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OBITUARY

LEE W. ANDERSON

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Anderson's dissertation, written under A.D. Wallace at Tulane in 1956, studied topological lattices from the standpoint of topological algebra. Thus, he began with a lattice already equipped with any compatible topology rather than a lattice with an instrinsic topology.

One of his most important results in lattice theory was the following:

Theorem. *A locally compact connected topological lattice is locally convex and a locally convex topological lat-*

tice is locally connected ([1] and [2]).

Continuous homomorphisms of lattices were studied in [5]. With minor modifications one has the following.

Theorem. If L is a distributive topological lattice of finite breadth then L has enough homomorphisms into I-the unit interval.

In these early years, Anderson's main interest was locally compact connected lattices. The next theorem combines several results. The third part is an amalgamation of a result from [4] with one from [8], co-authored with L. E. Ward.

Theorem. Let L be a locally compact connected topological lattice.

(1) *If* dim L = 1, *then* L *is a chain* [1].

(2) *If* L *is planar, then it is simply connected and distributive* [2].

(3) *If* L *is distributive, then its breadth is bounded by its codimension.*

Subsequently, planar lattices were further studied by Clark and Eberhart [33]. Lawson, [38], showed that breadth and codimension were equal for locally compact connected lattices. Baker and Stralka, [30], showed that every compact and every locally compact connected distributive lattice of finite breadth n could be embedded in a product of n compact chains. Further results are given in [41].

Wallace had shown that the centre of a compact lattice is peripheral and totally disconnected. A contribution of Anderson's in this area is the following.

Theorem. If L *is a compact connected distributive lattice with* $\dim L \leq n$, *then* $\text{card(Centre)} \leq 2^n - 2$.

Seeking to characterize I^n by its lattice properties he obtained the following rather technical result:

Theorem. Let L *be a locally compact connected separable metric distributive lattice with* 0 *and* 1. *Suppose that the centre of the sublattices* $b \vee (C \wedge L)$ *contains no more elements than the centre of* $a \vee L$ *whenever* $a \leq b \leq c$. *Then, if the centre of* L *is finite,* L *is isomorphic with* I^n *for some* n.

Further investigations along the lines of the last two results were done by Choe in [31] and [32].

As its title bluntly states, [9] is a study of homomorphisms and dimension. There were several starting points for this theory; an example of a dimension raising homomorphism due to Koch, the well known fact that a locally compact group has no dimension raising homomorphisms, and an earlier result that a compact connected one dimensional locally connected monoid has no dimension raising homomorphisms.

A semigroup is termed n-stable if some homomorphism can raise its dimension by n but none can raise it by $k > n$. The 0-stable semigroups are called stable.

Certain classes of compact semigroups are shown to be stable. Others are found to exhibit stability with respect to special sorts of homomorphisms. In particular, a compact connected lattice admits no dimension raising homomorphism onto a finite dimensional lattice. (After this last result, Anderson was never again to write in lattice theory.)

Later, [39], Lawson was to establish the dimensional stability of compact connected locally connected semi-lattices. An appropriate notion of co-stability was later formulated by Lau, [37].

[10]; [11] and [13] begin a systematic investigation of the Green equivalences in a compact semigroup. The Schützenberger groups, the nature of $S/\mathcal{H}$, fiberings of parts of S over an $\mathcal{H}$-class, minimal idempotents, fiberings of $\mathcal{L}$ by $\mathcal{H}$ and so forth are studied. Details of this theory and its present rôle are given in [34]. Further developments have justified this emphasis on these decompositions.

[12] studied the accessibility of an orbit of a compact group operating on a continuum. This was motivated by the position of H_1 in a compact connected monoid. The condition that assures accessibility, namely the 1-semi-local connectedness of the hyperspace at the orbit, is taken from the behavior of H_1. Apparently, it remains unknown if this last condition is required.

[17] is, in a way, dual to [9], [10] and [13]. Here, certain dimension lowering homomorphisms are considered. Specifically, homomorphisms onto a thread are studied along with variants of cross-sections to light open mappings.

The problem of which semigroups can be found in compact semigroups is considered in [22] after several preliminary papers. The problem is viewed in terms of the Bohr compactification, which is perhaps most appropriate. Completely simple semigroups which are to be found in compact semigroups are characterized in terms of a rather technical condition which, however, is quite appropriate for the setting. (This is the so-called condition of Frolik.) A substantial part of this work is a collection of examples which indicates numerous questions of possible interest. Another item of interest is the construction of a non-stable $\mathcal{D}$-trivial simple semigroup in any compact connected non abelian group. This area has as yet not provoked the same degree of interest as the earlier papers.

[19] deals with re-constructing a semigroup S from $S/\mathcal{H}$, the Schützenberger groups and the homomorphisms induced on certain subgroups by left and right multiplication. If $\mathcal{H}$ is a congruence and has an algebraic cross section this can be done in a very clean manner. Even in the general case some of this can be recovered using endomorphisms on a partial group.

[23] and [14] are to some extent derivitives of [22]. The first concerns the Van der Waerden property; each algebraic automorphism is continuous. The second describes finite dimensional connected group compactifications. (Lawson has reported that a compact connected locally connected semi-lattice has the Van der Waerden property.)

[25] and [26] do the centralizer conjecture when the orbit space of H_1 is one dimensional or planar.

The starting point for [27] is the result, given in [29], that an $\mathcal{H}$-class of a stable semigroup is the class of some (well determined) congruence. It is shown that an $\mathcal{H}$-class in any profinite semigroup is the class of a closed congruence. It follows that an appropriate $\mathcal{D}$-class formulation of a result of Rhodes [40] on factoring homomorphisms on finite semigroups holds for profinite semigroups.

Finally, [28] establishes dim S - dim H_1 is an upper bound for the dimension of an algebraic irreducible semigroup near the identity.

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