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Matrix transformations involving analytic sequence spaces

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In *Theorem 3.2* one has to assume that *E is solid*, for in general (b) only implies that there are sequences $\xi^{(1)}, \ldots, \xi^{(m)} \in E$ with $\sup_{x \in B} \left| \sum_{n=0}^{\infty} x_n y_n \right| \leqq \sum_{k=1}^{m} \sum_{n=0}^{\infty} |\xi_n^{(k)}| \, |y_n|$ for $y \in F$, so that if E is solid we may take $\xi = \left(\sum_{k=1}^{m} |\xi_n^{(k)}| \right)_n$.

To give a concrete counter-example, let (q_n) be an enumeration of $\mathbb{Q}$, and let $x = (1, 0, 1, 0, 1, \ldots)$ and $y = (q_1, 1, q_2, 1, q_3, \ldots)$. We put $E = \varphi \oplus \langle \{x, y\} \rangle$ and $F = E^{\times}$. The solid span $|E|$ of E is a diagonal transform of l^{∞}, so that $F = |E|^{\times}$ is a diagonal transform of l^1 and $(F, \nu(F, E)) = (F, \nu(F, |E|))$ is barrelled. But we show that $(E, \sigma(E, F))$ is not simple. Else there would exist scalars α and β and some $z \in \varphi$ such that $|x_n| \leqq |z_n + \alpha x_n + \beta y_n|$ and $|y_n| \leqq |z_n + \alpha x_n + \beta y_n|$ for $n \in \mathbb{N}_0$. This implies that $\beta \neq 0$, so that we can find a sequence (n_k) with $\alpha + \beta q_{n_k} \to 0$, while we have $|x_{2n}| = 1$ for $n \in \mathbb{N}_0$. This is a contradiction.

As a consequence, in *Theorem 6.1(1)* and *Theorem 6.2* one also has to assume that *E is solid*.