# A Strict Phenomenological Connection between Dips and Resonances in Pion-Nucleon Scattering.

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On p. 656 *delete* from « A reason for this fact ... » to « ... with constant residues » (12 lines) and *substitute* with

« A reason for this fact is not difficult to understand. The number of zeros which a pole can cross is fixed by its total and orbital angular momenta. Therefore the residue of the ρ pole must have no zero in $B$ and one zero in $A$; that of the nucleon must have no zero in $B$ ($\mathcal{N}$ is absent from $A$ because of its parity); that of the Δ must have one zero in $B$ and one zero in $A$. However, the zero of the Δ in $B$ occurs rather far, at $t \simeq +2.4$ (GeV)$^2$. Therefore when the $\mathcal{N}$, Δ and ρ poles cross the $u \simeq -0.15$ and $t \simeq -0.6$ zeros in the amplitude $B$, other poles are needed at the intersection points in order to remove the zeros from their residues.

The situation can also be looked in another way, realizing that the intersection point of two poles must be crossed by a zero. Indeed near the intersection point one can approximate the amplitude by

$$\frac{a}{s-m_a^2}+\frac{b}{u-m_b^2}=\frac{a(u-m_b^2)+b(s-m_a^2)}{(s-m_b^2)(u-m_a^2)},$$

*i.e.* near the intersection point the amplitude is represented by the product of two poles times the zero $[a(u-m_b^2)+b(s-m_a^2)]$. If $a=b$ the zero takes the form $[t-t_0]$, with $t_0=m_a^2+m_b^2-\sum_i m_i^2$, $\sum_i m_i^2$ being the sum of the squares of the external masses. »

On p. 657 (18th line) *delete* the phrase

« However it then remains to explain why the $M=1700$ bump, as it results from Fig. 2 has two zeros both in $A$ and in $B$, instead of having in $A$ one zero more than in $B$. » (4 lines).

On p. 658 (5th line) *delete* from « On the contrary ... » to « ... from such zero. » (7 lines) and *substitute* with

« The g-meson ($\rho_N(1650)$ in the notation of the tables of the Particle Data Group) fits more nicely. In $A$ (Fig. 2$a$) it crosses the zero at $s \simeq -1.5$ (indicated by phase-shifts), that at $u \simeq -0.15$, and the zero at $u \simeq -2.3$ required by the intersection of the ρ with the $\sqrt{s} = 1.930$ bump. In the amplitude $B$ the zero at $u \simeq -2.3$ must be present in the Δ residue (see above) and therefore must not be removed by the g; this together with the evidence from phase-shifts against a zero at $s \simeq -1.5$ in $B$ suggests a decoupling of the g-meson from the amplitude $B$. *The dip at* $u \simeq -2.3$, anyway, should be present both in $A$ and in $B$ and therefore it *should be clearly detected in backward* $\pi^+$p *scattering* ».

On p. 658 (27th line) *delete* from « We do not know ... » to « ... cannot be solved by $N$ » (5 lines).

In Fig. 1 and 2 *correct* for the ρ mass ($\sqrt{t} = 0.760$ instead of $\sqrt{t} = 0.700$) and for the position of the zero at $s \simeq +0.45$, the correct value being $s \simeq +0.30$. In Fig. 2*b* (corresponding to amplitude $B$) the zero at $s \simeq -1.5$ and the pole at $\sqrt{t} = 1.650$ should be cancelled.

*The conclusions contained in the original version of the paper remain essentially unaffected by the above modifications. They are moreover considerably enriched.* Indeed, as we have seen above (erratum to p. 656), at the intersection point of two poles the orientation of the direction of the zero line with respect to those of the two pole-lines determines and is vice versa determined by the ratio of the residues of the two poles at the intersection point. *In the case of* πp *scattering*, that we are here considering, *all the zeros are oriented in such a way that the residues of the two poles must be equal at the interaction point.* This equality must be exact for true poles, and should be expected to be approximate for quasi-poles like the resonances. In the amplitude $B$, for instance, one can compare $2G^2_{\pi\mathcal{N}} = +370$, the residue of the nucleon pole, with the residue of the Δ at the $\mathcal{N}\Delta$ intersection which is $\sim +300$ (derived from phase shifts). As to the ρ, its residue (constant in the amplitude $B$) has been evaluated through backward dispersion relations (C. LOVELACE: *Invited paper at the Irvine Conference* (1967)), finding a value $\sim +110 \times 2 = +220$, the factor 2 being a crude correction for the fact that in the fit used for this determination $m^2_\rho = 0.35$ (GeV)$^2$ is taken instead of the experimental value $\sim 0.6$ (GeV)$^2$. This number for the ρ-residue should be interpreted of course only as an order-of-magnitude estimation, an error by a factor 2 being perfectly allowed. The Δ residue at the Δρ intersection is $\sim +170$.

The comparison of the above numbers gives an idea of the quantitative imprecision introduced in the above-mentioned equalities by the finite widths of the resonances. In definitive it is not so bad, and we can say that analiticity opens to us a completely new and promising way of correlating coupling constants in strong-interaction physics.

A more extended and detailed version of the paper is in preparation.