quired to cause this change in curvature from an original out-of-roundness (ϵ_o) to an out-of-roundness (ϵ) is:[9]*

$$M = (3EI/2a)(\epsilon_o - \epsilon) \quad (5)$$

where E is Young's modulus and I is the cross-sectional rigidity per unit length.

The incremental change in bending moment (dM/dP) with a change in pressure can be obtained from the Mayer-Mita solution for rigid-specimen geometry, which for $\epsilon < 0.05$ in the present notation can be written:

$$M = 0.5\epsilon Pa^2 \quad (6)$$

from which

$$dM/dP = 0.5\epsilon a^2 \quad (7)$$

Substitution of eq (7) for ϵ in eq (5) and solving for M yields the required solution:

$$M = \frac{3EI\epsilon_o}{2a}[1 - \exp(-Pa^3/3EI)] \quad (8)$$

from which the resulting bending stresses are obtained from $S_b = Md/2I$, where d is the specimen wall thickness. For $P \to 0$, solution (8) approaches eq (6) for rigid-specimen geometry.

Equation (8) may be compared with the H.L.S. solution for a circle with nominal radius (r_o), with the out-of-roundness described by: $r = r_o + \delta \cos 2\theta$, given by:

$$M = 3EI\delta/r_o^2[1 + 3EI/Pr_o^3]^{-1} \quad (9)$$

With the bending moments proportional to Young's modulus of elasticity, the present solution is of special interest in the strength testing of refractory oxides,[10] carbides and similar hard materials. For instance, for specimens of aluminum oxide with $E = 60 \times 10^6$ psi with nominal radius of 1.000 in. and a wall thickness of 0.05 in., an internal pressure of approximately 1,500 psi is required to cause tensile fracture at 30,000 psi. For an out-of-roundness $\epsilon_o = 0.001$, the bending stresses at this pressure amount to 1,240 psi. These stresses represent an appreciable fraction of the fracture stress and, if neglected, may cause an appreciable scatter in the strength data obtained. On the other hand, for similar specimens of graphite with $E \approx 1.5 \times 10^6$, at an internal pressure of 250 psi to give tensile failure at about 5,000 psi, the bending stresses amount to only 38 psi, and can be neglected to a first approximation.

For purposes of comparison, the L.H.S. solution for $\delta = 0.0005$ in. for the above alumina and graphite specimens yields values for the bending stresses of 1,000 psi and 18 psi, respectively. These values are somewhat lower than those based on the present solution, presumably because of the basic difference in specimen shape.

* *This approximation is valid for a wall-thickness <10 percent radius of curvature. For long specimens E is replaced by E/(1 − v²) where v is Poisson's ratio.*

Conclusion

It is concluded that strength testing by means of internal pressurization of thick-walled tubular or ring specimens made of materials with high values of Young's modulus of elasticity requires close dimensional control in specimen manufacture.

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References

1. *Marin, G., Mechanical Behavior of Engineering Materials, Prentice-Hall, Inc. (1962).*
2. *Mayer-Mita, R., "Die Berechnung dünnwandiger ovaler (in besonderem elliptischer) Röhren gegen gleichformigen Normaldruck," Z. Ver. Deut. Ing.,* **17**, *649–654 (1914).*
3. *Timoshenko, S., Strength of Materials, Part I, 3rd ed., D. Van Nostrand Co., 389–390 (1955).*
4. *Haigh, P. B., "An Estimate of the Bending Stresses Induced by Pressure in a Tube that is not Initially Quite Circular," Proc. Inst. Mech. Engrs.,* **133**, *96–98 (1936).*
5. *Laasonen, P., "Problems Connected with the Strength of Sulphite Cellulose Digesters," Papper och Tra,* **37**, *383–387 (1955).*
6. *Schmidt, K., "Stress in an Out-of-Round Pressure Vessel," Z. Ver. Deut. Ing.,* **102** *(11) (1960).*
7. *Wylie, C. R., Advanced Engineering Mathematics, McGraw-Hill (1954).*
8. *Handbook of Chemistry and Physics, 47th ed., The Chemical Rubber Co., A247 (1966–67).*
9. *Roark, R. J., Formulas for Stress and Strain, 3rd ed., McGraw-Hill (1954).*
10. *Sedlacek, R., and Halden, F. A., "Method of Tensile Testing of Brittle Materials," Rev. Sci. Instr.,* **33**, *298–300 (1962).*

ERRATA:

A Photoviscoelastic Analysis of Time-dependent Stresses in a Polyphase System

by R. M. Hackett and E. M. Krokosky

It has been brought to our attention that eq (11) on page 541 of the December 1968 issue of E/M is incomplete. Equation (11) should read as follows:

$$\gamma_j F_o + \sum_{i=1}^{n} \frac{F_i}{(1/\gamma_i) + (1/\gamma_j)} = \sigma_1(p) - \sigma_2(p)\big|_{p=1/\gamma}$$

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