RAYLEIGH WAVES IN AN ORTHOTROPIC THERMOELASTIC MEDIUM UNDER GRAVITY FIELD AND INITIAL STRESS

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(Received 21 March, 1997; Accepted 28 March, 1997)

Abstract. The aim of the present paper is to investigate the influence both of gravity field and initial stress on the propagation of Rayleigh waves in an orthotropic thermoelastic medium subject to certain boundary conditions. We suppose that the body is under initial stress alonq x_1-direction and incremental thermal stresses. The wave velocity equation has been obtained. Many special cases and comparison with the previous results have been studied.

1. Introduction

The problem of propagation in an orthotropic thermoelastic medium is very important for the possibility of its extensive application in various branches of science and techonology, particularly in optics, acoustics, geophysics and plasma physics. Many authors such as El-Naggar et al. (1994); Love (1957); Das et al. (1992), and others studied the effect of gravity on the propagation of Rayleigh waves in an elastic solid medium. However, none of the authors considered the effect of gravity on the propagation of Rayleigh waves in an orthotropic medium.

The propagation of thermoelastic waves discussed by Abd-Alla (1991); El-Naggar and Abd-Alla (1989). Bouden and Datta (1990) studied Rayleigh and Love waves in cladded anisotropic medium. Dey and Addy (1979) studied the influence of initial stress on the propagation of Rayleigh waves in a half-space under incremental thermal stresses. Dey and Sengupta (1975) discussed the influence of gravity under the assumption of Biot (1965) on thermoelastic Rayleigh waves. Thermoelastic Rayleigh waves were investigated by Sengupta and Acharya (1979) under the influence of gravity in an elastic layer with the assumption that the heat is radiated from the free plane boundary surface of the layer, and the gravity field produces a type of initial stress of a hydrostatic nature.

In this paper, the ifluence of gravity and an initial stress on the propagation of Rayleigh waves in an orthotropic thermoelastic solid medium has been investigated using the wave equations which are satisfied by the displacement potentials ϕ and ψ.

2. Formulation of the Problem

Consider an orthotropic elastic solid medium, the boundary of which $x_3 = 0$ is free of stress and extending to infinity throughout the half-space $x_3 \leq 0$, x_3-axis being taken positive in the direction towards the exterior of the body and the x_1-axis taken positive along the direction of Rayleigh wave propagation, the origin 0 being any point on the middle plane of the infinite. Let the medium be under initial compression stress P_0 along the x_1-direction, with the influence of gravity and at initial temperature T_0. It is assumed that, the elastic medium exchanges heat freely with its surroundings, an initial stress is produced by a slow process of creep where the shear stresses tend to become small or vanish after a long interval of time.

In view of the two-dimensional nature of the problem we may assume that the state of initial stress as

$$\tau_{11} = \tau_{33} = \tau; \quad \tau_{13} = 0, \tag{1}$$

where τ is a function of depth.

The equilibrium conditions of the initial stress field are given by Bouden and Daatta (1990)

$$\frac{\partial \tau}{\partial x_1} = 0; \quad \frac{\partial \tau}{\partial x_3} - \rho g = 0, \tag{2}$$

where g is the acceleration due to gravity and ρ is the density of the medium.

The dynamical equations of the two-dimensional problem under initial stress field and initial compression stress P_0 in the direction of the x_1-axis are given by (Bouden and Daatta, 1990; Dey and Chakaraborty, 1984)

$$\begin{aligned} &\frac{\partial \tau_{11}}{\partial x_1} + \frac{\partial \tau_{13}}{\partial x_3} - P_0 \frac{\partial \omega}{\partial x_3} - \rho g \frac{\partial u_3}{\partial x_1} = \rho \frac{\partial^2 u_1}{\partial t^2}, \\ &\frac{\partial \tau_{13}}{\partial x_1} + \frac{\partial \tau_{33}}{\partial x_3} - P_0 \frac{\partial \omega}{\partial x_1} - \rho g \frac{\partial u_1}{\partial x_1} = \rho \frac{\partial^2 u_3}{\partial t^2}, \end{aligned} \tag{3}$$

where $(u_1, 0, u_3)$ are the components of displacement, τ_{ij} are the stress components and

$$\omega = \frac{1}{2}\left(\frac{\partial u_3}{\partial x_1} - \frac{\partial u_1}{\partial x_3}\right), \tag{4}$$

denotes the component of rotation.

The thermoelastic stress components for orthotropic body, under the effect of an initial compression stress P_0 in terms of displacement components u_1 and u_3, are given by El-Naggar and Abd-Alla (1989)

$$\tau_{11} = (c_{11} + P_0)\frac{\partial u_1}{\partial x_1} + (c_{13} + P_0)\frac{\partial u_3}{\partial x_3} - \nu_1 T,$$

$$\tau_{33} = c_{13}\frac{\partial u_1}{\partial x_1} + c_{33}\frac{\partial u_3}{\partial x_3} - \nu_3 T, \tag{5}$$

$$\tau_{13} = c_{44}\left(\frac{\partial u_1}{\partial x_3} + \frac{\partial u_3}{\partial x_1}\right),$$

where c_{ij} are the elastic constants,

$$\begin{aligned} \nu_1 &= (c_{11} + c_{12})\alpha_1 + c_{13}\alpha_2, \\ \nu_3 &= 2c_{13}\alpha_1 + c_{33}\alpha_2, \end{aligned} \tag{6}$$

α_1 is the thermal expansion coefficient in the planes of orthotropy and α_2 is the thermal expansion coefficient along the x_3-axis.

Equations (3), with help of (4) and (5), change to

$$(c_{11} + P_0)\frac{\partial^2 u_1}{\partial x_1^2} + \left(c_{44} + \frac{P_0}{2}\right)\frac{\partial^2 u_1}{\partial x_3^2} + \left(c_{13} + c_{44} + \frac{P_0}{2}\right)\frac{\partial^2 u_3}{\partial x_1 \partial x_3}$$

$$-\frac{\partial}{\partial x_1}(\nu_1 T) - \rho g\frac{\partial u_3}{\partial x_1} = \rho\frac{\partial^2 u_1}{\partial t^2}, \tag{7}$$

$$\left(c_{44} + c_{13} + \frac{P_0}{2}\right)\frac{\partial^2 u_1}{\partial x_1 \partial x_3} + \left(c_{44} + \frac{P_0}{2}\right)\frac{\partial^2 u_3}{\partial x_1^2} + c_{33}\frac{\partial^2 u_3}{\partial x_3^2}$$

$$-\frac{\partial}{\partial x_3}(\nu_3 T) + \rho g\frac{\partial u_1}{\partial x_1} = \rho\frac{\partial^2 u_3}{\partial t^2}. \tag{8}$$

3. Solution of the Problem

Let $\phi(x_1, x_3, t)$ and $\psi(x_1, x_3, t)$ be the displacement potentials. They are related to the displacement components u_1 and u_3 by the following relations

$$u_1 = \frac{\partial \phi}{\partial x_1} - \frac{\partial \psi}{\partial x_3}, \quad u_3 = \frac{\partial \phi}{\partial x_3} + \frac{\partial \psi}{\partial x_1}. \tag{9}$$

Substituting from Equation (9) in Equations (7) and (8), we see that ϕ and ψ satisfy the wave equations

$$(c_{11} + P_0)\frac{\partial^2 \phi}{\partial x_1^2} + (c_{13} + 2c_{44} + P_0)\frac{\partial^2 \phi}{\partial x_3^2} - \rho g\frac{\partial \psi}{\partial x_1} - \nu_1 T = \rho\frac{\partial^2 \phi}{\partial t^2}, \tag{10}$$

$$\left(c_{33} - c_{44} - c_{13} - \frac{P_0}{2}\right)\frac{\partial^2 \psi}{\partial x_3^2} + \left(c_{44} - \frac{P_0}{2}\right)\frac{\partial^2 \psi}{\partial x_1^2} + \rho g\frac{\partial \phi}{\partial x_1} = \rho\frac{\partial^2 \psi}{\partial t^2}. \tag{11}$$

The temperature T is determined by the generalised Fourier's law of heat conducting, as follows El-Naggar and Abd-Alla (1989)

$$\nabla^2 T - \frac{1}{\chi}\frac{\partial T}{\partial t} - \eta\nabla^2\frac{\partial \phi}{\partial t} = 0, \tag{12}$$

where

$$\chi = \frac{\delta_1 + \delta_2}{2\rho s}, \quad \eta = \frac{T_0(\nu_1 + \nu_3)}{(\delta_1 + \delta_2)},$$

δ_1 and δ_2 being the thermal conductivity in the plane of orthotropy and in the x_3-direction, respectively, s being the specific heat per unit mass and T_0 is the initial temperature.

Eliminating T from Equations (10) and (12), and retaining Equation (11), we have

$$\left(\chi\nabla^2 - \frac{\partial}{\partial t}\right)\left\{(c_{11} + P_0)\frac{\partial^2 \phi}{\partial x_1^2} + (c_{13} + 2c_{44} + P_0)\frac{\partial^2 \phi}{\partial x_3^2} - \rho g\frac{\partial \psi}{\partial x_1} - \rho\frac{\partial^2 \phi}{\partial t^2}\right\}$$

$$-\chi\nu_1\eta\nabla^2\frac{\partial \phi}{\partial t} = 0, \tag{13}$$

$$\left(c_{33} - c_{44} - c_{13} - \frac{P_0}{2}\right)\frac{\partial^2 \psi}{\partial x_3^2} + \left(c_{44} - \frac{P_0}{2}\right)\frac{\partial^2 \psi}{\partial x_1^2} + \rho g\frac{\partial \phi}{\partial x_1} = \rho\frac{\partial^2 \psi}{\partial t^2}. \tag{14}$$

For a plane harmonic wave propagation in the x_1-direction, we seek solution of Equations (13) and (14) in the following forms

$$\begin{aligned} \phi &= f(x_3)\mathrm{e}^{i\alpha(x_1 - ct)}, \\ \psi &= h(x_3)\mathrm{e}^{i\alpha(x_1 - ct)}, \end{aligned} \tag{15}$$

where c is the speed of Rayleigh waves and α is the wave number.

Substituting from Equations (15) into equations (13) and (14), we get

$$\frac{\mathrm{d}^4 f}{\mathrm{d}x_3^4} + \alpha\left\{\frac{(ic - \alpha\chi)}{\chi} + \frac{\alpha(\rho c^2 - c_{11} - P_0) + ic\nu_1\eta}{c_{13} + 2c_{44} + P_0}\right\}\frac{\mathrm{d}^2 f}{\mathrm{d}x_3^2}$$

$$-\alpha^3\left\{\frac{(ic - \alpha\chi)(c_{11} + P_0 - \rho c^2) + ic\chi\nu_1\eta}{\chi(c_{13} + 2c_{44} + P_0)}\right\}f - \frac{i\alpha\rho g}{(c_{13} + 2c_{44} + P_0)}\frac{\mathrm{d}^2 h}{\mathrm{d}x_3^2}$$

$$-\frac{i\alpha^2\rho g(ic - \alpha\chi)}{\chi(c_{13} + 2c_{44} + P_0)}h = 0, \tag{16}$$

$$\frac{d^2h}{dx_3^2} - \frac{\alpha^2\left(c_{44} - \frac{P_0}{2} - \rho c^2\right)}{\left(c_{33} - c_{44} - c_{13} - \frac{P_0}{2}\right)}h + \frac{i\alpha\rho g}{\left(c_{33} - c_{44} - c_{13} - \frac{P_0}{2}\right)}f = 0 \tag{17}$$

The solution of Equations (16) and (17) which satisfies the condition that, the corresponding stresses vanish as $x_3 \to -\infty$ and $\lambda_1, \lambda_2, \lambda_3$ are taken to be imaginary, has the form

$$\begin{aligned} f(x_3) &= A_1 e^{-i\alpha\lambda_1 x_3} + A_2 e^{-i\alpha\lambda_2 x_3} + A_3 e^{-i\alpha\lambda_3 x_3}, \\ h(x_3) &= B_1 e^{-i\alpha\lambda_1 x_3} + B_2 e^{-i\alpha\lambda_2 x_3} + B_3 e^{-i\alpha\lambda_3 x_3}, \end{aligned} \tag{18}$$

where the constants B_1, B_2, B_3 are related with the constants A_1, A_2, A_3; respectively, by means of Equation (16) or Equation (17).

Equating the coefficients of $e^{-i\alpha\lambda_1 x_3}$, $e^{-i\alpha\lambda_2 x_3}$ and $e^{-i\alpha\lambda_3 x_3}$ to zero, we have using Equation (17).

$$B_j = m_j A_j, \quad j = 1, 2, 3 \tag{19}$$

where

$$m_j = \frac{i\rho g}{\alpha\left[\left(c_{44} - \frac{P_0}{2} - \rho c^2\right) + \left(c_{33} - c_{44} - c_{13} - \frac{P_0}{2}\right)\lambda_j^2\right]},$$

$\lambda_1, \lambda_2, \lambda_3$ are the roots of the equation

$$\lambda^6 + K_1\lambda^4 + K_2\lambda^2 + K_3 = 0, \tag{20}$$

$$K_1 = \left[1 + \frac{c_{44} - \frac{P_0}{2} - \rho c^2}{c_{33} - c_{44} - c_{13} - \frac{P_0}{2}} + \frac{c_{11} + P_0 - \rho c^2}{c_{13} + 2c_{44} + P_0}\right] - \frac{ic}{\alpha\chi}\left[\frac{c_{13} + 2c_{44} + P_0 + \chi\nu_1\eta}{c_{13} + 2c_{44} + P_0}\right],$$

$$K_2 = \left[\frac{\alpha^2(c_{11} + P_0 - \rho c^2)(\left(c_{44} - \frac{P_0}{2} - \rho c^2\right) - \rho^2 g^2}{\alpha^2(c_{13} + 2c_{44} + P_0)\left(c_{33} - c_{44} - c_{13} - \frac{P_0}{2}\right)} + \frac{\left(c_{44} - \frac{P_0}{2} - \rho c^2\right)}{c_{33} - c_{44} - c_{13} - \frac{P_0}{2}} + \frac{(c_{11} + P_0 - \rho c^2)}{c_{13} + 2c_{44} + P_0}\right]$$

$$-\frac{ic}{\alpha\chi}\left[\frac{c_{11}+P_0-\rho c^2+\chi\nu_1\eta}{c_{13}+2c_{44}+P_0}\right.$$

$$\left.+\frac{\left(c_{44}-\frac{P_0}{2}-\rho c^2\right)(c_{13}+2c_{44}+P_0+\chi\nu_1\eta)}{c_{13}+2c_{44}+P_0)\left(c_{33}-c_{44}-c_{13}-\frac{P_0}{2}\right)}\right],$$

$$K_3=\left[\frac{\alpha^2(c_{11}+P_0-\rho c^2)\left(c_{44}-\frac{P_0}{2}-\rho c^2\right)-\rho^2g^2}{\alpha^2(c_{13}+2c_{44}+P_0)\left(c_{33}-c_{44}-c_{13}-\frac{P_0}{2}\right)}\right]-$$

$$-\frac{ic}{\alpha\chi}\left[\frac{\alpha^2\left(c_{44}-\frac{P_0}{2}-\rho c^2\right)(c_{11}+P_0-\rho c^2+\chi\nu_1\eta)-\rho^2g^2}{\alpha^2(c_{13}+2c_{44}+P_0)\left(c_{33}-c_{44}-c_{13}-\frac{P_0}{2}\right)}\right].$$

Using Equations (15), (10), (18) and (19), we get

$$\begin{aligned}\phi &= (A_1e^{-i\alpha\lambda_1x_3}+A_2e^{-i\alpha\lambda_2x_3}+A_3e^{-i\alpha\lambda_3x_3})\exp[i\alpha(x_1-ct)],\\ \psi &= (m_1A_1e^{-i\alpha\lambda_1x_3}+m_2A_2e^{-i\alpha\lambda_2x_3}+m_3A_3e^{-i\alpha\lambda_3x_3})\\ &\quad\times\exp[i\alpha(x_1-ct)],\end{aligned}\tag{21}$$

and the temperature T has the form

$$T=\frac{1}{\nu_1}\left\{(c_{11}+P_0)\frac{\partial^2\phi}{\partial x_1^2}+(c_{13}+2c_{44}+P_0)\frac{\partial^2\phi}{\partial x_3^2}-\rho g\frac{\partial\psi}{\partial x_1}-\rho\frac{\partial^2\psi}{\partial t^2}\right\}.\tag{22}$$

Introducing Equations (21) into Equation (22), we have

$$\begin{aligned}T &= \frac{\alpha^2}{\nu_1}\left\{\left[\left(\rho c^2-c_{11}-P_0-\frac{i\rho g}{\alpha}m_j\right)-(c_{13}+2c_{44}+P_0)\lambda_j^2\right]A_j\right\}\\ &\quad\times\exp[i\alpha(x_1-\lambda_jx_3-ct)],\quad j=1,2,3.\end{aligned}\tag{23}$$

In order to determine the arbitrary constants A_1, A_2, A_3 in Equation (23), we consider the following boundary conditions.

4. Boundary Conditions and Frequency Equation

The boundary conditions on the plane $x_3=0$ are given by (Bouden and Daata, 1990; Sengupta and Acharya, 1979)

$$\tau_{13}=\tau_{33}=0,\quad \frac{\partial T}{\partial x_3}+\theta T=0\quad \text{on } x_3=0,\tag{24}$$

where

$$\tau_{13} = c_{44}\left(2\frac{\partial^2\phi}{\partial x_1 \partial x_3} - \frac{\partial^2\psi}{\partial x_3^2} + \frac{\partial^2\psi}{\partial x_1^2}\right),$$

$$\tau_{33} = c_{13}\frac{\partial^2\phi}{\partial x_1^2} + c_{33}\frac{\partial^2\phi}{\partial x_3^2} + (c_{13} - c_{33})\frac{\partial^2\psi}{\partial x_1 \partial x_3} - \nu_3 T,$$

θ is the ratio of the coefficients of heat transfer to the thermal conductivity.

In view of Equations (21), (23) and (24), we then obtain

$$\begin{aligned}
&(2\lambda_1 + m_1\lambda_1^2 - m_1)A_1 + (2\lambda_2 + m_2\lambda_2^2 - m_2)A_2 \\
&\quad +(2\lambda_3 + m_3\lambda_3^2 - m_3)A_3 = 0, \\
&\left\{\left[c_{33} + \frac{\nu_3}{\nu_1}(c_{13} + 2c_{44} + P_0)\right]\lambda_1^2 + (c_{33} - c_{13})m_1\lambda_1 + c_{13} - \frac{\nu_3}{\nu_1}\right. \\
&\quad \left.\times\left(\rho c^2 - c_{11} - P_0 - \frac{i\rho g}{\alpha}m_1\right)\right\}A_1 + \left\{\left[c_{33} + \frac{\nu_3}{\nu_1}(c_{13} + 2c_{44} + P_0)\right]\lambda_2^2\right. \\
&\quad \left. +(c_{33} - c_{13})m_2\lambda_2 + c_{13} - \frac{\nu_3}{\nu_1}\left(\rho c^2 - c_{11} - P_0 - \frac{i\rho g}{\alpha} - m_2\right)\right\}A_2 \\
&\quad +\left\{\left[c_{33} + \frac{\nu_3}{\nu_1}(c_{13} + 2c_{44} + P_0)\right]\lambda_3^2 + (c_{33} - c_{13})m_3\lambda_3 + c_{13}\right. \\
&\quad \left. -\frac{\nu_3}{\nu_1}\left(\rho c^2 - c_{11} - P_0 - \frac{i\rho g}{\alpha}m_3\right)\right\}A_3 = 0, \qquad (25) \\
&(\theta - i\alpha\lambda_1)\left[c_{11} + P_0 - \rho c^2 + \frac{i\rho g}{\alpha}m_1 + (c_{13} + 2c_{44} + P_0)\lambda_1^2\right]A_1 \\
&\quad +(\theta - i\alpha\lambda_2)\left[c_{11} + P_0 - \rho c^2 + \frac{i\rho g}{\alpha}m_2 + (c_{13} + 2c_{44} + P_0)\lambda_2^2\right]A_2 \\
&\quad +(\theta - i\alpha\lambda_3)\left[c_{11} + P_0 - \rho c^2 + \frac{i\rho g}{\alpha}m_3 + (c_{13} + 2c_{44} + P_0)\lambda_3^2\right]A_3 = 0.
\end{aligned}$$

Eliminating the constants A_1, A_2 and A_3 from Equations (25), we get

$$\det(a_{ij}) = 0; \quad i, j = 1, 2, 3 \qquad (26)$$

where

$$\alpha_{1j} = \frac{i\rho g(\lambda_j^2 - 1)}{\alpha\left[\left(c_{44} - \frac{P_0}{2} - \rho c^2\right) + \left(c_{33} - c_{44} - c_{13} - \frac{P_0}{2}\right)\lambda_j^2\right]} + 2\lambda_j,$$

$$\alpha_{2j} = \left[c_{33} + \frac{\nu_3}{\nu_1}(c_{13} + 2c_{44} + P_0)\right]\lambda_j^2 + \frac{\nu_3}{\nu_1}(c_{11} + P_0 - \rho c^2) + c_{13} - \frac{\nu_3 \rho^2 g^2}{\alpha^2 \nu_1 \left[\left(c_{44} - \frac{P_0}{2} - \rho c^2\right) + \left(c_{33} - c_{44} - c_{13} - \frac{P_0}{2}\right)\lambda_j^2\right]} + \frac{i\rho g (c_{33} - c_{13})\lambda_j}{\alpha \left[\left(c_{44} - \frac{P_0}{2} - \rho c^2\right) + \left(c_{33} - c_{44} - c_{13} - \frac{P_0}{2}\right)\lambda_j^2\right]},$$

$$\alpha_{3j} = (\theta - i\alpha\lambda_j)\left[(c_{13} + 2c_{44} + P_0)\lambda_j^2 + c_{11} + P_0 - \rho c^2 - \frac{\rho^2 g^2}{\alpha^2 \left[\left(c_{44} - \frac{P_0}{2} - \rho c^2\right) + \left(c_{33} - c_{44} - c_{13} - \frac{P_0}{2}\right)\lambda_j^2\right]}\right].$$

Equation (26) determines the wave velocity equation for the Rayleigh waves in an orthotropic elastic medium under the influence of gravity, initial compression stress and incremental thermal stresses, in the determinantal form.

5. Particular Cases

We consider the following particular cases:

Case (I): When the initial stress is absent and the medium is isotropic.

In this case, Equation (26) takes the form

$$\begin{vmatrix} 2y_1 + \frac{ig(y_1^2 - 1)}{\alpha[\delta^2(y_1^2 + 1) - c^2]} \\ 2\gamma^2(1 + y_1^2) - 2\delta^2 - c^2 - \frac{g^2}{\alpha^2[\delta^2(y_1^2 + 1) - c^2]} + \frac{i(2\alpha g \delta y_1)^2}{\alpha^2[\delta^2(y_1^2 + 1) - c^2]} \\ (\theta - i\alpha y_1)\left[c^2 - \gamma^2(1 + y_1^2) + \frac{g^2}{\alpha^2[\delta^2(y_1^2 + 1) - c^2]}\right] \end{vmatrix}$$

$$2y_2 + \frac{ig(y_2^2 - 1)}{\alpha[\delta^2(y_2^2 + 1) - c^2]}$$

$$2\gamma^2(1 + y_2^2) - 2\delta^2 - c^2 - \frac{g^2}{\alpha^2[\delta^2(y_2^2 + 1) - c^2]} + \frac{i(2\alpha g \delta y_2)^2}{\alpha^2[\delta^2(y_2^2 + 1) - c^2]}$$

$$(\theta - i\alpha y_2)\left[c^2 - \gamma^2(1 + y_2^2) + \frac{g^2}{\alpha^2[\delta^2(y_2^2 + 1) - c^2]}\right]$$

$$\left|\begin{matrix} 2y_3 + \dfrac{ig(y_3^2-1)}{\alpha[\delta^2(y_3^2+1)-c^2]} \\ 2\gamma^2(1+y_3^2) - 2\delta^2 - c^2 - \dfrac{g^2}{\alpha^2[\delta^2(y_3^2+1)-c^2]} + \dfrac{i(2\alpha g\delta y_3)^2}{\alpha^2[\delta^2(y_3^2+1)-c^2]} \\ (\theta - i\alpha y_3)\left[c^2 - \gamma^2(1+y_3^2) + \dfrac{g^2}{\alpha^2[\delta^2(y_3^2+1)-c^2]}\right] \end{matrix}\right| = 0 \tag{27}$$

where

$$\gamma^2 = \frac{\lambda+2\mu}{\rho}, \quad \delta^2 = \frac{\mu}{\rho},$$

λ and μ being Lame's constants, y_1, y_2, y_3 are the roots of the equation

$$\begin{aligned} y^6 &+ \left\{\left(3 - \frac{c^2}{\gamma^2} - \frac{c^2}{\delta^2}\right) - \frac{ic}{\alpha\chi}\left(1 + \frac{\chi\nu_0\eta}{\rho\gamma^2}\right)\right\} y^4 \\ &+ \left\{\left[\left(1-\frac{c^2}{\gamma^2}\right)\left(2-\frac{c^2}{\delta^2}\right) + 1 - \frac{c^2}{\delta^2} - \frac{g^2}{\alpha^2\gamma^2\delta^2}\right]\right. \\ &\left. - \frac{ic}{\alpha\chi}\left[1 + \frac{\chi\nu_0\eta}{\rho\gamma^2} + \left(1-\frac{c^2}{\delta^2}\right)\left(1+\frac{\chi\nu_0\eta}{\rho\gamma^2}\right)\right]\right\} y^2 \\ &+ \left\{\left[\left(1-\frac{c^2}{\gamma^2}\right)\left(1-\frac{c^2}{\delta^2}\right) - \frac{g^2}{\alpha^2\gamma^2\delta^2}\right] - \frac{ic}{\alpha\chi}\left[\left(1-\frac{c^2}{\delta^2}\right)\right.\right. \\ &\left.\left. \times\left(1 - \frac{c^2}{\gamma^2} + \frac{\chi\nu_0\eta}{\rho\gamma^2}\right) - \frac{g^2}{\alpha^2\gamma^2\delta^2}\right]\right\} = 0. \end{aligned}$$

(in the isotropic case $\nu_1 = \nu_3 = \nu_0$).

Case (II): When there is no coupling between the temperature and strain fields

$$\left(\gamma_3 = \gamma_1 = 0, \quad \eta = \frac{\alpha c}{\chi} = 0\right)$$

and h vanishes.

Equation (26), in this case becomes

$$\left|\begin{matrix} 2z_1 + \dfrac{i\rho g(z_1^2-1)}{\alpha\left[\left(c_{44} - \frac{P_0}{2} - \rho c^2\right) + \left(c_{33} - c_{44} - c_{13} - \frac{P_0}{2}\right) z_1^2\right]} \\ c_{33}z_1^2 + c_{13} + \dfrac{i\rho g(c_{33}-c_{13})z_1}{\alpha\left[\left(c_{44} - \frac{P_0}{2} - \rho c^2\right) + \left(c_{33} - c_{44} - c_{13} - \frac{P_0}{2}\right) z_1^2\right]} \end{matrix}\right.$$

$$\begin{vmatrix} 2z_2 + \dfrac{i\rho g(z_2^2 - 1)}{\alpha\left[\left(c_{44} - \dfrac{P_0}{2} - \rho c^2\right) + \left(c_{33} - c_{44} - c_{13} - \dfrac{P_0}{2}\right) z_2^2\right]} \\ c_{33} z_1^2 + c_{13} + \dfrac{i\rho g(c_{33} - c_{13}) z_2}{\alpha\left[\left(c_{44} - \dfrac{P_0}{2} - \rho c^2\right) + \left(c_{33} - c_{44} - c_{13} - \dfrac{P_0}{2}\right) z_2^2\right]} \end{vmatrix} = 0, \tag{28}$$

where z_1, z_2 are the roots of the equation

$$z^4 + \left[\frac{c_{44} - \dfrac{P_0}{2} - \rho c^2}{c_{33} - c_{44} - c_{13} - \dfrac{P_0}{2}} + \frac{c_{11} + P_0 - \rho c^2}{c_{13} + 2c_{44} + P_0}\right] z^2$$

$$+ \frac{\alpha^2\left(c_{44} - \dfrac{P_0}{2} - \rho c^2\right)(c_{11} + P_0 - \rho c^2) - \rho^2 g^2}{\alpha^2\left(c_{33} - c_{44} - c_{13} - \dfrac{P_0}{2}\right)(c_{13} + 2c_{44} + P_0)} = 0.$$

In the absence of initial stress and when the medium is isotropic, Equation (28) takes the form

$$\left[2V_1 + \frac{ig(V_1^2 - 1)}{\alpha[\delta^2(V_1^2 + 1) - c^2]}\right]\left[\gamma^2(V_2^2 + 1) - 2\delta^2 + \frac{2ig\delta^2 V_2}{\alpha[\delta^2(V_2^2 + 1) - c^2]}\right]$$

$$= \left[2V_2 + \frac{ig(V_2^2 - 1)}{\alpha[\delta^2(V_2^2 + 1) - c^2]}\right]\left[\gamma^2(V_1^2 + 1) - 2\delta^2 + \frac{2ig\delta^2 V_1}{\alpha[\delta^2(V_1^2 + 1) - c^2]}\right], \tag{29}$$

where V_1 and V_2 are the roots of the equation

$$V^4 + \left[2 - \frac{c^2}{\gamma^2} - \frac{c^2}{\delta^2}\right] V^2 + \left(1 - \frac{c^2}{\gamma^2}\right)\left(1 - \frac{c^2}{\delta^2}\right) - \frac{g^2}{\alpha^2\gamma^2\delta^2} = 0.$$

Case III: If the influence of gravity field is neglected.

In this case, putting $g = 0$ in Equations (16) and (17), one can obtain

$$f(x_3) = A_1 e^{-i\alpha\xi_1 x_3} + A_2 e^{-i\alpha\xi_2 x_3},$$

$$h(x_3) = B_3 e^{-i\alpha\xi_3 x_3},$$

where ξ_1, ξ_2 are the roots of the equation

$$\xi^4 + \left[\left(1 + \frac{c_{11} + P_0 - \rho c^2}{c_{13} + 2c_{44} + P_0}\right) - \frac{ic}{\alpha\chi}\left(1 + \frac{\chi\nu_1\eta}{c_{13} + 2c_{44} + P_0}\right)\right]\xi^2$$

$$+ \left[\frac{c_{11} + P_0 - \rho c^2}{c_{13} + 2c_{44} + P_0} - \frac{ic}{\alpha\chi}\frac{c_{11} + P_0 - \rho c^2 + \chi\nu_1\eta}{c_{13} + 2c_{44} + P_0}\right] = 0,$$

$$\xi_3^2 = \left(\frac{\rho c^2 - c_{44} + \dfrac{P_0}{2}}{c_{33} - c_{44} - c_{13} - \dfrac{P_0}{2}}\right).$$

The relations (21) and (23), in this case, reduced to

$$\phi = (A_1 e^{-i\alpha\xi_1 x_3} + A_2 e^{-i\alpha\xi_2 x_3}) e^{-i\alpha(x_1 - ct)},$$

$$\psi = B_3 e^{i\alpha(\xi_3 x_3 - x_1 + ct)}, \tag{30}$$

$$T = \frac{\alpha^2}{\nu_1}\{[(\rho c^2 - c_{11} - P_0) - (c_{13} + 2c_{44} + P_0)\lambda_j^2]A_j\}$$

$$\times \exp[i\alpha(x_1 - \xi_j x_3 - ct)], \qquad j = 1, 2 \text{ (only)}.$$

In view of Equations (24), (30) and eliminating the constants A_1, A_2, B_3, we get the frequency equation in the form

$$\begin{vmatrix} 2\xi_1 & 2\xi_2 & (\xi_3^2 - 1) \\ \left[c_{13} + \frac{\nu_3}{\nu_1}(c_{13} + 2c_{44} + P_0)\right]\xi_1^2 + c_{13} - \frac{\nu_3}{\nu_1}(\rho c^2 - c_{11} - P_0) & \left[c_{33} + \frac{\nu_3}{\nu_1}(c_{13} + 2c_{44} + P_0)\right]\xi_2^2 + c_{13} - \frac{\nu_3}{\nu_1}(\rho c^2 - c_{11} - P_0) & c_{33}\xi_3^2 + c_{13} \\ (\theta - i\alpha\xi_1)[\rho c^2 - c_{11} - P_0 - (c_{13} + 2c_{44} + P_0)\xi_1^2] & (\theta - i\alpha\xi_2)[\rho c^2 - c_{11} - P_0 - (c_{13} + 2c_{44} + P_0)\xi_2^2] & 0 \end{vmatrix} = 0. \tag{31}$$

Where the medium is isotropic, Equation (30) becomes

$$\begin{vmatrix} 2\xi_1 & 2\xi_2 & (\xi_3^2-1) \\ (2\rho\gamma^2+P_0)(\xi_1^2+1)-\rho(c^2+2\delta^2) & (2\rho\gamma^2+P_0)(\xi_2^2+1)-\rho(c^2+2\delta^2) & \rho\gamma^2\xi_3^2+\rho(\gamma^2-2\delta^2) \\ (\theta-i\alpha\xi_1)[\rho c^2-P_0-(\rho\gamma^2+P_0)(\xi_1^2+1)] & (\theta-i\alpha\xi_2)[\rho c^2-P_0-(\rho\gamma^2+P_0)(\xi_2^2+1)] & 0 \end{vmatrix} = 0, \tag{32}$$

where ξ_1, ξ_2 are the roots of the equation

$$\xi^4 + \left[2 - \frac{\rho c^2}{\rho\gamma^2+P_0} - \frac{ic}{\alpha\chi}\left(\frac{\chi\nu_1\eta}{\rho\gamma^2+P_0}\right)\right]\xi^2 + \left(1-\frac{\rho c^2}{\rho\gamma^2+P_0}\right)$$

$$-\frac{ic}{\alpha\chi}\left(1-\frac{\rho c^2+\chi\nu_1\eta}{\rho\gamma^2+P_0}\right) = 0,$$

while

$$\xi_3^2 = \left(\frac{\rho c^2}{\rho\delta^2-\frac{P_0}{2}} - 1\right).$$

Case IV: When the initial stress is absent, there is no coupling between the temperatue and strain field and the influence of gravity field is neglected.

Equation (26), in this case, simplifies to

$$2(c_{33}-c_{11})\left[\left(\frac{\rho c-c_{11}}{c_{13}+2c_{44}}\right)\left(\frac{\rho c-c_{44}}{c_{33}-c_{44}-c_{13}}\right)\right]^{1/2}$$

$$= \left[\frac{\rho c-c_{44}}{c_{33}-c_{44}-c_{13}} - 1\right]\left[\frac{c_{33}(\rho c^2-c_{11})}{c_{13}+2c_{44}} + c_{13}\right]. \tag{33}$$

In the isotropic case, Equation (32) takes the form

$$4\left[\left(1-\frac{c^2}{\delta^2}\right)\left(1-\frac{c^2}{\delta^2}\right)\right]^{1/2} = \left(2-\frac{c^2}{\delta^2}\right)^2. \tag{34}$$

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