manuscripta math. 71, 223 (1991)

manuscripta mathematica

Addendum: **Les extensions quadratiques du corps des rationnels, ou du corps de Gauss, ou du corps des racines cubiques de l'unité de calibres 1**

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We would like to advise the reader that we need not assume any Riemann hypothesis in theorem 5 and theorem 9 of our paper [1]. Indeed, H. M. Stark's results provide us with the following unconditional statement:

Theorem 5'. *Let* $\mathbf{K} = \mathbf{Q}(i, \sqrt{\delta})$ *be a biquadratic field with relative discriminant* $\delta_{\mathbf{K}/\mathbf{Q}(i)} = \delta$ *such that* $\delta = m^2 + 4i$, $\delta \neq \pm 3$, $m = a + ib \in \mathbf{Z}[i]$, $a > b \geq 0$, *a and b of opposite parity. Then, the field* $\mathbf{K}$ *is principal if and only if* $D \stackrel{def}{=} |\delta|^2 = 17, 73, 97, 281, 641,$ *or* 1481; *i.e. there exists exactly* 7 *quadratic extensions of the Gauss field* $\mathbf{Q}(i)$ *of caliber* 1*: the six ones given above and the one with* $\delta = 4(1+i)$, *i.e. with* $D = 32$.

Proof. We only give what has to be altered in the proof of theorem 5 of [1]. Since $\mathbf{Q}(i)$ is the only quadratic subfield of $\mathbf{K}$, and since the Dedekind zeta function of $\mathbf{Q}(i)$ does not vanish on $]0,1[$, lemma 10 of H.M. Stark implies that the Dedekind zeta function of $\mathbf{K}$ does not vanish on $[1 - \frac{1}{16Log(16D)}, 1[$ (note that $|D_{\mathbf{K}/\mathbf{Q}}| = 16D$). We now apply lemma 11.7 of Washington with $\alpha = 1 - \frac{1}{16Log(16D)}$. In the same way, theorem 9 of [1] is true unconditionaly.

[1] S. Louboutin, *Les extensions quadratiques du corps des rationnels, ou du corps de Gauss, ou du corps des racines cubiques de l'unité de calibres* 1, Manuscripta Math. **69**, 387-410 (1990).
[2] H.M. Stark, *Somme effective cases of the Brauer-Siegel theorem*, Inventiones Mathematicae, Vol. **23**, Fasc. 2 (1974), 135-152.

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(Received February 13, 1991)