Correction to a Paper by Srivastava, Garg, Malhotra, and Kumar

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In the paper "Stochastic Behaviour of a Parallel Redundant Complex System with Three Types of Components", that appeared in Computing 7, 83—90 (1971), the authors considered a complex system with three classes of components L_1 , L_2 and L_3 . The failed components of these classes follow some particular type of repair discipline. Their framing of one of the equations i.e. for $P_{R_j, k}(t)$ is found to be erroneous, in the sense that the relation they have obtained for $P_{R_j, k}(t)$ is true only for $k=0$ whereas they have assumed that k varies from 0 to $K-1$. Briefly, the fault lies in the fact that their equation for $P_{R_j, k}(t)$ does not recognize the effect of a failure of a component of class L_3 on the probability state $P_{R_j, k-1}(t)$. Here in this note we have tried to find the correct relation for $P_{R_j, k}(t)$ and i.e.,

$$P_{R_j, k}(t + \Delta) = P_{R_j, k}(t) [1 - (K - k) \alpha \Delta - \lambda' \Delta - \mu'_j \Delta] + P_{0, k}(t) \lambda'_j \Delta + P_{R_j, k-1}(t) (K - k + 1) \alpha \Delta; \quad 0 \leq k \leq K - 1. \quad (3^*)$$

Clearly

$$P_{R_j, -1}(t) = 0$$

With the result the mode of finding the solution of the model changes completely, method of Recurrence no longer holds as is done by the authors. To solve the model we take the help of Discrete Transforms discussed earlier in [1].

The equations (10) and (17) of the original paper take the following form

$$\frac{\delta}{\delta t} P_{R_j, k}(t) + P_{R_j, k}(t) [(K - k) \alpha + \lambda + \mu'_j] = \alpha (K - k + 1) P_{R_j, k-1}(t) + \lambda'_j P_{0, k}(t) \quad (10^*)$$

and

$$[s + (K - k) \alpha + \lambda + \mu'_j] \bar{P}_{R_j, k}(s) = \alpha (K - k + 1) \bar{P}_{R_j, k-1}(s) + \lambda'_j \bar{P}_{0, k}(s) \quad (17^*)$$

Rewriting equation (16) of the original paper,

$$[s + (K - k) \alpha + \lambda + \lambda'] \bar{P}_{0, k}(s) = \alpha (K - k + 1) \bar{P}_{0, k-1}(s) + \sum_{j=1}^m \mu'_j \bar{P}_{R_j, k}(s) \quad (16)$$

(The equations numbering in sequel starts with number (18), the rest of the first seventeen equations are as in the original paper.)

Let us define the following Discrete Transforms:

$$\bar{A}_p(s) = \sum_{r=p}^{K-1} \binom{r}{p} \bar{P}_{0, K-r}(s) \quad (p=1, 2, \dots, K-1) \quad (18)$$

$$\bar{A}_0(s) = \sum_{r=1}^{K-1} \bar{P}_{0, K-r}(s) + \bar{P}_K(s) \quad (19)$$

and

$$\bar{B}_p^j(s) = \sum_{r=p}^{K-1} \binom{r}{p} \bar{P}_{R_j, K-r}(s) \quad (p=1, 2, \dots, K-1) \quad (20)$$

Where the inverse relation for (18) and (20) are given by

$$\bar{P}_{0, k}(s) = \sum_{r=0}^{K-1} (-1)^r \binom{K-k+r}{r} \bar{A}_{K-k+r}(s); \quad 1 \leq k \leq K-1 \quad (21)$$

and

$$\bar{P}_{R_j, k}(s) = \sum_{r=0}^{K-1} (-1)^r \binom{K-k+r}{r} \bar{B}_{K-k+r}^j(s); \quad 1 \leq k \leq K-1 \quad (22)$$

Now changing k to $(K-r)$ in equations (16) and (17*), multiplying their both sides by rC_p and summing $r=p$ to $(K-1)$ we obtain

$$[s+p\alpha+\lambda+\lambda'] \bar{A}_p(s) = K\alpha \bar{P}_{0,0}(s) + \sum_{j=1}^m \mu'_j \bar{B}_p^j(s) \quad (23)$$

and

$$[s+p\alpha+\lambda+\mu'_j] \bar{B}_p^j(s) = \frac{\lambda'_j}{s+K\alpha+\lambda+\mu'_j} [K\alpha \bar{P}_{0,0}(s) + (s+K\alpha+\lambda+\mu'_j) \bar{A}_p(s)] \quad (24)$$

From relations (23) and (24), one obtains

$$\bar{A}_p(s) = \varphi_p \bar{P}_{0,0}(s) \quad (25)$$

and

$$\bar{B}_p^j(s) = \psi_p^j \bar{P}_{0,0}(s) \quad (26)$$

Where

$$\varphi_p = K\alpha \left[s+p\alpha+\lambda+\lambda' - \sum_{j=1}^m \frac{\lambda'_j \mu'_j}{s+p\alpha+\lambda+\mu'_j} \right]^{-1} \cdot \left[1 + \sum_{j=1}^m \frac{\lambda'_j \mu'_j}{(s+p\alpha+\lambda+\mu'_j)(s+K\alpha+\lambda+\mu'_j)} \right]$$

and

$$\psi_p^j = \frac{\lambda'_j K\alpha}{(s+p\alpha+\lambda+\mu'_j)(s+K\alpha+\lambda+\mu'_j)} + \lambda'_j \varphi_p$$

Employing relations (25) and (26) in relations (21) and (22), respectively, one could get $\bar{P}_{0,k}(s)$ and $\bar{P}_{R_j,k}(s)$ in terms of $\bar{P}_{0,0}(s)$.

Now, simplifying relation (15) of the original paper, and utilizing the values of $\bar{P}_{0,k}(s)$ and $\bar{P}_{R_j,k}(s)$ one could easily see that

$$\bar{P}_{0,0}(s) = \left[s + K\alpha + \lambda + \lambda' - \sum_{j=1}^m \frac{\lambda'_j \mu'_j}{s + K\alpha + \lambda + \mu'_j} - \alpha \bar{K}(s) \cdot \right. \\ \cdot \left(\sum_{r=0}^{K-1} [(-1)^r \binom{r+1}{r} \{\varphi_{r+1} + \sum_{j=1}^m \psi_{m,j}^i\}] \right) - \\ \left. - \left(\sum_{r=0}^{K-1} [(-1)^r \binom{K-k+r}{r} \{\varphi_{K-k+r} + \sum_{j=1}^m \psi_{K-k+r,j}^i\}] \right) \left(\sum_{i=1}^m \lambda_i \bar{S}_i(s) \right) \right]^{-1}$$

Now one could easily evaluate the other state probabilities.

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References

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