tioned above, both of these quantities are sensitive to the cyclically stabilized mechanical properties within the reversed plastic zone.

J. J. P. Hickerson and R. W. Hertzberg: *Met. Trans.*, 1972, vol. 3, pp. 179-89.

Authors' Reply

C. BATHIAS AND R. M. PELLOUX

The authors agree with Dr. Hertzberg that the fatigue crack growth rates should be in some way related to the cyclic stress-strain behavior of the material within the reversed plastic zone and near the crack tip. However, at this time there is no sound theory which correlates n' , the cyclic work hardening coefficient with m , the slope of the $(dI/dn, \Delta K)$ curve. Furthermore, it will be necessary to measure n' for a cyclic deformation history defined by the plastic strain distribution within the reversed plastic zone. This value of n' could be quite different from the n' measured on a stabilized cyclic stress-strain curve. At this point we do not think that n' martensitic is greater than n' austenitic. We have measured some very high cyclic strain hardening coefficients in a 301 austenitic stainless steel in the $M_D - M_S$ temperature range (see *Scripta Metallurgica*, vol. 7, 1973, pp. 657-60) However, the slope, m , of these austenitic steels was always greater than the slope, m , of different martensitic steels. It appears from some of our other work that the slope, m , may be more dependent on the level of residual stresses on the flank of the crack and near the crack tip than on the cyclic work hardening coefficient n' .

Yield Locus of Random-Textured fcc Metals Calculated with Lower-Bound Assumptions

W. F. HOSFORD

An upper bound yield locus calculation was recently made for random-textured fcc metals deforming by $\{111\}\langle 110 \rangle$ slip.¹ The method of the calculation followed that of Bishop and Hill² and therefore utilized the assumption that the strains in each grain were the same.

This note concerns the corresponding lower-bound yield locus calculation for random-textured fcc metals deforming by $\{111\}\langle 110 \rangle$ slip. The $\sigma_3 = 0$ section of the locus is calculated assuming that the ratio of principal stresses $\alpha = \sigma_2/\sigma_1$ to be the same in each grain. The levels of σ_1/τ , (τ is the critical shear stress for slip), necessary to activate the most highly stressed slip system are computed for each orientation and averaged.

Each orientation is described by the three Euler an-

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Table I. Lower Bound Calculations for σ_1/τ

$\alpha = \sigma_2/\sigma_1$	σ_1/τ
0	2.2320 $\pm$.0017
0.125	2.3578 $\pm$.0008
0.25	2.4602 $\pm$.0011
0.375	2.5305 $\pm$.0007
0.500	2.5638 $\pm$.0011

Table II. Values of σ_1 Normalized by Tensile Yield Stress

$\alpha = \sigma_2/\sigma_1$	σ_1/Y Lower Bound	von Mises
0	1	1
0.125	1.056	1.0596
0.250	1.102	1.1094
0.375	1.134	1.1428
0.500	1.148	1.1547

gles θ , ϕ and ψ . The angles θ and ϕ are the longitude and latitude of the 1 axis with respect to a standard cubic orientation with $[100]$ at the center and $[001]$ at the pole. The angle ψ is the rotation of the 2 and 3 axes about the 1 axis. For each computer run values θ and ϕ were chosen to lie in each of 414 grid sections of equal spherical angle filling a spherical triangle bounded by $\phi = 0$, $\theta = 0$ and $\theta = 45$ deg. Within each grid section the values of θ and $\sin \phi$ were randomly chosen. For each θ - ϕ combination eight values of ψ were analyzed; the first being chosen randomly between 0 and $22\frac{1}{2}$ deg and the remaining seven equaling the sum of the first and $22\frac{1}{2}$, $45 \dots 157\frac{1}{2}$ deg. Thus a single computer run determined the average value of σ_1/τ for 3312 orientations. Six duplicate runs were made for each stress ratio, α .

The results of the computations are summarized in Table I. The values of σ_1/τ are the averages from the results of the six duplicate runs listed with 90 pct confidence interval calculated from the individual runs. The calculations are in excellent agreement with the values of 2.232 ± 0.002 for $\alpha = 0$ and 2.562 ± 0.006 for $\alpha = \frac{1}{2}$ reported by Cox and Sopwith,³ and 2.238 for $\alpha = 0$ reported by Sachs.⁴

In Table II the lower bound results are listed in normalized form, σ_1/Y , where Y is the yield stress in uniaxial tension. Also shown are the corresponding values predicted from the von Mises criterion

$$\sigma_1/Y = [1 - \alpha + \alpha^2]^{-1/2}$$

It is interesting that although the lower bound values of σ_1/Y are lower than those according to von Mises, the difference is less than 1 pct for each case. If the basic assumption that the stress state in each grain is the same were reasonable, the von Mises criterion would be a very close approximation for random-textured fcc metals. Taylor⁵ and Kocks,⁶ however, argue convincingly that the assumptions for upper bound are much better than those for lower bound calculations. Therefore, the close agreement of the lower bound and von Mises is probably fortuitous.

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