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A Correction to the Article: “Some Theoretical Considerations with Regard to the Homogeneity-Criteria of Helmert and Abbe¹”

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R. SNEYERS was so kind to draw my attention to a mistake I made in my article. Although this mistake does not change my general conclusions, a “complete” correct mathematical treatment of the problem requires correction. First, see pg. 263, at the bottom, it is obvious that $\underline{B} = 2 \underline{A} - 2 \underline{F}$ and not $2 \underline{A} - \underline{F}$. Secondly the variance of the quantity $2 \underline{A}/\underline{B}$ can be computed in two ways 1) as done on pg. 264, using $\varrho(\underline{A}, \underline{B})$ and 2) with the equalities $2 \underline{A}/\underline{B} = 1 + 2 \underline{F}/\underline{B}$ and $\sigma^2(2 \underline{A}/\underline{B}) = 4\sigma^2(\underline{F}/\underline{B})$. Of course I was fully aware (in method 1) of the assumption $\varrho(\underline{A}, \underline{B}) \rightarrow 1$ for $n \rightarrow \infty$ and the approximate nature of the expressions for $E(\underline{A}/\underline{B})$ and $\sigma^2(\underline{A}/\underline{B})$ at the bottom of pg. 264. The second method 2 requires computation of $\sigma^2(\underline{F}/\underline{B})$. Approximately $\sigma^2(\underline{F}/\underline{B}) \cong \sigma^2(\underline{F})/E^2 \underline{B}$, using $E \underline{F} = 0$. Further $\sigma^2(\underline{F}) = E \underline{F}^2$, since $E \underline{F} = 0$ and $E^2 \underline{F} = n \sigma^4$, since $E \varepsilon_i^2 \varepsilon_{i+1}^2 = \sigma^4$, $E \varepsilon_i \varepsilon_{i+1}^2 \varepsilon_{i+2} = 0$ and $E \varepsilon_i \varepsilon_{i+1} \varepsilon_{i+2} \varepsilon_{i+3} = 0$ for $i = 1, 2, \dots, n$, with three facts are included in the null-hypothesis: “Each ε_i is independent of each ε_j , for $i \neq j$, and each ε_i is distributed normally”. Consequently $\sigma^2(2 \underline{A}/\underline{B}) \cong \sigma^2(\underline{F})/E^2 \underline{B} = 4n \sigma^4/4n^2 \sigma^4 = 1/n$, using $E \underline{B} = 2 n \sigma^2$ (pg. 264). But this holds only approximately. In this way the value of $\varrho(\underline{A}, \underline{B})$ (which I assumed to be nearly 1) follows from $2 \underline{F} = 2 \underline{A} - \underline{B}$ and $\sigma^2(2 \underline{F}) = \sigma^2(2 \underline{A}) + \sigma^2(\underline{B}) - 2 \varrho(\underline{A}, \underline{B}) \sigma(2 \underline{A}) \sigma(\underline{B})$, substituting $\sigma^2(\underline{A}) = 2 n \sigma^4$, $\sigma^2(\underline{B}) = 12 n \sigma^4$ (pg. 264) and $\sigma^2(\underline{F}) = n \sigma^4$ (see above). Hence $\varrho = 2/\sqrt{6} = 0.82$.

The consequence of this correction is that

$$P \left[1 - \frac{1}{\sqrt{n}} \leq 2 \frac{\underline{A}}{\underline{B}} \leq 1 + \frac{1}{\sqrt{n}} \right] = 0.68$$
 if it is allowed (again an approximation, see pg. 265) to assume that the quantity $2 \underline{A}/\underline{B}$ satisfies a normal distribution with mean 1 and standard deviation $1/\sqrt{n}$ (under the null-

¹ Archiv Met. Geoph. Biokl. Serie B, 8, 260 (1957)

hypothesis). It seems more senseful now to consider the limits $1 - \frac{2}{\sqrt{n}}$ and $1 + \frac{2}{\sqrt{n}}$. Then $P = 0.95$ and if the computed $2A/B$ is situated between these limits we conclude that the null-hypothesis is true (of course this statement has a chance 0.05 of being false). I am lucky to have given a more theoretical base to PUTNINS' suggestion, made empirically, to change $1/\sqrt{n}$ into $2/\sqrt{n}$.

As I said already, although it was necessary to make this correction, my general conclusions rest the same: the interpretation of the results of application of the HELMERT — and ABBE — criteria is fairly difficult. I myself prefer the homogeneity definition, pg. 262 at the top.