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Erratum

Estimates for the asymptotic behavior of solutions of the Helmholtz equation, with an application to second order elliptic differential operators with variable coefficients

H.D. Alber

Technische Hochschule Darmstadt, Fachbereich Mathematik, Schlossgartenstrasse 7, W-6100 Darmstadt, Federal Republic of Germany

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It has been pointed out to the author by N. Weck that the proof of the estimates (A6) and (A7) in the case $r>p>0$ is not correct, since the estimates (A4) and (A5) yield a remainder term $r^{1/2}O(p^{-4/3})$, which is not uniformly bounded in r. Instead, the estimates (A6) and (A7) must be proved as follows:

The proof given in the paper is correct for all r and p with $p<r\leqq C(p+p^3)$. For $r\geqq\frac{1}{2}(p+\frac{3}{2})(p-\frac{1}{2})>0$ we use the integral representation

$$H_p^{(1)}(r)=\frac{1}{\Gamma(p+1/2)}\sqrt{\frac{2}{\pi r}}\,e^{i(r-(p+1/2)\frac{\pi}{2})}\int_0^\infty e^{-t}t^{p-1/2}\left(1+\frac{it}{2r}\right)^{p-1/2}dt$$

(cf. [1, p. 6 and p. 23]), and obtain

$$\begin{aligned}|r^{1/2}H_p^{(1)}(r)|&\leqq\left(\frac{2}{\pi}\right)^{1/2}\frac{1}{\Gamma(p+1/2)}\int_0^\infty e^{-t}t^{p-1/2}\left(1+\frac{t}{2r}\right)^{p-1/2}dt\\&\leqq\left(\frac{2}{\pi}\right)^{1/2}\frac{1}{\Gamma(p+1/2)}\int_0^\infty e^{-t}t^{p-1/2}e^{(p-1/2)\frac{t}{2r}}dt\\&\leqq\left(\frac{2}{\pi}\right)^{1/2}\frac{1}{\Gamma(p+1/2)}\left[\frac{1}{1-\frac{p-1/2}{2r}}\right]^{p+1/2}\int_0^\infty e^{-\tau}\tau^{p-1/2}d\tau\\&=\left(\frac{2}{\pi}\right)^{1/2}\left[\frac{1}{1-\frac{p-1/2}{2r}}\right]^{p+1/2}\leqq\left(\frac{2}{\pi}\right)^{1/2}\left(1+\frac{1}{p+1/2}\right)^{p+1/2}\leqq\left(\frac{2}{\pi}\right)^{1/2}e,\end{aligned}$$

since

$$\frac{1}{1-\frac{p-1/2}{2r}}\leqq1+\frac{1}{p+1/2}\qquad\text{for } r\geqq\frac{1}{2}\left(p+\frac{3}{2}\right)\left(p-\frac{1}{2}\right).$$

For these r and p we also have

$$|r^{1/2} J_p(r)| \leqq \left(\frac{2}{\pi}\right)^{1/2} e,$$

which is obtained from a similar estimate for $H_p^{(2)}$ and from $J_p(r) = \frac{1}{2}(H_p^{(1)}(r) + H_p^{(2)}(r))$.

Reference

1. Erdélyi, A., Magnus, W., Oberhettinger, F., Tricomi, F.: Higher transcendental functions, vol. 2. New York: McGraw Hill 1953