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Correction to "Some remarks on weakly compactly generated Banach spaces", by W. B. Johnson and J. Lindenstrauss, Israel Journal of Mathematics, Vol. 17, No. 2, 1974, pp. 219–230.

David Yost has pointed out that the norm constructed for the space U^* in part (e) of example 1 in our paper is not locally uniformly convex. A dual norm on $U^* = l_1 \oplus l_2(\Gamma)$ which has this property is given by

$$|||(y,z)||| = \| y \|_{l_1} + (\| y \|_{l_1}^2 + \| y \|_{l_2}^2 + \| z \|_{l_2(\Gamma)}^2)^{1/2}.$$

To see that $|||\cdot|||$ is the dual to a norm on U, let (y_α, z_α) be a net in the $|||\cdot|||$-unit ball which weak* converges to (y,z); we need to check that $|||(y,z)||| \leqq 1$. The net (y_α) in l_1 must converge coordinatewise to the vector y, so we can write $y_\alpha = y_\alpha^1 + y_\alpha^2$, where the supports of y_α^1 and y_α^2 are disjoint for each fixed α, and $\| y - y_\alpha^1 \|_{l_1} \to 0$. By passing to a subnet, we can assume that $(y_\alpha^2, 0)$ weak* converges in U^*, necessarily to an element of the form $(0, z_1)$, and, *a fortiori*, $(0, z_\alpha)$ weak* converges to $(0, z_2) \equiv (0, z - z_1)$. This last statement just means that (z_α) converges weakly in $l_2(\Gamma)$ to z_2.

We thus have the following inequalities:

$$\| y_\alpha \|_{l_1} = \| y_\alpha^1 \|_{l_1} + \| y_\alpha^2 \|_{l_1},$$

$$\| y_\alpha \|_{l_2}^2 = \| y_\alpha^1 \|_{l_2}^2 + \| y_\alpha^2 \|_{l_2}^2,$$

$$\lim_\alpha \| y_\alpha^1 \|_{l_1} = \| y \|_{l_1},$$

$$\lim_\alpha \| y_\alpha^1 \|_{l_2} = \| y \|_{l_2},$$

$$\| z_2 \|_{l_2(\Gamma)} \leqq \liminf_\alpha \| z_\alpha \|_{l_2(\Gamma)}.$$

Moreover — and this is the main point — one has from the form of the duality between U and U^* that

$$\| z_1 \|_{l_2(\Gamma)} \leqq \liminf_\alpha \| y_\alpha^2 \|_{l_1}.$$

Therefore,

$$\begin{aligned}
|||(y,z)||| &= \|y\|_{l_1} + (\|y\|_{l_1}^2 + \|y\|_{l_2}^2 + \|z_1 + z_2\|_{l_2(\Gamma)}^2)^{1/2} \\
&\leqq \|y\|_{l_1} + \|z_1\|_{l_2(\Gamma)} + (\|y\|_{l_1}^2 + \|y\|_{l_2}^2 + \|z_2\|_{l_2(\Gamma)}^2)^{1/2} \\
&\leqq \liminf_{\alpha} \{\|y_\alpha^1\|_{l_1} + \|y_\alpha^2\|_{l_1} + (\|y_\alpha^1\|_{l_1}^2 + \|y_\alpha^1\|_{l_2}^2 + \|z_\alpha\|_{l_2(\Gamma)}^2)^{1/2}\} \\
&\leqq \liminf_{\alpha} |||(y_\alpha, z_\alpha)||| = 1.
\end{aligned}$$

Now we check that $|||\cdot|||$ is locally uniformly convex and hence is dual to a Frechét differentiable norm on U. Suppose that $|||(y_\alpha, z_\alpha)||| = 1$, $|||(y,z)||| = 1$, and $|||(y_\alpha + y, z_\alpha + z)||| \to 2$. It follows that

$$(\|y_\alpha + y\|_{l_1}^2 + \|y_\alpha + y\|_{l_2}^2 + \|z_\alpha + z\|_{l_2(\Gamma)}^2)^{1/2} \sim$$

$$(\|y_\alpha\|_{l_1}^2 + \|y_\alpha\|_{l_2}^2 + \|z_\alpha\|_{l_2(\Gamma)}^2)^{1/2} + (\|y\|_{l_1}^2 + \|y\|_{l_2}^2 + \|z\|_{l_2(\Gamma)}^2)^{1/2}$$

(where "$s_\alpha \sim t_\alpha$" means "$s_\alpha - t_\alpha$"$\to 0$). That is, in the locally uniformly convex space $(\mathbf{R} \oplus l_2 \oplus l_2(\Gamma))_{l_2^3}$, we have

$$\|(\|y_\alpha + y\|_{l_1}, y_\alpha + y, z_\alpha + z)\| \sim \|(\|y_\alpha\|_{l_1}, y_\alpha, z_\alpha)\| + \|(\|y\|_{l_1}, y, z)\|$$

which implies that $\|y_\alpha\|_{l_1} \to \|y\|_{l_1}$, $\|y_\alpha - y\|_{l_2} \to 0$, and $\|z_\alpha - z\|_{l_2(\Gamma)} \to 0$. Hence also $\|y_\alpha - y\|_{l_1} \to 0$, whence $|||(y_\alpha - y, z_\alpha - z)||| \to 0$. Therefore $|||\cdot|||$ is locally uniformly convex.

In conclusion, it should be noted that a similar correction should be made in example 2, part (b) of our paper.

THE OHIO STATE UNIVERSITY
COLUMBUS, OHIO, USA

AND

THE HEBREW UNIVERSITY OF JERUSALEM
JERUSALEM, ISRAEL