VISCOSITY OF THE MOON* †

I: *After Mare Formation*

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Abstract. Using data from the present gravitational potential and surface topography of the Moon, it is possible to determine a lower limit of about 5 b.y. for the relaxation time of the mascons. Assuming that the Moon has behaved as a Maxwellian viscoelastic body since the formation of the mascons, this relaxation time indicates a value of about 10^{27} poise for the viscosity of the lunar interior. Such a high viscosity implies that there has been no convection current inside the upper 800 km of the Moon since the formation of the mascons.

1. Introduction

Different values have been suggested for the viscosity of the lunar interior. Urey (1968), Kopal (1969), and Baldwin (1970) using different techniques have derived values of about 10^{26} poise, for the average viscosity of the Moon, which led them to conclude that the lunar interior is mechanically strong. On the other hand, Runcorn (1967) has proposed convection in the Moon to explain the departure of the lunar surface from hydrostatic equilibrium. Such convection requires that the viscosity of the lunar interior be less than 10^{24} poise (Turcotte and Oxburgh, 1969). It seems therefore that a more careful calculation of the lunar viscosity is in order.

Recent dating implies that the mascons have existed for more than 3 billion years (Wetherill, 1971). This is a useful criterion which must be fulfilled by any model proposed for the viscosity of the Moon. In the present paper we find the relaxation time of the mascons using the present gravitational potential and the present surface topography of the Moon. Using this relaxation time we, then, determine the average viscosity of the lunar interior.

2. Relaxation Time of the Mascons

It has been pointed out that the lunar mascons create stress differences on the order of 70 bars inside the Moon (Arkani-Hamed, 1972). These stress differences appear to be larger than the threshold stress of the lunar interior (Baldwin, 1968a). In this case, they can not have been supported elastically by the Moon. Neither can they have been supported by any kind of lunar-wide convection currents, because such currents are characterized by the low-degree spherical harmonics which would be

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un ble to support any high-degree harmonic loads, such as the mascons. On the other hand, it is implausible to assume that the local convection currents can have existed continuously for the past 3 billion years (b.y.), and could therefore have supported the mascons. Another alternative is that the present stress differences are the remainder of the larger ones which have steadily decayed through viscous deformation of the lunar interior. Viscous flow of the lunar craters has already been suggested by Scott, 1967 and Baldwin, 1970. The following calculations are based on this alternative.

In this section we estimate the relaxation times of the above-mentioned stress differences from the present gravitational and topographical data of the Moon. This estimate determines the viscosity of the lunar interior. The following assumptions and constraints are made throughout this paper.

(1) The Moon was a spherical body with spherically symmetric physical properties before the formation of the mascons.

(2) The formation of the mascons, i.e. the formation of the mare basins and subsequent filling of the basins with mare basalts, was completed 3 b.y. ago. This is a lower value than the values deduced from the isotopic dating (Wetherill, 1971).

(3) The formation of the mascons introduced density anomalies within the lunar crust which is assumed to be 50 km thick. The results of the calculations are, however, nearly identical if the mascons are regarded as surface mass distributions. This is because the thickness of the crust is very small in comparison to the radius of the Moon and to the dimensions of the mascons.

During the mascon formation, a lateral mass transfer from beneath the highland sites to beneath the mare sites appears to have taken place, because the highlands, such as the areas north of Mare Serenitatis, north of Mare Nectaris, and northwest of Mare Imbrium, are presently associated with negative gravitational potentials despite their elevation. In other words, the mare sites associated with positive gravitational potentials appear to be surrounded by the rings of negative potentials. Bearing in mind that viscous flow tends to diminish lateral variations of the gravitational potential, this implies that after the formation of the mascons there were larger mass deficiencies beneath the highlands than those inferred from the present gravitational potential.

(4) Just after the formation of the mascons the surface of the Moon acquired a topography, produced by (a) the impacting planetesimals, which scattered material from the impact sites to the neighboring regions, and the subsequent filling of the impacted sites by mare basalts. After the completion of this process, the impact sites were below the surrounding highlands (Baldwin, 1968b), and (b) the instantaneous elastic deformation of the Moon under the influence of the mascons. This process also yields depressions at the mascon sites and elevations at the surrounding areas. It is difficult to determine the initial topography of the lunar surface which is produced by the first processes, because it depends on knowledge about the nature of impacts and on the response of the Moon to such impacts. We disregard this part of the topography and assume an initial topography which is produced by the

second process only. Neglecting the first part, however, strengthens our arguments.

(5) The crustal density anomalies have not changed since their formation. In other words, after the formation of the mascons, there has been no mass transfer into or out of the lunar crust.

(6) The viscous flow has taken place in the lunar interior since the formation of the mascons. Therefore, there has been mass transfer below the crust and subsequent subsidence of the lunar crust.

(7) At the present time, after 3 b.y. from the formation of the mascons, the Moon's surface topography is the same as that deduced from the laser altimetry data (Wollenhaupt and Sjogren, 1972; and Wollenhaupt *et al.*, 1972), and the gravitational potential at the surface of the Moon is equal to Michael *et al.* 's (1969) spherical harmonic presentation of the lunar gravitational potential. Table I shows the fully normalized spherical harmonic coefficients of this potential and Figure 1 illustrates its lateral variations. The potential is based on the data obtained from the front side of the Moon and it may not represent the actual gravitational potential of the back side. However, it has been shown that the short-wave variations of the back side potential have negligible effect on the inferred stress differences on the front side (Arkani-Hamed, 1972). Therefore, the conclusions of this paper are valid only for the front side of the Moon.

Therefore, just after the formation of the mascons, the Moon had a crust, 50 km thick and with laterally varying density. The instantaneous elastic deformation of the Moon, under the influence of the crustal density anomalies, created stress differences inside it as well as topography at its surface. Furthermore, since the formation of the mascons, the stress differences have been steadily decaying, through the viscous deformation of the Moon. The viscous flow has been removing materials from beneath the mare sites and placing them under surrounding highlands. The vacancies, thus produced beneath the mare sites, tend to be filled up by the subsidence of the overlying layers and thus the surface topography of the Moon has been growing.

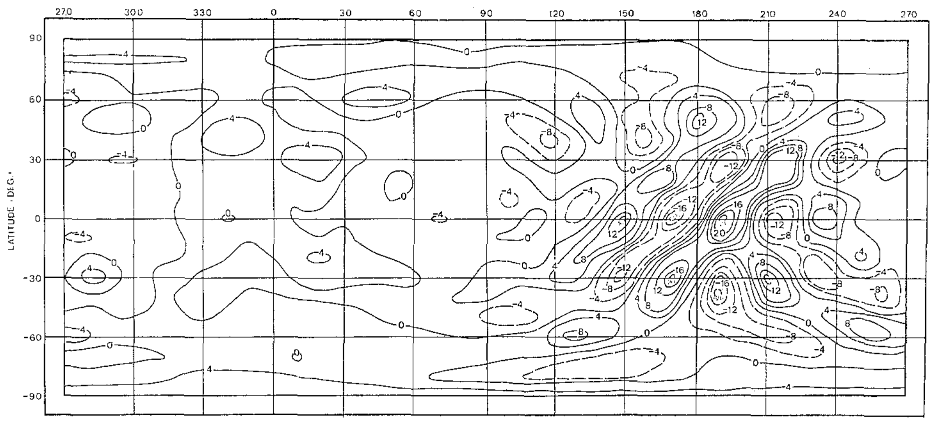

Fig. 1. The lateral variations of the lunar gravitational potential deduced from Michael *et al.*'s (1969) coefficients. Units are in 10^6 ergs.

TABLE I

Coefficients of the fully normalized spherical harmonic expansion of the lunar gravitational potential, in ergs (after Michael *et al.*, 1969). A fully normalized spherical harmonic, S_{nm}, is defined such that

$$\int_0^{\pi} \int_0^{2\pi} \sin\theta \, d\theta \, d\varphi (S_{nm})^2 = 4\pi.$$

n	m	Odd	Even	n	m	Odd	Even
2	0	0.00000000	0.00000000	9	9	−0.48013270 + 006	0.35278199 + 006
2	1	−0.10009670 + 006	−0.96857181 + 004	10	0	0.00000000	0.34741624 + 006
2	2	0.92764008 + 004	0.98148611 + 006	10	1	−0.20062371 + 006	0.45118612 + 005
3	0	0.00000000	−0.67319551 + 005	10	2	0.74234798 + 005	0.21419733 + 004
3	1	0.60198567 + 005	0.63856587 + 006	10	3	0.29564668 + 005	0.25541934 + 006
3	2	0.16802715 + 006	0.41497990 + 006	10	4	0.16337038 + 005	−0.60598177 + 005
3	3	−0.13776101 + 006	0.33578993 + 006	10	5	−0.86756168 + 005	−0.15139088 + 006
4	0	0.00000000	0.18254831 + 006	10	6	0.24691829 + 006	0.38871819 + 006
4	1	0.20260880 + 006	−0.17073708 + 006	10	7	−0.78289850 + 006	−0.54118027 + 006
4	2	−0.19019295 + 006	−0.12167294 + 006	10	8	−0.16183693 + 007	0.39201049 + 006
4	3	−0.26484240 + 006	0.15755309 + 006	10	9	−0.33365852 + 006	−0.29959311 + 005
4	4	0.19205312 + 006	−0.14671468 + 006	10	10	0.31666026 + 006	−0.25007727 + 006
5	0	0.00000000	−0.63551703 + 005	11	0	0.00000000	−0.14494863 + 006
5	1	−0.39961001 + 005	0.44910753 + 005	11	1	−0.55623345 + 006	−0.14446752 + 006
5	2	−0.39095370 + 005	0.20272320 + 006	11	2	−0.78593231 + 005	−0.26765362 + 006
5	3	0.27475929 + 006	0.31453609 + 005	11	3	−0.35514951 + 006	−0.10200517 + 006
5	4	−0.21974917 + 006	−0.15993965 + 006	11	4	−0.35549093 + 005	−0.30730469 + 006
5	5	−0.25810865 + 006	0.12222575 + 006	11	5	−0.29673993 + 006	−0.36080394 + 006
6	0	0.00000000	−0.84487024 + 004	11	6	−0.12330580 + 006	0.31188693 + 006
6	1	0.89393470 + 005	−0.12958282 + 005	11	7	−0.72881679 + 006	−0.20455866 + 006
6	2	−0.14862361 + 006	−0.20200364 + 006	11	8	0.51126044 + 006	0.60392565 + 006
6	3	−0.16818114 + 006	−0.41064002 + 006	11	9	0.16184552 + 007	−0.72053881 + 006
6	4	−0.68669422 + 005	−0.73006516 + 005	11	10	0.30108362 + 006	−0.24164119 + 006
6	5	−0.29194161 + 005	0.40615528 + 006	11	11	−0.15658600 + 006	0.47003736 + 005
6	6	0.47290943 + 006	−0.24585468 + 006	12	0	0.00000000	0.18644693 + 006
7	0	0.00000000	0.17569273 + 006	12	1	0.86899844 + 005	−0.37395384 + 006
7	1	0.31164306 + 006	0.30210691 + 006	12	2	0.18693896 + 006	0.13602318 + 006
7	2	0.11581004 + 006	−0.81594723 + 004	12	3	0.10666244 + 006	0.13404875 + 006
7	3	0.38617924 + 006	0.19162515 + 006	12	4	0.34584122 + 006	−0.34666053 + 005
7	4	0.33866656 + 006	0.46920955 + 006	12	5	0.27517686 + 006	0.17124239 + 006
7	5	0.33046937 + 006	0.27362129 + 006	12	6	0.44414173 + 006	0.50622862 + 006
7	6	−0.12323523 + 006	−0.42584754 + 006	12	7	−0.29404551 + 006	−0.19696342 + 006
7	7	−0.49172241 + 006	0.18310110 + 006	12	8	0.57116846 + 006	0.13232684 + 006
8	0	0.00000000	−0.18196296 + 006	12	9	−0.41433294 + 006	−0.82458436 + 006
8	1	−0.79323514 + 005	0.16099096 + 006	12	10	−0.10762287 + 007	0.47018687 + 006
8	2	0.35127276 + 005	−0.14573863 + 006	12	11	−0.15928749 + 006	0.48165350 + 005
8	3	−0.23744011 + 006	−0.35720831 + 006	12	12	−0.48885742 + 004	−0.11417166 + 005
8	4	−0.52458968 + 006	0.97361940 + 004	13	0	0.00000000	−0.31389715 + 006
8	5	−0.49116422 + 006	−0.53565939 + 006	13	1	−0.29871250 + 006	−0.62875921 + 005
8	6	−0.89948600 + 006	−0.33055352 + 006	13	2	−0.33485235 + 006	−0.18771498 + 006
8	7	0.19911799 + 006	0.33356280 + 006	13	3	−0.60637508 + 006	−0.28596675 + 005
8	8	0.66568473 + 006	−0.37684450 + 006	13	4	0.30519453 + 004	−0.81469484 + 005
9	0	0.00000000	−0.10003041 + 005	13	5	−0.27107167 + 006	−0.27827369 + 006
9	1	−0.23431493 + 006	0.13537906 + 006	13	6	−0.28980338 + 006	0.82520285 + 005
9	2	−0.31490152 + 005	−0.13866437 + 005	13	7	−0.53737390 + 006	−0.33284099 + 006
9	3	0.26479470 + 006	−0.38586293 + 005	13	8	0.13981008 + 006	−0.53118798 + 005
9	4	0.10236756 + 006	−0.25076750 + 005	13	9	−0.50579800 + 006	0.12323892 + 005
9	5	−0.18999786 + 005	−0.16289478 + 006	13	10	0.37155700 + 006	0.53966635 + 006
9	6	0.69637450 + 006	0.37087185 + 006	13	11	0.36817963 + 006	−0.43222019 + 006
9	7	0.15657047 + 007	−0.26717801 + 006	13	12	−0.12296214 + 006	−0.11928191 + 006
9	8	0.68151734 + 004	−0.24708570 + 006	13	13	−0.13174514 + 005	0.73780365 + 005

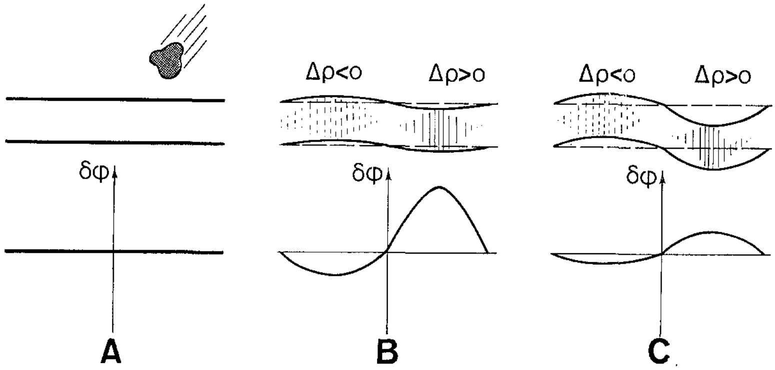

Fig. 2. The lunar crust at different times: (A) before the formation of the mascons, (B) just after the formations of the mascons, (C) a long time after the formation of the mascons.

Figure 2 illustrates these processes schematically. Consequently, the viscous flow has tended to reduce the lateral undulations of the gravitational potential as well as the associated stress differences. This implies that the relaxation time of the gravitational potential is almost the same as that of the mascons as well as that of the stress differences. In the present calculations we consider the relaxation time of the gravitational potential.

In the following calculations we suppose that the relaxation time of the lunar gravitational potential is equal to 3 b.y. The other two possibilities (longer than or shorter than 3 b.y.) will be discussed on the basis of the results obtained from these calculations. Let us assume that just after the formation of the mascons the lateral undulations of the lunar gravitational potential and surface topography, which are specified by a spherical harmonic of degree n, and order m are Φ_{nm}^0 and T_{nm}^0 respectively. Then, the corresponding gravitational potential at the present time, Φ_{nm}, is

$$\Phi_{nm} = \Phi_{nm}^0 / e \, . \tag{1}$$

The difference Ψ_{nm}; between Φ_{nm}^0 and Φ_{nm},

$$\Psi_{nm} = (1 - e)\, \Phi_{nm} \tag{2}$$

is associated with the surface topography, T_{nm}, and the topography at the bottom of the crust, T'_{nm}, which have been created by the subsidence of the lunar surface through the viscous deformation of the Moon. Gravitational potentials associated with the topographies of the interfaces of the deeper layers ,with different densities, are small in comparison to those associated with T_{nm} and T'_{nu}. They are thus neglected in this calculation. Ψ_{nm} can be expressed in terms of T_{nm} and T'_{nm} as

$$\Psi_{nm} = 4\pi Ga \left[\varrho_c T_{nm} + (R/a)^{n+2} \Delta\varrho T'_{nm}\right] / (2n + 1) \, , \tag{3}$$

where G = gravitational constant, a = radius of the Moon, ϱ_c = density of the crust, R = radial distance to the bottom of the crust ($a - 50$ km), and $\Delta\varrho$ = difference between the density of the crust and that of the upper part of the lunar interior. But, the thickness of the crust is much smaller than the wavelengths of the harmonics we are concerned with (the wavelengths of the harmonics are within 4450 km (for the 2nd degree harmonic) and 800 km (for the 13th degree harmonic)). For this reason, the crust behaves as a thin shell and T'_{nm} is almost the same as T_{nm}. Therefore, using Equations (2) and (3), T_{nm} can be expressed in terms of Φ_{nm} as

$$T_{nm} = (2n + 1)(1 - e)\ \Phi_{nm}/[4\pi Ga\{\varrho_c + (R/a)^{n+2}\ \Delta\varrho\}]. \qquad (4)$$

Tables II and III display the coefficients of the lunar surface topographies which are produced by instantaneous elastic deformation, T^0_{nm}, and viscous deformation, T_{nm}, of the Moon, respectively. T^0_{nm}'s are determined by solving the elastic equations throughout the Moon (Arkani-Hamed, 1972), while T_{nm}'s are computed from Equation (4), where we have used a value of 3.0 g/cc for the density of the lunar crust, and a value of 0.5 g/cc for $\Delta\varrho$. These values are reasonable from the viewpoint of the geochemical properties of the lunar rocks (Wood *et al.*, 1970).

Figures 3 and 4 display the lateral variations of the surface topography constructed by the harmonic coefficients given in Tables II and III respectively. It is clear from the figures that the elastic deformation is only about 2% of the viscous deformation. We therefore ignore the former in the following calculations. Figure 4 illustrates the surface topography of the Moon which we would expect to have at the present time if the relaxation time of the mascons were 3 b.y. Note that the mare sites have sunk

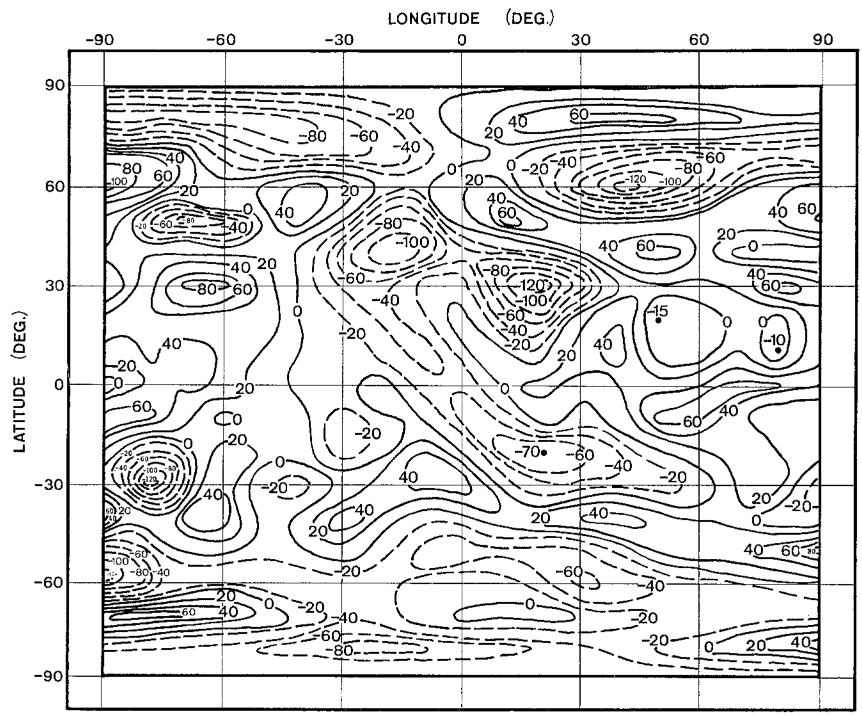

Fig. 3. The surface topography of the front side of the Moon produced by the elastic deformation of the Moon just after the formation of the mascons. Units are in meters.

TABLE II

Coefficients of the fully normalized spherical harmonic expansion of the surface topography of the Moon just after the formation of the mascons, $T_{nm}°$. Units are $\times$ 2.718 cm.

n	m	Odd	Even	n	m	Odd	Even
2	0	0.00000000	0.00000000	9	9	0.33940783 + 003	−0.24938308 + 003
2	1	0.46986976 + 002	0.45465295 + 001	10	0	0.00000000	−0.25083947 + 003
2	2	−0.43544894 + 001	−0.46072512 + 003	10	1	0.14485317 + 003	−0.32576280 + 002
3	0	0.00000000	0.36545275 + 002	10	2	−0.53598581 + 002	−0.15465352 + 001
3	1	−0.32679558 + 002	−0.34611075 + 003	10	3	−0.21346111 + 002	−0.18441640 + 003
3	2	−0.91215676 + 002	−0.22527712 + 003	10	4	−0.11795574 + 002	0.43752746 + 002
3	3	0.74785318 + 002	−0.18228784 + 003	10	5	0.62639188 + 002	0.10930637 + 003
4	0	0.00000000	−0.10768067 + 003	10	6	−0.17827852 + 003	−0.28066007 + 003
4	1	−0.11951450 + 003	0.10071407 + 003	10	7	0.56526386 + 003	0.39073985 + 003
4	2	0.11219067 + 003	0.71772209 + 002	10	8	0.11684857 + 004	−0.28303715 + 003
4	3	0.15622474 + 003	−0.92937126 + 002	10	9	0.24090620 + 003	0.21631049 + 002
4	4	−0.11328794 + 003	0.86543782 + 002	10	10	−0.22863321 + 003	0.18055935 + 003
5	0	0.00000000	0.39634328 + 002	11	0	0.00000000	0.10667974 + 003
5	1	0.24921872 + 002	−0.28003809 + 002	11	1	0.40937842 + 003	0.10632565 + 003
5	2	0.24382017 + 002	−0.12642931 + 003	11	2	0.57843290 + 002	0.19698854 + 003
5	3	−0.17135497 + 003	−0.19616196 + 002	11	3	0.26138404 + 003	0.75074078 + 002
5	4	0.13704764 + 003	0.99747142 + 002	11	4	0.26163532 + 002	0.22617106 + 003
5	5	0.16097071 + 003	−0.76226682 + 002	11	5	0.21839558 + 003	0.26554560 + 003
6	0	0.00000000	0.54913880 + 001	11	6	0.90750989 + 002	−0.22954351 + 003
6	1	−0.58102914 + 002	0.84224714 + 001	11	7	0.53639684 + 003	0.15055172 + 003
6	2	0.96600624 + 002	0.13129594 + 003	11	8	−0.37627904 + 003	−0.44447907 + 003
6	3	0.10931240 + 003	0.26690296 + 003	11	9	−0.11911557 + 004	0.53030439 + 003
6	4	0.44632941 + 002	0.47451915 + 002	11	10	−0.22159245 + 003	0.17784383 + 003
6	5	0.18975277 + 002	−0.26398802 + 003	11	11	0.11524465 + 003	−0.34593955 + 002
6	6	−0.30737609 + 003	0.15979773 + 003	12	0	0.00000000	−0.13966787 + 003
7	0	0.00000000	−0.11800934 + 003	12	1	−0.65096894 + 002	0.28012977 + 003
7	1	−0.20932451 + 003	−0.20291926 + 003	12	2	−0.14003645 + 003	−0.10189531 + 003
7	2	−0.77787321 + 002	0.54805569 + 001	12	3	−0.79901109 + 002	−0.10041626 + 003
7	3	−0.25938900 + 003	−0.12871084 + 003	12	4	−0.25907054 + 003	0.25968429 + 002
7	4	−0.22747566 + 003	−0.31515882 + 003	12	5	−0.20613569 + 003	−0.12827811 + 003
7	5	−0.22196977 + 003	−0.18378603 + 003	12	6	−0.33270770 + 003	−0.37921715 + 003
7	6	0.82774677 + 002	0.28603341 + 003	12	7	0.22027024 + 003	0.14754580 + 003
7	7	0.33028026 + 003	−0.12298540 + 003	12	8	−0.42786374 + 003	−0.99126374 + 002
8	0	0.00000000	0.12561697 + 003	12	9	0.31037786 + 003	0.61769824 + 003
8	1	0.5476483 + 002	−0.11113908 + 003	12	10	0.80620563 + 003	−0.35221818 + 003
8	2	−0.24249892 + 002	0.10060974 + 003	12	11	0.11932266 + 003	−0.36080786 + 002
8	3	0.16391527 + 003	0.24659649 + 003	12	12	0.36620434 + 001	0.85526281 + 001
8	4	0.36214715 + 003	−0.67213196 + 001	13	0	0.00000000	0.23904824 + 003
8	5	0.33907209 + 003	0.36978905 + 003	13	1	0.22748439 + 003	0.47883132 + 002
8	6	0.62095444 + 003	0.22819552 + 003	13	2	0.25500667 + 003	0.14295427 + 003
8	7	−0.13745984 + 003	−0.23027296 + 003	13	3	0.46178469 + 003	0.21777786 + 002
8	8	−0.45955122 + 003	0.26015221 + 003	13	4	−0.23242077 + 001	0.62043052 + 002
9	0	0.00000000	0.70711919 + 001	13	5	0.20643452 + 003	0.21191921 + 003
9	1	0.16563821 + 003	−0.95700030 + 002	13	6	0.22069965 + 003	−0.62843289 + 002
9	2	0.22260521 + 002	0.93022428 + 001	13	7	0.40923688 + 003	0.25347492 + 003
9	3	−0.18718449 + 003	0.27276813 + 002	13	8	−0.10647231 + 003	0.40452599 + 002
9	4	−0.72364060 + 002	0.17726860 + 002	13	9	0.38519026 + 003	−0.93852548 + 001
9	5	0.13431029 + 002	0.11515100 + 003	13	10	−0.28295908 + 003	−0.41098269 + 003
9	6	−0.49227007 + 003	−0.26217087 + 003	13	11	−0.28038705 + 003	0.32915711 + 003
9	7	−0.11068032 + 004	0.18886926 + 003	13	12	0.93641771 + 002	0.90839092 + 002
9	8	−0.48176748 + 001	0.17466592 + 003	13	13	0.10033045 + 002	−0.56187407 + 002

TABLE III

Coefficients of the fully normalized spherical harmonic expansion of the expected surface topography of the Moon at the present time for a relaxation time of 3 b.y. Units are in cm.

n	m	Odd	Even	n	m	Odd	Even
2	0	0.00000000	0.00000000	9	9	0.32057264 + 005	−0.23554374 + 002
2	1	0.17167028 + 004	0.16611436 + 003	10	0	0.00000000	−0.25717708 + 005
2	2	−0.15909439 + 003	−0.16832922 + 005	10	1	0.14851297 + 005	−0 33399339 + 004
3	0	0.00000000	0.16224215 + 004	10	2	−0.54952780 + 004	−0.15856094 + 003
3	1	−0.14508036 + 004	−0.15365530 + 005	10	3	−0.21885433 + 004	−0.18907579 + 005
3	2	−0.40495050 + 004	−0.10001141 + 005	10	4	−0.12093596 + 004	0.44858185 + 004
3	3	0.33200819 + 004	−0.80926387 + 004	10	5	0.64221804 + 004	0.11206806 + 005
4	0	0.00000000	−0.56770227 + 004	10	6	−0.18278283 + 005	−0.28775111 + 005
4	1	−0.63009130 + 004	0.53097373 + 004	10	7	0.57954558 + 005	0.40061213 + 005
4	2	0.59147938 + 004	0.37838960 + 004	10	8	0.11980081 + 006	−0.29018825 + 005
4	3	0.82363103 + 004	−0.48997297 + 004	10	9	0.24699283 + 005	0.22177570 + 004
4	4	−0.59726430 + 004	0.45626667 + 004	10	10	−0.23440977 + 005	0.18512129 + 005
5	0	0.00000000	0.24241574 + 004	11	0	0.00000000	0.11787486 + 005
5	1	0.15242983 + 004	−0.17131049 + 004	11	1	0.45233913 + 005	0.11748361 + 005
5	2	0.14912791 + 004	−0.77328051 + 004	11	2	0.63913441 + 004	0.21766078 + 005
5	3	−0.10480597 + 005	−0.11997868 + 004	11	3	0.28881403 + 005	0.82952451 + 004
5	4	0.83822549 + 004	0.61008419 + 004	11	4	0.28909167 + 004	0.24990575 + 005
5	5	0.98454637 + 004	−0.46622582 + 004	11	5	0.24131430 + 005	0.29341231 + 005
6	0	0.00000000	0.38218484 + 003	11	6	0.10027452 + 005	−0.25363211 + 005
6	1	−0.40437960 + 004	0.58617983 + 003	11	7	0.59268704 + 005	0.16635082 + 005
6	2	0.67231261 + 004	0.91378206 + 004	11	8	−0.41576626 + 005	−0.49112329 + 005
6	3	0.76078291 + 004	0.18575680 + 005	11	9	−0 13161571 + 006	0.58595523 + 005
6	4	0.31063246 + 004	0.33025170 + 004	11	10	−0.24484666 + 005	0.19650700 + 005
6	5	0.13206248 + 004	−0.18372808 + 005	11	11	0.12733857 + 005	−0.38224291 + 004
6	6	−0.21392494 + 005	0.11121463 + 005	12	0	0.00000000	−0.16529387 + 005
7	0	0.00000000	−0.92012336 + 004	12	1	−0.77040749 + 004	0.33152745 + 005
7	1	−0.16321112 + 005	−0.15821692 + 005	12	2	−0.16573007 + 005	−0.12059087 + 005
7	2	−0.60651070 + 004	0.42732111 + 003	12	3	−0.94561214 + 004	−0.11884044 + 005
7	3	−0.20224659 + 005	−0.10035633 + 005	12	4	−0.30660431 + 005	0.30733067 + 004
7	4	−0.17736364 + 005	−0.24573053 + 005	12	5	−0.24395707 + 005	−0.15181434 + 005
7	5	−0.17307067 + 005	−0.14329867 + 005	12	6	−0.39375228 + 005	−0.44879519 + 005
7	6	0.64539731 + 004	0.22302134 + 005	12	7	0.26068501 + 005	0.17461722 + 005
7	7	0.25752078 + 005	−0.95892186 + 004	12	8	−0.50636738 + 005	−0.11731390 + 005
8	0	0.00000000	0.10835688 + 005	12	9	0.36732540 + 005	0.73103235 + 005
8	1	0.47236251 + 004	−0.95868286 + 004	12	10	0.95412672 + 005	−0.41684251 + 005
8	2	−0.20917894 + 004	0.86785700 + 004	12	11	0.14121576 + 005	−0.42700820 + 004
8	3	0.14139289 + 005	0.21271349 + 005	12	12	0.43339482 + 003	0.10121848 + 004
8	4	0.31238720 + 005	−0.57977929 + 003	13	0	0.00000000	0.30141272 + 005
8	5	0.29248271 + 005	0.31897908 + 005	13	1	0.28683201 + 005	0.60375199 + 004
8	6	0.53563370 + 005	0.19684086 + 005	13	2	0.32153449 + 005	0.18024912 + 005
8	7	−0.11857250 + 005	−0.19863286 + 005	13	3	0.58225812 + 005	0.27459318 + 004
8	8	−0.39640770 + 005	0.22440662 + 005	13	4	−0.29305623 + 003	0.78229252 + 004
9	0	0.00000000	0.66787812 + 003	13	5	0.26029051 + 005	0.26720609 + 005
9	1	0.15644624 + 005	−0.90389226 + 004	13	6	0.27827722 + 005	−0.79238260 + 004
9	2	0.21025190 + 004	0.92582744 + 003	13	7	0.51600128 + 005	0.31960313 + 005
9	3	−0.17679682 + 005	0.25763106 + 004	13	8	−0.13424951 + 005	0.51006139 + 004
9	4	−0.68348269 + 004	0.16743121 + 004	13	9	0.48568123 + 005	−0.11833742 + 004
9	5	0.12685683 + 004	0.10876078 + 005	13	10	−0.35677931 + 005	−0.51820255 + 005
9	6	−0.46495189 + 005	−0.24762189 + 005	13	11	−0.35353627 + 005	0.41502978 + 005
9	7	−0.10453820 + 006	0.17838810 + 005	13	12	0.11807165 + 005	0.11453779 + 005
9	8	−0.45503214 + 003	0.16497296 + 005	13	13	0.12650532 + 004	−0.70845947 + 004

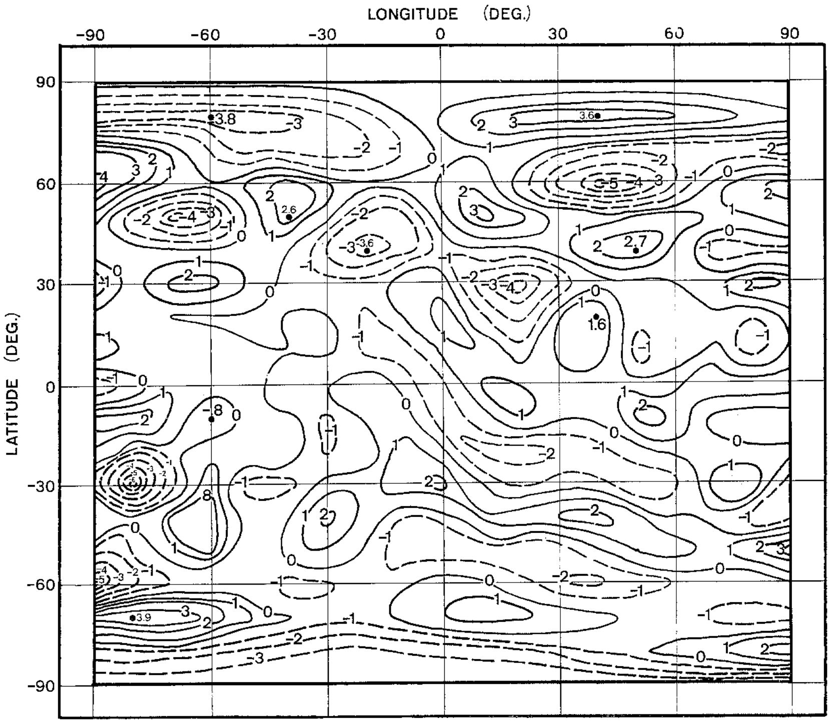

Fig. 4. The expected surface topography of the front side of the Moon at the present time for a relaxation time of 3 b.y. Units are in km.

while the highland areas have displaced upward. For example the heights of the rims of maria Imbrium and Serenitatis from their floors are about 7 km.

In the foregoing calculations we assumed that the relaxation times of all of the harmonics of the gravitational potential are the same as that of the high-degree harmonics. But in viscous deformation, the relaxation times of low-degree harmonics tend to be shorter than those of the high-degree ones. However, it will be shown in the next section that for the harmonics we are concerned with (harmonics with degrees 2–13) the difference of the relaxation times is not more than a factor of 3, and the assumption implies that the viscosity of the deeper part of the lunar interior is greater (about 3 times) than that of the upper part.

The actual topography of the lunar surface, about ± 2 km (Sjogren and Wollenhaupt, 1972; Wollenhaupt *et al.*, 1972) is less than that shown in Figure 4, about ± 4 km. For example, the rims of maria Imbrium and Serenitatis are actually about 3 km above their floors. This indicates that the mascon sites have not actually sunk as much as predicted for the case of a relaxation time of 3 b.y.

If the relaxation time of the lunar gravitational potential was less than 3 b.y. the expected lunar surface topography would be even larger than that shown in Figure 4. This indicates that it is impossible to have such a short relaxation time.

The foregoing arguments lead us to conclude that the relaxation time of the lateral undulations of the lunar gravitational potential, and thus the mascons and the associated stress differences, is greater than 3 b.y. Bearing in mind that the viscous flow has an exponential behavior, the difference between the expected and the actual surface topography of the Moon implies that the relaxation time is at least 5 b.y. This is of course a lower limit because we have ignored the initial topography of the lunar surface which is associated with the impact of planetesimals and subsequent filling of the impact sites by the mare basalts.

3. Viscosity of the Moon

In the present section we determine the average viscosity of the lunar interior from the relaxation time obtained in the previous section. In carrying out the calculations we assume that the lunar interior has behaved as a uniform Maxwellian viscoelastic body, since the formation of the mascons. We also regard the crustal density anomalies as surface loads. It has already been mentioned that the responses of the Moon to crustal density anomalies and to the surface loads are almost identical.

The relaxation time, τ_n, of a load, specified by a spherical harmonic of degree n and located on the surface of a uniform Maxwellian viscoelastic sphere, is (cf. McKenzie, 1967) given by

$$\tau_n = [\{2(n+1)^2 + 1\}/nga + \varrho/\mu]\,\nu\,, \tag{5}$$

where g = gravitational acceleration at the lunar surface, 162.87 cm s^{-1}, ϱ = mean density of the Moon, 3.36 g/cc, μ = mean rigidity of the Moon, 4×10^{11} dyn/cm^2, and ν = average kinematic viscosity of the Moon. It is worthwhile mentioning that the viscosity of the upper part of the lunar interior, within 1000 km depth has strong influence on the relaxation times of the surface loads specified by the high-degree harmonics (harmonics with degrees greater than 8). This is because, in a uniform viscous medium, subjected to a surface load, the maximum flow occurs at a depth comparable to the size of the load (Haskell, 1939). Therefore, in the case of these harmonics one should use the values for ϱ, μ, and ν which are appropriate for the upper part of the Moon. Nevertheless, we assume that the above-mentioned mean values of these quantities are applicable to the entire lunar interior.

Table IV shows the relaxation times obtained from Equation (5). The values are normalized to correspond to a relaxation time of unity for the 2nd degree harmonic. It is clear from the table that the surface loads specified by the low-degree spherical harmonics decay faster than those specified by the high-degree spherical harmonics. However, in the case of the harmonics we are concerned with, the differences of the relaxation times is not more than a factor of 3. Assuming the same value for the relaxation times of all the harmonics implies that the viscosity of the lower part of the lunar interior is about 3 times greater than that of the upper part.

Using the value obtained in the previous section for the relaxation time of the mascons, about 5 b.y., Equation (5) yields a value of about 3×10^{26} stokes $\approx 10^{27}$ poise for the average viscosity of the lunar interior which is even higher than the values proposed by Urey (1968), Kopal (1969) and Baldwin (1970). Note that we have considered a uniform viscoelastic Moon with a constant viscosity. Therefore, the foregoing value presents the average viscosity of the Moon for this period of time. Such a high viscosity prevents any kind of convection inside the upper 800 km of the Moon, as already pointed out by Turcotte and Oxburgh (1969).

TABLE IV

Normalized relaxation times of the surface loads specified by different spherical harmonics.

n	τ_n/τ_2
2	1
3	1.15
4	1.33
5	1.52
6	1.72
7	1.92
8	2.12
9	2.32
10	2.52
11	2.72
12	2.92
13	3.13

The results we have obtained so far are based on the assumptions that the surface loads of the Moon have been decaying through viscous deformation of the Moon. Let us examine whether these loads can be supported by a convection current hypothesis. The simultaneous existence of the lunar mascons (surface loads specified by the high-degree spherical harmonics) and the departure of the lunar figure from a spherical body (surface loads specified by the low-degree spherical harmonics) yields a strong criterion which should be fulfilled by any convection hypothesis. Any simplified convection current, such as the one proposed by Runcorn (1967) requires a viscosity of less than 10^{24} poise for the lunar interior, (Turcotte and Oxburgh, 1969). Such a convection current, however, cannot support any surface load other than the one specified by the 2nd degree spherical harmonic, because the harmonics are independent of each other. In this case, the surface loads specified by the harmonics with degrees 3–13 would decay at most within 4–10 m. y. This contradicts the existence of the mascons (which depend on the viscosity of the upper part of the lunar interior) and the existence of the low-degree harmonics of the gravitational potential (which depends on the viscosity of the lower part of the lunar interior) for about 3 b.y. On the other hand, it is implausible to assume that local convection currents have existed since the formation of the mascons and could thus have supported all the surface loads, including the mascons as well as low-degree harmonics, simultaneously.

Therefore, we conclude that the viscosity of the lunar interior is at least 10^{27}

poise. Moreover, there has been no convection current inside the Moon since the formation of the mascons and the present mascons are the remainder of the larger ones which were created about 3 b.y. ago and which have been steadily decaying since.

4. Conclusions

Using data from the present surface topography and gravitational potential of the Moon we estimated the relaxation time of the mascons to be about 5 b.y. On the basis of this relaxation time, the viscosity of the lunar interior was found to be at least 10^{27} poise, which lead us to conclude that there has been no convection current inside the upper 800 km of the Moon since the formation of the mascons.

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