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ABSTRACT

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The structure of inverse semigroups. All inverse semigroups are constructed in terms of groups and semilattices. The construction is explicit in the sense that, e.g., Schreier's theorem for group extensions is explicit.

First, a fundamental inverse semigroup S is, thanks to Munn's theorem, completely determined by the semilattice $E = E_S$, the equivalence relation $\mathcal{D}_{|E}$ on E and a selection of maximal subgroups $H_\alpha = H_{e_\alpha}$ ($\alpha \in S/\mathcal{D}$, $e_\alpha \in E \cap D_\alpha$); explicit conditions tell how these can be chosen when E is known. If now S is any inverse semigroup with $E_S = E$, then each maximal subgroup $G_\alpha = G_{e_\alpha}$ is a group extension of $K_\alpha < G_\alpha$ by H_α, and S is completely determined by E, the groups K_α, the group extensions data and: i) for each $\alpha \in S/\mathcal{D}$ and $c \in E \cap D_\gamma$ with $c \leq e_\alpha$, a homomorphism $K_\alpha \rightarrow K_\gamma$; ii) a factor set τ, with $\tau_{c,h} \in K_\gamma$ defined whenever $h \in H_\alpha$ and c is as above; iii) a factor set ν, with $\nu_{c,a} \in K_\gamma$ defined whenever $a \in E \cap D_\alpha$ and c is as above. Again explicit conditions tell how these can be chosen. There is a simpler description in terms of E and the groups G_α and homomorphisms $G_\alpha \rightarrow H_\alpha$ (but with the inconvenient that the maps $G_\alpha \rightarrow G_\gamma$ need not be homomorphisms). In either case the multiplication, while too complicated to be given here, has essentially the same form as in the bisimple or 0-bisimple case. The difficult part of the proof is to show that the simple associativity conditions imposed on the construction data actually make this multiplication associative.

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