Sum Rules in Potential Scattering - II [1]. Addendum

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In the above-mentioned paper, it was shown that the zeros in the k of the reduced radial wave-function $\varphi_l(k, r)$ have the density r/π in each of the half-planes $\mathrm{Re}\, k > 0$ and $\mathrm{Re}\, k < 0$. The proof was based on three theorems of the theory of entire functions. Here, we give a simplified proof based on the following theorem of LEVINSON [2] and KOOSIS [3]:

Theorem. – Let $f(z)$ be an entire function of exponential type A in each of the half-planes $\mathrm{Im}\, z > 0$ and $\mathrm{Im}\, z < 0$. Let $f(z)$ be bounded on the real axis. If we denote by $n_+(R)$ and $n_-(R)$ the number of zeros of f of modulus less than R, counted according to their multiplicity, in $\mathrm{Re}\, z > 0$ and $\mathrm{Re}\, z < 0$ respectively, then

$$\lim_{R\to\infty} \frac{n_+(R)}{R} = \lim_{R\to\infty} \frac{n_-(R)}{R} = \frac{A}{\pi}\,.$$

In other words, the zeros have the density A/π in each half-plane.

Applying this theorem to the wave-function $\varphi_l(k, r)$, and remembering that

$$\varphi_l(k, r) \underset{|k|\to\infty}{=} \frac{\sin kr}{k} + o\left(\frac{\exp\left[|\mathrm{Im}\, k| r\right]}{k}\right),$$

we see at once that the total density of the zeros of φ in the k-plane, is

$$D = \lim_{R\to\infty} \frac{n_+(R) + n_-(R)}{R} = \frac{2r}{\pi}\,.$$

Therefore, to prove our sum rules (formulae (3.1)) we need only the above theorem and the theorem of Pfluger (theorem 2 of our paper).

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[2] N. LEVINSON: *Gap and Density Theorems*, Chap. III (New York, 1949).

[3] P. KOOSIS: *Bull. Soc. Math. France*, **86**, 27 (1958).

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