Semigroup Forum Vol. 6 (1973), 373.

EDITORIAL NOTE

Editorial Note - concerning the paper

Carruth, J. H., and C. E. Clark, Concerning restrictions of Green's $\mathcal{H}$ equivalence, Semigroup Forum 5(1972), 186-188.

The authors ask (Question 2) the following question: "If S is a compact semigroup on which $\mathcal{H}_S$ is a congruence and and T is a closed subsemigroup such that T contains each subgroup ($\mathcal{H}$-class) which it meets, must $\mathcal{H}_T$ be a congruence?"

The following example shows that the answer to the question is negative.

Let $S_o = SO(3) \times \mathbb{H}^* \times \{0,1\}$, and let S_1 be the subsemigroup $S_1 = SO(3) \times [1,\infty] \times \{1\} \cup G \times \mathbb{H} \times \{1\} \cup SO(3) \times \mathbb{H}^* \times \{0\}$; where G is the circle subgroup of SO(3) given by

$$G = \left\{ \begin{pmatrix} \cos t & \sin t & 0 \\ \sin -t & \cos t & 0 \\ 0 & 0 & 1 \end{pmatrix} = t \in \mathbb{R} \right\}.$$

Let R be the equivalence relation on S_1 which identifies the points (g,t,0) and (g,t,1) for $t \geq 1$, and let $S = S_1/R$. Now clearly $\mathcal{H}_S$ is a congruence relation on S since $\mathcal{H}$-classes are orbits (=translations) of $G \times \{0\} \times \{1\}$ for points (h,t,1), $h \in G$, $t < 1$, and otherwise are orbits (= translations) of $SO(3) \times \{0\} \times \{0\}$. Let T be the subsemigroup determined by $SO(3) \times [1,\infty] \times \{0,1\} \cup G \times \mathbb{H} \times \{1\}$ However, $\mathcal{H}_T$ is not a congruence on T, since, for t = 1, $\mathcal{R}$-classes are left translates of $G \times \{0\} \times \{1\}$ and $\mathcal{L}$-classes are right translates. Since G is not normal in SO(3), these translates do not coincide.