

Lettera del prof. G. A. Miller al prof. Luigi Bianchi.

UNIVERSITY OF ILLINOIS

October 17, 1925.

*My dear Professor Bianchi,*

In the *Annali di Matematica*, August 1925, volume 2, page 321, Pacifico Mazzoni published an article entitled « Sui gruppi transitivi. Totalità delle sostituzioni permutabili con tutte quelle di un dato gruppo ». The main theorem on page 323 is not new. It was first proved by one of my students and is commonly known as *Kuhn's theorem*. Cf. *Theory and Applications of Finite Groups*, Miller, Blichfeldt, Dickson, 1916, page 37.

The last part of the theorem on page 324 is not true. That is,  $K$  is not always holoedrally isomorphic with some subgroup of the entire transitive group  $G$ . This may be proved by extending the direct product of two quaternion groups by means of an operator of order 2 which interchanges these quaternion groups. If the group of order 128 thus obtained is represented as a transitive substitution group of degree 64 with respect to a subgroup of order 2 contained in one of these quaternion groups the group  $K$  of order 32, which is composed of all the substitutions on the letters of this transitive group  $G$  which are commutative with every one of its substitutions is not holoedrally isomorphic with a subgroup of  $G$ .

A proof of this interesting fact results directly if it is noted that  $K$  contains an abelian subgroup of order 8 and of type (1, 1, 1) while  $G$  does not contain such a subgroup. In fact, the subgroup of order 64 which is the direct product of two quaternion groups involves only one non-cyclic subgroup of order 4 and two of the substitutions of order 2 which are contained in this subgroup are not commutative with any of the other substitutions contained in  $G$ . It therefore follows that this transitive group of degree 64 and of order 128 does not contain a subgroup which is holoedrally isomorphic with the group of order 32 composed of all the substitutions on the letters of this transitive group which are commutative with every substitution of this transitive group. Hence the last part of the theorem in question is not correct, and the rest of the theorem is not new.

Very respectfully yours

G. A. MILLER

P. S. - In view of the delay in writing I would say that you have my permission to publish the above letter in order to avoid the spreading of these errors.

G. A. MILLER