VISCOSITY OF THE MOON*†

II: During Mare Formation

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Abstract. The Apollo 15 mission provided reliable data on the depths of maria Serenitatis and Smythii. Using the present depth values and the excess masses of the associated mascons of these maria together with the ages of their final fillings the average viscosity of the upper part of the lunar interior is determined for the mare formation period (from 3.8 to 3.3 b.y. ago) and the period after mare formation (since 3.3 b.y. ago). It is found that the lower limit of the average viscosity within the first period is about 10^{25} poise and within the second period is about 8×10^{26} poise.

1. Introduction

The Apollo 15 gravity and laser altimetry data yielded information about the nature and development of maria Serenitatis, Crisium and Smythii. In the present paper we focus our attention on the northeast section of the front side of the Moon where these data came from (Sjogren *et al.*, 1972; Wollenhaupt and Sjogren, 1972). Moreover, crater counting has yielded the relative ages of the formation of the maria in this region (Baldwin, 1970, 1971; Hartmann and Wood, 1971; Neukum *et al.*, 1972). Simultaneous study of these data together with the ages of the fillings of the maria provides an estimate of the viscosity of the Moon after the mare formation period as well as during the mare formation. The first part has already been presented in Part I of this paper (Arkani-Hamed, 1972a). In the present part, Part II, a lower limit is determined for the viscosity of the Moon during the mare formation period. Mare Crisium is disregarded because of the complexity of its filling.

2. Description of Data

This section is devoted to a brief description of the data which we used in this paper

A. TOPOGRAPHY

The topographic profile of the front side of the Moon (Figure 1) is deduced from Apollo 15 laser altimetry by Wollenhaupt and Sjogren (1972).†† It is clear from the figure that most of the front side of the Moon including the highlands are below the mean

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†† This is the profile beneath revolutions 15 and 16 of Apollo 15, the path of which is specified by Wollenhaupt and Sjogren (1972).

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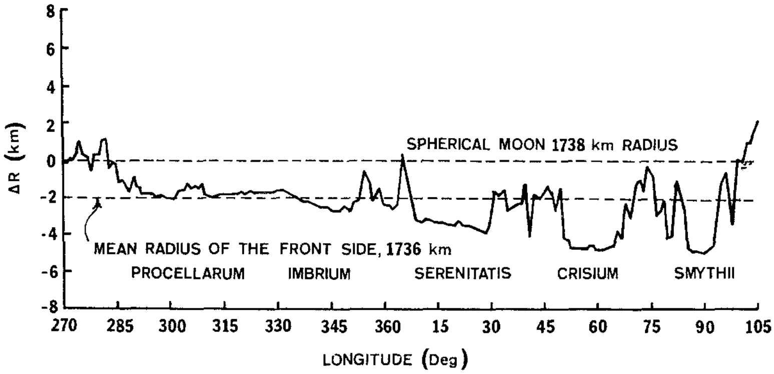

Fig. 1. The Apollo 15 laser altimetry profile of the front side of the Moon, presented by Wollenhaupt and Sjogren (1972). ΔR presents deviations from a mean radius of the Moon, 1738 km. The mean radius of the front side is also shown.

radius of the Moon, 1738 km. However, using their tabulated data the mean radius of the front side is found to be about 1736 km. The departure of the mean radius of the front side of the Moon from that of the total Moon, about 2 km, is also observed in Apollo 16 laser altimetry data (Wollenhaupt, 1972). This front side depression is a lunar-wide phenomenon and is probably not related to the mascons which are local anomalies. Therefore, we will assume that before the formation of the mare basins the surface of the front side of the Moon was spherical with a radius of 1736 km. Table I gives the average values of the diameters and depths of maria Serenitatis and Smythii deduced from Figure 1. (The value of the diameter of Mare Smythii is 1.5 times greater than that shown in the figure, because revolution 16 was not over the center.)

TABLE I

Parameters of Maria Serenitatis and Smythii.

	Diameter (km)	Depth (km)	Surface mass density (kg cm^{-2})	Age of the final filling, b.y.
Serenitatis	650	1.4	450	3.3
Smythii	300	2.8	420	3.8

B. GRAVITY

The gravity profile deduced from Apollo 15 gravity analysis conducted by Sjogren *et al.* (1972) indicates that Mare Serenitatis has a positive gravity anomaly of about 230 mgals. Sjogren *et al.* showed that this anomaly could be accounted for by a disk with a surface mass density of 800 kg/cm^2 and a diameter of 490 km. This implies an apparent surface mass density of about 450 km/cm^2 for the mare. In the case of Mare Smythii the apparent surface mass density is determined from the total excess mass of the mare which is taken to be about 3×10^{20} g. Wong *et al.* (1971) estimated the excess mass at Mare Smythii site to be 2.3 to 3.0×10^{20} g. The effect of these different values will be discussed later in this paper. Table I also includes the surface mass densities thus determined.

C. AGE

In considering the ages of the maria one should distinguish between the times of the formation of the basins and those of the floodings of the mare materials. In the case of maria Serenitatis, Crisium and Smythii the basin formations were by a single event while the filling processes have taken place by prolonged continuous or successive discrete events. Using a linear relationship; Age $= aR + b$, between the age of a mare filling and the crater density ratio, R, together with the known radiometric age determinations of samples from Mare Tranquillitatis, 3.65 b.y. (Lunatic Asylum, 1970) and Mare Procellarum, 3.3 b.y. (Papanastassiou and Wasserburg, 1970), Baldwin (1970, 1971) determined the age of the fillings of maria Serenitatis and Crisium to be about 3.26 and 3.39 b.y. respectively. Neukum *et al.* (1972) adopted an exponential law for Baldwin's data and concluded that the ages were about $3.3 \pm .07$ and $3.41 \pm .07$ b.y. respectively, which are close to the values obtained by Baldwin. Using Hartmann and Wood's (1971) data and adopting Baldwin's linear relationship we find the ages of the fillings of Maria Serenitatis and Smythii to be 3.22 and 3.78 b.y. respectively, which agrees with Baldwin's results. In the case of Mare Crisium, however, the data from entire mare yields an age of about 3.39 b.y. while those from the central part of the mare yields an age of about 3.14 b.y. This indicates that Hartmann and Wood's average data agrees with Baldwin's results. We disregard Mare Crisium in the present study because of its complicated filling process. Table I shows the ages of the fillings which are adopted in this paper.

3. Viscosity of the Moon During Mare Formation

In Part I we estimated the lower limit of the viscosity of the upper 800 km of the lunar interior averaged over the last 3 b.y. to be about 8×10^{26} poise. There we used Michael *et al.* 's (1969) spherical harmonic presentation of the lunar gravitational potential together with Wollenhaupt and Sjogren's (1972) laser altimetry data. Moreover, we assumed that the time of the completion of the fillings of the maria was 3 b.y. ago, and we neglected the surface topography of the Moon at that time. As discussed in the previous section, however, the completion times of the fillings of different maria are different. For example, the fillings at maria Smythii and Serenitatis were completed about 3.8 and 3.3 b.y. ago, respectively. Therefore, the relaxation time obtained in Part I is adequate for Mare Serenitatis.

The different filling times together with different depths of maria can be used to estimate the average viscosity of the lunar interior during filling as well as since filling. In the present section we pursue a comparative study of maria Smythii and Serenitatis which have a very old and a very young filling, respectively. The following assumptions and constraints are made throughout this section:

(1) The lunar model adopted is the same as that used in Part I (a uniform sphere

overlaid by a uniform surface layer with 50 km thickness). The density contrast at the bottom of the surface layer is 0.5 g/cc. The mascons are regarded as surface mass distributions.

(2) We consider only the northeast section of the front side of the Moon with emphasis on maria Smythii and Serenitatis.

(3) The last filling of maria Smythii and Serenitatis took place 3.8 and 3.3 b.y. ago respectively.

(4) Just after the completion of the fillings the topography at maria Smythii and Serenitatis were H'_0 and H_0 with respect to the mean radius of the front side of the Moon, 1736 km. This topography is due to a combination of impact processes, subsequent fillings of impact sites and instantaneous elastic deformation of the Moon under the influence of these maria. In Part I we assumed that H_0 is zero and obtained about 8×10^{26} poise as a lower limit for the viscosity of the Moon averaged over the last 3 b.y. In the present section, however, we treat H_0 and H'_0 as parameters and determine their effects on the inferred viscosity of the Moon.

(5) After the completion of a mare filling there has been no mass transfer to or from the mare site, and the mare site has sunk under the influence of its load through the viscous deformation of the lunar interior.

(6) At a given time the upper 800 km of the lunar interior has a uniform viscosity. The effects of the maria we are concerned with do not penetrate deeper than 800 km (Arkani-Hamed, 1972b). The value of the viscosity, however, changes with time. For simplicity, we regard its time dependence as a step function; having a value of ν_1 from 3.8 to 3.3 b.y. ago and a value of ν_2 from 3.3 b.y. ago to the present. ν_1 and ν_2, in fact, represent the average viscosity of the Moon in the two periods.

(7) At the present time the apparent surface mass densities and the depths of maria Smythii and Serenitatis are the same as those obtained in the previous section (See Table I).

Assumption (6) is a somewhat rough assumption. The viscosity of the lunar interior probably has radial as well as lateral variations. For example, the floor of Mare Serenitatis has a distinct slope toward Mare Crisium (Figure 1). This slope may be related to a lower viscosity at Mare Crisium side with respect to that at Mare Imbrium side. It can not be attributed to the shape of the equipotential surface at the time of the last filling of Mare Serenitatis. Figure 2 displays to what extent the pre-existing non-uniform equipotential surface affects the slope of the floor of Mare Serenitatis. Based on the following assumptions, we illustrate the shape of the equipotential surface at the time of the last filling of Mare Serenitatis. It is assumed that the filling took place in the presence of the mascons associated with maria Serenitatis and Imbrium (the filling of Mare Imbrium had, probably, been completed before the last filling of Mare Serenitatis (Baldwin, 1970, 1971; Hartmann and Wood, 1971). The mascons are assumed to be spherical bodies located at 200 km depth with masses of 3×10^{21} g respectively. These are the present values of the excess masses, corrected for the decay taken place within the last 3.3 b.y. with a relaxation time of about 5 b.y. It is worth mentioning that the assumption of spheres produces a steeper slope for the equipoten-

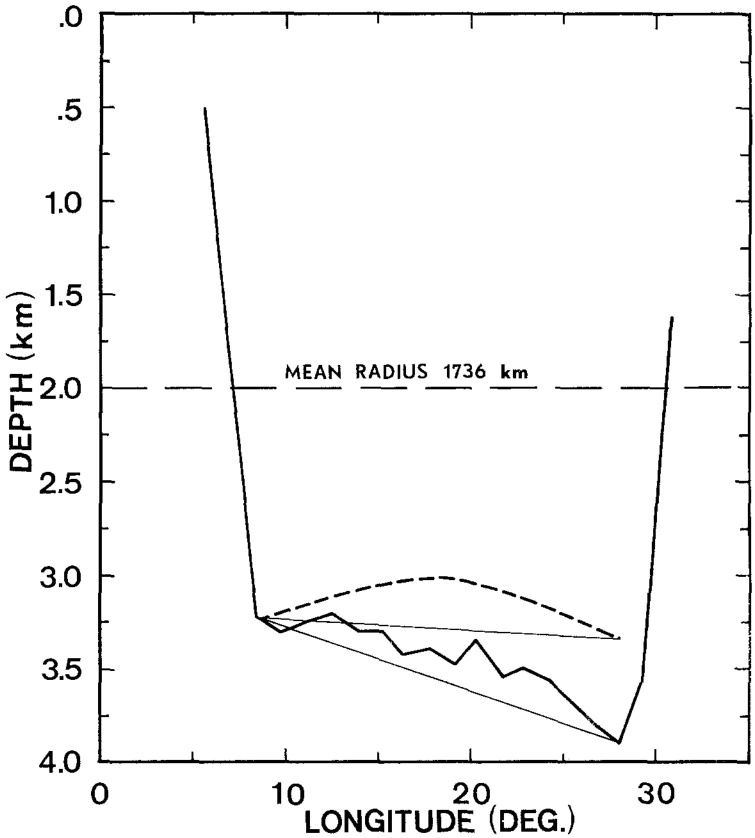

Fig. 2. The solid line denotes the floor of Mare Serenitatis deduced from the laser altimetry data, and the dashed line shows the equipotential surface at the time of the last filling of the mare.

tial surface at the Serenitatis site than the disk shaped mascons. We have ignored the effects of maria Crisium and Smythii on the equipotential surface, because the filling of the central part of Mare Crisium probably took place after the last filling of Mare Serenitatis, and more importantly, because the sizes of the mascons associated with Maria Crisium and Smythii are smaller, by factors of about 2 and 5, respectively, than that associated with Mare Imbrium (Wong *et al.*, 1971). Their distances from Mare Serenitatis are larger by factors of about 1.5 and 2 respectively, than that of Mare Imbrium. By considering these two maria, however, the slope of the equipotential surface at Mare Serenitatis site would be less steep than that shown in Figure 2. Figure 2 shows that the shape of the floor of Mare Serenitatis is similar to that of the equipotential surface but the former has tilted toward Mare Crisium more than the latter. On the other hand, it is implausible to relate the slope of the floor of Mare Serenitatis to the shape of the equipotential surface of the low degree harmonics of the lunar gravity which might have controlled the slope strongly at the time of the last filling of the mare, because such a distinct slope has not been observed on the floor of

Mare Crisium (Figure 1) and Mare Fecunditatis (Wollenhaupt, 1972). The foregoing discussion leads us to one possible explanation; the viscosity of the lunar interior below the eastern edge of Mare Serenitatis has been lower than the viscosity below the western edge. The lower altitude of highlands between maria Serenitatis and Crisium compared to other highlands (Figure 1) substantiates this suggestion. However, for the present order of magnitude calculations lateral variations of viscosity are disregarded. A detailed treatment of the subject will appear elsewhere.

In this section we determine the relaxation times of Mare Serenitatis assuming different initial topography at the mare site. Using these relaxation times, we then obtain the relaxation times of Mare Smythii for the last 3.3 b.y. and also for the period from 3.8 to 3.3 b.y. ago. These relaxation times are then used in order to estimate lower limits to the viscosity of the upper 800 km of the lunar interior within these periods.

Mare Serenitatis: It is assumed that the viscosity of the lunar interior has been constant since the last filling of Mare Serenitatis, 3.3 b.y. ago. This implies that the excess mass of the mare has decayed since with a constant relaxation time, τ. τ is determined by the following equation (a detailed formulation is presented in the appendix)

$$\tau = \left[\frac{\varrho_1}{\varrho_2 t_p} \ln\left(1 + \frac{\varrho_1 (H_0 - H)}{\sigma_p}\right)\right]^{-1}, \tag{1}$$

where ϱ_1 = density of the surface layer, 3 g/cc; ϱ_2 = density of the upper part of the lunar interior, 3.5 g/cc; t_p = present time, 3.3 b.y.; H = present topography at the mare site, -1.4 km; σ_p = apparent surface mass density associated with the mare, 450 kg/cm^2; and H_0 = initial topography at the mare site which depends on (a) – the initial depth of the corresponding basin, (b) – the total amount of filling, and (c) – the displacement of the floor of the basin from the time of the formation of the basin till the final filling. Therefore, H_0 can have a positive or negative value. It is, however, implausible to assume a large positive value for H_0 because at the time of the filling the mare basalt is fluid enough to spread on the Moon. In the present paper we consider only non-positive values of H_0. Figure 3 illustrates the values of τ versus H_0 obtained from Equation (1). A lower limit for the relaxation time coincides with the zero value for H_0, (the value adopted in Part I). Figure 3 also displays the average viscosity, ν_2, of the lunar interior within the last 3.3 b.y. where ν_2 is given by

$$\nu_2 = \frac{nga\tau}{2(n+1)^2 + 1}, \tag{2}$$

where n = the degree of the spherical harmonic having a wave length about twice the diameter of Mare Serenitatis, 9; g = gravitational acceleration at the surface of the Moon, 163 cm/s^2; and a = mean radius of the front side of the Moon, 1736 km. This equation is the same as Equation (5) of Part I but the elastic term is missing here, because it is negligible in comparison to the viscous term at the values of n considered

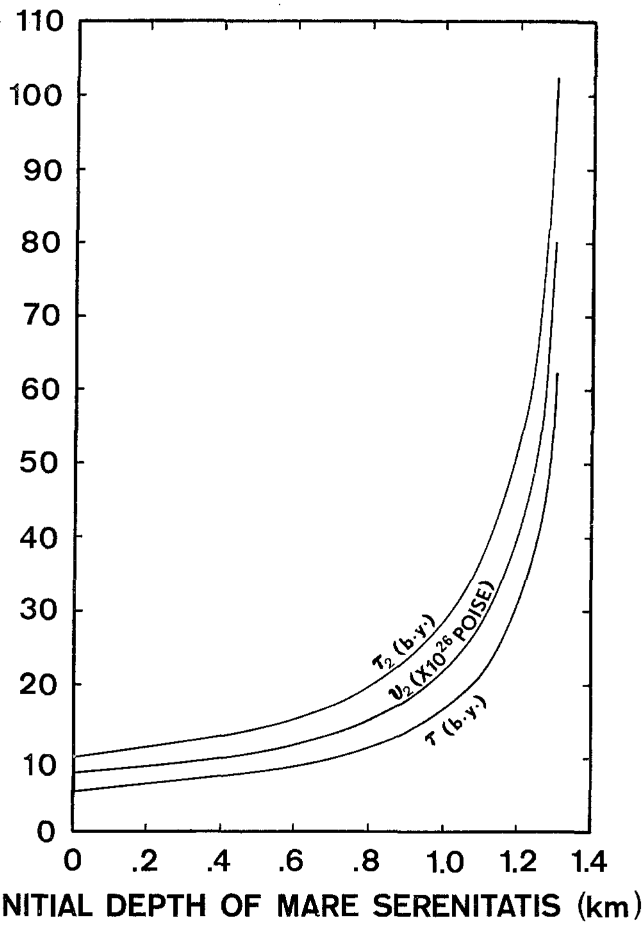

Fig. 3. Variations of the relaxation time of maria Serenitatis and Smythii, and the viscosity of the lunar interior as functions of the initial depth of Mare Serenitatis. τ and τ_2 are, respectively, the relaxation times of maria Serenitatis and Smythii for the last 3.3 b.y. and ν_2 is the average viscosity of the upper 800 km of the Moon during this period.

here. A lower value for viscosity is about 8×10^{26} poise, which is the one we determined in Part I. It grows exponentially as H_0 becomes closer to H.

Mare Smythii: It is assumed that the viscosity of the upper 800 km of the lunar interior changes from a value of ν_1 to a velue of ν_2 3.3 b.y. ago. This indicates that the excess mass of Mare Smythii has decayed with two different relaxation times, with a relaxation time of τ_1 during the period from 3.8 to 3.3 b.y. ago, and with a relaxation time of τ_2 since 3.3 b.y. ago. τ_2 is determined from the relaxation time of Mare Serenitatis through the equation

$$\frac{\tau_2}{\tau} = \frac{(2(n'+1)^2+1)\,n}{(2(n+1)^2+1)\,n'}, \tag{3}$$

which is derived from Equation (2). Here n' is the degree of the spherical harmonic having a wave length about twice the diameter of Mare Smythii, 18. τ_2 is also shown in Figure 3 for comparison with τ. τ_1, on the other hand, is expressed as (the detailed derivation of this equation is presented in the appendix)

$$\tau_1 = \left[\frac{\varrho_1}{\varrho_2 t_1} \ln\left(1 + \frac{\varrho_1 (H'_0 - H')}{\sigma'_p}\right) - \frac{t'_p - t_1}{t_1 \tau_2}\right]^{-1}, \tag{4}$$

where t_1 = the time when the viscosity of the Moon changed from ν_1 to ν_2, $3.8-3.3=$ $=0.5$ b.y.; H' = the topography at the mare site at the present time, -2.8 km; σ'_p = the apparent surface mass density associated with Mare Smythii at the present time 420 kg/cm^2; t'_p = the present time, 3.8 b.y.; and H'_0 = the initial topography at the mare site. As in the discussion of Mare Serenitatis we consider only non-positive values of H'_0 in the present study. Figure 4 shows the variations of τ_1 versus H'_0 for two different values of ν_2. For a given ν_2 the lower value of τ_1 corresponds to the zero value of H'_0 and for a given value of H'_0 lower value of τ_1 corresponds to the higher value of ν_2. Figure 4 also illustrates the viscosity of the lunar interior within the period from 3.8 to 3.3 b.y. ago, ν_1, as a function of H'_0 for the two values of ν_2. ν_1 is determined from an equation similar to Equation (2) for given values of τ_1. It is clear from

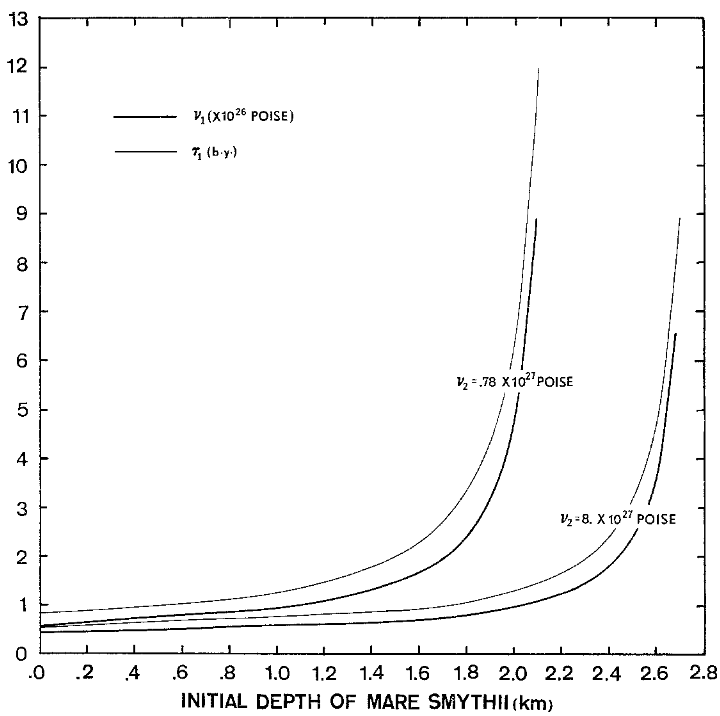

Fig. 4. Variations of the relaxation time of Mare Smythii and the viscosity of the first period as functions of the initial depth of the mare for given values of the viscosity of the Moon for the second period, ν_2. τ_1 is the relaxation time of the mare from 3.8 to 3.3 b.y. ago, and ν_1 is the average viscosity of the upper 800 km of the Moon during this period.

the figure that for a given value of H_0' a lower value of ν_1 corresponds to a higher value of ν_2 and vice versa. Moreover, a lower limit of the viscosity during the mare formation period (between 3.8 and 3.3 b.y. ago) is about 5×10^{25} poise.

4. Discussion

We concluded in the previous section that (1) – the lower limit to the average viscosity of the upper 800 km of the lunar interior within the last 3.3 b.y. (the second period), is about 8×10^{26} poise. The upper limit can, however, be as high as infinity (which would be the case if the Moon were an ideally elastic body); and (2) – the lower limit to the average viscosity of this region, between 3.8 and 3.3 b.y. ago (the first period), is about 5×10^{25} poise which is about one order of magnitude less than that of the second period. In this section we discuss briefly the effects of different parameters on the value of the viscosity of the first period.

(1) *Time of the last filling of Mare Smythii:* We estimated the age of the final filling of Mare Smythii from Hartmann and Wood's (1971) relative age of the mare and Baldwin's (1970, 1971) linear equation. This may introduce an error of a few percent. Assuming that the final filling of the mare took place 3.55 b.y. ago (about 0.25 b.y. younger than the one considered in this paper) lowers the viscosity of the first period by a factor of 2. This is probably an upper limit of the effect of the uncertainty of the age determination.

(2) *Initial topography at the Mare Smythii site:* Figure 4 shows that the lower value of the viscosity of the first period corresponds to a zero initial depth for Mare Smythii, but if the initial depth were as much as 1 km the value would not change appreciably.

(3) *Present excess mass at Mare Smythii site:* The present value of the excess mass at Mare Smythii site is estimated by Wong *et al.* (1971) to be between 2.3×10^{20} and 3.0×10^{20} g. The latter is adopted in the present study. It is clear from Equation (4) that adopting the former value would reduce the value of τ_1 and thus the inferred value of the viscosity of the first period by a factor of about 2.

(4) *Diameter of Mare Smythii:* We have taken the diameter of Mare Smythii to be 300 km. The actual diameter of the mare is between 200 and 400 km. Using these limiting values, Equations (3) and (4) show that the viscosity of the first period does not reduce by more than 20 percent, which is negligible in comparison to the effects of the other parameters.

Therefore the foregoing parameters may reduce the viscosity of the first period by about a factor of 5.

5. Conclusions

Using data available on the present topography and gravitational fields of maria Serenitatis and Smythii, together with the ages of their last fillings we conclude that a lower limit to the viscosity of the lunar interior within the last 3.3 b.y. is about 8×10^{26} poise which is the value determined in Part I. Furthermore a lower limit to the viscosity in the period from 3.8 to 3.3 b.y. ago is about 10^{25} poise.

It is worth pointing out that (1) – the above-mentioned values of the viscosity are average values for the corresponding periods, (2) – since the effects of maria Serenitatis and Smythii do not penetrate deeper than about 800 and 500 km respectively, these viscosity values are representative of the viscosity of the upper part of the Moon, and (3) – the results of this paper are applicable only to the northeast section of the front side of the Moon.

The viscosities obtained in this study are greater than the value required to maintain convection currents inside the Moon (Turcotte and Oxburgh, 1969). This implies that there has been no prolonged convection cells in the upper part of the lunar interior since 3.8 b.y. ago.

Acknowledgement

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Appendix

This appendix is devoted to the formulation of the relaxation time of a surface load which is placed on a spherical lunar model. The formulas are developed under a general condition and the results are reduced in order to present the condition considered in the present paper. The model we consider consists of a uniform viscous spherical body of radius R, viscosity ν, and density ϱ_2 overlaid by an elastic spherical shell with density ϱ_1 and outer radius a, the mean radius of the Moon, and with a thickness smaller than the dimension of the surface load. The viscosity is assumed to be time dependent.

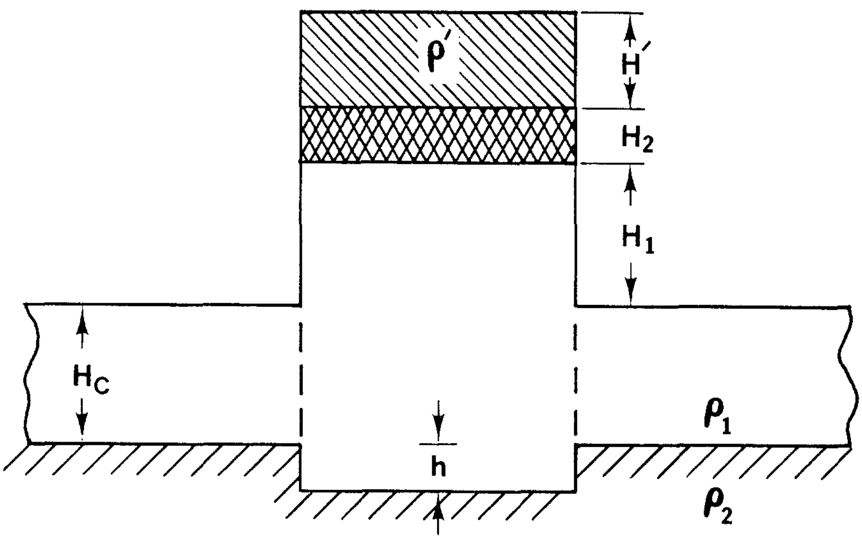

Fig. 5. The viscous deformation of the lunar model at a given time t.

Suppose at time $t=0$ an apparent surface mass density of σ_0 is introduced on the surface of this lunar model. σ_0 includes, in general, the actual surface mass density of the material placed on the surface, η, plus the surface mass density associated with the initial topography at the surface of the Moon. Suppose also that from time $t=0$ a flow of material with a rate of $\varepsilon(t)$ g/cm^2 added on to the surface of the Moon. Such a flow represents the mare filling process. The Moon undergoes a viscous deformation under the influence of these surface loads, such that at time t the apparent surface mass density is $\sigma(t)$ and also the bottom of the elastic layer has sunk into the inner viscous sphere by an amount of h. Figure 5 is a simplified illustration of the situation at this time. It is obvious from the figure that

$$\sigma(t) = H'\varrho' + \eta + H_1\varrho_1 - (\varrho_2 - \varrho_1)\, h\,, \tag{A-1}$$

where H' and H_1 are the thickness and the height (-depth) of the bottom of the filling with respect to the surface, respectively. Assuming that the densities are constant, the differentiation of Equation (A-1) yields

$$\frac{\mathrm{d}}{\mathrm{d}t}\sigma(t) = \varrho' \frac{\mathrm{d}}{\mathrm{d}t} H' + \varrho_1 \frac{\mathrm{d}}{\mathrm{d}t} H_1 - (\varrho_2 - \varrho_1) \frac{\mathrm{d}}{\mathrm{d}t} h\,, \tag{A-2}$$

but

$$\varrho' \frac{\mathrm{d}}{\mathrm{d}t} H' = \varepsilon(t)$$

and

$$\frac{\mathrm{d}}{\mathrm{d}t} h = -\frac{\mathrm{d}}{\mathrm{d}t} H\,.$$

This is because the delatation of the surface layer is negligible in comparison to the radial displacements at the surface and the bottom of the layer, which are due to the viscous deformation of the Moon. Therefore, Equation (A-2) can be written as

$$\frac{\mathrm{d}}{\mathrm{d}t}\sigma(t) = \varrho_2 \frac{\mathrm{d}}{\mathrm{d}t} H_1 + \varepsilon(t)\,. \tag{A-3}$$

On the other hand the conservation of mass in a viscous deformation process requires that

$$\frac{\mathrm{d}}{\mathrm{d}t}\sigma(t) = -\frac{1}{\tau}\sigma(t) + \varepsilon(t) + \xi(t)\,, \tag{A-4}$$

where τ is the relaxation time of the surface load at time t and $\xi(t)$ is the rate of the surface mass density created by the changes of the topography of the bottom of the surface layer, i.e.

$$\xi(t) = (\varrho_1 - \varrho_2)\frac{\mathrm{d}h}{\mathrm{d}t}\,. \tag{A-5}$$

Using Equations (A-3), (A-4) and (A-5) we conclude that

$$\varrho_1 \frac{\mathrm{d}}{\mathrm{d}t} H_1 = -\frac{1}{\tau}\sigma(t); \tag{A-6}$$

and finally

$$\frac{\mathrm{d}}{\mathrm{d}t}\sigma(t) = -\frac{1}{\tau}\left(\frac{\varrho_2}{\varrho_1}\right)\sigma(t) + \varepsilon(t). \tag{A-7}$$

The general solution of Equation (A-7) is

$$\sigma(t) = C\, e^{-(\varrho_2/\varrho_1)\int(\mathrm{d}t/\tau)} + e^{-(\varrho_2/\varrho_1)\int(\mathrm{d}t/\tau)} \times \int \varepsilon(t)\, e^{(\varrho_2/\varrho_1)\int(\mathrm{d}t/\tau)}\, \mathrm{d}t, \tag{A-8}$$

where C is a constant to be found from an initial condition and the integrals are indefinite integrals. Equation (A-8) can be used for the historical development of a mascon. In the present paper, however, we are concerned with the period after the filling. Therefore Equation (A-8) is reduced to the following form

$$\sigma(t) = C\, e^{-(\varrho_2/\varrho_1)\int(\mathrm{d}t/\tau)}. \tag{A-9}$$

Furthermore, a model we consider for the viscosity is a step function like

$$\nu(t) = \nu_1 \Theta(t_1 - t) + \nu_2 \Theta(t - t_1), \tag{A-10}$$

where Θ is the Heaviside function and t_1 is the time of the completion of the mare formation 3.3 b.y. ago. This means that in the case of Mare Serenitatis we have only the second term of Equation (A-10), i.e. the mare has decayed with a constant relaxation time, τ, since its final filling time. In the case of Mare Smythii, however, we include both terms which implies that Mare Smythii has decayed with two different relaxations times since its final filling with a relaxation time of τ_1 from 3.8 to 3.3 b.y. ago and with a relaxation time of τ_2 since 3.3 b.y. Therefore, the apparent surface mass density at Mare Serenitatis site is

$$\sigma(t) = \sigma_0\, e^{-(\varrho_2/\varrho_1)(t/\tau)} \tag{A-11}$$

and that at Mare Smythii site is

$$\sigma'_{\prime}(t) = \sigma'_0 \left[e^{-(\varrho_2/\varrho_1)(t/\tau_1)}\Theta(t_1 - t) + e^{(-(\varrho_2/\varrho_1)\{(1/\tau_1)-(1/\tau_2)\}t_1 - (t/\tau_2))}\Theta(t - t_1)\right], \tag{A-12}$$

where σ_0 and σ'_0 denote the initial values of the apparent surface mass densities, respectively. Using the present values of the apparent surface mass densities Equations (A-11) and (A-12) yield the following expressions for the relaxation times

$$\tau = \frac{\varrho_1}{\varrho_2 t_p}\left[\ln\left(1 + \frac{\varrho_1(H_0 - H)}{\sigma_p}\right)\right]^{-1}, \tag{A-13}$$

and

$$\tau = \frac{\varrho_1}{\varrho_2 t_p}\left[\ln\left(1 + \frac{\varrho_1(H'_0 - H')}{\sigma'_p}\right) - \frac{t'_p - t_1}{t_1 \tau_2}\right]^{-1}, \tag{A-14}$$

where H_0, H, t_p and σ_p are the initial topography, the present topography, the present time, and the present apparent surface mass density at Mare Serenitatis site, respectively. For Mare Smythii these values are given with primed symbols.

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