THE MOON'S PERMANENT MAGNETIC FIELD: A CRATERED-SHELL MODEL

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Abstract. As part of our study of the larger-scale remanent magnetic field of the Moon, we have examined the effects of cratering in an otherwise spherically symmetrical shell magnetized by a concentric dipolar magnetic field $\mathbf{H}_0$ to an intensity of magnetization $c\mathbf{H}_0$, where c is a constant. In our initial model, we assume that the material excavated from the craters is distributed with random orientation and thus does not contribute to the remanent dipole moment $\mathbf{M}_g$. We further assume that the mare fill does not contribute significantly to $\mathbf{M}_g$. We choose the magnetizing dipole moment $\mathbf{M}_0$ and the constant c such that the magnitude of the product $cH_0 \simeq 3 \times 10^{-4}\Gamma$ at the outer surface of the shell in the equatorial plane of the dipole. This value of the intensity of remanent magnetization was chosen to be within the range 10^{-7}–$10^{-3}\Gamma'$; the intensities of thermo-remanent magnetization exhibited by Apollo samples. Finally, we use the locations and diameters of the 10 largest craters on the Moon and the depth-to-diameter ratios of Pike's formulation to model approximately the excavation of the magnetized shell.

The remanent dipole moment $\mathbf{M}_g$ was calculated for each of three orthogonal orientations of the magnetizing dipole $\mathbf{M}_0$. The three magnitudes of $\mathbf{M}_g$ fall in the range 4×10^{18}–$1 \times 10^{19}\Gamma$ cm^3 which is close to the upper limit of $10^{19}\Gamma$ cm^3 estimated for M_g from the field measurements obtained with the Apollo subsatellites. Further, the distribution of the craters is such as to produce a significant transverse component of $\mathbf{M}_g$ with acute angles between the spin axis and $\mathbf{M}_g$ in the range 51°–77°.

1. Introduction

Measurements of the magnetic field of the Moon obtained with the Apollo subsatellites indicate that the lunar magnetic dipole moment is probably less than $10^{19}\Gamma$ cm^3 (Russell *et al.*, 1975b). These measurements and measurements of the field at the surface of the Moon (Dyal *et al.*, 1974; Russell *et al.*, 1975a) also show that the lunar crust produces appreciable fields on smaller scales. Lunar samples collected during the Apollo program exhibit remanent magnetization in the range 10^{-7}–$10^{-3}\Gamma$ cm^{3}cm^{-3} (cf. reviewed by Fuller, 1974).

At a magnetization of $3 \times 10^{-4}\Gamma$ cm^{3}cm^{-3}, a uniformly magnetized shell with an outer radius of $1R_m$ and 10 km thick would have a dipole moment of about $1.5 \times 10^{20}\Gamma$ cm^3. However, Runcorn (1975) has shown that a weakly permeable spherical shell, magnetized by a concentric dipolar field will have essentially no external magnetic field after the magnetizing field vanishes and that the material in the shell may nevertheless exhibit appreciable remanent magnetization. Coleman and Russell (1975) have pointed out that the properties of the larger-scale lunar field may therefore depend, in a rather straight forward manner, upon the geometrical properties of the distribution of the larger craters. In this paper, we consider this possibility in further detail.

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2. The Cratered-Shell Model

Let us consider a spherical shell which we assume to have been magnetized to an intensity $\mathbf{m}_s = c\mathbf{H}_0$ by a magnetizing field $\mathbf{H}_0$. This assumption implies that the crustal material is only weakly permeable. Thus, $c\mathbf{H}_0$ is the remanent magnetization of the shell produced by a field $\mathbf{H}_0$, which has since decreased to zero, and the coefficient c may be described as the coefficient of thermoremanent magnetization. Then the dipole moment of the magnetized shell is

$$\mathbf{M}_s = \iiint_{V_s} \mathbf{m}_s \, dV = \iiint_{V_s} c\mathbf{H}_0 \, dV,$$

where V_s is the volume of the shell.

As mentioned previously, Runcorn (1975) has shown that $\mathbf{M}_s = 0$ when $\mathbf{H}_0$ is the field of a dipole concentric with the spherical shell. Thus, we consider here the effects of the deviation from spherical symmetry that is associated with the lunar craters.

To determine the remanent dipole moment, $\mathbf{M}_g$, of our model Moon, we first calculate the dipole moment of the material to be removed to form the craters, or

$$\mathbf{M}_k = \sum_l \mathbf{M}_l = \sum_l \iiint_{V_l} \mathbf{m}_s \, dV,$$

where V_l is the volume of the lth crater and the summation is taken over all the craters of interest. Then, since $\mathbf{M}_s = \mathbf{M}_g + \mathbf{M}_k = 0$, we have $\mathbf{M}_g = -\mathbf{M}_k$.

Table I is a list of the craters used in the calculation and their respective locations and dimensions. Figure 1 illustrates the geometry used to estimate $\mathbf{M}_l$ for the craters. The model craters are conical sections and it is assumed that the excavated material was redeposited randomly so that it contributes nothing to $\mathbf{M}_g$. Only those

TABLE I

Characteristics of the major basins and craters used in the model discussed in the text. Both the latitude λ_c and longitude ϕ_c are in degrees and the diameters *Do* and depths *do* are in kilometers.

l	Name	λ_c	ϕ_c	*Do*	*do*
1	Imbrium [a]	38	−19	1340	9.12
2	Unnamed [c]	−10	−170	1200	8.82
3	Orientale [a]	−21	−96	930	8.17
4	Australe [b]	−45	90	900	8.09
5	Nectaris [a]	−15	34	890	7.92
6	Nubium [b]	−19	−17	750	7.65
7	Serenitatis [b]	26	19	680	7.44
8	Humboldtianum [b]	58	82	640	7.30
9	Unnamed [a]	18	167	600	7.16
10	Hertzsprung [a]	1	−129	490	6.74

[a] Hartmann and Wood (1971)
[b] Stuart-Alexander and Howard (1970)
[c] Kaula et al. (1975)

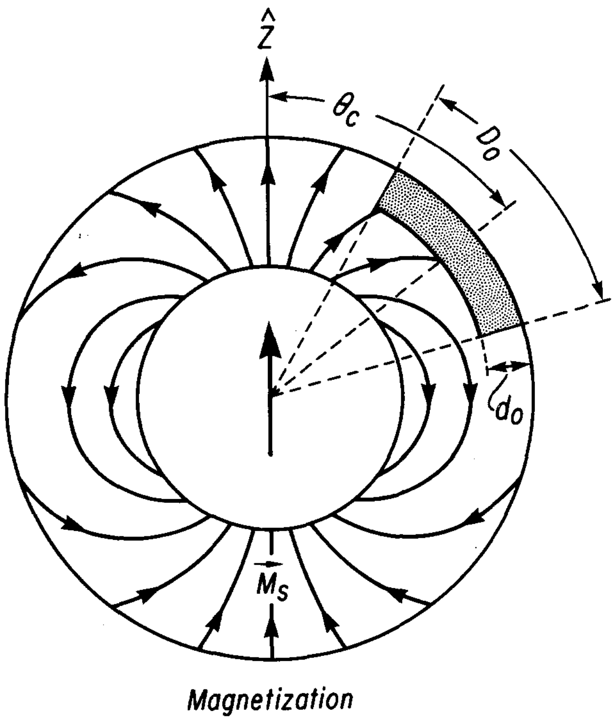

Fig. 1. Sketch showing the crater geometry used in the calculations. Each crater is a conical section bounded by two shells concentric with each other and centered at the apex of the cone. $\mathbf{M}_0$ is drawn along the lunar spin axis ($\hat{z}$) and $\theta_c = 90° - \lambda_c$.

TABLE II

Dipole moment associated with each of the basins or craters of Table I and the total dipole moment associated with the set for three orthogonal orientations of $\mathbf{M}_0$. Dipole moments are in $10^{18}\Gamma$ cm^3 and declination, θ_l, and azimuth, ϕ_l, are in degrees. The model is described in the text.

	$\hat{X}$			$\hat{Y}$			$\hat{Z}$		
l	θ_l	ϕ_l	$\lvert M_l \rvert$	θ_l	ϕ_l	$\lvert M_l \rvert$	θ_l	ϕ_l	$\lvert M_l \rvert$
1	32.5	−40.8	5.83	115.7	−125.5	3.91	84.6	−19.0	5.22
2	75.0	15.3	5.51	85.1	−61.4	2.94	150.6	10.0	2.94
3	84.1	164.4	1.63	58.2	80.4	3.05	121.5	84.0	1.89
4	90.0	−180.0	1.50	161.6	90.0	2.36	71.6	−90.0	2.36
5	111.3	54.6	2.19	107.8	−5.5	1.75	136.8	−146.0	1.40
6	118.4	−27.3	1.85	75.9	−134.2	1.10	126.4	163.0	1.14
7	51.1	32.6	1.42	69.9	−44.9	0.89	109.7	19.0	1.00
8	79.3	173.3	0.70	8.9	−56.3	0.94	49.4	82.0	1.23
9	117.0	−20.7	1.13	79.3	−124.6	0.67	129.0	167.0	0.68
10	91.3	82.7	0.56	91.4	29.0	0.63	177.0	−129.0	0.38
M_k	57.9	10.7	10.00	103.3	−68.4	4.40	128.8	−10.8	6.95

craters with diameters greater than 490 km were included and their depths were calculated using the formula for the depth-to-diameter ratio given by Pike (1974).

For this cratered shell model, we have calculated $\mathbf{M}_g$ for three mutually orthogonal orientations of $\mathbf{M}_0$: earthward ($\hat{x}$), solenographic eastward ($\hat{y}$), and northward ($\hat{z}$). We used $|\mathbf{M}_0| = 2 \times 10^{22}\Gamma$ cm^3 (which corresponds to $|\mathbf{H}_0| = 0.3$ G in the equatorial plane of $\mathbf{H}_0$ at the outer surface of the shell) and $c = 10^{-3}$. Thus, in this equatorial plane, $|\mathbf{m}_s| = |c\mathbf{H}_0| = 3 \times 10^{-4}\Gamma$ and is well within the range of remanent magnetizations exhibited by the Apollo samples. The assumption that this value for $\mathbf{m}_s$ corresponds to the values of c and $|\mathbf{H}_0|$, and thus to the value of $|\mathbf{M}_0|$ used here is based upon the studies of Collinson (1973), Gose *et al.* (1973), and Stephenson and Collinson (1974). However, any other pair of values with the same product could of course have been used.

The corresponding results for $\mathbf{M}_k$ (or $-\mathbf{M}_g$) are given in Table II, but we would emphasize that, as suggested above, these results are valid for *any* choices of $\mathbf{M}_0$ and c that give $\mathbf{m}_s = 3 \times 10^{-4}\Gamma$ in the equatorial plane at the outer surface of the shell.

3. Discussion

The values for $|\mathbf{M}_g|$ in Table II are comparable to the 10^{-19} G cm^3 estimated as an upper limit on $|\mathbf{M}_g|$ from the Apollo subsatellite measurements. Specifically for $\mathbf{M}_0 = M_0\hat{x}$, $|\mathbf{M}_g| = 1.0 \times 10^{19}$, for $\mathbf{M}_0 = M_0\hat{y}$, $|\mathbf{M}_g| = 4.4 \times 10^{18}$; and for $\mathbf{M}_0 = M_0\hat{z}$, $|\mathbf{M}_g| = 7.0 \times 10^{18}$. Thus, the results show that for a crust with a mean magnetization that is empirically reasonable, the craters can account for a dipole moment as great as that indicated by the estimated upper limit. They further suggest that, if the lunar crust is magnetized in the 'dipolar' pattern of our model to a depth of at least 9 km, the depth of the deepest crater in our set, then the mean magnetization of the crust cannot be much greater than the $3 \times 10^{-4}\Gamma$ cm^{3}cm^{-3} used in these calculations. On the other hand, if $\mathbf{M}_0$ varied as the crust cooled so that alternate layers with magnetizations of opposite polarities were produced in the upper 9 km, then the mean magnetization could be substantially greater than 3×10^{-4} G.

Another result of interest is that the orientation of $\mathbf{M}_g$ is significantly different from that of $\mathbf{M}_0$. Specifically, the angles between them are, for $\mathbf{M}_0 = M_0\hat{x}$, 81°; for $\mathbf{M}_0 = M_0\hat{y}$, 69°; and for $\mathbf{M}^0 = M_0\hat{z}$, 51°. It is perhaps especially noteworthy that for $\mathbf{M}_0$ along the present axis of rotation of the Moon, i.e., for $\mathbf{M}_0 = M_0\hat{z}$, the remanent dipole moment is inclined at 51° to the spin axis and lies very nearly in the xz plane, i.e., in the plane formed by the line along the Moon's spin axis and the Earth-Moon line. However, we have not attempted to determine the $\mathbf{M}_0$ that best fits the $\mathbf{M}_g$ estimated from the subsatellite data because of the uncertainties associated with the Apollo estimate of $\mathbf{M}_g$.

In closing, we would emphasize that these calculations do not include the effects of the near side-far side asymmetry of the thickness of the lunar crust, the filling of the basins and the uplift, folding, or any other nonrandom distribution and orientation of any magnetized ejecta. Nevertheless, we believe that the results reported here indicate the potential of this approach to the determination of $\mathbf{M}_0$ given an appropriately accurate measurement of $\mathbf{M}_g$.

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