются объектами, которые описывают внутреннюю энергию (массу) физических полей (частиц), именно так, как спиноры, векторы и т. д. описывают спин. После короткого обзора математической теории массоров трактуется общерелятивистский квантованный свободный унитарный массорный модель. Масса покоя поля появляется спонтанно как в лагранжиане, так и в уравнениях движения системы. Поле имеет заряд, который является скаляром, и энергию, которая является инвариантным имплетом. Но энергия может быть комплетизована зарядом. Квантованные величины заряда суть $Q = n$, а величины энергии суть $T = \kappa n$, где к является массой покоя кванта поля, а n равняется 0, 1, 2, . . . в случае Бозе—Эйнштейновской статистики, или 0, или 1 в случае Ферми—Дираковской статистики.