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Correction to "Spectral theory of Laplacians for Hecke groups with primitive character"

by

ERIK BALSLEV and ALEXEI VENKOV

University of Aarhus
Aarhus, Denmark

University of Aarhus
Aarhus, Denmark

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In [1] we considered character perturbations of the automorphic Laplacian $A=A(\Gamma_0,\chi)$ for the Hecke group $\Gamma_0(N)$ with primitive character χ. We assume that $N=4N_2$ or $N=4N_3$, where N_2 and N_3 are products of distinct primes and $N_2\equiv 2 \bmod 4$, $N_3\equiv 3 \bmod 4$. In these cases we are dealing with regular perturbations of A, which allows for a rigorous analysis of the problem of stability of embedded eigenvalues. The perturbation is represented on the form $\alpha M+\alpha^2 N$, where M is a first order differential operator and N is a multiplication operator. In order to prove instability of an embedded eigenvalue λ we prove that the Phillips–Sarnak integral $I(\Phi,\lambda)=\langle M\Phi, E\rangle\neq 0$ for a common eigenfunction Φ of A with eigenvalue λ and all Hecke operators, where E is a generalized eigenfunction with eigenvalue λ. We consider only the operator A_{odd} acting on odd eigenfunctions, since $\langle M\Phi, E\rangle=0$ for Φ even. Let $\lambda=\frac{1}{4}+r^2$ be an eigenvalue of A_{odd}, and $\varrho(q)$ the eigenvalues of the exceptional Hecke operators $U(q)$, $q|N$, with the common eigenfunction Φ. The operators $U(q)$ are unitary ([1, Theorem 4.1]), so the eigenvalues $\varrho(q)$ lie on the unit circle. The basic result on the Phillips–Sarnak integral follows from [1, (7.23), (7.24)]. We formulate this in the following theorem.

THEOREM 1. *Let $\varepsilon_q\neq 0$, $q|N$, $q>2$, be fixed parameters of the perturbation* ([1, Theorem 6.2)], *and let Φ_n be a common eigenfunction of A_{odd} with eigenvalue λ_n and $U(q)$ with eigenvalues $\varrho_n(q)$, $q|N$. Then $I(\Phi_n,\lambda_n)\neq 0$ if and only if*

$$\varrho_n(2)\neq 2^{ir_n} \quad \text{and} \quad \varrho_n(q)\neq \frac{q^{ir_n}}{\varepsilon_q} \quad \text{for } q>2.$$

In [1, Theorem 4.3] it is stated that for all $q|N$, $\varrho_n(q)=\pm 1$. This gives rise only to the exceptional sequences $r_n=n\pi/\log 2$ and $r_{n,q}=n\pi/\log q$, $n\in\mathbf{Z}$, $q|N$, $q>2$, if $\varepsilon_q=\pm 1$ as stated in [1, Theorem 7.1].

This lemma, however, is not correct. The eigenvalues of $U(q)$ may lie anywhere on the unit circle. Consequently [1, Theorem 7.1] should be replaced by Theorem 1. This leaves us with the problem of analyzing the conditions of Theorem 1. For $q>2$ we can obtain $\varrho_n(q)\neq q^{ir_n}/\varepsilon_q$ by choosing $\varepsilon_q\neq\pm 1$. For $q=2$ there is no such freedom. We might a priori have $\varrho_n(2)=2^{ir_n}$ for all eigenvalues λ_n or for no such λ_n. It is a delicate problem to establish that $\varrho_n(2)\neq 2^{ir_n}$ for at least a certain proportion of the eigenvalues λ_n. This is the subject of a separate paper [2]. We prove the Weyl law for a certain operator T ([2, Theorem 5]) whose eigenvalues in average measure the distance $|\varrho_n(2)-2^{ir_n}|$, and obtain from this that $\varrho_n(2)\neq 2^{ir_n}$ asymptotically for at least $\frac{1}{4}$ of all eigenvalues λ_n, counted with multiplicity ([2, Theorem 6]). Together with the Weyl law for A_{odd} ([2, Theorem 4]) this implies the following result, replacing [1, Theorem 8.5].

THEOREM 2. *It holds that*

$$\liminf_{\lambda\to\infty}\frac{\#\{\lambda_n\leqslant\lambda\,|\,I(\Phi_n,\lambda_n)\neq 0\}}{\lambda}\geqslant\frac{A(F)}{32\pi},$$

where the eigenvalues λ_n are counted with multiplicity.

Assuming further that the dimensions of all odd eigenspaces are bounded, we obtain the following result, replacing [1, Corollary 8.7 (c)].

COROLLARY 1. *Suppose that* $\dim N(A_{\mathrm{odd}}-\lambda_n)\leqslant m$ *for all n. Let $\tilde\lambda_n$ be any eigenvalue of A_{odd} such that for some $\tilde\Phi_n\in N(A_{\mathrm{odd}}-\tilde\lambda_n)$, $\tilde\Phi_n(\alpha)$ is a resonance function for small $\alpha\neq 0$. Then*

$$\liminf_{\lambda\to\infty}\frac{\#\{\tilde\lambda_n\leqslant\lambda\}}{\lambda}\geqslant\frac{A(F)}{32\pi m},$$

where $\tilde\lambda_n$ is not counted with multiplicity. Thus, asymptotically at least $1/4m$ of the eigenfunctions become resonance functions for $\alpha\neq 0$.

Our results remain qualitatively the same as in [1], but the number of eigenvalues which are proved unstable is reduced. Similar results can be obtained for $\varepsilon_q=\pm 1$, but with reduction by additional factors.

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References

[1] BALSLEV E. & VENKOV A., Spectral theory of Laplacians for Hecke groups with primitive character. *Acta Math.*, 186 (2001), 155–217.

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ERIK BALSLEV
Department of Mathematical Sciences
University of Aarhus
Ny Munkegade
Bygning 530
DK-8000 Aarhus C
Denmark
balslev@imf.au.dk

ALEXEI VENKOV
Department of Mathematical Sciences
University of Aarhus
Ny Munkegade
Bygning 530
DK-8000 Aarhus C
Denmark
venkov@imf.au.dk

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